Lecture 7
Relational Model

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Some parts have been revised and indicated.

Week	Tue Class	Wed Class	Lecture Sequence	Lab /Tutorial Sequence	Venue (Tue Class)	Venue (Wed Class)
7	Oct. 15	Oct. 16	7) Relational model	Lab 7: Relational Model	N001	PQ306
8	Oct. 22	Oct. 23	8) SQL I		N001	PQ306
9	Oct. 29	Oct. 30	9) SQL II	Labs 6, 8 & 9: SQL	6:30-7:30: N001; 7:30-9:30: PQ 604ABC	PQ 604ABC
10	Nov. 5	Nov. 6	10) Entity-relationship model	Lab 10: E-R Diagrams	N001	PQ306
11	Nov. 12	Nov. 13	11) Database normalization	Lab 11: Database normalization	N001	PQ306
12	Nov. 19	Nov. 20	12) Data storage and indexing	Lab 12: Storage and indexing	N001	PQ306
13	Nov. 26	Nov. 27	13) Query processing [Quiz 2]	Lab 13: Query processing	N001	PQ306

Outline

- Relational model
- Relational algebra: basic operations
- Relational algebra: other operations
- Null value

Scenario: online shopping

- Customer

- Product

- Invoice

- Others: Payment, Delivery, ……

cust-id	name	email	address
1	James	james@yahoo.com	AB
2	Mary	mary@gmail.com	CD
3	Peter	peter@yahoo.com	EF
4	Peter	peter@gmail.com	**null**
…	…	…	…

Basic Concepts

- Attribute / column
  - E.g., name

- Schema

  - Defines a table by a list of attributes (with types)
  - E.g., Customer-schema=(cust-id, name, email, address)

cust-id	name	email	address
…	…	…	…
…	…	…	…
…	…	…	…
…	…	…	…
…	…	…	…

Basic Concepts

- **Tuple** (also called as **row**)
  - E.g., (1, James, james@yahoo.com, AB, 159)

- **Relation** (also called as **table**)
  - A **set** of tuples (no duplicates); tuples are *unordered*
  - E.g., Customer table

cust-id	name	email	address
1	James	james@yahoo.com	AB
2	Mary	mary@gmail.com	CD
3	Peter	peter@yahoo.com	EF
4	Peter	peter@gmail.com	**null**
…	…	…	…

indicates a missing value

Keys

- Superkey
  - A set of attributes that **uniquely identify** a tuple
- Candidate key
  - A **minimal** superkey (such that none of its subsets can be superkey)
- Primary key
  - A candidate key chosen by database designer (if there are multiple choices)

Exercises on Keys

Relation **Customer**:

cust-id	name	email	address
1	James	james@yahoo.com	AB
2	Mary	mary@gmail.com	CD
3	Peter	peter@yahoo.com	EF
4	Peter	peter@gmail.com	**null**
…	…	…	…

Which one is a superkey?

