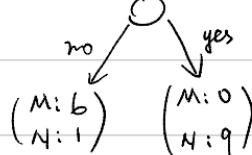


日期:

1. (a) $G_{ini}(t) = 1 - \sum_{i=1}^t p(i|t)^2$

1 step: Attribute (Lay Eggs)



$$\begin{aligned} I &= \frac{7}{16} \cdot G_{ini}(no) + \frac{9}{16} \cdot G_{ini}(yes) \\ &= \frac{7}{16} \left(1 - \left(\frac{6}{7}\right)^2 - \left(\frac{1}{7}\right)^2\right) + \frac{9}{16} \left(1 - 1 - 0\right) \\ &= \frac{3}{28} \end{aligned}$$

Attribute: (Can Fly)

```

graph TD
    A(( )) -- no --> B["(M: 5)  
N: 7"]
    A -- yes --> C["(M: 1)  
N: 3"]
  
```

$$\begin{aligned} I &= \frac{12}{16} \cdot \left(1 - \left(\frac{5}{12}\right)^2 - \left(\frac{7}{12}\right)^2\right) + \frac{4}{16} \cdot \left(1 - \left(\frac{1}{4}\right)^2 - \left(\frac{3}{4}\right)^2\right) \\ &= \frac{13}{30} \end{aligned}$$

Attribute: (Have Legs)

```

graph TD
    A(( )) -- no --> B["(M: 2)  
N: 3"]
    A -- yes --> C["(M: 4)  
N: 7"]
  
```

$$\begin{aligned} I &= \frac{5}{16} \cdot \left(1 - \left(\frac{2}{5}\right)^2 - \left(\frac{3}{5}\right)^2\right) + \frac{11}{16} \cdot \left(1 - \left(\frac{4}{11}\right)^2 - \left(\frac{7}{11}\right)^2\right) \\ &= \frac{3}{20} + \frac{7}{22} \end{aligned}$$

First (Lay Eggs) attribute, $I = \frac{3}{28} \approx 0.1071$

(b)

Step 2:

Attribute: (Can Fly)

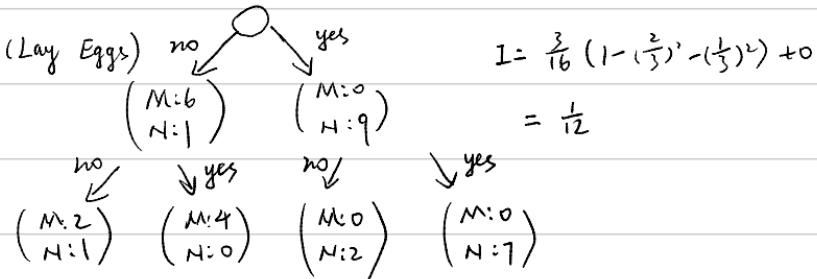
```

graph TD
    A["(Lay Eggs)"] -- no --> B["(M: 6)  
N: 1"]
    A -- yes --> C["(M: 0)  
N: 9"]
    B -- no --> D["(M: 5)  
N: 1"]
    B -- yes --> E["(M: 1)  
N: 0"]
    C -- no --> F["(M: 0)  
N: 6"]
    C -- yes --> G["(M: 0)  
N: 3"]
  
```

$$\begin{aligned} I &= \frac{6}{16} \cdot \left(1 - \left(\frac{5}{6}\right)^2 - \left(\frac{1}{6}\right)^2\right) + 0 \\ &= \frac{5}{48} \end{aligned}$$

Attribute: (Have Legs)

日期:



$$I = \frac{3}{16} \left(1 - \left(\frac{2}{3}\right)^2 - \left(\frac{1}{3}\right)^2\right) + 0 \\ = \frac{1}{12}$$

2nd step: (Have Legs) attribute, $I = \frac{1}{12} \approx 0.083$

$$(C) g(l, c, h) = \log \frac{P(\text{Mammal} | l, c, h)}{P(\text{Non-Mammal} | l, c, h)} \\ = \log \frac{P(l|M) \cdot P(c|M) \cdot P(h|M) \cdot P(M)}{P(l|N) \cdot P(c|N) \cdot P(h|N) \cdot P(N)}$$

$$\lambda_0 = \log \frac{P(M)}{P(N)} = \log \frac{6}{10}$$

$$g_1(l) = \log \frac{P(l|M)}{P(l|N)} = \begin{cases} \log \frac{(0+1)/(6+2)}{(9+1)/(10+2)} & l=1 \\ \log \frac{(6+1)/(6+2)}{(1+1)/(10+2)} & l=-1 \end{cases}$$

