Preferences and Utilities

The Toolbox

October 9, 2023

(Micro)Economics

In Robbins definition, Economics is *the science which studies human behaviour as a relationship between ends and scarce means which have alternative uses.*

People make **trade-offs** and consider opportunity costs in their decisions.

(Efficient) Allocation

- Allocating resources implies choosing. This is the central topic of Microeconomics: create models that explain choices.
- Typical Framework: Individuals are optimisers. Therefore, optimisation will be our key technical tool to analyse choices.
  - Find the objective

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  - Find a solution

Preferences

The fundamental concept that founds economics is the one of preferences, i.e. a relation between bundle of goods.
Formally, we write

$$X \succ Y\ ,\ \ X \sim Y\ ,\ \ X \prec Y$$

X here represents a general bundle with as many goods as we want, but usually we will work in 2 dimensional frameworks and therefore $X = (x_1, x_2) \in \mathbb{R}^2_+$

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  - For each X, Y, Z, if $X \succsim Y$ and $Y \succsim Z$, then $X \succsim Z$
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- **Convexity**: averages are better than extreme
  - For each X, Y with $X \succsim Y$, $\alpha X + (1 - \alpha) Y \succsim Y$

From preferences to utility

If preferences are **rational** (i.e. complete and transitive) and continuous[1], then we can attach to each bundle a number, therefore constructing the a so-called **utility function** that represents the preferences, i.e.

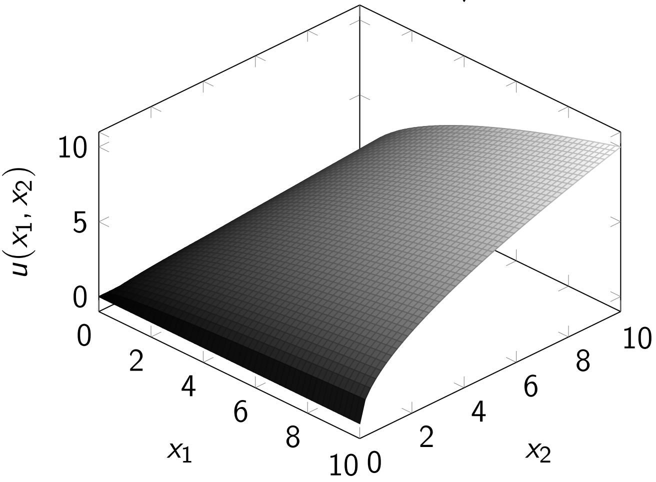
$$X \succsim Y \iff u(X) \geq u(Y)$$

This seems a simple trick, but will simplify a lot handling our problems, and we will mostly drop preferences and use directly utility functions. This is why defining the most preferred object might be harder than defining the highest utility bundle.

[1] continuity of preferences guarantee continuity of the utility function

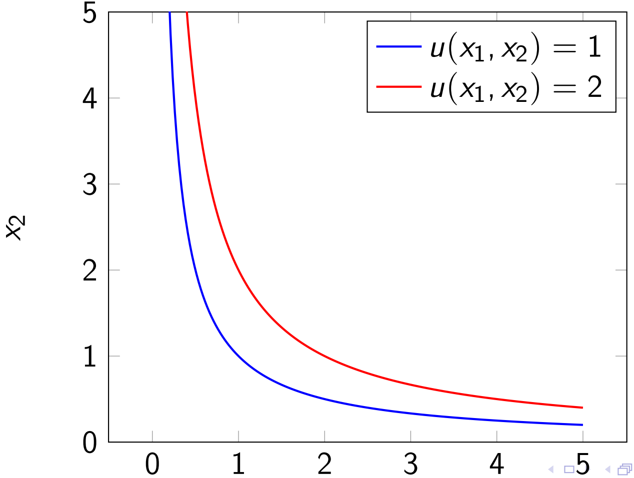
Utility: a graphical perspective

In our 2 dimensional world, utilities are 3-dimensional objects: the function picks a bundle of 2 goods and attaches to that a number. For example: $u(x_1, x_2) = \sqrt{x_1 \cdot x_2}$



Indifference curves

Imagine slicing the utility function horizontally, you obtain the so called level curves of a function, i.e. the set of point such that $u(x_1, x_2) = \bar{u}$. Since all of the points on the curve have the same utility, we call them **indifference curves**.



Notable preferences

In economic theory, there are a few preferences/utilities which are widely used:

- Cobb Douglas $u(x_1, x_2) = x_1^a x_2^b$
- Prefect Complements $u(x_1, x_2) = \min[x_1, x_2]$
- Perfect Substitutes $u(x_1, x_2) = x_1 + x_2$
- Quasi Linear $u(x_1, x_2) = v(x_1) + x_2$, where $v(\cdot)$ is a generic function, can be a log, a square root or a power
- Homothetic: family of preferences that satisfy: if$X \succsim Y$, then $tX \succsim tY$

Try and draw indifference curves for each of this and ask yourself what assumptions the underlying preferences satisfy.

Monotonic transformation

One thing to remind about utilities is that they represent preferences and therefore they are an **ordinal** concept: we only care about ordering bundles, not their actual levels of utility. Then, moving the utility of *all* bundles up or down in a smooth way would preserve the order. This is called a monotonic transformation, i.e. a function $g(\cdot)$ such that

$$X \succsim Y \iff u(X) \geq u(Y) \iff g(u(X)) \geq g(u(Y))$$

For example, $v(x_1, x_2) = a\, log x_1 + b\, log x_2$ is a monotonic transformation of a Cobb Doublas because

$$log\ u(x_1, x_2) = log(x_1^a x_2^b) = a\, log x_1 + b\, log x_2$$

Can you think of other possible monotonic transformations?

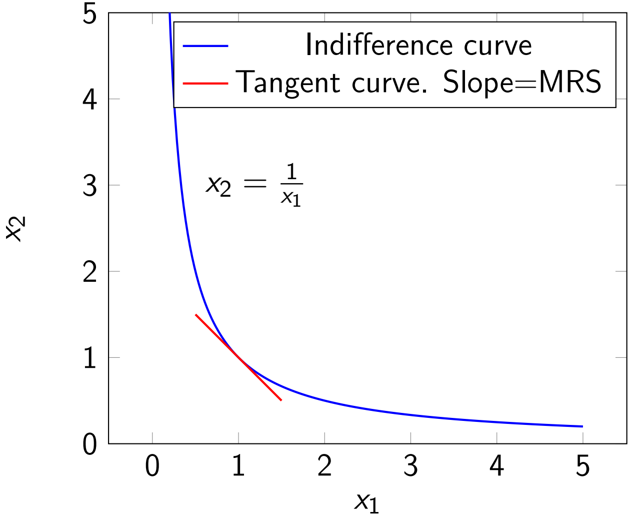
Marginal rate of Substitution

Using utilities and indifference curves, we can define a very important concept, which is the slope of the indifference curve at a point. In other terms, how we need to swap/trade goods to remain indifferent. Formally, this object is called the Marginal rate of substitution and it is computed as follows:

$$MRS = -\frac{d\ x_2}{d\ x_1} = -\frac{\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}}$$

We refer to $\frac{\partial u}{\partial x_1}$ as the Marginal utility for good 1, i.e. the additional utility gained by increasing marginally the quantity of good 1.

MRS - visualisation



Lecture 2 - (Marshallian) Demand

October 2023

Reminder

Typical Framework: Individuals are optimisers. Therefore, optimisation will be our key technical tool to analyse choices.

- Find the objective
- Define constraints
- Find a solution

We introduced the notion of utility. How can we apply it to explain choices?

Utility Optimisation

- Find the objective $\rightarrow$ Utility
- Define constraints $\rightarrow$??
- Find a solution $\rightarrow$??

In mathematical terms:

$$\begin{aligned} \max_{x_1, x_2} \quad & u(x_1, x_2) \\ s.t. \quad & ??? \end{aligned}$$

Budget Constraint

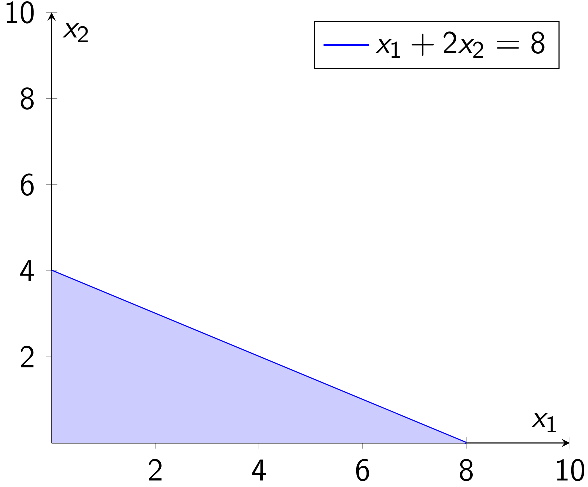
Constraints usually represent the scarsity of resources.
Simplest resource which is scarse is money, in our terminology, **income**.
Assigning prices to goods, we obtain the following *budget constraint*

$$p_1x_1 + p_2x_2 \leq m$$

This represents the set of bundles which are affordable at income m. Given utility comes only from these two goods, there is no incentive to save, and therefore we will often consider the budget constraint with equality, i.e. spending all the income.

What would happen to the set of affordable bundles (and graph) if a price increase/decrease? If income increases/decreases?

