Johns Hopkins
Engineering for Professionals

605.767 Applied Computer Graphics

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Module 12D
Rigid Body Physics

Rigid Body Object Properties

- Already have
  - Position - Point3
  - Orientation - SRT
  - Velocity - Vector3
  - Linear acceleration - Vector3
- Must add
  - Mass - float32
  - Center of Mass - Point3
  - Angular velocity - Quaternion
  - Angular acceleration - Quaternion

Center of Mass

- Critical for performing rigid-body collisions
- Typically chosen as the origin
  - Models are defined such that the local coordinates have the center of mass as the origin
- Object is made up of *n* particles, then center of mass is:
  - $$p_c = \frac{1}{m}\sum_{n} m_i p_i$$
  - m - mass of the entire object
    - m_i - mass of particle *i*
  - p_i - position of particle *i*

Angular Components

- Rigid bodies add to particles
  - Orientation or rotation (also known as attitude)
  - **Angular velocity** - the change in orientation over time
  - **Angular acceleration** - the change in angular velocity over time
  - **Moment of Inertia** - angular equivalent for mass
- Force for rotation is known as **torque**
  - Angular acceleration times moment of inertia equal torque
  - $\theta'' I = \tau$ or $\theta'' = \frac{1}{I}\tau$

Moment of Inertia

- **Inertia Tensor** - in 3D, a 3x3 matrix representing the moment of inertia about each principal axis on the diagonal

- $$I = \begin{bmatrix} I_x & & \\ & I_y & \\ & & I_z \end{bmatrix}$$

- The remaining values are called **products of inertia** and contribute negative values

- $$I = \begin{bmatrix} I_x & -I_{xy} & -I_{xz} \\ -I_{xy} & I_y & -I_{yz} \\ -I_{xz} & -I_{yz} & I_z \end{bmatrix}$$

Inertia Tensor - Cuboid (Rectangular Block)

- Given mass **m**, and dimensions d_x, d_y, d_z aligned along the X, Y, and Z axes

$$I = \begin{bmatrix} \frac{1}{12}m\left(d_y^2 + d_z^2\right) & 0 & 0 \\ 0 & \frac{1}{12}m\left(d_x^2 + d_z^2\right) & 0 \\ 0 & 0 & \frac{1}{12}m\left(d_x^2 + d_y^2\right) \end{bmatrix}$$

Inertia Tensor - Sphere

- Given mass **m**, and radius **r**
  - With constant density
- $$I = \begin{bmatrix} \frac{2}{5}mr^2 & 0 & 0 \\ 0 & \frac{2}{5}mr^2 & 0 \\ 0 & 0 & \frac{2}{5}mr^2 \end{bmatrix}$$
  - Hollow sphere — mass only around the shell
- $$I = \begin{bmatrix} \frac{2}{3}mr^2 & 0 & 0 \\ 0 & \frac{2}{3}mr^2 & 0 \\ 0 & 0 & \frac{2}{3}mr^2 \end{bmatrix}$$

Inertia Tensor - Ellipsoid

- Given mass **m**, and radii $\mathbf{r}_x$, $\mathbf{r}_y$, $\mathbf{r}_z$

$$I = \begin{bmatrix} \frac{1}{5}m\left(r_y^2 + r_z^2\right) & 0 & 0 \\ 0 & \frac{1}{5}m\left(r_x^2 + r_z^2\right) & 0 \\ 0 & 0 & \frac{1}{5}m\left(r_x^2 + r_y^2\right) \end{bmatrix}$$

Inertia Tensor - Cylinder

- Given mass **m**, radius **r**, and height **h** along the Z-axis
  - With constant density

$$I = \begin{bmatrix} \frac{1}{12}mh^2 + \frac{1}{4}mr^2 & 0 & 0 \\ 0 & \frac{1}{12}mh^2 + \frac{1}{4}mr^2 & 0 \\ 0 & 0 & \frac{1}{2}mr^2 \end{bmatrix}$$

  - Hollow tube — mass only around the shell
    - with outer radius $\mathbf{r}_o$ and inner radius $\mathbf{r}_i$

$$I = \begin{bmatrix} \frac{1}{12}mh^2 + \frac{1}{4}m\left(r_o^2 + r_i^2\right) & 0 & 0 \\ 0 & \frac{1}{12}mh^2 + \frac{1}{4}m\left(r_o^2 + r_i^2\right) & 0 \\ 0 & 0 & \frac{1}{2}m\left(r_o^2 + r_i^2\right) \end{bmatrix}$$

