Johns Hopkins
Engineering for Professionals

605.767 Applied Computer Graphics

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Module 12C
Force Physics

Physics Topics

- Particle Physics
- Motion Evaluation
- Forces
  - Gravity
  - Springs
  - Rigid Body Dynamics
- Other Physics

Newton’s Laws of Motion

1. An object continues with a constant velocity unless a force acts upon it
2. A force acting on an object produces acceleration that is proportional to the object’s mass

Particle Physics

- Not to be confused with particle systems or physics used in particle systems
- Assumptions using infinitely small particles
  - Simplifies many calculations
  - Useful for sampling and solving more-complex problems
- **Basic components**
  - ***p*** - position in world space
  - ***v*** - velocity, change in position over time $v = \dfrac{p' - p}{t' - t}$
  - ***a*** - acceleration, change in velocity over time $a = \dfrac{v' - v}{t' - t}$
  - Therefore, acceleration is the second derivative of the position with respect to time
    - $a = \dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{\mathrm{d}}{\mathrm{d}t}\dfrac{\mathrm{d}p}{\mathrm{d}t} = \dfrac{\mathrm{d}^2p}{\mathrm{d}t^2}$

Particle Physics (cont.)

- **Momentum** - mass times velocity
- **Force** - mass times acceleration $f = ma$
  - Useful variant:
    - $a = \frac{1}{m} f$
- Updating values
  - Update position:
    - $p' = p + vt + \frac{1}{2}at^2$
  - Update velocity:
    - $v' = v + at$
    - velocity usually has a dampening factor (d), thus making the update function:
      - $v' = vd + at$

Evaluating Motion

- Update object state
  - $\mathbf{X}_i$ - current state of object (position, velocity, etc.)
  - t_i - current time
  - h - change in time
  - $\mathbf{F}(t, \mathbf{X}_i)$ - update function
- **Euler's Method**
  - Simplest form, but often unstable
  - $\mathbf{X}_{i+1} = \mathbf{X}_i + h\mathbf{F}(t, \mathbf{X}_i)$
    - derived from $(\mathbf{X}_{i+1} - \mathbf{X}_i)/h = \mathbf{F}(t, \mathbf{X}_i)$
  - $t_{i+1} = t_i + h$

Evaluating Motion (cont.)

- Midpoint Method - Runge-Kutta (second-order)
  - $\mathbf{A}_1 = \mathbf{F}(t, \mathbf{X}_i)$
  - $\mathbf{A}_2 = \mathbf{F}(t + h/2, \mathbf{X}_i + h\mathbf{A}_1/2)$
  - $\mathbf{X}_{i+1} = \mathbf{X}_i + h\mathbf{A}_2$
  - $t_{i+1} = t_i + h$
- **Runge-Kutta** (fourth-order)
  - $\mathbf{A}_1 = \mathbf{F}(t, \mathbf{X}_i)$
  - $\mathbf{A}_2 = \mathbf{F}(t + h/2, \mathbf{X}_i + h\mathbf{A}_1/2)$
  - $\mathbf{A}_3 = \mathbf{F}(t + h/2, \mathbf{X}_i + h\mathbf{A}_2/2)$
  - $\mathbf{A}_4 = \mathbf{F}(t + h, \mathbf{X}_i + h\mathbf{A}_3)$
  - $\mathbf{X}_{i+1} = \mathbf{X}_i + (h/6)(\mathbf{A}_1 + 2\mathbf{A}_2 + 2\mathbf{A}_3 + \mathbf{A}_4)$
  - $t_{i+1} = t_i + h$

Force of Gravity

- General formula
  - $f = G\dfrac{m_1 m_2}{r^2}$
    - G - gravitational constant
    - m_i - mass of each object
    - r - distance between the objects' centers
  - Earth's average gravitation at sea level
    - $f = gm$
    - where $g \approx 10\dfrac{\mathrm{m}}{\mathrm{s}^2}$
    - games often use a reduced gravitational factor producing a “bouncy” effect

Spring Force

- Hook's Law
  - Force exerted by a spring depends on the distance the spring is extended
  - Formula
    - $f_{\text{spring}} = -k\left(|d| - l_i\right)d$
    - k - spring constant
    - $\boldsymbol{d}$ - vector from one end of the spring to the other
      - $|\boldsymbol{d}|$ - norm of $\boldsymbol{d}$
    - l_i - current length of the spring
  - Observations
    - If ($l_i == |\boldsymbol{d}|$), the spring is at rest — no force is exerted
    - If ($l_i < |\boldsymbol{d}|$), the spring pushes outward on the two connected objects
    - If ($l_i > |\boldsymbol{d}|$), the spring draws the two connected objects inward

Spring Force (cont.)

- **Limit of elasticity**
  - There are practical limits to the amount a spring can stretch and contract
    - Too much contraction
      - The spring coils touch — **connected objects cannot be any closer together**
      - l_i - current spring length cannot be less than zero
    - Too much extension
      - The spring loses its elasticity
      - Unlikely you will need this level of detail
- Rarely do applications use spring objects (coils)
  - Spring force usually referred to as ***spring-like*** forces

Spring Force Applications

- Tethered Objects
  - **Camera attached to character**
    - Third-person view
- One-dimensional Array
  - Hair
  - **Rope**
- Two-dimensional Array
  - **Cloth**
  - Water Surface
- Mimic Buoyancy
- **Stiff Springs**
  - Can mimic connected joints (bones) in rigid hierarchies
    - Useful for procedural animations