$$g_2(c) = \log \frac{P(c|M)}{P(c|N)} = \begin{cases} \log \frac{(1+1)/(6+2)}{(5+1)/(10+2)} & c=1 \\ \log \frac{(5+1)/(6+2)}{(7+1)/(10+2)} & c=-1 \end{cases}$$

$$g_3(h) = \log \frac{P(h|M)}{P(h|N)} = \begin{cases} \log \frac{(4+1)/(6+2)}{(7+1)/(10+2)} & h=1 \\ \log \frac{(2+1)/(6+2)}{(3+1)/(10+2)} & h=-1 \end{cases}$$

$$\lambda_0 = \log \frac{3}{5}, \quad g_1(1) = \log \frac{3}{20}, \quad g_1(-1) = \log \frac{21}{4}, \quad g_2(1) = \log \frac{3}{4}, \quad g_2(-1) = \log \frac{9}{8} \\ \approx -0.5108 \quad \approx -0.8239 \quad \approx 1.6592 \quad \approx -0.1249 \quad \approx 0.0512$$

日期:

$$g_3(1) = \log \frac{15}{16}, \quad g_3(-1) = \log \frac{9}{8}$$

$$\approx -0.0280 \quad \approx 0.0512$$

2. (a) truth

P N

predicted P 4(TP) 2(FP)

N 1(FN) 7(TN)

AP=5 AN=9

positive spam, negative legitimate, $\theta = 20$

$$TPR = \frac{TP}{AP} = \frac{4}{5} = 0.8$$

$$FPR = \frac{FP}{AN} = \frac{2}{9} = 0.2222$$

$$(b) AUC = \frac{\sum_{x \in AP} \sum_{y \in FN} 1_{f(y) > f(x)}}{\# AP \cdot \# AN}$$

$$= \frac{9+9+9+8+4}{5 \times 9}$$

$$= \frac{39}{45} = 0.8667$$

3.

$$\frac{\partial f}{\partial (x+y)} = \frac{\partial f}{\partial (x+y)} \cdot \frac{\partial (x+y)}{\partial (x+y)} = 1$$

$$\frac{\partial f}{\partial z^2} = \frac{\partial f}{\partial (x+y)} \cdot \frac{\partial (x+y)}{\partial (z^2)} = 0$$

$$\frac{\partial f}{\partial (x+w)^2} = 1$$

$$\text{日期: } \frac{\partial(x+y)}{\partial x} = 1, \quad \frac{\partial(x+y)}{\partial y} = 1, \quad \frac{\partial z^2}{\partial z} = 2z = 4$$

$$\frac{\partial(x+w)^2}{\partial x} = 2(x+w) = 18, \quad \frac{\partial(x+w)^2}{\partial w} = 18$$

$$\frac{\partial f}{\partial x} = 1 \cdot 1 + 1 \cdot 18 = 19, \quad \frac{\partial f}{\partial y} = 1 \cdot 1 = 1$$

$$\frac{\partial f}{\partial z} = 0, \quad \frac{\partial f}{\partial w} = 1 \cdot 18 = 18, \quad \boxed{\nabla f = (19, 1, 0, 18)}$$

4. (a) consider a3,

$$\begin{array}{c} \text{yes} \swarrow \text{o} \searrow \text{no} \\ \left(\begin{array}{l} \text{d2 yes: 9} \\ \text{d2 no: 4} \end{array} \right) \quad \left(\begin{array}{l} \text{d2 yes: 1} \\ \text{d2 no: 6} \end{array} \right) \end{array} \quad Gini = \frac{13}{20} \left(1 - \left(\frac{9}{13} \right)^2 - \left(\frac{4}{13} \right)^2 \right) + \frac{7}{20} \left(1 - \left(\frac{1}{7} \right)^2 - \left(\frac{6}{7} \right)^2 \right) = \frac{33}{91} = 0.3626$$

consider a5 :

$$\begin{array}{c} \text{yes} \swarrow \text{o} \searrow \text{no} \\ \left(\begin{array}{l} \text{d2 yes: 1} \\ \text{d2 no: 3} \end{array} \right) \quad \left(\begin{array}{l} \text{d2 yes: 9} \\ \text{d2 no: 7} \end{array} \right) \end{array} \quad Gini = \frac{4}{20} \left(1 - \left(\frac{1}{4} \right)^2 - \left(\frac{3}{4} \right)^2 \right) + \frac{16}{20} \left(1 - \left(\frac{9}{16} \right)^2 - \left(\frac{7}{16} \right)^2 \right) = \frac{15}{32} = 0.4681$$

a3 better.