Budget Constraint - Graph



Utility Maximisation Problem (UMP)

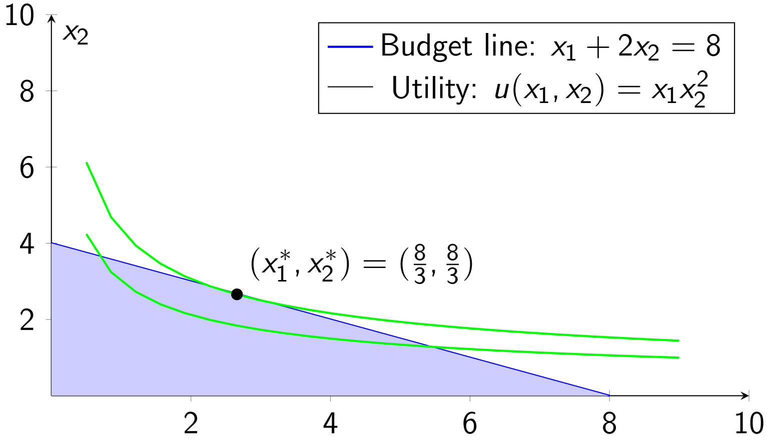
The standard optimisation problem for utility will then look like

$$\begin{aligned} \max_{x_1, x_2} \quad & u(x_1, x_2) \\ s.t. \quad & p_1 x_1 + p_2 x_2 \leq m \end{aligned}$$

It means we are looking for an affordable bundle which gives the highest utility. This bundle will clearly depend on prices and income.

We call the solution to this problem the **Marshallian demand**, as it will represent the optimal quantity demanded at given prices and income $x(p, m)$.

UMP - Graphical solution



The optimal point lies on the highest indifference curve touching the budget line.

Computing Marshallian Demand

1- Equimarginal principle

Most of the cases we will work with involves utilities whose indifference curves are smooth and convex (e.g. Cobb Douglas).

In this case we can exploit the fact that, at the optimum, the indifference curve and the budget line will be **tangent**. Therefore, their slopes will coincide, giving us the condition

$$MRS(x_1^*, x_2^*) = -\frac{p_1}{p_2} \text{ or } \frac{\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}} = \frac{p_1}{p_2}$$

And together with this condition, we use the budget constraint equation (with $=$).

What if the MRS is greater than the price ratio?

Computing Marshallian Demand

2- Setting up the Lagrangian

A more robust (but longer) method is borrowed from constrained optimization, in which we construct the Lagrangian function, i.e.

$$L = \textit{objective function} - \lambda(\textit{constraint}),$$

where λ is the so called Lagrangian multiplier. Therefore,

$$L(x_1, x_2, \lambda) = u(x_1, x_2) - \lambda(p_1 x_1 + p_2 x_2 - m)$$

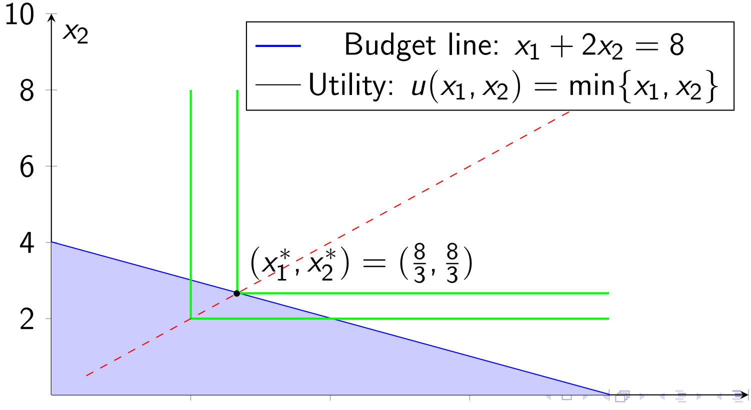
And then compute first order conditions as if this was an unconstrained optimisation.

Computing Marshallian Demand

3- Other cases

There are cases in which neither of the previous cases work: what to do then? You can always rely on intuition.

For example, with perfect complements,
$u(x_1, x_2) = \min\{x_1, x_2\}$



Testing the Model

Clearly, this is theoretical model that aims at explaining choices, assuming the existence of such thing as a utility function underlying individual decisions.

We would ideally want to look at data to see if decision would be consistent with some generic prediction of the model.

Upon observing prices, choice and other bundles (income is usually not observed as well), we will test two hypothesis on the Revealed Preferences.

Revealed Preferences

Suppose you can observe prices and choice. What can I say about the underlying preferences?

The simplest implication is: if I have two available bundles and I observe that the first is chosen when the second was affordable, we can say that it is *revealed preferred* to the second.
In other terms, if $p_1x_1 + p_2x_2 \geq p_1y_1 + p_2y_2$ and x is chosen, it must be that x is (revealed) preferred to y.
If then, at prices q we choose y over z, we say that x is *indirectly* preferred to z.

WARP

Weak Axiom of Revealed Preferences

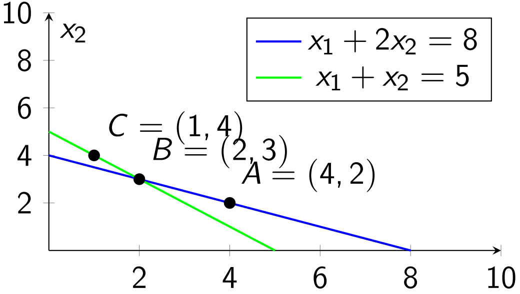
As a fairly mild condition to check our observed choice are in some manner *rational*, or more precisely consistent with our model, we simply demand that if we choose x over y at some prices p when both are affordable, we can never reverse the choice at any other price in which both are affordable.

In other terms, if x is chosen over y with $p_1x_1 + p_2x_2 \geq p_1y_1 + p_2y_2$, then if at prices q we observe y being chosen, it must be that $q_1x_1 + q_2x_2 > q_1y_1 + q_2y_2$.

SARP

Strong Axiom of Revealed Preferences

As a stronger condition, for a choice to be consistent, we might require that if we observe A being chosen while B is affordable, B can never be revealed preferred to A. If also B is chosen over a third bundle C when both affordable, then C can never be (indirectly or not) revealed preferred to A.
In a way, we are checking observed choices respect transitivity.



Lecture 3 - Demand: Comparative Statics

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Comparative statics

Economists are interested in capturing evolving phenomena. Therefore, it is very interesting to study what would happen if we change some parameters of a model in terms of our expected outcome, **ceteribus paribus**.

In our framework, we might want to understand what would happen to the optimal bundle $x^*(p, m)$ if prices or income changes. Here, we will study mainly 2 aspects:

- Change in Income
- Change in **own** price

A taxonomy of goods

- Income change
  - $\frac{\partial x_1^*}{\partial m} > 0$: **Normal Good**
  - $\frac{\partial x_1^*}{\partial m} < 0$: **Inferior Good**
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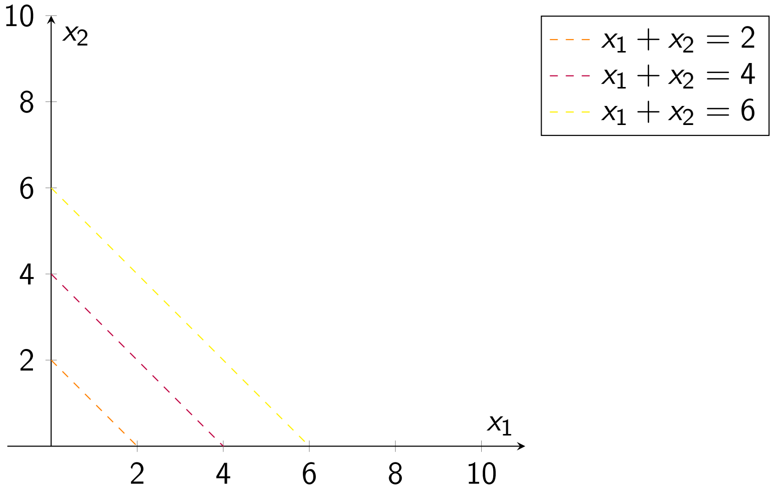
Can you provide examples for each of the goods? What about combinations of the two?

Normal Good - Income expansion path



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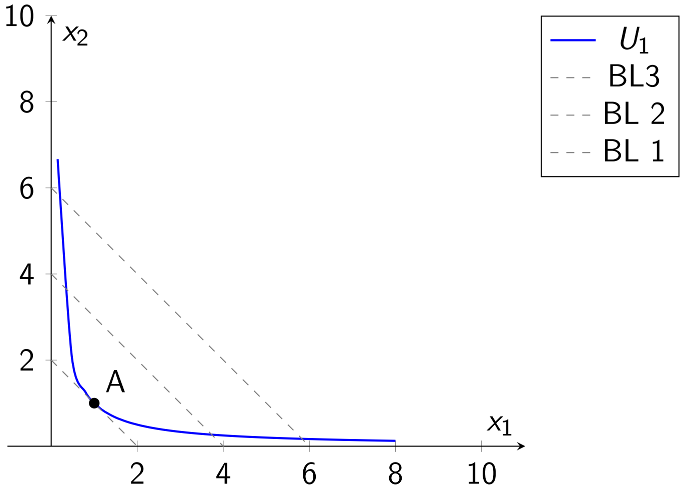

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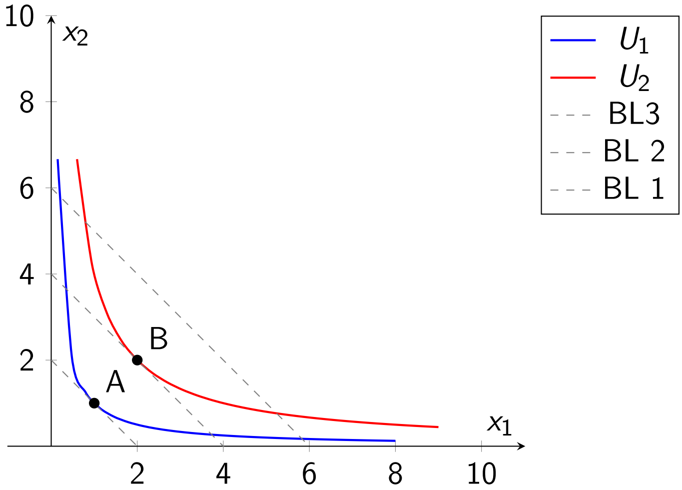