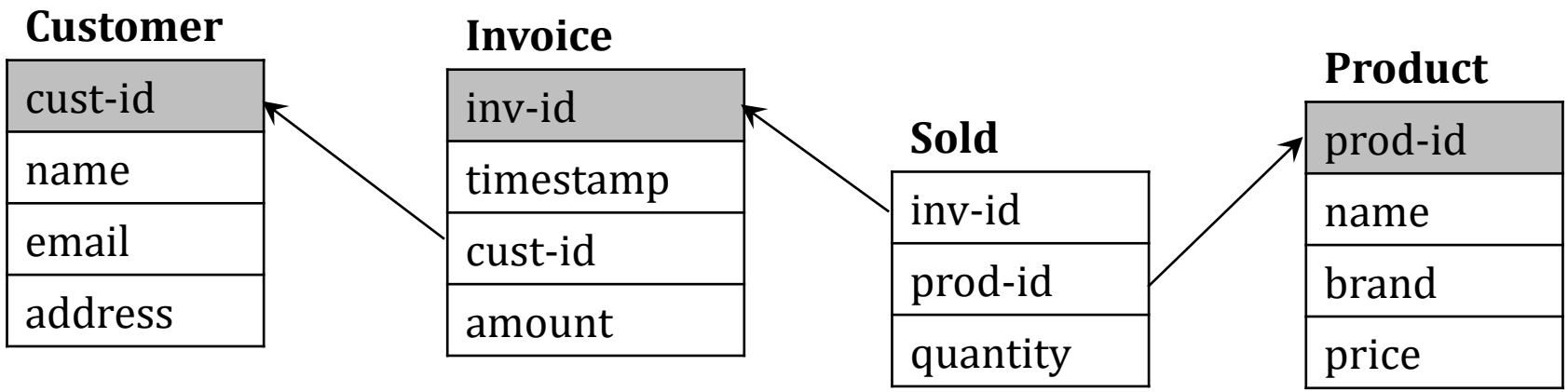
- {cust-id}
- {name}
- {cust-id, name}

Which one is a candidate key?

- {cust-id}
- {cust-id, name}

We choose {cust-id} as the primary key.

Schema Diagram



- Schema diagram
  - Express the schemas of tables and their relationships
- Primary key (highlighted in gray)
- Foreign key
  - An attribute that uniquely identifies a tuple in another table
  - E.g., The attribute *cust-id* in *Invoice* is a foreign key to *Customer*
- Why is the table ***Sold*** useful? What happens if we remove it?

Outline

- Relational model
- 
  Relational algebra: basic operations
- Relational algebra: other operations
- Null value

Relational Algebra

- A language for expressing queries
  - E.g., "find products that have price > 8.0"

$$\Downarrow$$

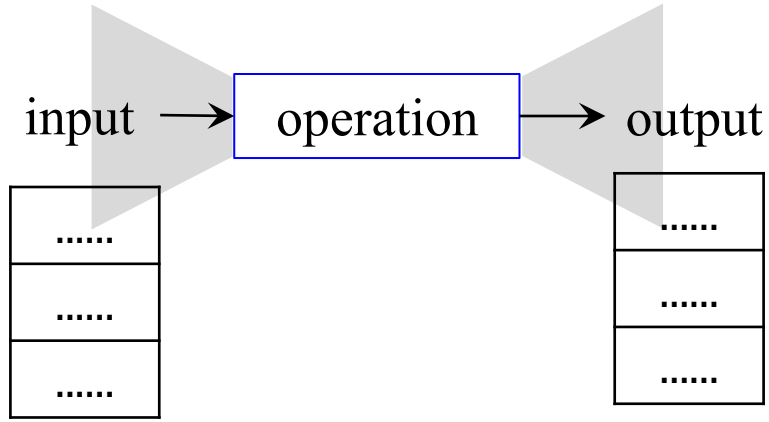
$$\sigma_{price > 8.0}(Product)$$

- It forms the foundation of SQL

- It is used within DBMS (for query processing & optimization)

Relational algebra: Basic Operations

- Basic operations
  - select: σ
  - project: Π
  - Cartesian product: $\times$
  - union: $\cup$
  - set difference: $-$
  - rename: ρ



- Each operation takes table(s) as input, then returns a table as output

Relational algebra: expression vs. tree

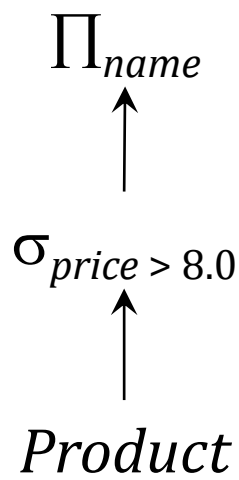
- Relational algebra **expression**
  - The output of a sub-expression can be used as the input of a larger expression

$$\Pi_{name}\ (\ \sigma_{price\ >\ 8.0}\ (Product)\ )$$

② ①

  - The innermost operation is applied first

- Relational algebra **tree**
  - A graphical representation of relational algebra expression (easy to understand)



How to express the following queries in relational algebra?