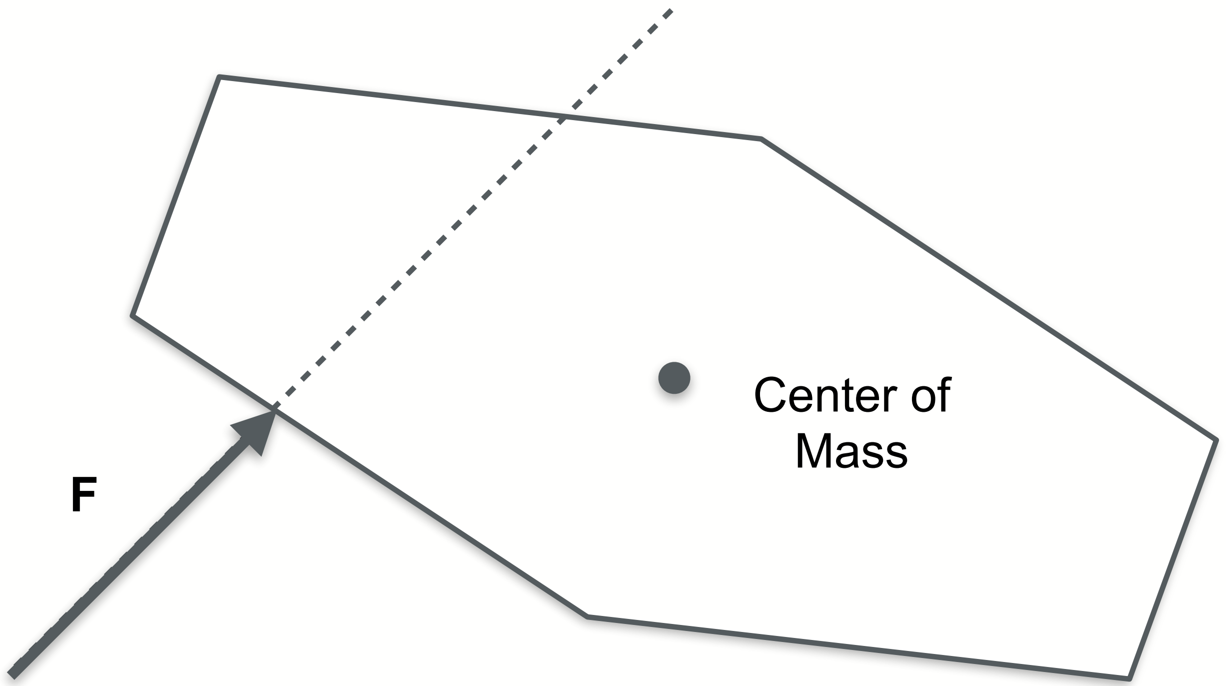
Inertia Tensor - Cone

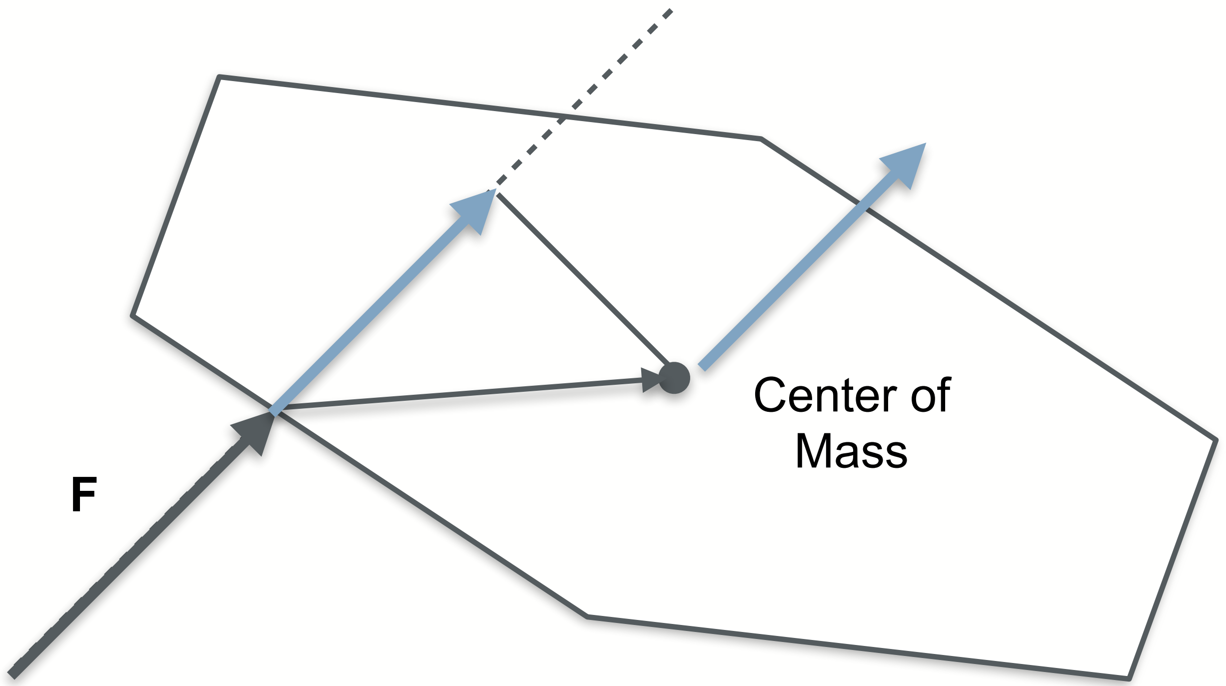
- Given mass **m**, radius at the base **r**, and height **h** along the Z-axis
  - Note: the center of mass is 1/4 the distance from the center of its base to the tip

$$I = \begin{bmatrix} \frac{3}{5}mh^2 + \frac{3}{20}mr^2 & 0 & 0 \\ 