Johns Hopkins
Engineering for Professionals

605.767 Applied Computer Graphics

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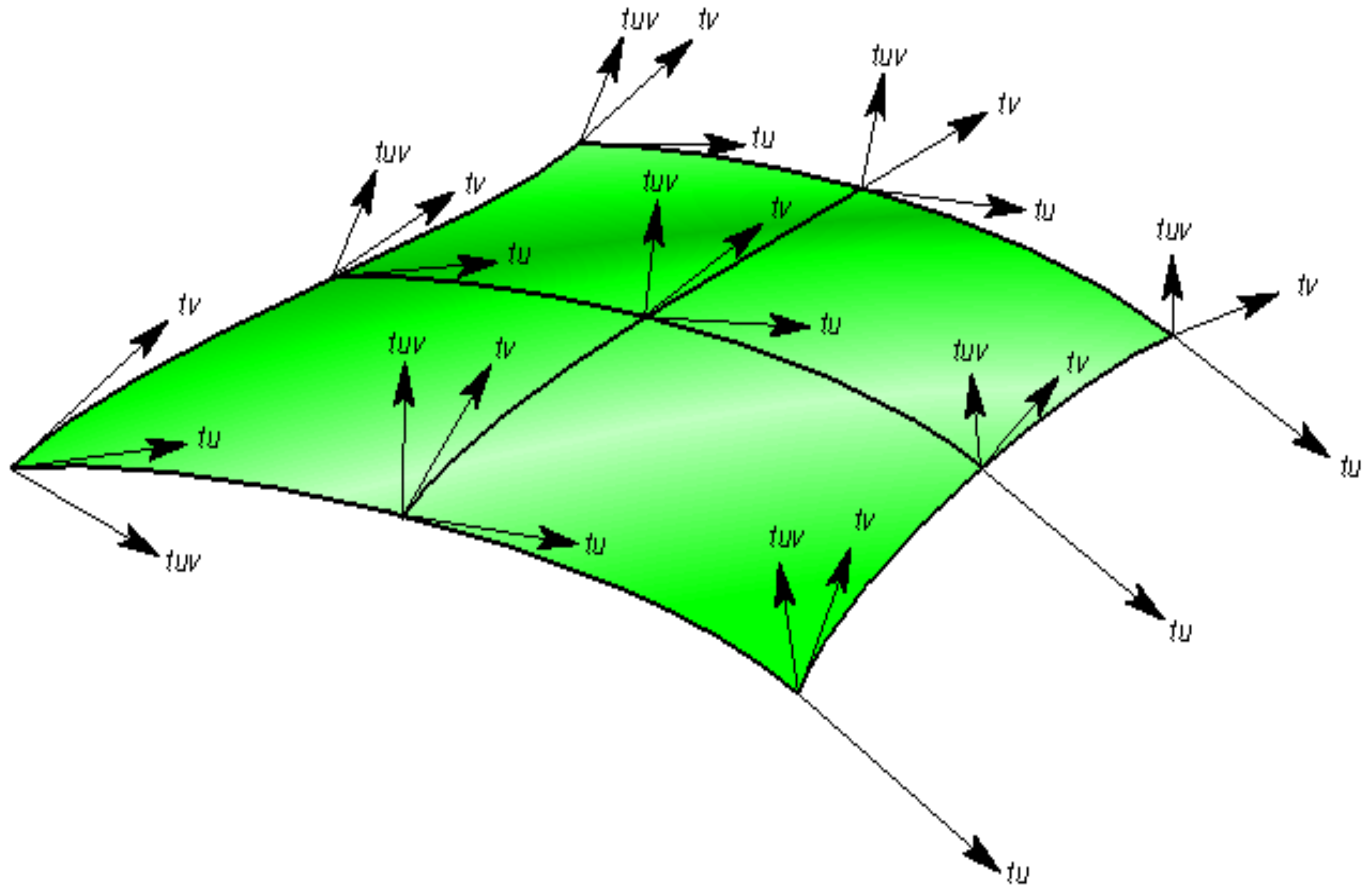
Module 6A
Introduction to Parametric Curves

