1. $p(x|y,\theta) = \binom{y_1}{x_1}\left(\frac{\frac{1}{2}}{\frac{1}{2}+\frac{\theta}{4}}\right)^{x_1}\left(\frac{\frac{\theta}{4}}{\frac{1}{2}+\frac{\theta}{4}}\right)^{y_1-x_1}$

$$Q(\theta|\theta^{(k)}) = E\left(\log p(x,y|\theta) \,\middle|\, Y, \theta^{(k)}\right)$$

$$p(x,y|\theta) = \left(\tfrac{1}{2}\right)^{x_1}\left(\tfrac{\theta}{4}\right)^{x_2}\left(\tfrac{1-\theta}{4}\right)^{x_3}\left(\tfrac{1-\theta}{4}\right)^{x_4}\left(\tfrac{\theta}{4}\right)^{x_5}\binom{x_1+x_2+x_3+x_4+x_5}{x_1,\ x_2,\ x_3,\ x_4,\ x_5}$$

set $\log P(x,y|\theta) \triangleq c_1 + x_1\log\left(\tfrac{1}{2}\right) + (x_2+x_5)\log\left(\tfrac{\theta}{4}\right) + (x_3+x_4)\log\left(\tfrac{1-\theta}{4}\right)$

where $c_1 = \log\left(\frac{(x_1+x_2+x_3+x_4+x_5)!}{x_1!\cdot x_2!\cdot x_3!\cdot x_4!\cdot x_5!}\right)$

substitute into $Q(\theta|\theta^{(k)})$, we have

$$Q(\theta|\theta^k) = E_{x_1}\left[c_1 + x_1\log\left(\tfrac{1}{2}\right) + (x_2+x_5)\log\left(\tfrac{\theta}{4}\right) + (x_3+x_4)\log\left(\tfrac{1-\theta}{4}\right) \,\middle|\, Y, \theta^{(k)}\right)$$

$$= E\left(c_1 \,\middle|\, Y, \theta^{(k)}\right) + E\left(x_1\log\left(\tfrac{1}{2}\right) \,\middle|\, Y, \theta^{(k)}\right) +$$

$$E\left((y_1-x_1+y_4)\log\left(\tfrac{\theta}{4}\right) + (y_2+y_3)\log\left(\tfrac{1-\theta}{4}\right) \,\middle|\, Y, \theta^{(k)}\right)$$

We find that 1^{st} and 2^{nd} term in RHS is no relevant with θ

so we consider 3^{rd} term only.

3^{rd} term $= (y_1+y_4)\log\left(\tfrac{\theta}{4}\right) + (y_2+y_3)\log\left(\tfrac{1-\theta}{4}\right) - E\left(x_1 \,\middle|\, Y, \theta^{(k)}\right)\cdot\log\left(\tfrac{\theta}{4}\right)$

$$= (y_1+y_4)\log\left(\tfrac{\theta}{4}\right) + (y_2+y_3)\log\left(\tfrac{1-\theta}{4}\right) - \left(\tfrac{2}{2+\theta^{(k)}}\right)y_1\cdot\log\left(\tfrac{\theta}{4}\right)$$

if $\frac{\partial Q(\theta,\theta^{(k)})}{\partial\theta} = \frac{y_1+y_4}{\theta} + \frac{y_2+y_3}{\theta-1} - \frac{\frac{2}{2+\theta^{(k)}}\cdot y_1}{\theta} = 0$,

$\left.\frac{\partial Q(\theta,\theta^{(k)})}{\partial\theta}\right|_{\theta^{(k+1)}} = 0$, $\theta^{(k+1)}$ is equal to $M(\theta^{(k)})$.

that is $M(\theta^{(k)}) = \frac{\frac{\theta^{(k)}}{2+\theta^{(k)}}y_1 + y_4}{\frac{\theta^{(k)}}{2+\theta^{(k)}}y_1 + y_4 + y_2 + y_3}$

$$= \frac{\theta^{(k)}\cdot y_1 + (2+\theta^{(k)})y_4}{\theta^{(k)}y_1 + (2+\theta^{(k)})(y_2+y_3+y_4)}$$

$$= \theta^{(k+1)}.$$

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$$\theta^{(k+1)} = M(\theta^{(k)}) = \frac{230\theta^{(k)}+110}{2850\theta^{(k)}+220}$$

$$\boxed{= \frac{46\theta^{(k)}+22}{570^{(k)}+44}} . \quad \square$$

2. single linkage : $dist(C_1, C_2) = \min_{x \in C_1, y \in C_2} dist(x, y)$

step 1: $\boxed{level = 0.47}$,

set $\boxed{C_1 = \{x_1\},\ C_2 = \{x_2, x_5\},\ C_3 = \{x_3\},\ C_4 = \{x_4\}}$

	C_1	C_2 ✓	C_3
C_2	2.33		
C_3 ✓	3.15	1.30	
C_4	1.90	1.50	3.70

step 2: $\boxed{level = 1.30}$,

$\boxed{C_1 = \{x_1\},\ C_5 = \underbrace{\{x_2, x_3, x_5\}}_{= C_2 \cup C_3},\ C_4 = \{x_4\}}$

	C_1	C_5 ✓
C_5	2.33	
C_4 ✓	1.90	1.50

step 3: $\boxed{level = 1.50}$

$\boxed{C_1 = \{x_1\},\ C_6 = C_4 \cup C_5 = \{x_2, x_3, x_4, x_5\}}$

	C_1
C_6	1.90

step 4: $\boxed{level = 1.90}$

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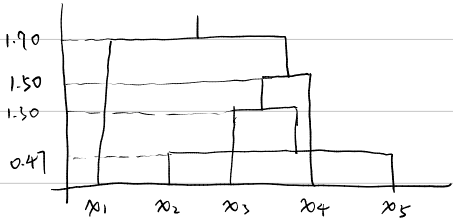
$\boxed{\text{cluster tree:}}$



3. (a) $\text{support}\{e\} = \frac{8}{10} = \boxed{\frac{4}{5}}$,

$\text{support}\{b,d\} = \frac{2}{10} = \boxed{\frac{1}{5}}$,

$\text{support}\{b,d,e\} = \frac{2}{10} = \boxed{\frac{1}{5}}$

(b) $\text{confidence}(\{b,d\} \to \{e\}) = \frac{\text{supp}\{b,d,e\}}{\text{supp}\{b,d\}} = \boxed{1}$

$\text{confidence}(\{e\} \to \{b,d\}) = \frac{\text{supp}\{b,d,e\}}{\text{supp}\{e\}} = \boxed{\frac{1}{4}}$

$\boxed{\text{not symmetric}}$

(c) confidence $\geq 50\%$.

$\boxed{c(\{b,d\} \to \{e\}) = 1}$

$\cancel{c(\{b\} \to \{d,e\}) = \frac{2}{6}}$

$\cancel{c(\{b,e\} \to \{d\}) = \frac{2}{5}}$

$\cancel{c(\{d\} \to \{b,e\}) = \frac{2}{6}}$

$\cancel{c(\{d,e\} \to \{b\}) = \frac{2}{5}}$

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