# Test 1 Part 2: Triathlon

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**Abstract**

The completion time of a schedule is the earliest time at which all contestants will be finished with all three legs of the triathlon. How would you determine the best order for sending n persons out if you want the whole competition to be over as early as possible?

## 1 Manual Enumeration of Problem Instance 1

Manually enumerate all the possible orders of sending out these three persons and find out what is the best order to minimize the completion time.

### 1.1 Permutation 1

$$1 : 65 + 0 = 65$$

$$2 : 70 + 20 = 90$$

$$3 : 90 + 20 + 35 = 145$$

### 1.2 Permutation 2 - Optimal

$$1 : 65 + 0 = 65$$

$$3 : 90 + 20 = 110$$

$$2 : 70 + 20 + 40 = 130$$

### 1.3 Permutation 3

$$2 : 70 + 0 = 70$$

$$1 : 65 + 35 = 100$$

$$3 : 90 + 35 + 20 = 145$$

### 1.4 Permutation 4

$$2 : 70 + 0 = 70$$

$$3 : 90 + 35 = 125$$

$$1 : 65 + 35 + 40 = 140$$

### 1.5 Permutation 5 - Optimal

$$3 : 90 + 0 = 90$$

$$1 : 65 + 40 = 105$$

$$2 : 70 + 40 + 20 = 130$$

### 1.6 Permutation 6

$$3 : 90 + 0 = 90$$

$$2 : 70 + 40 = 110$$

$$1 : 65 + 40 + 35 = 140$$

## 2 Exhaustive Search for Optimality

### 2.1 Problem Instance 1

Finishing time solution: 130

Optimal order: 1 3 2

Search time: 0.015s

### 2.2 Problem Instance 2

Finishing time solution: 349

Optimal order: 1 2 3 4 5 6 7 8 10 9

Search time: 148.279s

# 3 Greedy Search for Optimality

## 3.1 Methods

- descending sort by (running + biking) / swimming
- descending sort by running + biking time
- position the athlete with the lowest running + biking time in the last position

## 3.2 Problem Instance 1

Finishing time solution: 130

Optimal order: 1 3 2

Search time: 0.014s

## 3.3 Problem Instance 2

Finishing time solution: 349

Optimal order: 8 4 3 7 10 1 5 2 6 9

Search time: 0.014s

# 4 Proof of Optimality Greedy Criterion

Objective: Demonstrate the optimality of the greedy algorithm
Notation: B biking time, R running time, Sswimming time, T start time.

Criterion:

$$B_j + R_j \leq B_{j-1} + R_{j-1}$$

Derivation of Criterion:

$$\sum_{n=1}^{j} S_n + B_j + R_j \leq \sum_{n=1}^{j} S_n + B_{j-1} + R_{j-1}$$

$$\sum_{n=1}^{j-2} S_n + S_{j-1} + S_j + B_j + R_j \leq \sum_{n=1}^{j-2} S_n + S_j + S_{j-1} + B_{j-1} + R_{j-1}$$

$$S_{j-1} + S_j + B_j + R_j \leq S_j + S_{j-1} + B_{j-1} + R_{j-1}$$

$$B_j + R_j \leq B_{j-1} + R_{j-1}$$

Proof:

Suppose:

$$B_j + R_j \leq B_i + R_i$$

$$S_i + S_j + B_j + R_j \leq S_j + S_i + B_i + R_i$$

Let T = $S_i$ , $T^* = S_j$

$$T + S_j + B_j + R_j \leq T^* + S_i + B_i + R_i$$

Alternate:

Suppose:

$$B_j + R_j \leq B_{j-1} + R_{j-1}$$

Add swim times

$$S_{j-1} + S_j + B_j + R_j \leq S_j + S_{j-1} + B_{j-1} + R_{j-1}$$

Add cumulative time sum

$$\sum_{n=1}^{j-2} S_n + S_{j-1} + S_j + B_j + R_j \leq \sum_{n=1}^{j-2} S_n + S_j + S_{j-1} + B_{j-1} + R_{j-1}$$

Simplify

$$\sum_{n=1}^{j} S_n + B_j + R_j \leq \sum_{n=1}^{j} S_n + B_{j-1} + R_{j-1}$$