Johns Hopkins
Engineering for Professionals

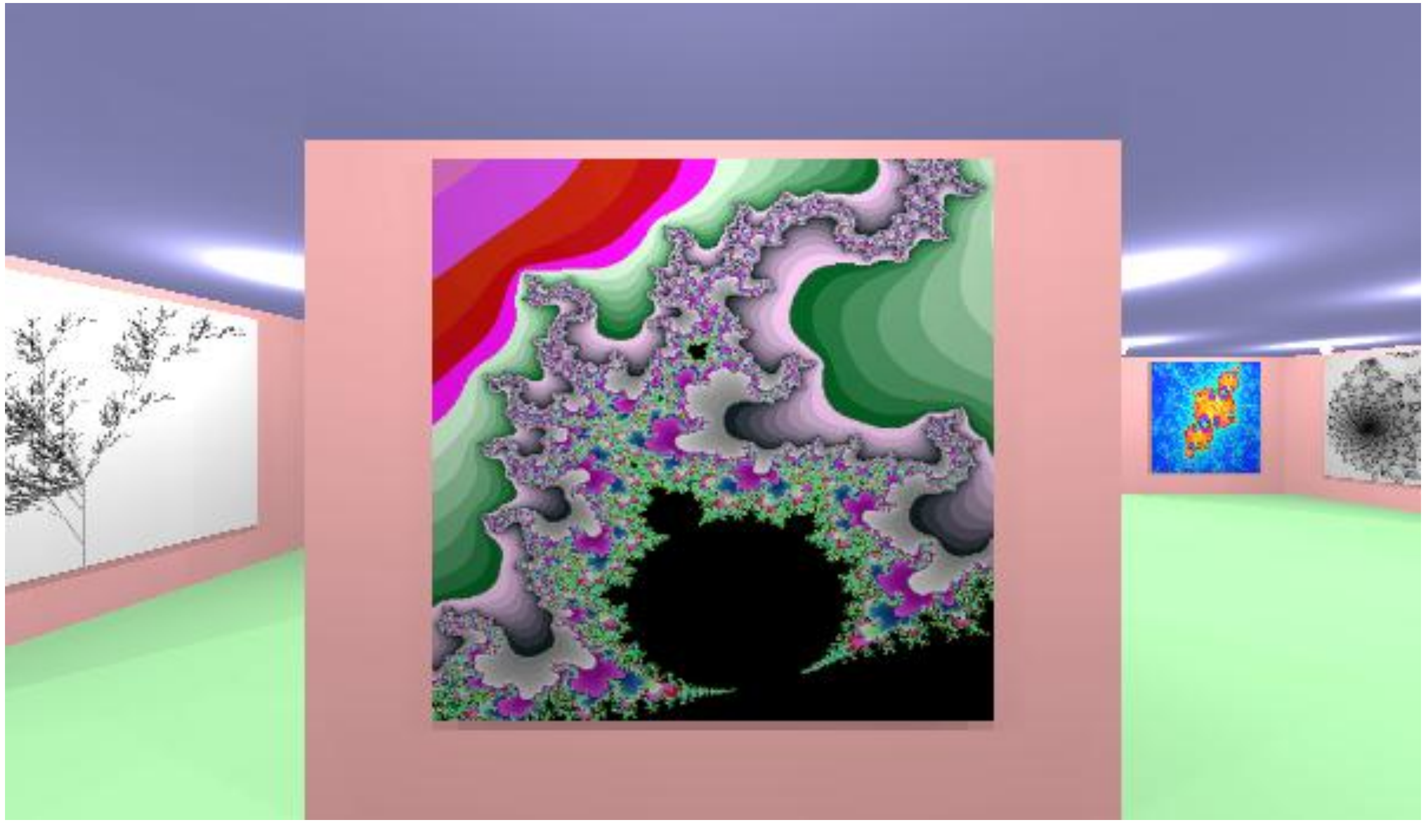
605.767 Applied Computer Graphics

Brian Russin

JOHNS HOPKINS
WHITING SCHOOL
of ENGINEERING

Module 7G
Fractals and Shape Grammars

Fractals



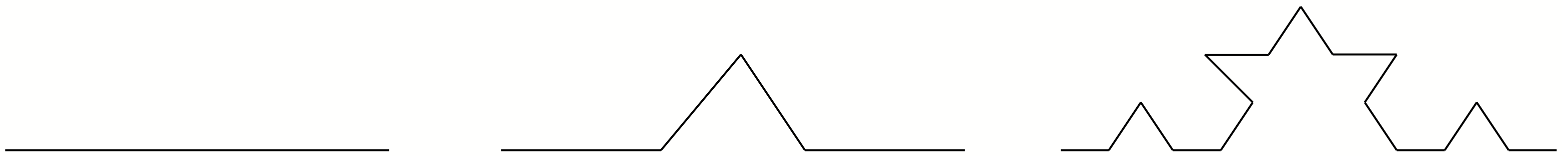
- Fractal gallery
  - http://www.mhri.edu.au/~pdb/fractals/fractalgallery
  - Site does not appear to be active: searches for Fractal Gallery lead to many others, like this: https://scratch.mit.edu/studios/33129

Fractal Geometry

- **Fractal geometry** - term defined by Benoit Mandelbrot
  - Euclidean geometry is insufficient to describe natural objects which have irregular or fragmented features
  - In computer graphics: the term fractal has been generalized to include objects and techniques outside Mandelbrot's original definition
- Properties of fractal objects
  - Infinite detail at every point
  - Self similarity
    - Precise definition requires statistical self-similarity at all levels of detail
- Classification of fractals
  - Self-similar fractals
    - Have parts that are scaled down versions of the entire objects
  - Self-affine fractals
    - Have parts that are formed with different scaling parameters in different directions
    - Can include random variations to obtain statistically self-affine fractals

Successive Refinement

- Complex curves can be constructed by repeatedly “refining” a simple curve
- von Koch snowflake is such a construction
  - Exhibits self similarity
  - Start with a line segment with a bump on it
  - Replace each segment of the line with a figure exactly like the original line
  - Repeat the process
  - If the process is repeated infinite times the object becomes self similar
    - Entire object is similar to a sub-portion of itself



Replace each line with
this pattern