- (Q1) Find products that have price > 8.0
- (Q2) Find the names of products
- (Q3) Find the customer name and inv-id (invoice id) of each invoice
- (Q4) Find the highest price in the table *Product*
- (Q5) Find the total amount in the table *Invoice*

Relational algebra: Select

- **Select** σ: retrieve tuples that satisfy a condition
  - Example: Find products that have price > 8.0

$$\sigma_{price > 8.0}(Product)$$

condition *input*

Relation **Product**:

prod-id	name	brand	price
1	Coca Cola	CO	7.8
2	Pepsi	PE	8.9
3	7 Up	DP	6.5
4	Sprite	CO	8.3

prod-id	name	brand	price
2	Pepsi	PE	8.9
4	Sprite	CO	8.3

Relational algebra: Select

- In the condition, we may use
  - Comparisons: =, ≠, <, >, ≤, ≥
  - Connectives: and (∧), or (∨), not (¬)

- Examples:

Relation **Product**:

prod-id	name	brand	price
1	Coca Cola	CO	7.8
2	Pepsi	PE	8.9
3	7 Up	DP	6.5
4	Sprite	CO	8.3

$$\sigma_{price > 8.5 \vee brand=\text{“DP”}} (Product)$$

prod-id	name	brand	price
2	Pepsi	PE	8.9
3	7 Up	DP	6.5

$$\sigma_{price > 8.0 \wedge brand=\text{“CO”}} (Product)$$

prod-id	name	brand	price
4	Sprite	CO	8.3

Relational algebra: Project

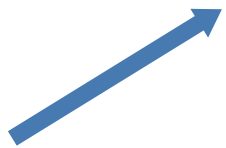
- **Project** $\prod$: choose attributes in the output

  Examples *attribute(s)* *input*

  - 1) Find the names of products: $\prod_{name}(Product)$
  - 2) Find the brands of products: $\prod_{brand}(Product)$

Relation **Product**:

prod-id	name	brand	price
1	Coca Cola	CO	7.8
2	Pepsi	PE	8.9
3	7 Up	DP	6.5
4	Sprite	CO	8.3



name
Coca Cola
Pepsi
7 Up
Sprite

brand
CO
PE
DP

Why only 1 "CO"?

Composite Operations

- Apply operations one-by-one
  - The innermost operation is applied first
  - E.g., Find the names of products that have price > 8.0

$$\Pi_{name} (\sigma_{price > 8.0} (Product))$$

② ①

prod-id	name	brand	price
1	Coca Cola	CO	7.8
2	Pepsi	PE	8.9
3	7 Up	DP	6.5
4	Sprite	CO	8.3

name
Pepsi
Sprite

- Does the following expression make sense? Why?

$$\sigma_{price > 8.0} (\Pi_{name} (Product))$$

Query (Q3)

- "Find the customer name and inv-id (invoice id) of each invoice"

- How to express this in relational algebra?
  - *Invoice* contains inv-id but not customer name
  - *Customer* contains customer name but not inv-id
  - How to combine these two tables (correctly)?

Relation **Customer**:

cust-id	name	email	address
1	James	james@yahoo.com	AB
2	Mary	mary@gmail.com	CD
3	Peter	peter@yahoo.com	EF
4	Peter	peter@gmail.com	**null**

Relation **Invoice**:

inv-id	timestamp	cust-id	amount
1	101	3	8.9
2	102	2	7.8

Relational algebra: Cartesian Product

- **Cartesian product** ×: given two relations *R* and *S*, this operation returns a relation *T* containing every possible pair of tuples from *R* and *S*
  - Example: *Customer* × *Invoice*

Relation **Customer**:

cust-id	name	email	address
1	James	james@yahoo.com	AB
2	Mary	mary@gmail.com	CD
3	Peter	peter@yahoo.com	EF
4	Peter	peter@gmail.com	**null**

Relation **Invoice**:

inv-id	timestamp	cust-id	amount
1	101	3	8.9
2	102	2	7.8

Relational algebra: Cartesian Product

- Result of $Customer \times Invoice$
  - When there are different "cust-id" attributes, we distinguish them by using the table name as **prefix**