Parametric Curves and Surfaces

- Parametric curves
  - Introduction
  - Continuity conditions
  - Bezier cubic curves
  - B-splines
  - NURBS and other splines
- Parametric surfaces
  - Bezier patches
- Rendering parametric curves and surfaces
  - Forward differences
  - Recursive subdivision
- OpenGL Evaluators and NURBS support
- Uses of parametric curves and surfaces

Parametric Curves and Surfaces



Introduction

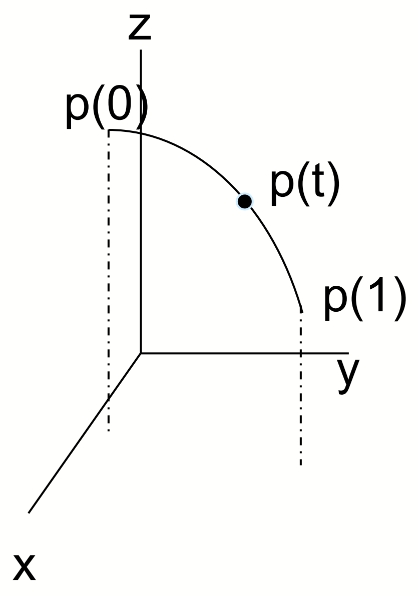
- Polylines are linear approximations to curves
  - Piecewise linear
  - Require many vertices to accurately represent a continuous curve or surface
- Polygons can only approximate a curved surface
- Can be difficult to edit shape of polygons and polylines
- Can describe curves and curved surfaces using a mathematical representation
  - Produce piecewise smooth curves and surfaces
  - Use functions with higher degree than linear functions
- Advantages
  - Storage efficiency
  - Easily transformed
  - Scalable - a form of LOD modeling
  - Easier interaction / shape editing
  - Can join curves / surfaces to form complex surfaces and curves
    - Apply appropriate boundary conditions to join the pieces

Parametric Representation

- **Parametric** representation of curves and surfaces is a common tool in computer graphics
  - Also called **spline** curves or surfaces
    - Splines are used in drafting - a flexible strip used to produce smooth curves through a set of points
  - Surfaces are constructed from orthogonal curves
- Commonly found in CAD systems
  - Original techniques were developed to model car and aircraft bodies
    - Bezier patch developed in 1960s for use in design of Renault car bodies
- Can be used to define the path of an object through defined points
  - Interpolate a parametric curve through these points
- Will describe parametric curves and then generalize to surfaces
  - Curves are easier to describe and notate
  - Properties extend to surfaces

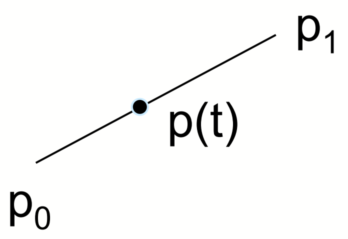
Parametric Representation of 3D Curves

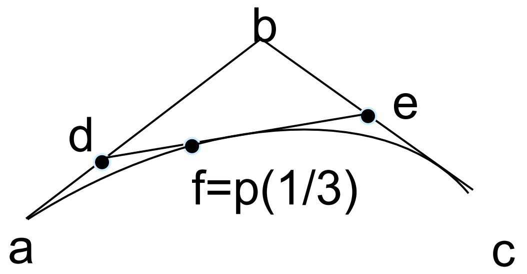
- Example: a point moving in space as a function of time
  - Parametric description of the path of the point
    - p(t) – function computes a point for each value of t
  - Example use: smooth motion of camera
- Cubic curves can be written as a single vector equation:
  - $p(t) = at^3 + bt^2 + ct + d$
  - x,y,z components are:
    - $x(t) = a_x t^3 + b_x t^2 + c_x t + d_x$
    - $y(t) = a_y t^3 + b_y t^2 + c_y t + d_y$
    - $z(t) = a_z t^3 + b_z t^2 + c_z t + d_z$
    - $(0 <= t <= 1)$
    - x, y, z are cubic polynomials of the parameter t