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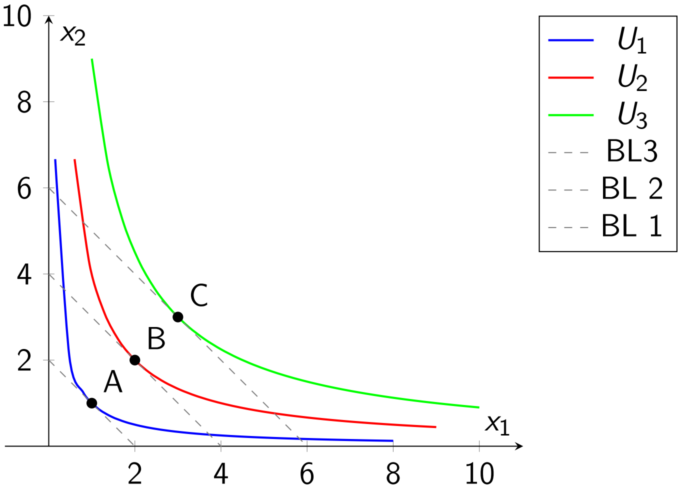
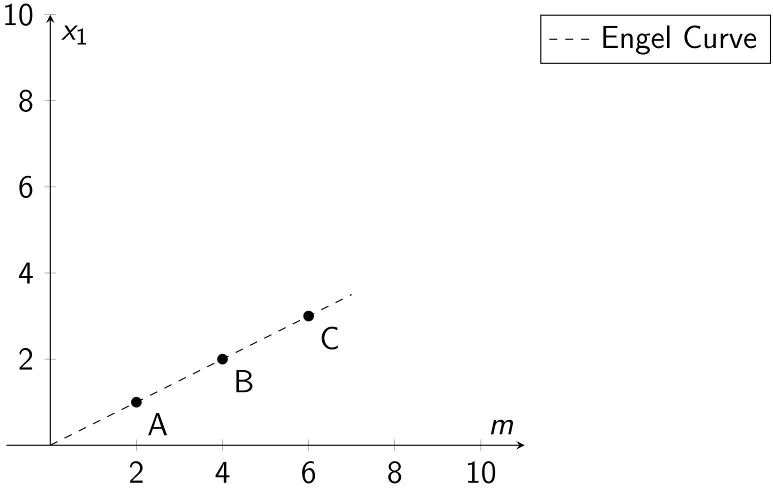
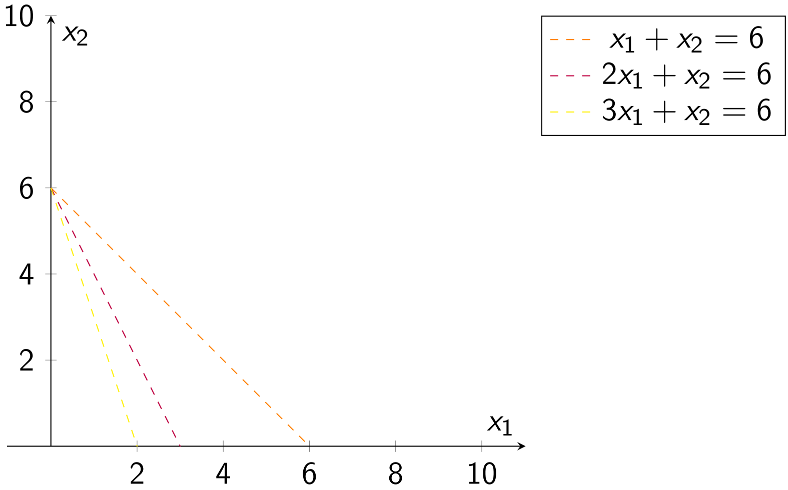


Engel Curve

If we plot all the income-demand combination we obtain the so called **Engel Curve**



Ordinary Good



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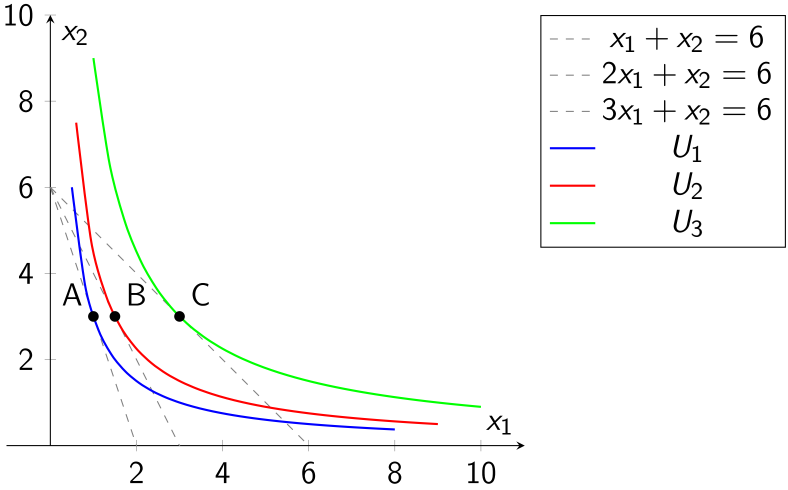
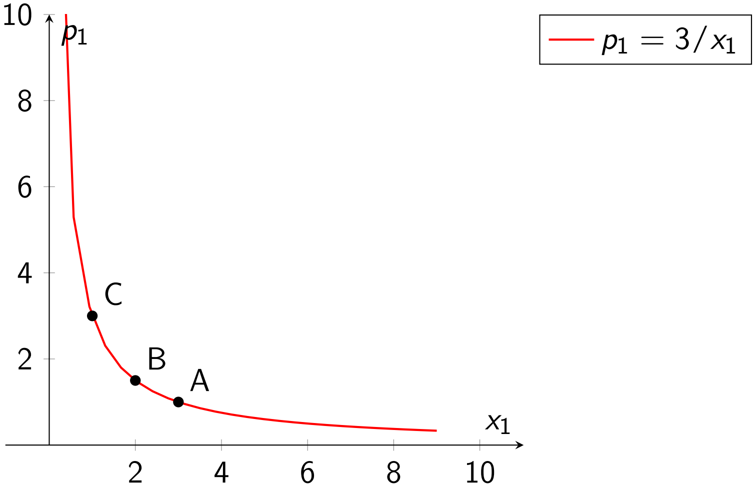


Why is x_2 not changing?

Individual Demand function

If we plot all the price-demand combination we obtain the so called **Demand Curve**



How do we interpret this function?

Price Elasticity of Demand

An interesting part of the demand function is its slope, i.e. how fast the demand reacts to a price change.

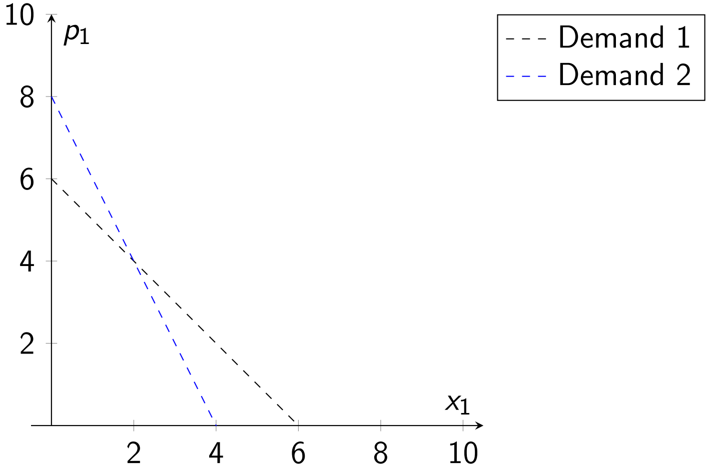
In particular, we define the **Price Elasticity of Demand** (PED) as the ratio of relative changes, (usually referred to in absolute terms)

$$\epsilon = \frac{\frac{\partial x}{x}}{\frac{\partial p}{p}} = \frac{\partial x}{\partial p}\frac{p}{q}$$

Usually a demand with elasticity less than one is called *inelastic*, while if the PED is greater than unity is called *elastic*.

Constructing Market Demand

We showed what is an individual demand. What if we have many individuals with different demands? How do we "sum" individual demands?



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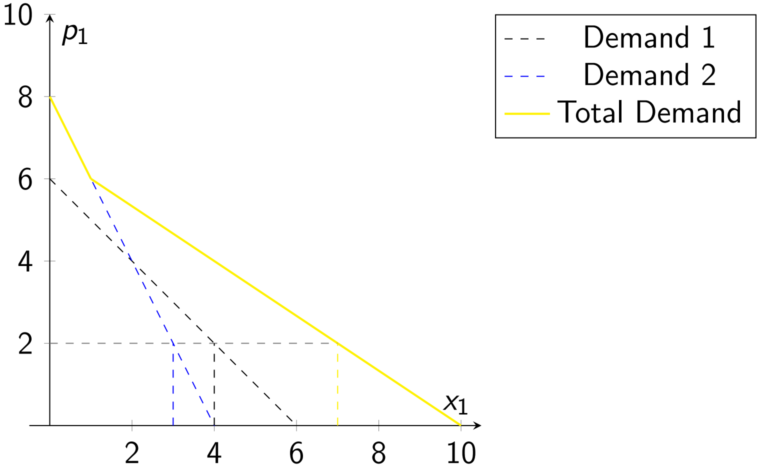


Slutsky Decomposition

Let's look back at the effects of a price change. We can ideally think to separate two different effect (for a price increase):

- Switch consumption towards the second good
  $\rightarrow$**Substitution Effect**
- Loss of purchase power
  $\rightarrow$ **Income Effect**

This idea is called *decomposition* of the effect on demand of a price change.
Ideally we want to separate the two effects, so that in the first only price change and in the second only income change.