Customer. cust-id	name	email	address	inv-id	timestamp	Invoice. cust-id	amount
1	James	james@yahoo.com	AB	1	101	3	8.9
2	Mary	mary@gmail.com	CD	1	101	3	8.9
3	Peter	peter@yahoo.com	EF	1	101	3	8.9
4	Peter	peter@gmail.com	**null**	1	101	3	8.9
1	James	james@yahoo.com	AB	2	102	2	7.8
2	Mary	mary@gmail.com	CD	2	102	2	7.8
3	Peter	peter@yahoo.com	EF	2	102	2	7.8
4	Peter	peter@gmail.com	**null**	2	102	2	7.8

- But some result tuples are not meaningful

Back to query (Q3)

- (Q3) "find the customer name and inv-id of each invoice"
- Step 1: apply selection to match keys
  - $\sigma_{Customer.cust\text{-}id=Invoice.cust\text{-}id}\ (\ Customer \times Invoice\ )$

Customer. cust-id	name	email	address	inv-id	timestamp	Invoice. cust-id	amount
3	Peter	peter@yahoo.com	EF	1	101	3	8.9
2	Mary	mary@gmail.com	CD	2	102	2	7.8

- Step 2: apply projection to keep the required attributes
  - $\prod_{inv\text{-}id,name}\ (\sigma_{Customer.cust\text{-}id=Invoice.cust\text{-}id}\ (\ Customer \times Invoice\ )\ )$

inv-id	name
1	Peter
2	Mary

Relational algebra: Union

- **Union** $\cup$: given two relations R and S with compatible schema, it returns a relation containing tuples in either R or S

Example:

prod-id	name	brand	price
1	Coca Cola	CO	7.8
2	Pepsi	PE	8.9
3	7 Up	DP	6.5
4	Sprite	CO	8.3

$$\prod_{prod\text{-}id}(\sigma_{prod\text{-}id\leq 2}(Product)) \cup \prod_{prod\text{-}id}(\sigma_{prod\text{-}id=4}(Product))$$

prod-id
1
2

$\cup$

prod-id
4

prod-id
1
2
4

Relational algebra: Set Difference

- **Set difference** –: given two relations R and S with compatible schema, it returns a relation containing tuples in R but not in S
- Example:

prod-id	name	brand	price
1	Coca Cola	CO	7.8
2	Pepsi	PE	8.9
3	7 Up	DP	6.5
4	Sprite	CO	8.3

$$\Pi_{prod\text{-}id}(Product) - \Pi_{prod\text{-}id}(\sigma_{prod\text{-}id=4}(Product))$$

prod-id
1
2
3
4

$-$

prod-id
4

$\Rightarrow$

prod-id
1
2
3

Relational algebra: Rename

- **Rename** ρ: assign a name (e.g., *Temp*) to the result of relational-algebra expression E

Example:

$$\rho_{Temp}\left(\prod_{name}(\sigma_{price > 8.0}(Product))\right)$$

Relation **Product**:

prod-id	name	brand	price
1	Coca Cola	CO	7.8
2	Pepsi	PE	8.9
3	7 Up	DP	6.5
4	Sprite	CO	8.3

Relation **Temp**:

name
Pepsi
Sprite

Exercises on relational algebra tree

Draw the relational algebra tree for each expression below

(1) $\rho_{Temp}\left(\prod_{name}\left(\sigma_{price > 8.0}(Product)\right)\right)$

(2) $\sigma_{Customer.cust\text{-}id=Invoice.cust\text{-}id}\left(Customer \times Invoice\right)$

(3) $\prod_{prod\text{-}id}(Product) - \prod_{prod\text{-}id}(\sigma_{prod\text{-}id=4}(Product))$

Exercise: Find the highest price in Product

- Idea:
  - Step (1) finds prices that are not the highest
  - Step (2) computes the set difference between $\prod_{price}(Product)$ and the above intermediate result

- Expression for step (1):

$$\prod_{Temp.price} (\sigma_{Temp.price < Product.price} (Product \times \rho_{Temp}(Product)))$$

Relation **Product**:

prod-id	name	brand	price
1	Coca Cola	CO	7.8
2	Pepsi	PE	8.9
3	7 Up	DP	6.5
4	Sprite	CO	8.3

price
7.8
6.5
8.3

Outline

- Relational model
- Relational algebra: basic operations
-  Relational algebra: other operations
- Null value