Fractal Dimension

- **Fractal dimension** is associated with self-similarity
  - Related to intuitive notion of dimension
    - Line segment is one dimensional with self similarity
      - Can be divided into N equal parts each scaled down 1/n from the whole
    - Square is two dimensional
      - Can be divided into N identical parts - each scaled by $N^{1/2}$
    - Cube can be divided into N smaller cubes each scaled by $N^{1/3}$
    - D-dimensional self-similar object can be divided into N smaller copies of itself each scaled down by factor r = 1/N 1/D
- Fractal or similarity dimension is given by
  - D = log(N) / log(1/r)
    - Need not be an integer
  - von Koch snowflake: any segment is composed of 4 sub-segments each scaled down by a factor of 1/3 from its parent
    - Fractal dimension is D = log(4) / log(3) = 1.26
  - Non-integer dimension can be interpreted as:
    - Fills more of space than line (1-D) but less than Euclidean area of plane (2- D)

Statistical Self-Similarity

- Objects in nature rarely exhibit exact self- similarity
  - Often exhibit **statistical self-similarity**
- Example is a coastline
  - Closer one looks at a coastline the more detail one sees
    - Measuring length of coastline - the more carefully the small wiggles are followed the longer the coastline becomes
  - Each small section of the coastline looks similar (but not exactly like) a larger portion
- Concept of statistical self-similarity can also be quantified in terms of fractal dimension
  - Using a ruler size r times the number of steps taken of size r, N(r) taken in tracing the coast:
    - Length = r * N(r)
    - N(r) varies on the average as $1/r^D$
  - Coastlines can be characterized by fractal dimensions of 1.15 to 1.25
  - Terrain has fractal dimension of 2.15 to 2.25