(b) a1 \rightarrow d1

$$AUC = \frac{\sum_{x \in AN} \sum_{y \in AP} 1_{f(y) > f(x)}}{\# AP \cdot \# AN} = \frac{1+2 \times 4 + 3 \times 2 + 5 + 8}{9 \times 11} = \boxed{\frac{28}{99} = 0.2828}$$

日期: / a1

(c) $\theta_1 = 37.95 \rightarrow a_2, a_3 \rightarrow d_2$

$P(d_2 = \text{yes} | a_1 \neq \theta_1, a_2 = \text{no}, a_3 = \text{no})$

$P(d_2 = \text{no} | a_1 \neq \theta_1, a_2 = \text{no}, a_3 = \text{no})$

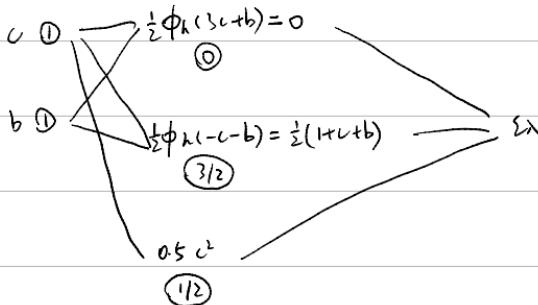
$$\approx \frac{P(d_2 = \text{yes}) \cdot P(a_1 \neq \theta_1 | \text{yes}) \cdot P(a_2 = \text{no} | \text{yes}) \cdot P(a_3 = \text{no} | \text{yes})}{P(d_2 = \text{no}) \cdot P(a_1 \neq \theta_1 | \text{no}) \cdot P(a_2 = \text{no} | \text{no}) \cdot P(a_3 = \text{no} | \text{no})}$$

$$= \frac{10}{10} \cdot \frac{9/10}{11/10} \cdot \frac{10/10}{10/10} \cdot \frac{1/10}{6/10}$$

$$= \frac{3}{2} > 1$$

$$\arg \max_{d_2} \sim = \boxed{\text{yes}}$$

5. (a) $\sum_{\lambda}(c=1, b=1) = \frac{1}{2}\phi_h(3c+b) + \frac{1}{2}\phi_h(-c-b) + 0.5c^2$



$$\frac{\partial \sum_{\lambda}}{\partial c} = \frac{\partial \frac{1}{2}(4c+b)}{\partial c} + \frac{\partial (0.5c^2)}{\partial c} = \frac{1}{2} + c = 1.5$$

$$\frac{\partial \sum_{\lambda}}{\partial b} = \frac{\partial \frac{1}{2}(4c+b)}{\partial b} + 0 = 0.5$$

$$\boxed{\nabla \sum_{\lambda} = \left(\frac{\partial \sum_{\lambda}}{\partial c}, \frac{\partial \sum_{\lambda}}{\partial b} \right) = (1.5, 0.5)}$$

日期:

$$(b) \quad s_\lambda = \frac{1}{2} \max\{0, 1-3c-b\} + \frac{1}{2} \max\{0, 1+c+b\} + \frac{1}{2} c^2.$$

① if $1-3c-b > 0, 1+c+b > 0, \quad s_\lambda = 1-c + \frac{1}{2} c^2$

$$\nabla s_\lambda = 0 \Rightarrow c=1, b<-2, b>-2, \text{ contradictory.}$$

② if $1-3c-b > 0, 1+c+b < 0, \quad s_\lambda = \frac{1}{2}(1-3c-b) + \frac{1}{2} c^2$

$$\nabla s_\lambda \neq 0$$

③ if $1-3c-b < 0, 1+c+b > 0, \quad s_\lambda = \frac{1}{2}(1+c+b) + \frac{1}{2} c^2$

$$\nabla s_\lambda \neq 0$$

④ if $1-3c-b < 0, 1+c+b < 0, \quad s_\lambda = \frac{1}{2} c^2.$

$$\nabla s_\lambda = 0 \Rightarrow c=0, b \geq 1, b \leq -1, \text{ contradiction.}$$

Since s_λ is convex, so that if s_λ is differentiable,

$$\nabla s_\lambda = 0.$$

if s_λ is not differentiable,

⑤ if $1-3c-b=0, \quad 1+c+b > 0, \quad \nabla s_\lambda \neq 0$

⑥ if $1-3c-b=0, \quad 1+c+b < 0, \quad \text{then } c=0, \text{ contradictory}$

⑦ if $1-3c-b=0, \quad 1+c+b=0,$

then $c=1, b=-2, \quad s_\lambda = 0.5$

$$(c^* = 1)$$

日期: Fix $0 < q < 1$, $\gamma > 0$.

