

# Lecture 11

## Database normalization

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Acknowledgement: Slides were offered from Prof. Ken Yiu.  
Some parts might be revised and indicated.

# Outline



- ◆ Motivation

- ◆ Concepts: functional dependency, closure, cover

- ◆ Properties of normalization

- ◆ Normal forms: BCNF vs. 3NF

# First Normal Form

- ◆ An attribute is **atomic** if it cannot be divided into parts

## Examples of non-atomic attributes

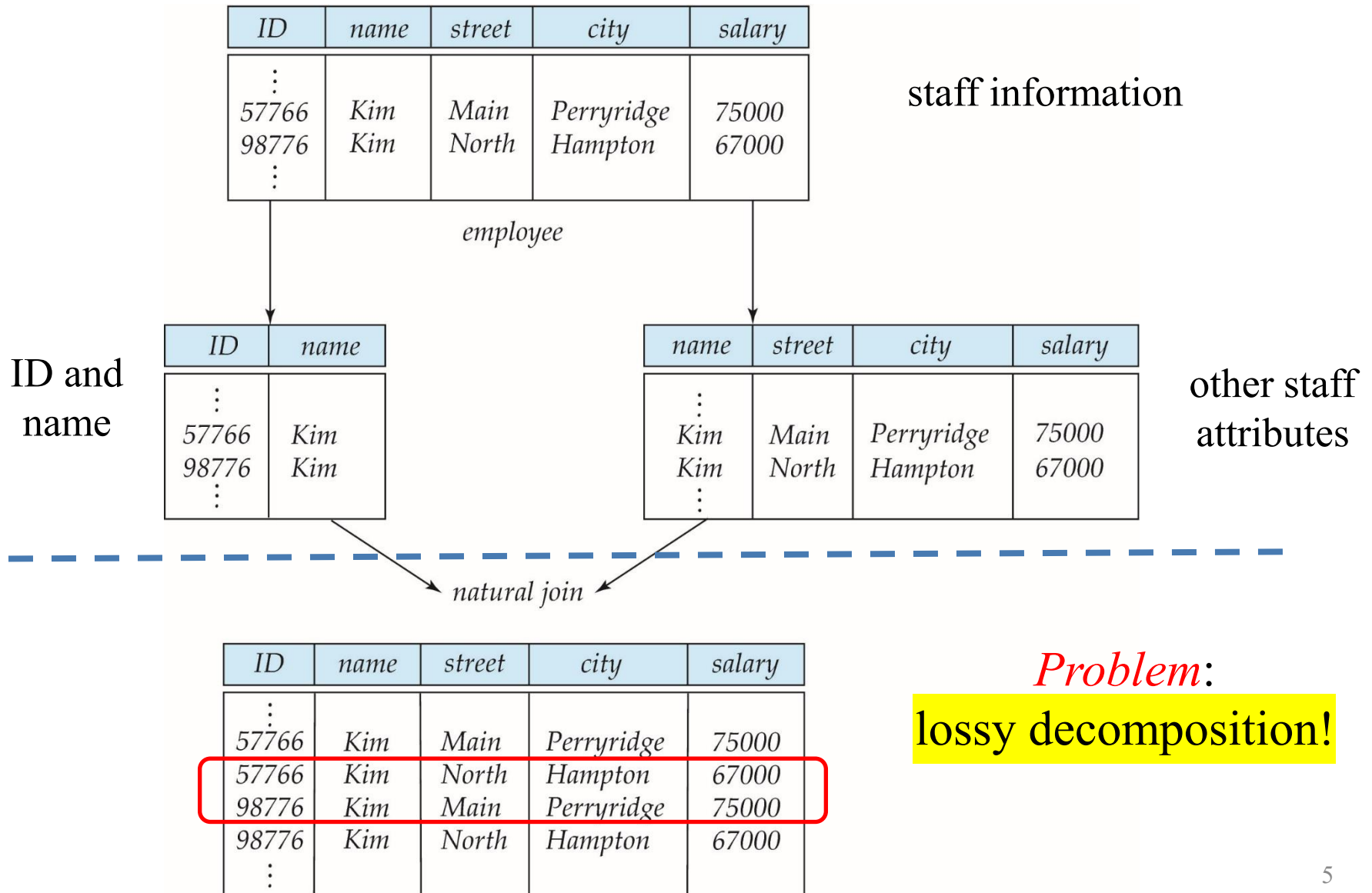
- ◆ Set of names, composite attributes
- ◆ Course code (CS101) that can be divided into “CS” and “101”
  - programmer may extract “CS” to find the department
  - bad idea: encode data in program rather than in DBMS
- ◆ **Drawback** of non-atomic attributes
  - ◆ Complicated to store them
  - ◆ Redundant storage of data → may cause data inconsistency
- ◆ In this lecture, we assume all attributes are atomic, i.e., each relation schema is in **first normal form**

# Example Application: University Information Management System

- What if we use one (large) table only?
- Problem:** redundant information

staff name		staff salary	dept. location		dept. budget
ID	name	salary	dept_name	building	budget
22222	Einstein	95000	Physics	Watson	70000
12121	Wu	90000	Finance	Painter	120000
32343	El Said	60000	History	Painter	50000
45565	Katz	75000	Comp. Sci.	Taylor	100000
98345	Kim	80000	Elec. Eng.	Taylor	85000
76766	Crick	72000	Biology	Watson	90000
10101	Srinivasan	65000	Comp. Sci.	Taylor	100000
58583	Califieri	62000	History	Painter	50000
83821	Brandt	92000	Comp. Sci.	Taylor	100000
15151	Mozart	40000	Music	Packard	80000
33456	Gold	87000	Physics	Watson	70000
76543	Singh	80000	Finance	Painter	120000

