1) $m(X) = \frac{1}{N}\sum^{N}(x_i)$

$m(a+bX) = \frac{1}{N}\sum(a+bX) = \cancel{\frac{1}{N}}\sum^{N} a + \frac{1}{N}\sum b x_i$ ← constant

$= a + b \cdot \frac{1}{n}\sum x_i = a + b\,m(x)$

2) $cov(X, a+bY) = \frac{1}{N}\sum^{N}(X - M(x))(a+bY - m(a+bY))$

$= \frac{1}{N}\sum^{n}(x - M(x))(a+bY-(a+b\,m(Y)))$

$= \frac{1}{N}\sum(x - M(x))(\cancel{a-a} + bY - b\,M(Y))$

$= \frac{1}{N}\sum b(x - M(x))(Y - M(Y)) = b \cdot \frac{1}{n}\sum(x - m(x))(y - m(y))$

$= b\,cov(X, Y)$

3) $Cov(a+bx, a+bx) = \frac{1}{N}\sum(a+bx - M(a+bx))^2$

from (2): $(a+bY - m(a+bY)) = b(Y - m(y))$

$\therefore Cov(a+bx, a+bx) = \frac{1}{N}\sum b^2(x - M(x))^2 = b^2\underbrace{\frac{1}{N}\sum(x-m(x))^2}_{s^2}$

$= b^2 cov(x, x) = b^2 s^2$

4) When the number of data points are odd, yes the transformed median would be the median of the transformed data since in that case the median only concerns about ordinal position. If there are an odd # of data points, the median is the average of the 2 centermost points, so it would not hold in that case. A quantile that is transformed will be equal to the quantile of the transformation so long as no averages of data points are used in its calculation. However, any of these will always be true with an affine transformation, Range & IQR will be true iff. the transformation is affine.

5) $g(x) = \sqrt{x}$

$m(g(x)) = \frac{1}{N}\sum^{N}\sqrt{x_i} \neq g(m(x)) = \sqrt{\frac{1}{N}\sum^{N} x_i}$

Easy to see w/ example data: $\{1, 4, 9\}$

$m(g(x)) = \frac{1}{3}(1+2+3) = 2 \neq g(m(x)) = \sqrt{\frac{1}{3}(14)} = \sqrt{\frac{14}{3}}$