Johns Hopkins
Engineering for Professionals

605.767 Applied Computer Graphics

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Module 6B
Continuity Considerations

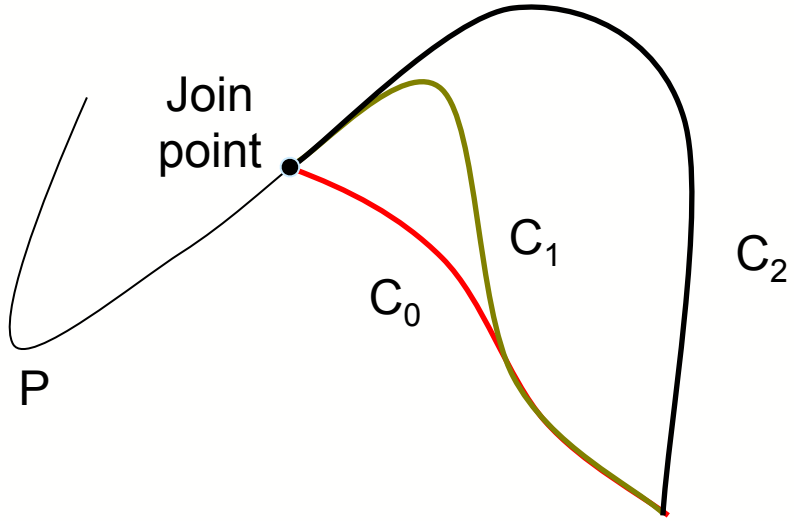
Joining Parametric Curves

- A parametric curve may consist of a number of curve segments joined together
  - Continuity constraints used to join them
  - Also called **piecewise** construction of a curve
    - Each curve segment is defined by a set of control points
- **Parametric** or **curve continuity** depends on how the derivatives of adjoining curve sections compare at the boundary
  - **Zero-order parametric continuity C^0**
    - Parametric components are equal at a common endpoint
  - **First-order parametric continuity C^1**
    - First derivatives of the parametric equations match at the join point
      - Tangent vectors or slope of the curves are equal (direction and magnitude)
  - **Second-order parametric continuity C^2**
    - Second derivatives of the parametric equations match at the join point
      - Curvature (rate of change of the tangent vectors) matches at the join point

Geometric Continuity Properties

- Alternative set of joining conditions are called **geometric continuity** conditions
- **Zero-order geometric continuity G^0**
  - Simply joining two curve segments at a common endpoint
- **First-order geometric continuity G^1**
  - In addition, first derivatives at the common endpoint match to within a constant
    - Tangent vectors have equal direction, possibly different magnitude
- **Second-order geometric continuity G^2**
  - In addition, second derivatives at the common endpoint match to within a constant
    - Curvature of the two curve sections are proportional
- Figure 17.10 (13.9 3rd Edition) illustrates C^0, G^1 and C^1 continuity
  - Note: C^1 continuity implies G^1, but not vice-versa

Continuity



Curve segment P joined to 3 other curves: C_0, C_1, C_2
C^0 parametric continuity between P and C_0
C^1 parametric continuity between P and C_1
C^2 parametric continuity between P and C_2
Visual difference between C_1 and C_2 is slight near the join point, but obvious further away

Continuity Conditions

- Second-order continuity is required for precise CAD and animation
  - Tangent line transitions smoothly from one curve segment to the next
  - First-order continuity - tangents match but the curvature can be very different
- Example: camera motion
  - Moving a camera along piecewise curves in equal time steps
  - First derivative is the camera velocity
  - Second derivative is the acceleration
  - Want to match velocity and acceleration at the join points
    - To avoid any abrupt changes in the animation sequence
- First-order continuity adequate for digitizing drawings and some lower precision design applications