Compensated Demand

To eliminate any possible purchase power "interference", we construct a compensated budget line.

Let's assume the original prices are (p_1, p_2), the original demand $(x_1^*(p, m), x_2^*(p, m))$ and the new price for good 1 p_1'. Then

$$p_1 x_1^* + p_2 x^* = m$$

$$p_1' x_1^* + p_2 x_2^* = m'$$

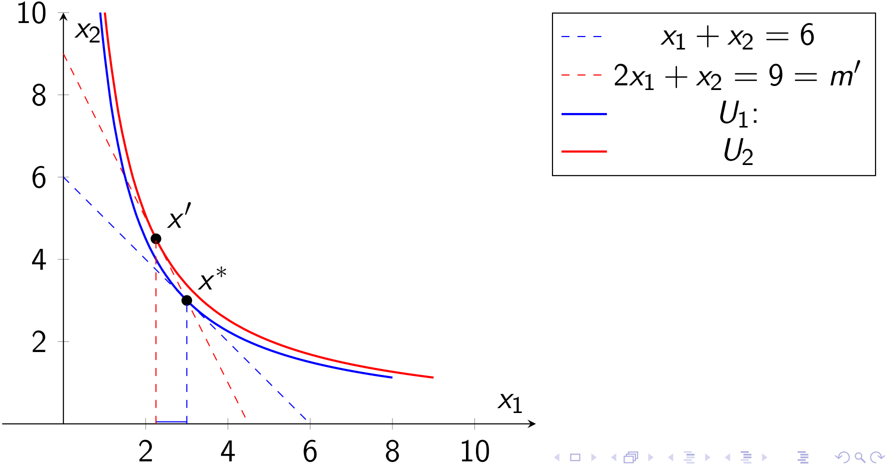
m' represents the income needed to be able to purchase the <u>initial</u> quantities at <u>new</u> prices. Let (x_1', x_2') be the demand at prices (p_1', p_2) and income m', i.e. the solution to

$$\max_{(x_1, x_2)} \; u(x_1, x_2)$$

$$s.t. \; p_1' x_1 + p_2 x_2 = m'$$

Substitution Effect

The difference between the bundle x^* and x' is the substitution effect, as the only thing impacting this change is due to the price change.



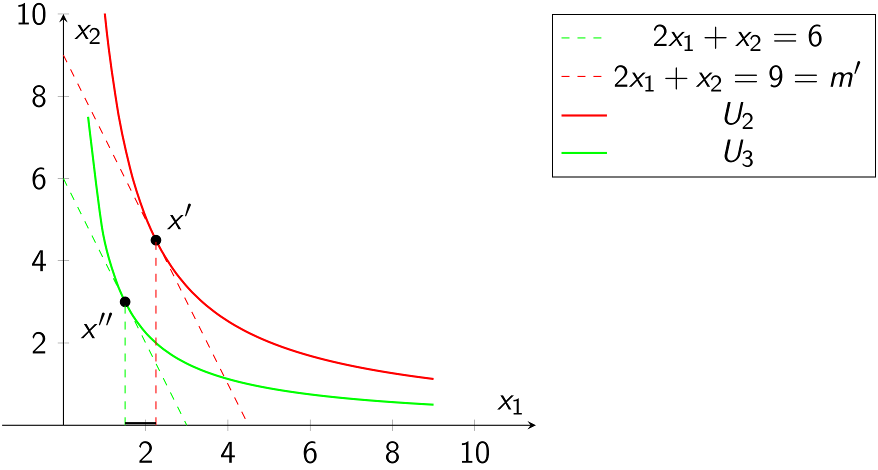
Income Effect

As a second step, we can move from x', i.e. the optimal bundle at new price and adjusted income, to x'', the final bundle obtained by only moving back the income to its "natural" level. Therefore, x'' is the solution to

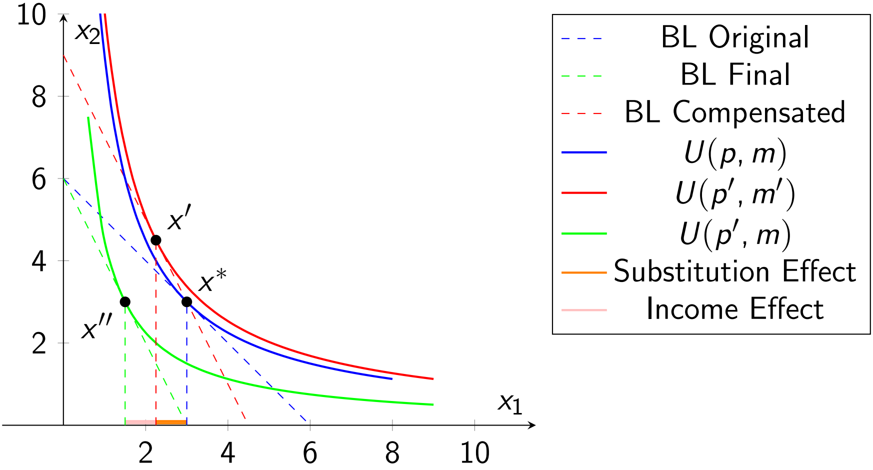
$$\max_{(x_1,x_2)} \; u(x_1, x_2)$$
$$s.t. \; p_1' x_1 + p_2 x_2 = m$$

Then, we define the Income Effect as the difference between x'' and x', as this only depends on the (fictitious) change in income.

Income Effect



Slutsky Decomposition- Connecting the pieces



Slutsky Decomposition - Mathematical approach

We are decomposing $x'' - x^*$ or $x(p', m) - x(p, m)$.

$$x'' - x^* = \underbrace{(x'' - x')}_{\textit{Income effect}} + \underbrace{(x' - x^*)}_{\textit{Substitution Effect}}$$

$$x(p', m) - x(p, m) = \underbrace{(x(p', m) - x(p', m'))}_{\textit{Income effect}} + \underbrace{(x(p', m') - x(p, m))}_{\textit{Substitution Effect}}.$$

Let's focus on the Substitution Effect. First, recall that $m' - m = \Delta m = x^* \Delta p$. Now let's compute its derivative with respect to p:

$$\frac{dx^S}{dp} = \frac{dx}{dp} + \frac{dx}{dm}\frac{dm}{dp} = \frac{dx}{dp} + \frac{dx}{dm}x^*$$

Slutsky Decomposition - Mathematical approach

Finally, rearranging the last expression we have:

$$\frac{dx}{dp} = \underbrace{\frac{dx^S}{dp}}_{Substitution\ Effect} - \underbrace{\frac{dx}{dm}x^*}_{Income\ Effect}$$

Law of Demand

The Substitution Effect is negative, i.e.

$$\frac{dx^S}{dp} = \frac{dx}{dp} + \frac{dx}{dm}x^* < 0$$

Application: what is the relation between normal and ordinary goods?
What can we say about the size of SE and IE for an inferior good?

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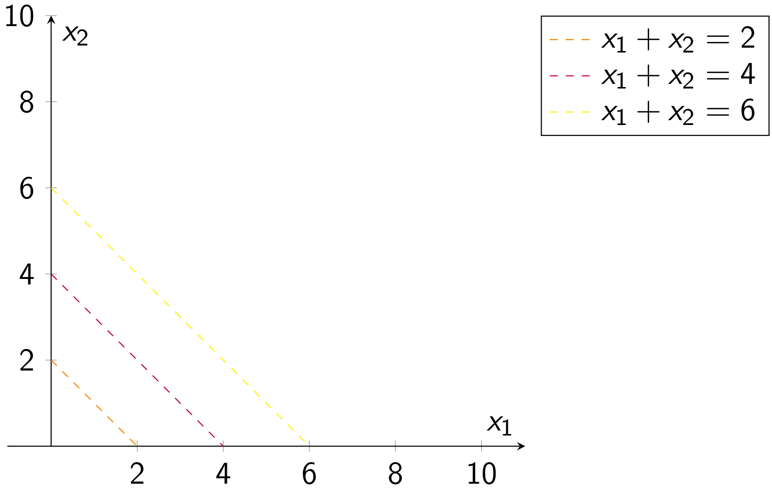

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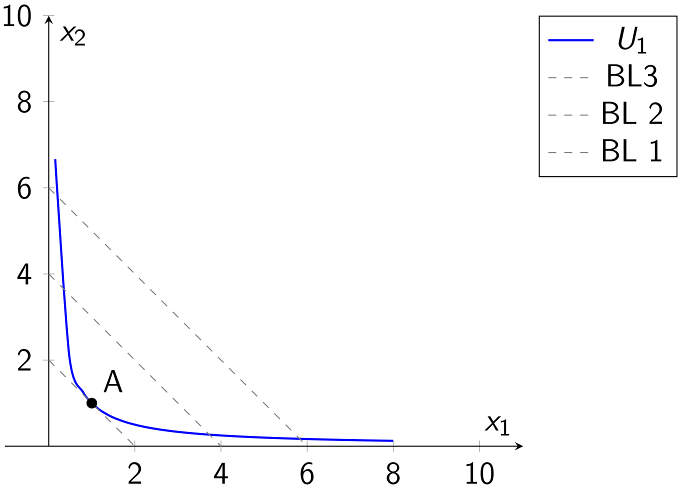