Linear Interpolation

- Linear interpolation defines a straight line path between 2 points
  - $p(t)=p_0+t(p_1-p_0)= (1-t)p_0+tp_1$
- Linear interpolation may not be acceptable when tracing a path through several points
  - Sudden changes at the joints
- A solution is to repeatedly perform linear interpolation
  - Produces geometric interpretation of Bezier curve
  - Use 3 points a,b,c – called control points
  - Create 2 new points d and e
    - By interpolating a,b and bc
  - Compute final point f
    - Interpolating between d and e
  - p(t) is quadratic
    - Produces a parabola



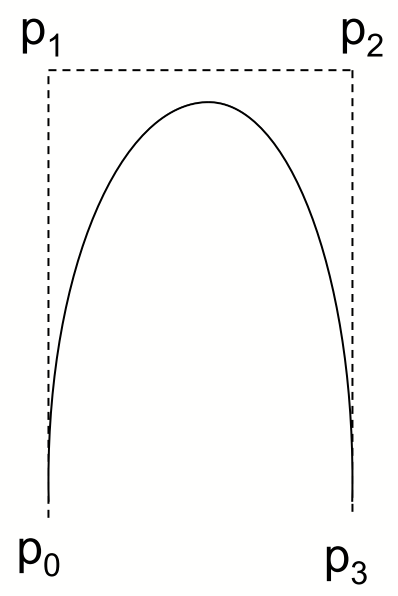


Using t = 1/3

$$
\begin{aligned}
p(t)&=(1-t)d+te \\
&=(1-t)[(1-t)a+tb]+t[(1-t)b+tc] \\
&=(1-t)^2a+2(1-t)b+t^2c
\end{aligned}
$$

Overview of Parametric Curves

- A **parametric curve** is defined by a set of points known as **control points** together with a set of **basis or blending functions**
- Control points indicate the general shape of the curve
  - Basis functions can **interpolate** the control points or **approximate** the set of control points
    - Interpolate - the curve passes through the control points
    - Approximate - the curve approaches the control points
  - Set of control points specified interactively
    - Move/adjust individual control points to control the curve
- The polygon formed by joining the control points is called the **characteristic** or **control polygon**



4 control points
Bezier basis functions

Basis Functions

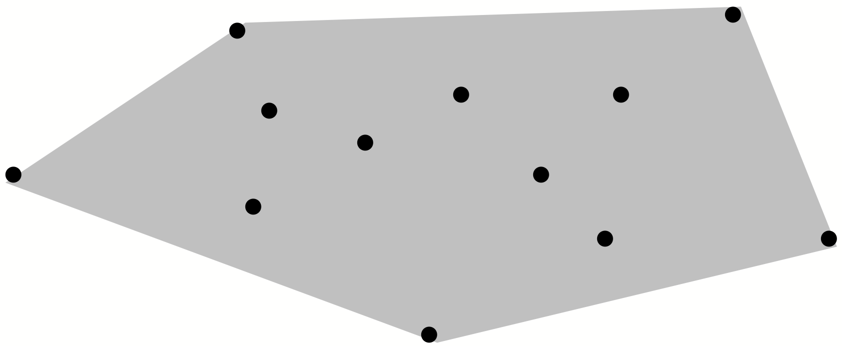
- **Basis functions** or **blending functions** blend or scale the control points into a curve segment
  - Several different basis functions have evolved in CAD and computer graphics applications
  - Bezier and B-spline basis functions are 2 of the most popular
- Basis functions are defined over the range of a parameter (t) for each control point
  - Derive the x,y,z components of the curve p(t) by summing the component of each control point p scaled by the value of the basis function at the parameter t: B(t)

$$p(t) = \sum_i B_i(t) p_i$$

$$x(t) = \sum_i B_i(t) p_{ix} \qquad y(t) = \sum_i B_i(t) p_{iy} \qquad z(t) = \sum_i B_i(t) p_{iz}$$

Convex Hull

- **Convex hull** is the convex polygon boundary that encloses a set of points
  - Imagine the region formed by stretching a rubber band around the control points
    - Each point is either on the perimeter of the convex hull or inside it
  - Not the same as the control polygon
- Convex hulls can be useful
  - Most parametric curves are bounded by their convex hull
  - Can be used as bounding volume when ray tracing surface patches



Convex hull of a set of points