Johns Hopkins
Engineering for Professionals

605.767 Applied Computer Graphics

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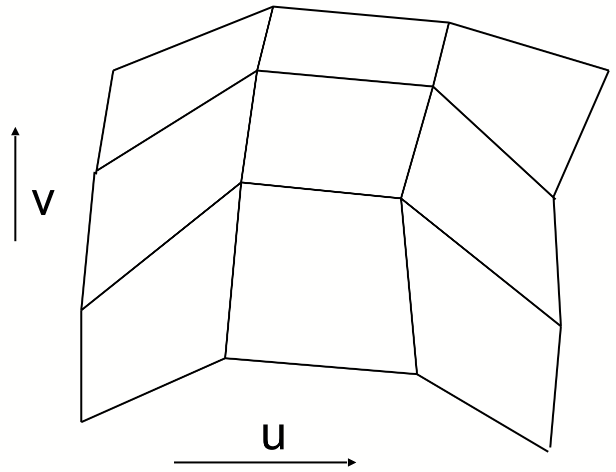
Module 6F
Parametric Surfaces

Parametric Curved Surfaces

- Can extend treatment of parametric cubic curves to **biparametric cubic surface patches**
  - Foley and van Dam call them **parametric bicubic surfaces**
  - A point on a surface patch is given by a biparametric function
    - A set of basis functions is used for each parameter
  - Two sets of orthogonal curves define a surface
- Bicubic Bezier patch is defined as:

$$p(u,v) = \sum_{i=0}^{3} \sum_{j=0}^{3} B_i(u) B_j(v) p_{ij}$$

  - p_{ij} is an array of 16 control points - 12 boundary/edge and 4 interior
  - $B_i(u)$ and $B_j(v)$ are basis functions



Bezier Surface Patches (cont.)

- Properties of the Bicubic Bezier patch
  - Bezier patch goes through the 4 corner points $p_{0,0}$, $p_{n,0}$, $p_{0,n}$, $p_{n,n}$
  - Each boundary is described by a cubic Bezier curve formed by the control points along that boundary
  - Patch lies within the convex hull of its control points
  - Invariant under affine transformation
    - Transforming control points and then generating points on the patch is equivalent to generating points on the path and transforming them
- See Figure 17.21 (13.18 in 3rd Edition) for example of moving a control point
- Normal vectors found by taking cross product of the derivatives with respect to u and v

$$\frac{\delta p(u,v)}{\delta u} = m \sum_{j=0}^{n} \sum_{i=0}^{m-1} B_i^{m-1}(u) B_j^n(v) \left[p_{i+1,j} - p_{i,j} \right]$$

$$\frac{\delta p(u,v)}{\delta v} = n \sum_{i=0}^{m} \sum_{j=0}^{n-1} B_i^{m}(u) B_j^{n-1}(v) \left[p_{i,j+1} - p_{i,j} \right]$$

$$n(u,v) = \frac{\delta p(u,v)}{\delta u} \times \frac{\delta p(u,v)}{\delta v}$$

(Not unit length)

Normals at the corner points can calculated from the cross product of tangent vectors at the vertex:
a = (p_{01} - p_{00}), b = (p_{10} - p_{00}) n = a X b

Joining Bezier Surface Patches

- Often want to "stitch" together several Bezier surfaces to form a more complex, composite surface
  - As with curves, need to apply some continuity conditions
- For positional continuity (G^0) along the "right" boundary
  - $a_{3j} = b_{0j}$
    - 2 patches must have a common boundary edge control polygon:
      - $a_{30} = b_{00}$, $a_{31} = b_{01}$, $a_{32} = b_{02}$, and, $a_{33} = b_{03}$
- For G^1 continuity the four pairs of control polyhedra edges that straddle the boundary must be collinear
  - To produce same tangent vector directions
  - For C^1 continuity must have same ratio between segment lengths
    - See Figures 17.24 (13.25 in 3rd Edition)
    - C^1 continuity is generally required for texture mapping continuity
- When developing successive composite surface patches, this **fixes** several of the control points in the new patch