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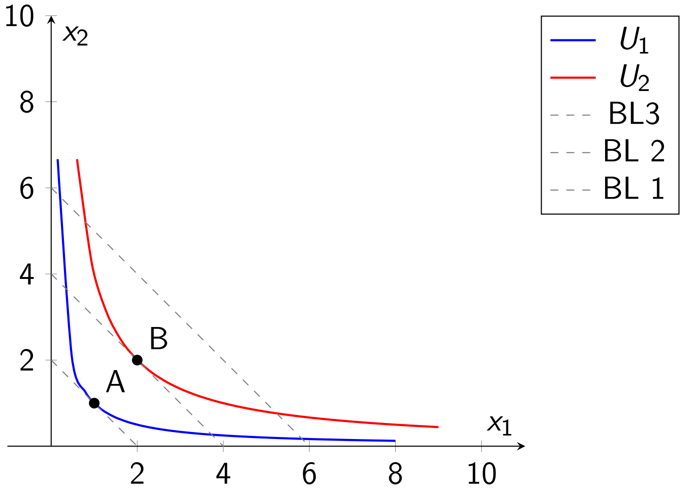

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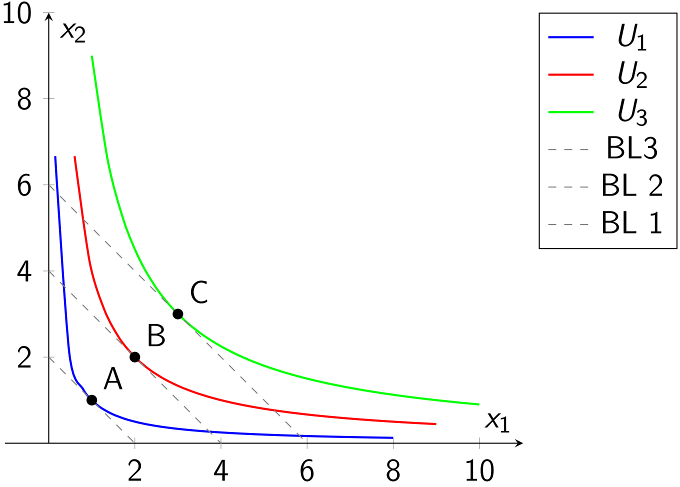

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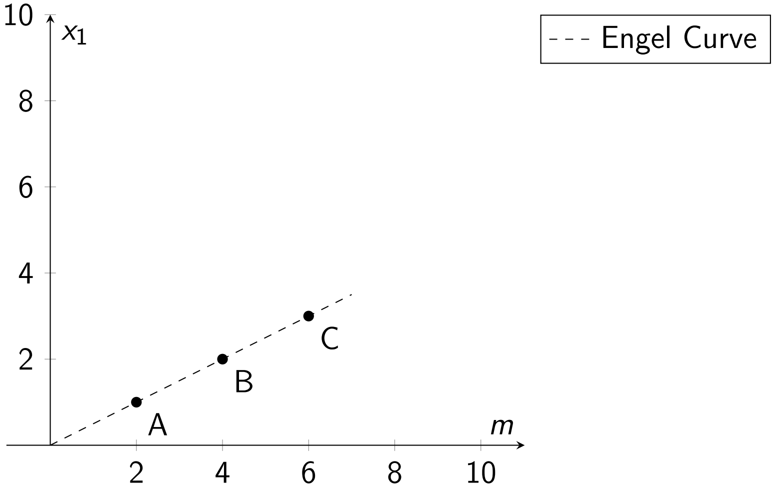
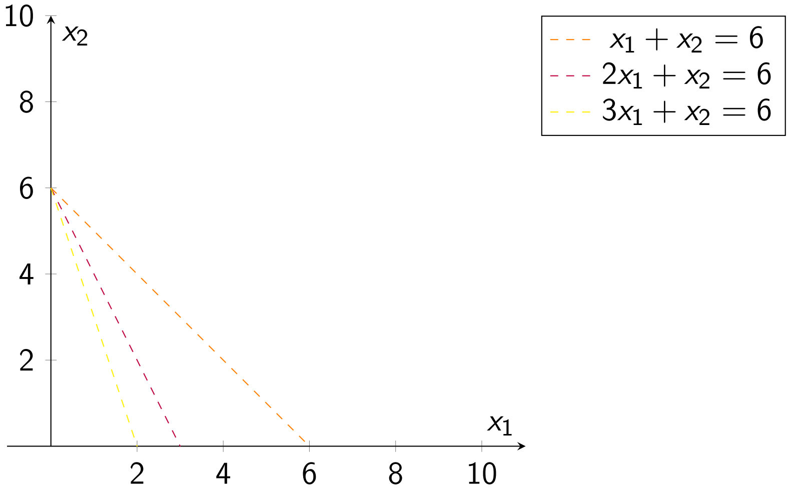


Ordinary Good



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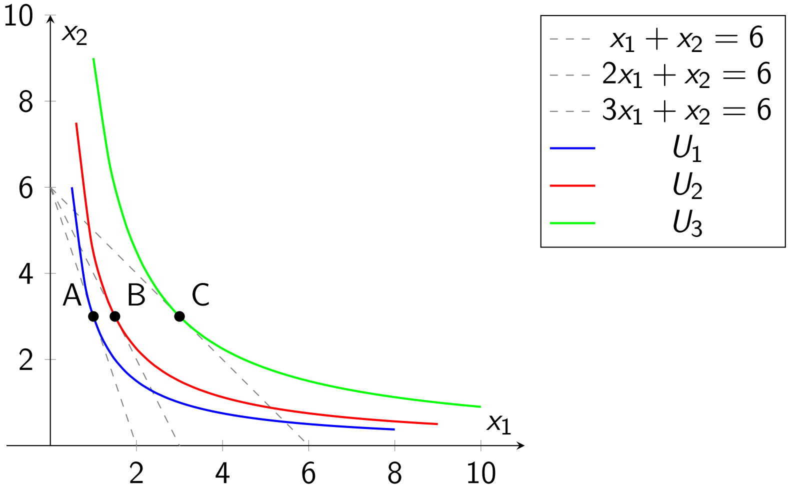

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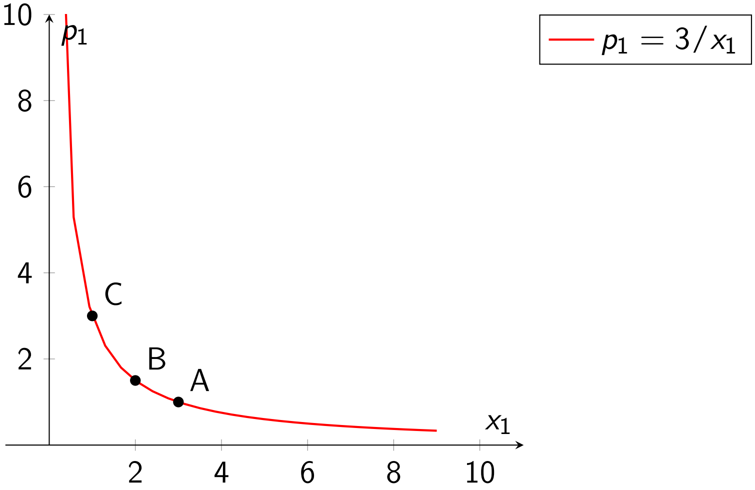
Individual Demand function

If we plot all the price-demand combination we obtain the so called **Demand Curve**



How do we interpret this function?

Price Elasticity of Demand

An interesting part of the demand function is its slope, i.e. how fast the demand reacts to a price change.

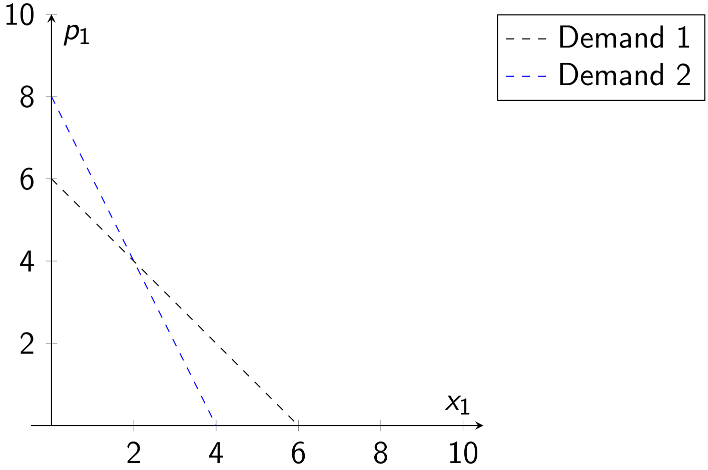
In particular, we define the **Price Elasticity of Demand** (PED) as the ratio of relative changes, (usually referred to in absolute terms)

$$\epsilon = \frac{\frac{\partial x}{x}}{\frac{\partial p}{p}} = \frac{\partial x}{\partial p}\frac{p}{q}$$

Usually a demand with elasticity less than one is called *inelastic*, while if the PED is greater than unity is called *elastic*.

Constructing Market Demand

We showed what is an individual demand. What if we have many individuals with different demands? How do we "sum" individual demands?



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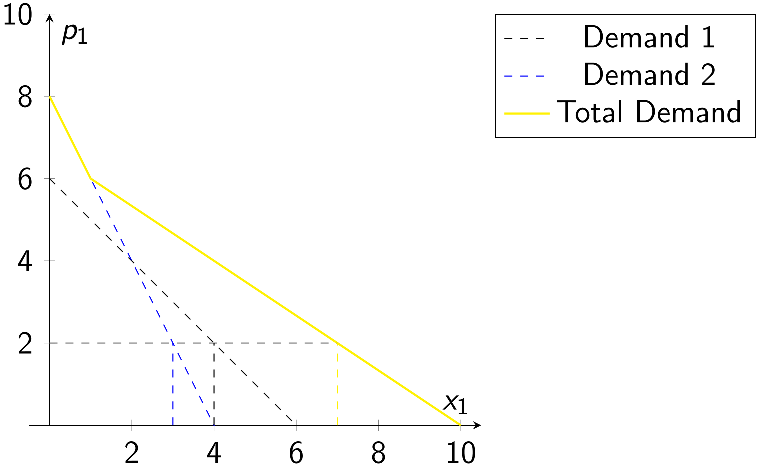


Slutsky Decomposition

Let's look back at the effects of a price change. We can ideally think to separate two different effect (for a price increase):

- Switch consumption towards the second good
  $\rightarrow$**Substitution Effect**
- Loss of purchase power
  $\rightarrow$ **Income Effect**

This idea is called *decomposition* of the effect on demand of a price change.
Ideally we want to separate the two effects, so that in the first only price change and in the second only income change.