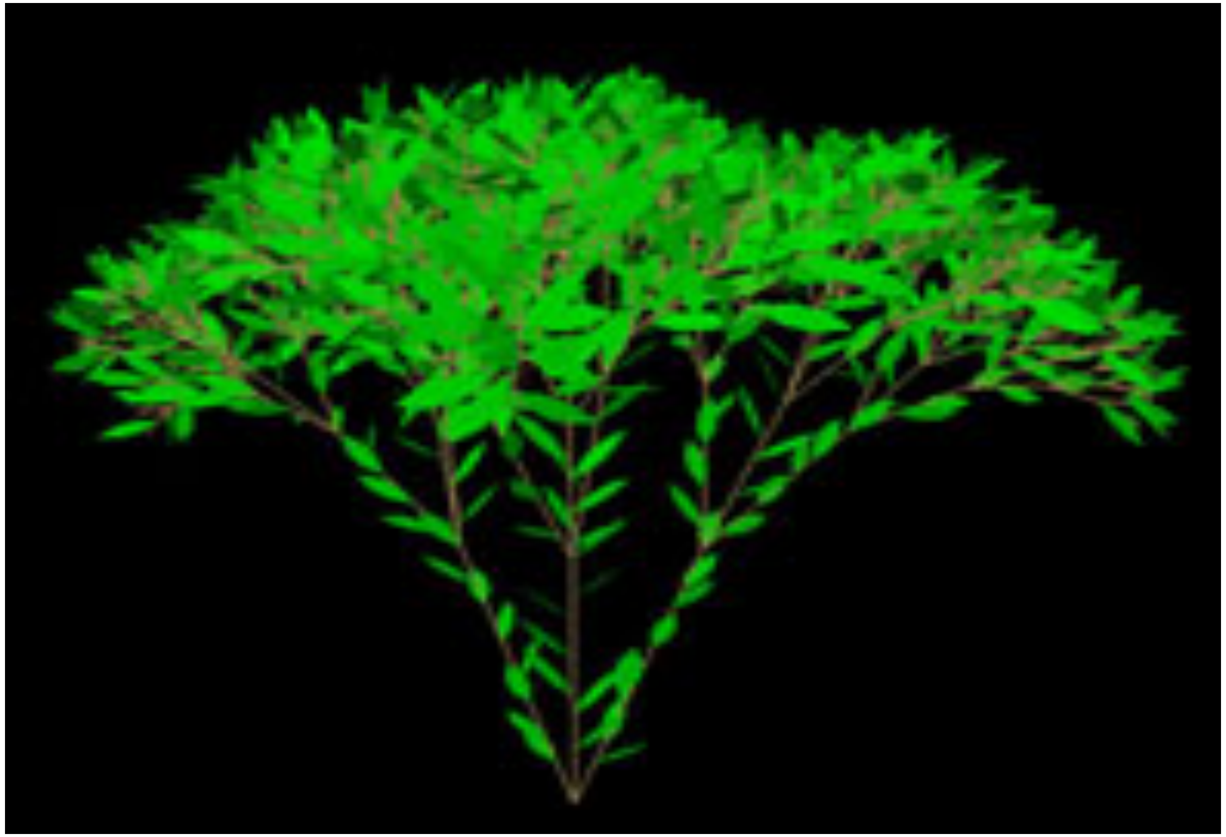
Julia and Mandelbrot Sets

- The two most famous fractal objects are the Julia set and the Mandelbrot set
- Objects are generated from the study of the rule $x \rightarrow x^2 + c$
  - x is a complex number: $x = a + bi$
  - Modulus < 1, repeated squaring will approach 0
    - Modulus is the real number $(a^2 + b^2)^{1/2}$
  - Modulus > 1, repeated squaring will approach infinity
  - Modulus = 1, repeated squaring stays at 1
  - Julia sets plot set of points going to 0, infinity, or neither for various complex numbers c
    - Shape of the set depends on the complex number c chosen
    - Generally plot regions of space that approach 0 vs those that approach infinity by performing an iteration $x \rightarrow x^2 + c$ a specified number of times
  - Can choose colors based on how rapidly the point diverges
    - This produces the pretty graphs of regions of the Julia or Mandelbrot set

Some Uses of Fractals

- Fractal terrain generation or coastline generation is common
- Fractals have also been used to generate other objects
  - Clouds, water, trees, feathers, surface textures
  - Distributing craters on lunar landscape
  - Creating a fractal planet: surface generated by Brownian motion or random walk on a sphere
    - Brownian motion is a form of random walk - describes erratic movement of gas particles
    - Given a starting position generate a straight line segment in a random direction with random distance - then repeat
  - Fractal flakes and clouds
  - Fractal music
  - Many others - including producing pretty patterns

Shape Grammars



L-system plant geometry to model a shrub

Grammar-Based Modeling

- Grammar based modeling techniques have mainly been used to model trees and plants
  - Also called **shape grammars**
  - Generally use parallel graph grammar languages called L-grammars or L-systems
    - Invented by A. Lindenmayer for modeling plants and trees
  - Foley and van Dam have a good description of the technique
  - Primarily used where object being modeled exhibits regularity
- Sets of production rules applied to an initial object
  - Add layers of detail that are harmonious with initial shape
  - Can alter geometry or surface color/texture
- Given a set of production rules a shape designer can experiment and apply different rules at each step of the transformation
  - Rules can be described graphically showing initial and final shapes
  - Trees: can be written as Tree:: = branch + tree
    - 'Tree is a branch with a tree on the end of it'