Other Relational Algebra Operations

- **Convenient** operations for simplifying expressions; have the same expressive power as basic operations
  - Natural join
  - Set intersection
  - Assignment
  - Division (we skip this)
- **Extended** operations that have more expressive power than basic operations
  - Generalized projection
  - Aggregation functions
  - Outer join (we skip this)

Natural join

⋈:\bowtie , join

- **Natural join ⋈**
  - Given two relations *R* and *S*, let the union schema be A, let the common attributes be a1, a2, …, ak
  - The natural join $R \bowtie S$ returns:

$$\Pi_A \left(\sigma_{R.a1=S.a1,\ R.a2=S.a2,\ \ldots,\ R.ak=S.ak} (R \times S) \right)$$

  - Example: *Customer* ⋈ *Invoice*
    - Common attribute: cust-id
    - Schema: (cust-id, name, email, address, inv-id, timestamp, amount)

cust-id	name	email	address	inv-id	timestamp	amount
3	Peter	peter@yahoo.com	EF	1	101	8.9
2	Mary	mary@gmail.com	CD	2	102	7.8

cust-id	name	email	address
1	James	james@yahoo.com	AB
2	Mary	mary@gmail.com	CD
3	Peter	peter@yahoo.com	EF
4	Peter	peter@gmail.com	**null**

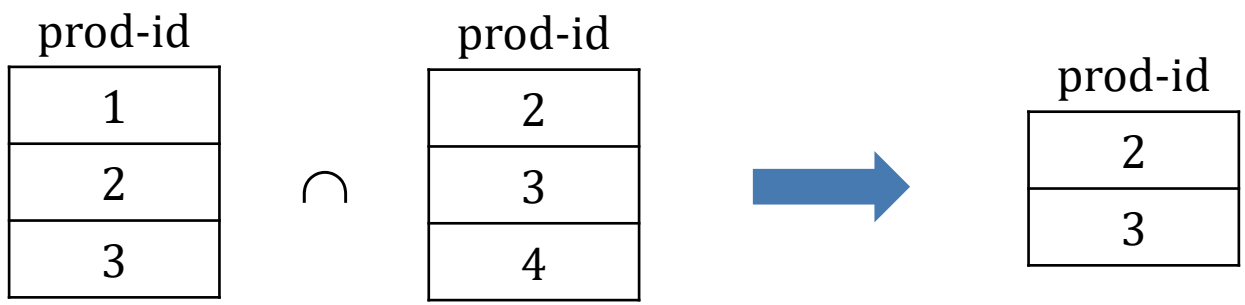
inv-id	timestamp	cust-id	amount
1	101	3	8.9
2	102	2	7.8

Set intersection

- **Set intersection** $\cap$: given two relations R and S with compatible schema, it returns a relation containing tuples that are in both R and S

Example:

$$\prod_{prod\text{-}id}(\sigma_{prod\text{-}id \leq 3}(Product)) \cap \prod_{prod\text{-}id}(\sigma_{prod\text{-}id \geq 2}(Product))$$



Assignment

- **Assignment** ←: assign temporary result to a variable
  - Improve the readability of relational algebra
  - For example, the following expression

$$\prod_{prod\text{-}id}(\sigma_{prod\text{-}id \leq 2}(Product)) \cup \prod_{prod\text{-}id}(\sigma_{prod\text{-}id=4}(Product))$$

can be rewritten as:

$$Temp1 \leftarrow \prod_{prod\text{-}id}(\sigma_{prod\text{-}id \leq 2}(Product))$$

$$Temp2 \leftarrow \prod_{prod\text{-}id}(\sigma_{prod\text{-}id=4}(Product))$$

$$Temp1 \cup Temp2$$

Assignment: Database Modification

- Insertion
  - Insert tuples into a relation
  - E.g., insert a new product "Soda"

    $Product \leftarrow Product \cup \{ (5, \text{“Soda”}, \text{“AB”}, 7.5) \}$

prod-id	name	brand	price
1	Coca Cola	CO	7.8
2	Pepsi	PE	8.9
4	Sprite	CO	8.3
5	Soda	AB	7.5