Compensated Demand

To eliminate any possible purchase power "interference", we construct a compensated budget line.

Let's assume the original prices are (p_1, p_2), the original demand $(x_1^*(p, m), x_2^*(p, m))$ and the new price for good 1 p_1'. Then

$$p_1 x_1^* + p_2 x^* = m$$

$$p_1' x_1^* + p_2 x_2^* = m'$$

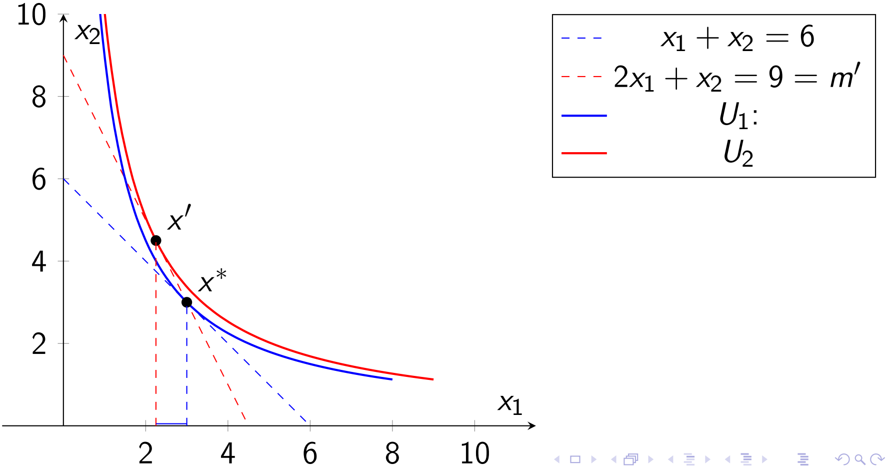
m' represents the income needed to be able to purchase the <u>initial</u> quantities at <u>new</u> prices. Let (x_1', x_2') be the demand at prices (p_1', p_2) and income m', i.e. the solution to

$$\max_{(x_1, x_2)} \; u(x_1, x_2)$$

$$s.t. \; p_1' x_1 + p_2 x_2 = m'$$

Substitution Effect

The difference between the bundle x^* and x' is the substitution effect, as the only thing impacting this change is due to the price change.



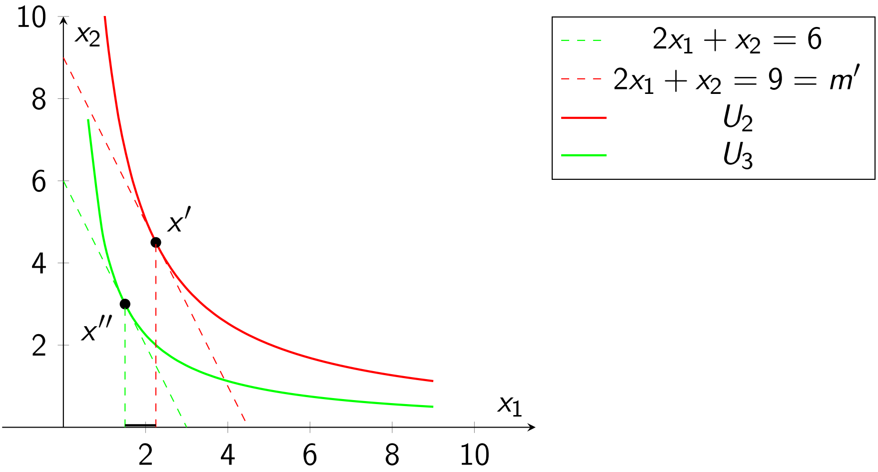
Income Effect

As a second step, we can move from x', i.e. the optimal bundle at new price and adjusted income, to x'', the final bundle obtained by only moving back the income to its "natural" level. Therefore, x'' is the solution to

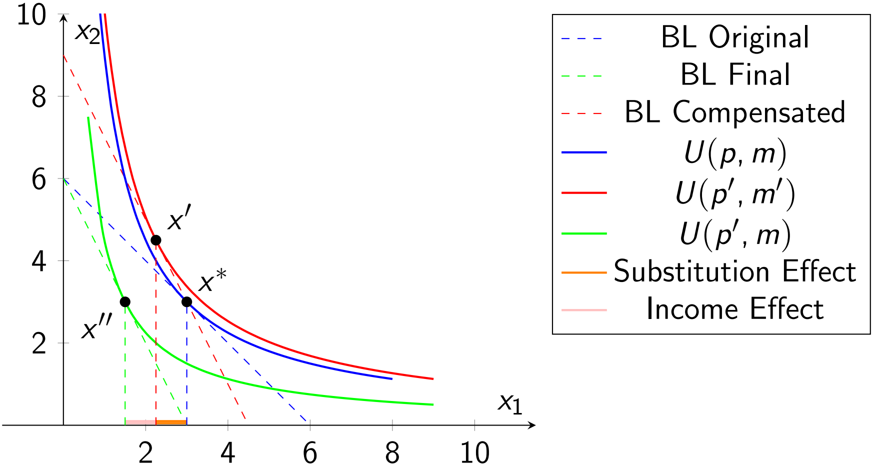
$$\max_{(x_1,x_2)} \ u(x_1,x_2)$$
$$s.t. \ p_1' x_1 + p_2 x_2 = m$$

Then, we define the Income Effect as the difference between x'' and x', as this only depends on the (fictitious) change in income.

Income Effect



Slutsky Decomposition- Connecting the pieces



Slutsky Decomposition - Mathematical approach

We are decomposing $x'' - x^*$ or $x(p', m) - x(p, m)$.

$$x'' - x^* = \underbrace{(x'' - x')}_{\textit{Income effect}} + \underbrace{(x' - x^*)}_{\textit{Substitution Effect}}$$

$$x(p', m) - x(p, m) = \underbrace{(x(p', m) - x(p', m'))}_{\textit{Income effect}} + \underbrace{(x(p', m') - x(p, m))}_{\textit{Substitution Effect}}.$$

Let's focus on the Substitution Effect. First, recall that $m' - m = \Delta m = x^* \Delta p$. Now let's compute its derivative with respect to p:

$$\frac{dx^S}{dp} = \frac{dx}{dp} + \frac{dx}{dm}\frac{dm}{dp} = \frac{dx}{dp} + \frac{dx}{dm}x^*$$

Slutsky Decomposition - Mathematical approach

Finally, rearranging the last expression we have:

$$\frac{dx}{dp} = \underbrace{\frac{dx^S}{dp}}_{Substitution\ Effect} - \underbrace{\frac{dx}{dm}x^*}_{Income\ Effect}$$

Law of Demand

The Substitution Effect is negative, i.e.

$$\frac{dx^S}{dp} = \frac{dx}{dp} + \frac{dx}{dm}x^* < 0$$

Application: what is the relation between normal and ordinary goods?

What can we say about the size of SE and IE for an inferior good?

Lecture 4 - Welfare Anaysis

October 2, 2023

Beyond comparative statics

We studied the implications of parameters change on the optimal choices.

Now we move to the implications in terms of utility, trying to establish measures for consumers welfare and its changes. But since utility is only an ordinal concept, a decrease in utility by 50 might not be that meaningful. Therefore, we need some more objective measure for it, and in particular in **monetary** terms

Substitution effect - A different perspective

In the Slutsky decomposition, we constructed a compensated budget line and demand by adjusting the income of the consumer by keeping their purchase power constant.

We can also think of a different compensated scenario, in which we can find the compensated demand by computing the optimal point keeping fixed the utility. This is called the Hicksian (compensated) demand or **Hicksian Substitution Effect**.

Hicksian Demand

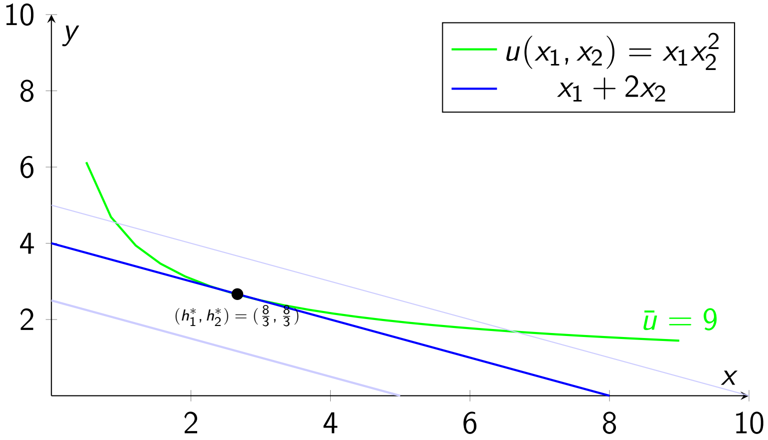
Computing Hicksian Demand implies creating a new framework: we can't maximise utility if we keep utility fixed!

We can think of "inverting" the problem, defining it in terms of **expenditure minimisation** (EMP): what is the minimum expense I need to make to secure a certain level of utility $\bar{u}$?

$$\min_{x_1,x_2} \; p_1x_1 + p_2x_2$$
$$s.t. \; u(x_1, x_2) = \bar{u}$$

We call the solution $h(p, \bar{u})$.
Computation: Equimarginal principle (with different constraint) or Lagrangian.

