Johns Hopkins
Engineering for Professionals

605.767 Applied Computer Graphics

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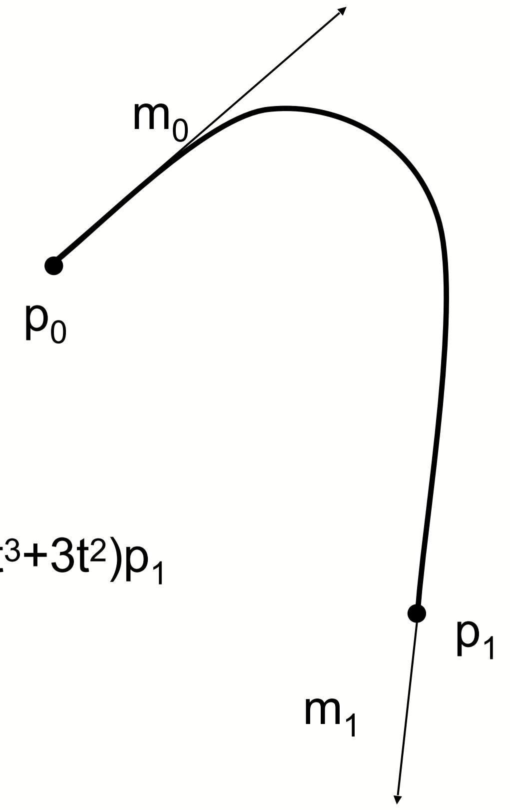
Module 6C
Cubic Curves

Advantages of Cubic Curves

- Cubic curves are commonly used in computer graphics
  - Exhibit sufficient shape flexibility for most applications
  - Higher order curves are more costly and complex
- Cubic curves are true space curves - not necessarily planar
  - Quadratic curves are functions of 3 control points
    - For Bezier curves
    - Confined to a plane in 3 space
  - Curves made up of quadratic segments reduces to a set of piecewise 2D segments
    - Not generally acceptable when modeling 3D shapes

Cubic Hermite Interpolation

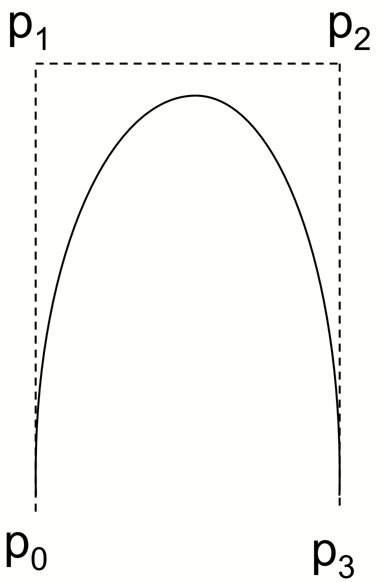
- Cubic Hermite curves
  - Defined by start and end points p0, p1 and tangents at these points m0 and m1
  - Simple curves to control
- Blending functions: see Figure 17.11 (13.10 in 3rd Edition)



$$p(t)=(2t^3-3t^2+1)p_0+(t^3-2t^2+t)m_0+(t^3-t^2)m_1+(-2t^3+3t^2)p_1$$

Cubic Bezier Curves

- Bezier formulation involves specifying a set of control points from which the cubic polynomial is derived
  - Foley provides a derivation showing the relationship between Bezier formulation and Hermite and Ferguson cubics
    - Bezier curves - reformulation of Hermite cubic polynomial forms
- The Bezier basis functions are:
  - $(1 - t)^3$
  - $3t(1 - t)^2$
  - $3t^2(1 - t)$
  - t^3
  - Known as the Bernstein polynomials
    - See Figure 17.7 (13.6 in 3rd Edition) far right - for plot of the Bernstein polynomials
- The polynomial is expressed in terms of these functions and 4 control points (p_0, p_1, p_2, p_3):
  - $p(t) = (1 - t)^3 p_0 + 3t(1 - t)^2 p_1 + 3t^2(1 - t)\, p_2 + t^3 p_3$
  - p_0 and p_3 are the control endpoints
    - Moving control points influences the shape of the curve



4 control points
Bezier basis functions

Cubic Bezier Curves (cont.)

- Position of the 4 control points completely determines the shape
  - No need to specify or control tangent vectors
- The tangent vectors at the endpoints are:
  - $p(0) = 3(p_1 - p_0)$, $p(1) = 3(p_3 - p_2)$
    - Differentiate basis functions with respect to t , solve at 0 and 1
  - Note: p_1 and p_2 lie on the tangent vectors
  - Moving a control point changes either an endpoint or a tangent vector
    - Middle 2 control points 'pull' the curve
- Can be expressed in matrix notation
  - M is the **basis** matrix

$$p(t) = TMP = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{bmatrix}$$

Joining Bezier Curve Segments

- Can join multiple curve segments to create piecewise polynomial curve
  - Create a more complex curve than that obtainable by cubic polynomial
- Must apply constraints at the joining endpoints
- G^1 continuity between 2 cubic curve segments Q_i and R_i requires:
  - $(Q_3 - Q_2) = k(R_1 - R_0)$
  - Tangent vectors at the end of Q match direction to the beginning of R
- Composite Bezier curve can be built one segment at a time
  - Joining conditions reduce local control of the curve
    - Can be alleviated by increasing the degree of the polynomial (complexity) or further splitting the curve segment

Bezier Curve Properties

- Polynomial degree is 1 less than the number of control points
  - Generally use cubic polynomials (4 control points)
- Curve follows the shape of the control polygon and is constrained within the convex hull formed by the control points
- Control points do **not** exert **local control**
  - Moving a control point changes all of the curve to some degree
- The first and last control points are the curve end points
- Tangent vectors to the curve at the endpoints are coincident with the first and last edges of the control point polygon
- The curve does not oscillate about any straight line more often than the control polygon
  - Known as variation diminishing property
- Curve is transformed by applying combination of linear transformations to its control point representation