Assignment: Database Modification

- Deletion
  - Delete tuples from a relation
  - E.g., remove products that have price < 7.0

$$Product \leftarrow Product - \sigma_{price<7.0}\ (Product)$$

Original

prod-id	name	brand	price
1	Coca Cola	CO	7.8
2	Pepsi	PE	8.9
3	7 Up	DP	6.5
4	Sprite	CO	8.3

After deletion

prod-id	name	brand	price
1	Coca Cola	CO	7.8
2	Pepsi	PE	8.9
~~3~~	~~7 Up~~	~~DP~~	~~6.5~~
4	Sprite	CO	8.3

Generalized projection

- **Generalized projection**
  - Functions may be used to compute derived attributes
  - Example: compute the price of each product at 20% discount
    - "special-price" is a derived attribute

$$\Pi_{\textit{prod-id, price*80\% as special-price}}(\textit{Product})$$

Relation **Product**:

prod-id	name	brand	price
1	Coca Cola	CO	7.8
2	Pepsi	PE	8.9
3	7 Up	DP	6.5
4	Sprite	CO	8.3

prod-id	special-price
1	6.24
2	7.12
3	5.2
4	6.64

Aggregate function

- **Aggregate function**
  - Apply on a collection of values to obtain a single result
  - Functions: sum, avg, count, min, max
- Examples
  - Find the total amount of *Invoice*: $\mathcal{G}_{\text{sum(amount)}}(Invoice)$
  - Find the highest price of *Product*: $\mathcal{G}_{\text{max(price)}}(Product)$

Relation **Invoice**:

inv-id	timestamp	cust-id	amount
1	101	3	8.9
2	102	2	7.8
3	103	2	6.5
4	104	3	8.3

sum(amount)
31.5

Aggregate function + grouping

- In some scenario, we wish to partition the table into groups, then use aggregate function on each group
  - E.g., find the total amount spent by each customer

$$\text{cust-id}\ \mathcal{G}\ \text{sum(amount)}(Invoice)$$

inv-id	timestamp	cust-id	amount
1	101	3	8.9
2	102	2	7.8
3	103	2	6.5
4	104	3	8.3

grouping

inv-id	timestamp	cust-id	amount
1	101	3	8.9
4	104	3	8.3
2	102	2	7.8
3	103	2	6.5

aggregation

cust-id	sum(amount)
3	17.2
2	14.3

Outline

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- Relational algebra: basic operations
- Relational algebra: other operations
- Null value

Null Value

- **Null** represents a missing value
- When we compare null and a known value (e.g., null > 8.0)
  - The result is **unknown**

Relation **Product**:

prod-id	name	brand	price
1	Coca Cola	CO	7.8
2	Pepsi	PE	8.9
3	7 Up	null	6.5
4	Sprite	CO	null

A and B

input		output
True	Unknown	Unknown
False	Unknown	False
Unknown	Unknown	Unknown

A or B

input		output
True	Unknown	True
False	Unknown	Unknown
Unknown	Unknown	Unknown

not A

input	output
Unknown	Unknown

Null Value

- Select operation
  - A tuple belongs to the result if it is *true* for the condition in select

Relation **Product**:

prod-id	name	brand	price
1	Coca Cola	CO	7.8
2	Pepsi	PE	8.9
3	7 Up	null	6.5
4	Sprite	CO	null

- Find the results of

$\sigma_{price > 8.0}(Product)$

prod-id	name	brand	price
2	Pepsi	PE	8.9

$\sigma_{price > 8.0 \vee brand="CO"}(Product)$

prod-id	name	brand	price
1	Coca Cola	CO	7.8
2	Pepsi	PE	8.9
4	Sprite	CO	null

$\sigma_{price > 7.0 \wedge brand="CO"}(Product)$

prod-id	name	brand	price
1	Coca Cola	CO	7.8

Summary

- After this lecture, you should be able to:
  1) Understand concepts in relational model (e.g., schema, relation, keys, schema diagram)
  2) Apply relational algebra to express queries

- Please read Chapter 2 in the book "*Database System Concepts*", 7th Edition

- Next lecture: SQL