EMP - Graphical solution



The optimal point, the hicksian demand, lies on the lowest budget line touching the Indifference curve constraint.

Duality

It seems there should be some connections between the solutions to UMP and EMP.
Suppose x is a solution to the UMP problem ant the utility at this point is $v = v(p, m) = u(x(p, m))$, also called the indirect utility as it depends directly on prices and income.

Then, if we set $\bar{u} = v$ and solve the EMP, we will find that $h(p, v) = x(p, m)$ and $e(p, v) = m$.

That is, if we set the constraint to the EMP equal to the maximum utility we could reach with some budget m, it must be that the bundle that achieves that utility with the smallest expenditure is the same that maximised utility.

Can you repeat the same argument starting from the hicksian demand?

Measures of welfare changes - Price change

As a result of a price change, we studied how the optimal bundle changes (and therefore its utility). How can we "normalize" the utility change in monetary terms?
We will use 3 ways:

- Compensating Variation
- Equivalent Variation
- Consumer Surplus (this is mostly used for market demands)

In the following, I will use the underlying case of a price <u>increase</u>. *As an exercise, try to think of the parallel case of a price decrease.*

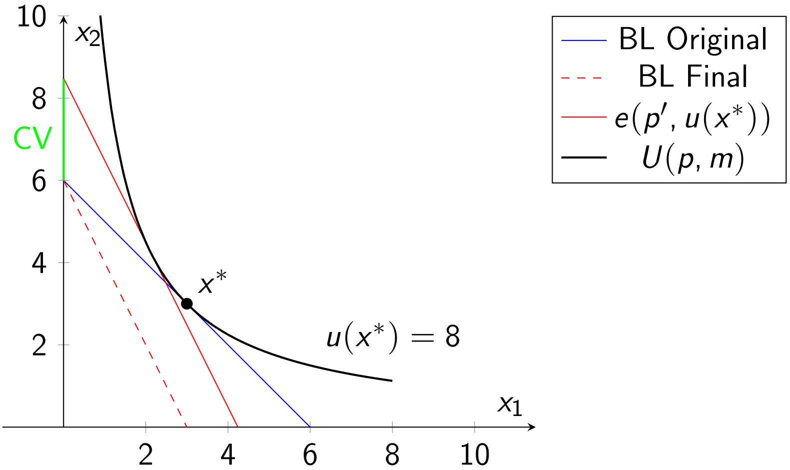
Compensating variation

In this case, we want to fix the utility at the original (before price change) one: $u(x^*(p, m))$.
Then, we can ask ourselves how we can adjust the income to make sure that **after** the price change the consumer enjoys the same level of utility as before. Let $e(p', u(x^*(p, m)))$ be the minimum expenditure needed to achieve the original utility at final prices, then the Compensating Variation is

$$CV(p, p', m) = |m - e(p', u(x^*(p, m)))|$$

Notice that income m can be also expressed as $e(p, u(x^*(p, m)))$.

Compensating variation

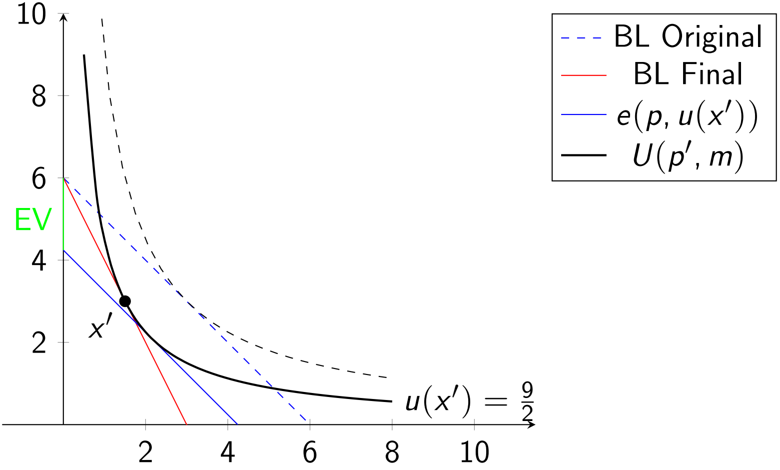


Equivalent Variation

In this case, we want to fix the utility at the final (post price change) one: $u(x^*(p', m))$.
Then, we can ask ourselves how we can adjust the income to make sure that **before** the price change the consumer enjoys the same level of utility as after. Let $e(p, u(x^*(p', m)))$ be the minimum expenditure needed to achieve the final utility at original prices, then the Equivalent Variation is

$$EV(p, p', m) = |e(p, u(x^*(p', m))) - m|$$

Equivalent Variation



Consumer's Surplus

A different approach to measuring welfare change is through demand function.
Imagine a scenario in which consumers have quasilinear preferences: $u(x_1, x_2) = v(x_1) + x_2$.

Given some prices p, we can define the consumer surplus as the net benefit from consuming q_1 units of good 1 as

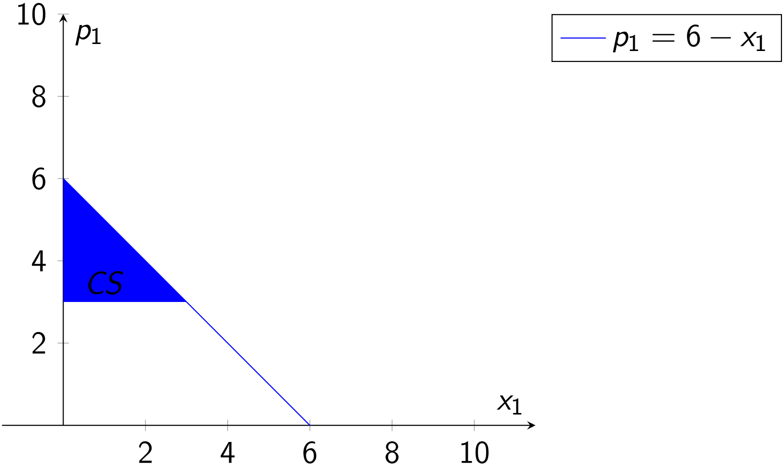
$$CS(q_1) = v(q_1) - p_1 q_1.$$

Then, moving from p_1 to p_1' implies a change in Consumer's Surplus

$$\Delta CS = |v(q_1(p_1')) - v(q_1(p_1))|$$

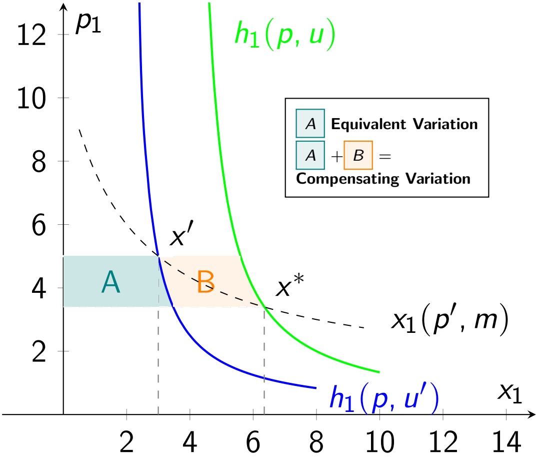
Consumer's Surplus

The Consumer's surplus can be portrayed as the area below the demand curve (whether it is a single consumer or demand curve).



CV-EV: A comparison

When goods are *normal* $CV \geq EV$ for a price increase, and viceversa for a price decrease (or for a price increase in the case of inferior goods).



Extra: Hicksian Decomposition

We can decompose a price change into hicksian substitution and income effects.

In this case, the "intermediate" bundle would be constructed at **new** price but keeping the utility *constant* at the *original* level.

Quantitatively it would change the size of the two components, substitution and income effect, but not the qualitative results (i.e. signs).

Can you replicate the graph from previous class to this scenario?

Applications

The usefulness of these measures is that they allow for a clear comparison between policy. Moreover, using money as a compensation is a standard method that is widely used.

The cleanest example was last years winter fuel "subsidy", in which housholds received a pay back of £66 per month during winter months to *compensate* for the gas price increase.

Would that constitute a payment that leans more towards a compensating or an equvalent variation?

Lecture 5 - Choice Under Uncertainty

November 2, 2023

A new framework?

We provided a framework to explain individual choices using utilities (and preferences).

Can we extend this framework to different scenarios, or is this framework consistent with different type of decisions?
Let's test the utility framework when DMs (Decision Makers) are faced with uncertainty in their outcomes.

What is uncertainty?

Uncertainty relates to missing information, in which the link between cause and consequence, choices and outcomes, is not deterministic.

We can work in settings in which DM wouldn't neither know the consequences of their actions or the probability attached to each attainable outcome.

We will focus on **risk**: situations in which the achievable outcomes and the probabilities are known by the agents.

Fitting risk into our utility framework

Can we extend preferences (and utilities) to a context of risky decisions?

The canonical example would be: would you enter a lottery in which you toss a coin, if it lands on tail you pay £10 and get £12 otherwise?

The role of expectations

One of the starting point to look at this decision problem would be the one of expectations. What is the expected wealth from the previous lottery?

$$
\begin{aligned}
E(w) = & \quad p_1 w_1 + p_2 w_2 \\
= & \quad \tfrac{1}{2}(-10) + \tfrac{1}{2}(12) = 1
\end{aligned}
$$

We can set $E(w) = 0$ for the so called outside option, i.e. not entering the lottery.

But then, since $0.2 > 0$, should we conclude that we expect agents to enter the lottery? Is there more to it?