0 & \frac{3}{5}mh^2 + \frac{3}{20}mr^2 & 0 \\ 0 & 0 & \frac{3}{10}mr^2 \end{bmatrix}$$

Linear Acceleration from Collision

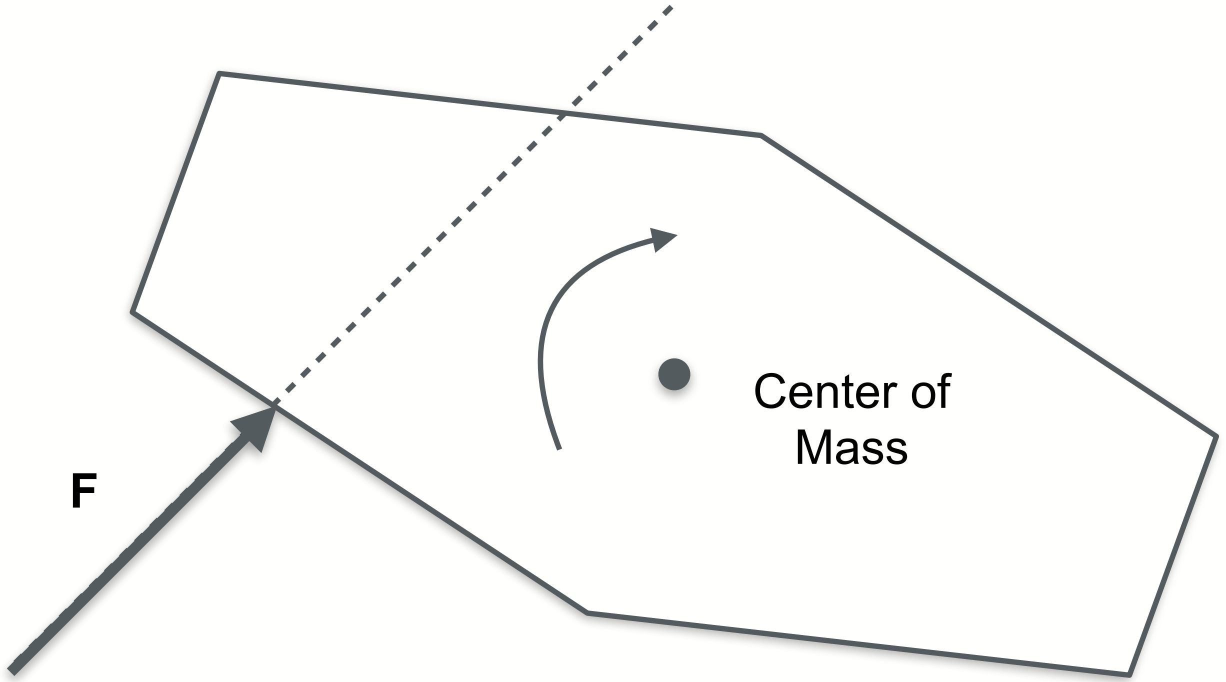
- The linear acceleration is in the direction of the force vector, but scaled by projection of the force vector onto the vector from the point of impact to the center of mass