6. $g^2 \leq (x - g)^2 + \gamma \cdot |x|^q$, $\forall x$, holds

find range of g .

assume that $f(x) \triangleq x^2 - 2xg + \gamma|x|^q \geq 0$

① if $\forall x \geq 0$, $f(x) = x^2 - 2xg + \gamma \cdot x^q$, $\frac{df}{dx} = 2(x - g) + \gamma q \cdot x^{q-1} = 0$

$\frac{dg}{dx} = 2 + \gamma q(q-1)x^{q-2}$, we know that f is concave \rightarrow convex.

so that if $f(x^*) \geq 0$, then $f(x) \geq 0, \forall x \geq 0$.

$$\Rightarrow g = x^* + \frac{1}{2}\gamma q x^{q-1}$$

$$\Rightarrow f(x^*) = -x^{*2} + \gamma(1-q)x^{*q} \geq 0, x^* \geq 0$$

$$\Rightarrow 0 \leq x^* \leq (\gamma(1-q))^{-\frac{1}{2-q}}$$

$$\frac{d^2f}{dx^2} \Big|_{x^*} = 1 + \frac{1}{2}\gamma q(q-1)x^{q-2} = 0, \text{ we know } f(x^*) \text{ is convex.}$$

$$\Rightarrow xg = \left(\frac{2}{\gamma q(1-q)}\right)^{\frac{1}{2-q}} = \left(\frac{\gamma q(1-q)}{2}\right)^{\frac{1}{2-q}} < (\gamma(1-q))^{\frac{1}{2-q}}$$

$$\Rightarrow g \geq g(x_0) = \left(\frac{\gamma q(1-q)}{2}\right)^{\frac{1}{2-q}} + \frac{1}{2}\gamma q \cdot \left(\frac{\gamma q(1-q)}{2}\right)^{\frac{q-1}{2-q}}$$

② if $\forall x \leq 0$, $f(x) = x^2 - 2xg + \gamma(-x)^q$,

$$\frac{df}{dx} = 2x - 2g - \gamma q(-x)^{q-1}, \quad \frac{d^2f}{dx^2} = 2 + \gamma q(q-1)(-x)^{q-2},$$

when $x \leq 0$, f is convex \rightarrow concave



$$\frac{dt}{dx} \Big|_{x^*} = 2x^* - 2g - \gamma q(-x^*)^{q-1} = 0 \Rightarrow g = x^* - \frac{1}{2}\gamma q(-x^*)^{q-1}$$

substitute g into f , $f(x^*) = -x^{*2} + (1-q)\gamma(-x^*)^q \geq 0$

$$\Rightarrow x^* \geq -\left((1-q)\gamma\right)^{\frac{1}{2-q}}$$

$$\text{日期: } \left. \frac{ds}{dx^q} \right|_{x_0} = 1 + \frac{1}{2} \gamma q (q-1) (-x_0)^{q-2} = 0$$

$$\Rightarrow x_0 = -\left(\frac{2}{\gamma q (1-q)} \right)^{\frac{1}{q-2}} = -\left(\frac{\gamma q (1-q)}{2} \right)^{\frac{1}{2-q}} = -\left((1-q)\gamma \right)^{\frac{1}{2-q}}$$

$$\Rightarrow g \leq -\left(\frac{\gamma q (1-q)}{2} \right)^{\frac{1}{2-q}} - \frac{1}{2} \gamma q \left(\frac{\gamma q (1-q)}{2} \right)^{\frac{q-1}{2-q}}$$

Hence $g \leq -\left(\frac{\gamma q (1-q)}{2} \right)^{\frac{1}{2-q}} - \frac{1}{2} \gamma q \left(\frac{\gamma q (1-q)}{2} \right)^{\frac{q-1}{2-q}}$

or $g \geq \left(\frac{\gamma q (1-q)}{2} \right)^{\frac{1}{2-q}} + \frac{1}{2} \gamma q \left(\frac{\gamma q (1-q)}{2} \right)^{\frac{q-1}{2-q}}.$