Expected utility

One possible extension would be to go beyond money per se, but attach a value/ utility to it. Therefore, we can rewrite the previous expectation as

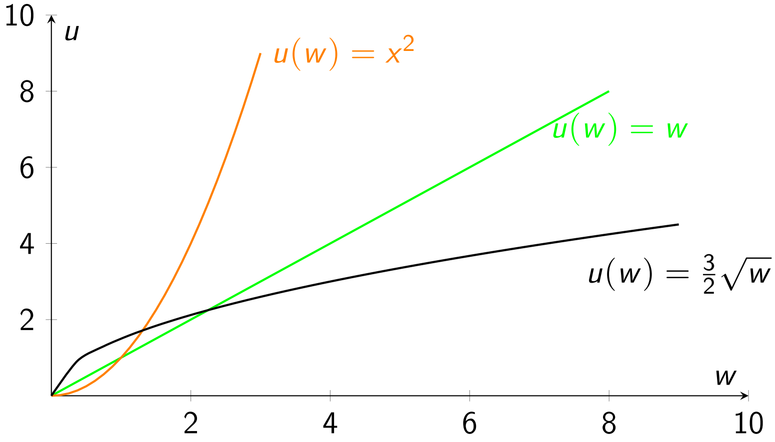
$$EU(w) = \frac{1}{2}u(-10) + \frac{1}{2}u(12) = ?$$

So, even if we set $u(0) = 0$, the shape or the function u will determine the choice.

This expression, called the **Expected Utility** attached to the previous lottery, can be seen as a utility function over lotteries. Von Neumann and Morgenstern defined a set of axioms that guarantee this is an appropriate representations for preferences over lotteries.

Utility of wealth

What can we say about the shape of the utility of wealth? We can assume it is an increasing function (monotonicity). In particular, the rate of increase will be very important.



Certain Equivalent

Another important concept is the one of the *Certain Equivalent*, i.e. the certain lottery which is equivalent to the risky one.

In our previous example, the amount of money to give (or take) that would make you indifferent between taking the gamble or that amount. In other words, the CE is an amount such that

$$u(CE) = EU(w) = pu(w_1) + (1-p)u(w_2)$$

The relation between the certain equivalent and the expected wealth will give us definitions of risky behaviours.

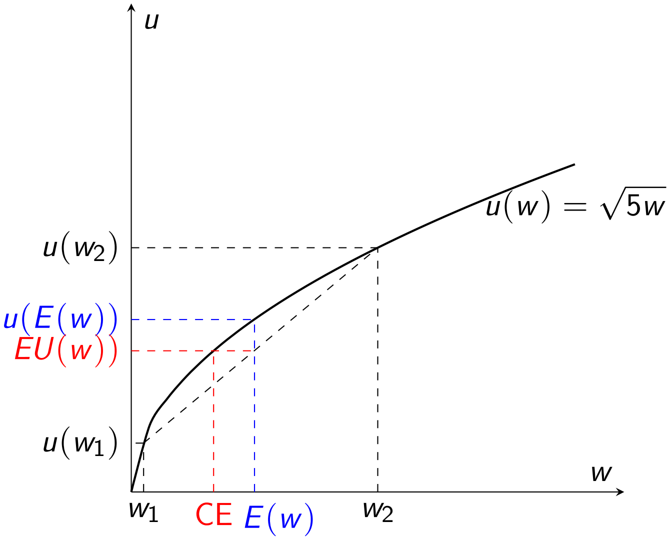
Risk attitudes

- **Risk Averse**: $EU(w) < u(E(w))$ or $CE < E(w)$
- **Risk Neutral**: $EU(w) = u(E(w))$ or $CE = E(w)$
- **Risk Lover**: $EU(w) > u(E(w))$ or $CE > E(w)$

Notice that these definition rely on the comparison between a lottery and its expected value. A risk averse agent will prefer certainty over risk in general, but this comparison holds only when comparing reasonably similar lotteries (in terms of their expected outcome).

Can you see the relation between concavity/convexity and risk attitude?

Risk Averse behaviour: visualisation



Notice that $E(w)$ and $EU(w)$ are both convex combinations of w and $u(w)$ respectively.

Lecture 6 - Insurance

November 4, 2023

An example

We saw that risk averse individuals would prefer, under some conditions, certainty over uncertainty.
Therefore there can be an incentive for individuals to trade risk.

Suppose we have an individual facing a lottery in which they lose £50 with 50% chance, the same win otherwise and own £50. Their utility for money it's $u(w) = \sqrt{w}$.

On the other hand, we have a risk neutral agent who owns £50 and faces no risk.

Is there a mutually beneficial trade?

An Example - 2

At the beginning,

$$EU_1(w) = 0.5u(w_b) + 0.5u(w_g) = 0.5u(0) + 0.5u(100) = 5$$

(I use w_b for wealth in bad state and w_g for wealth in good state)

$$EU_2(w) = w = 50$$

Suppose the two can devise a trade in which the lottery is transferred to the risk neutral agent in exchange for £10. Now we have that

$$EU_1(w) = \sqrt{40} > 5$$

$$EU_2(w) = 0.5(10) + 0.5(110) = 60$$

Then, **they are both better off!**

Insurance

The best real example of this trade is the insurance market. We can think of insurance firms as big risk neutral agents, offering contracts for risk averse individuals.

Let's try and find the optimal demand for insurance through an example. Suppose the insurance offers the following contract type: for each pound insured, the agent needs to pay α. In the case of an event occurring, the insurance will pay back the insured sum.

Optimal insurance contract

Let's fix the notation:

- k The amount insured
- w the value of the insured object
- A the potential damage
- α the price per insured pound
- p the probability of the event that would cause the damage happening

Therefore, we can find that $w_g = w - \alpha k$ and $w_b = w - A - \alpha k + k = w - A + (1 - \alpha)k$. We can set up our (expected) utility maximisation:

$$\max_k EU(k) = (1 - p)\, u(w - \alpha k) + pu(w - A + (1 - \alpha)k)$$

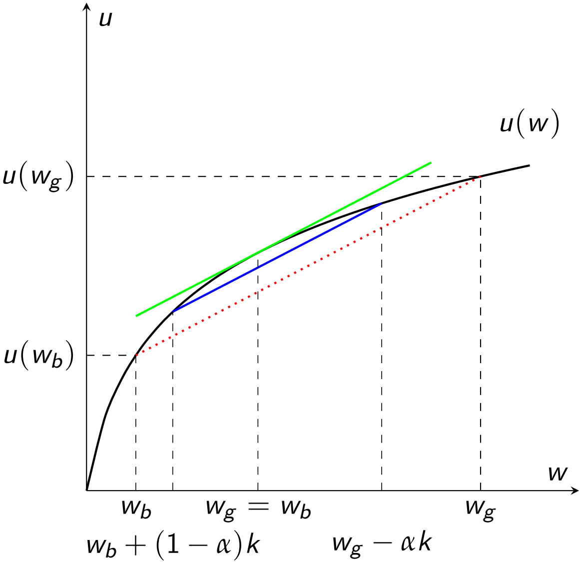
First Order Condition

$$\frac{dEU(k)}{dk} = \alpha(1-p)u'(w_g) - (1-\alpha)pu'(w_b) = 0$$

$$\frac{(1-p)u'(w_g)}{pu'(w_b)} = \frac{(1-\alpha)}{\alpha}$$

Notice that the only way we can have constant wealth, $w_b = w_g$ is when $k = A$. We call this situation **full insurance**, when the agent insures for the whole potential loss. To make sure the full insurance is optimal, we need to have that $p = \alpha$. We call this a *fair price*. If the price per unit is higher than the probability of the bad state, then the agent will insure less than the whole loss.

Full Insurance



Willingness to pay and Risk premium

Different insurance contract don't give the opportunity to choose the exact amount you are willing to insure. Instead, an agent is offered a contract in which there is an annual premium to be paid, which covers the damage if occurred.
In a simple scenario in which there are only two outcomes, an agent is **willing to pay** at most the difference between w_g and the certain equivalent *(why?)* .
Moreover, we define the **risk premium** as the the extra amount of money an agent is willing to pay with respect to a risk neutral person. This is quantified by the difference between $E(w)$ and the certain equivalent.

Asymmetric Information

In real world, the probability of the bad state occurring is not exogenous, but it is related to some factors specific to the individual seeking insurance.
The classical example is the one of car insurance: clearly the probability of an accident is higher for more reckless drivers. But then, the price they should pay for fair insurance is higher.

What if the insurer can't tell this when signing the contract?

This scenario implies that there is some information which is not transmitted from the insured agent to the insurer, such as a reckless driver trying to enjoy lower insurance prices by hiding their behaviour.

Adverse Selection

If the asymmetry in information arises before the contract is signed, we are in a situation called *adverse selection*. In this case, we are in presence of an **hidden characteristic**.

This problem might not only induce non efficient outcomes, but to the extreme this might prevent trade in some peculiar markets (see "Market for Lemons").
How is this problem actually solved?

- Signaling: the more informed agent might provide evidence to prove their type
- Screening: the less informed agent might "investigate" to learn those hidden characteristics

Moral Hazard

If the asymmetry in information arises after the contract is signed, we are in a situation called *moral hazard*. In this case, we are in presence of an **hidden action**.

For example, after hiring a worker, I might not be fully able to monitor whether they will do the work required and exert adequate effort.
How to solve this issue?

- Monitoring
- Limited liability

Signaling: more issues

As per now, we studied cases in which agents have information while others don't. When talking about solving this issue via signaling, we implicitly assumed agents share information plainly as they would benefit from it.

What if instead further manipulating the information can be even more beneficial? For example, a reckless driver might provide evidence that would portray them as virtuous driver. But then, the receiver might be induced to believe less in these signals...

Kamenica and Gentkow (2011) wrote a seminal paper in which they talk about (Bayesian) Persuasion explaining how information can be used to maximise someone's benefit.