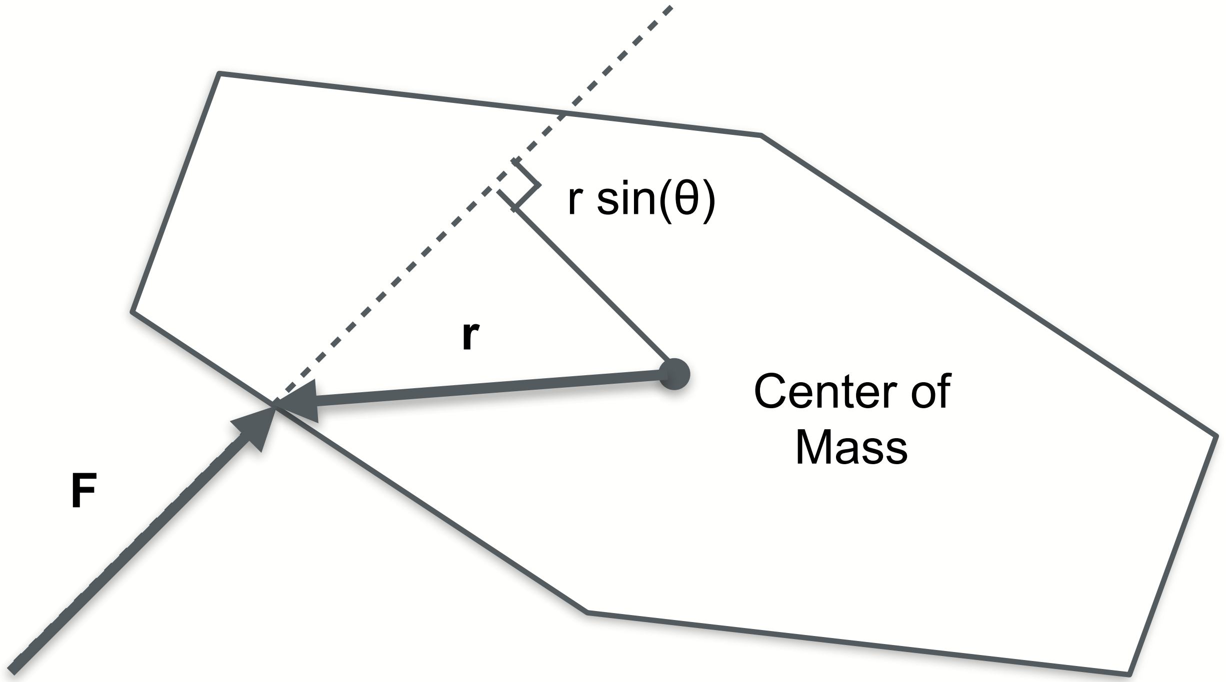




Torque From Collision

- Torque applied as a result of a force impacting a rigid body is the sine of the angle of the force at the point of impact
  - Torque is **r***sin(θ) - the cross product of the force vector and vector from the point of impact to center of mass
  - Torque increases as **r** increases
    - The greater the distance, the greater the torque





Putting It All Together

- Setup
  - Initiate any independent forces
    - e.g. Global gravity
  - Register objects with forces
    - Associate objects connected with spring forces
    - Each object
      - Influencing forces - Array<Force*>
- Update
  - Update each objects' position and velocities (linear and angular)
    - Also apply any registered forces
  - Perform collision tests
    - Collision handling phase applies rigid body impact forces

Additional Topics

- Friction
- Heat Transfer
- Deformable Bodies
- Fluid Dynamics
- Buoyancy