# What if we use many (small) tables?

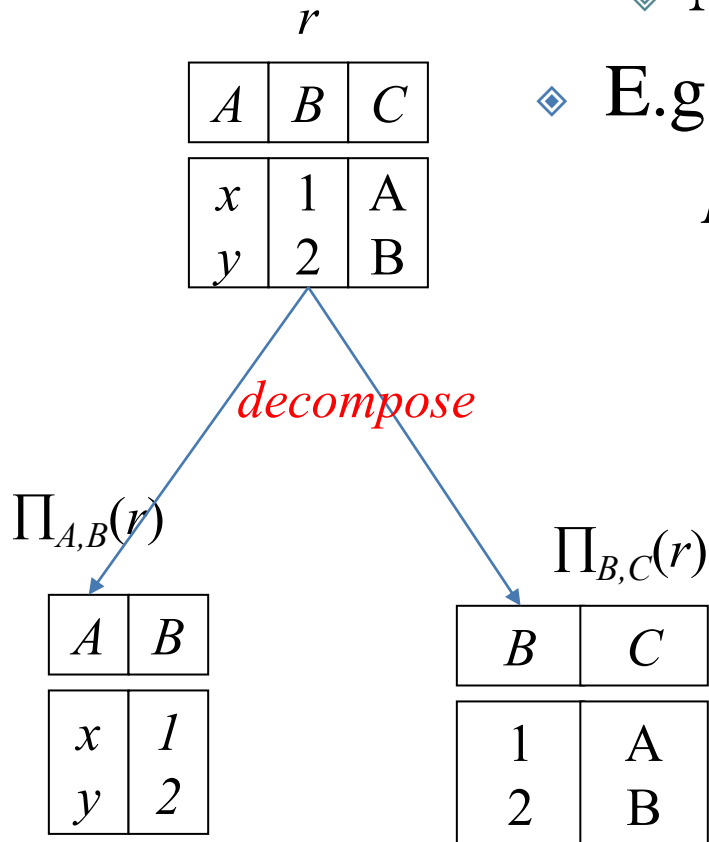


# Example of Lossless-Join Decomposition



## ◆ Lossless-join decomposition

- ◆ No information lost
- ◆ E.g., decompose  $R = (A, B, C)$  into  $R_1 = (A, B)$  and  $R_2 = (B, C)$



$$\Pi_{A,B}(r) \bowtie \Pi_{B,C}(r)$$

$A$	$B$	$C$
$x$	1	A
$y$	2	B

# How to design a good schema?

- ◆ **Decide** whether a particular relation  $R$  is in “good” form
  - ◆ E.g., BCNF, 3NF
- ◆ If a relation  $R$  is not in “good” form, **decompose** it into a set of relations  $\{R_1, R_2, \dots, R_n\}$  such that
  - ◆ each relation is in good form
  - ◆ the decomposition is a lossless-join decomposition
- ◆ Our theory is based *functional dependencies*

# Outline

- ◆ Motivation



- ◆ Concepts: functional dependency, closure, cover

- ◆ Properties of normalization

- ◆ Normal forms: BCNF vs. 3NF



# Functional dependency: $\alpha \rightarrow \beta$



generalize the concept of **key**

- ◆  $\alpha \rightarrow \beta$  is like a **rule**: the values of attributes in  $\alpha$  **determines uniquely** the values of attributes in  $\beta$

- ◆ *Example:* consider this **instance** of relation  $r(A,B)$

- ◆  $A \rightarrow B$  doesn't hold on  $r$

- ◆  $B \rightarrow A$  holds on  $r$



1	4
1	5
3	7

- ◆ *Definition:* Given a **relation schema**  $R$ , the **functional dependency**  $\alpha \rightarrow \beta$  holds on  $R$  means that:  
for any **legal relation**  $r(R)$ , whenever **any two tuples**  $t_1$  and  $t_2$  of  $r$  agree on attributes in  $\alpha$ , they also agree on attributes in  $\beta$ . That is,

$$t_1[\alpha] = t_2[\alpha] \Rightarrow t_1[\beta] = t_2[\beta]$$

# Express the concept of key by $\alpha \rightarrow \beta$

- ◆  $K$  is a **superkey** for relation schema  $R$  if and only if  $K \rightarrow R$
- ◆  $K$  is a **candidate key** for  $R$  if and only if
  - ◆  $K \rightarrow R$ , and
  - ◆ there exists no  $\alpha \subset K$  such that  $\alpha \rightarrow R$
- ◆ Functional dependencies allow us to express constraints that cannot be expressed using superkeys.

Consider the schema:

*inst\_dept* (ID, *name*, *salary*, dept\_name, *building*, *budget* )

We expect these **functional dependencies** to hold:

*dept\_name*  $\rightarrow$  *building*

and *ID*  $\rightarrow$  *building*

but would not expect the following to hold:

*dept\_name*  $\rightarrow$  *salary*

# Use of Functional Dependencies



- ◆ We use functional dependencies to:
  - ◆ **test a relation**  $r$  to see if it is legal under a given set  $\mathcal{F}$  of functional dependencies
    - ◆ If yes, then we say that:  **$r$  satisfies  $\mathcal{F}$**
  - ◆ **specify constraints** on the set of legal relations
    - ◆ We say that  **$\mathcal{F}$  holds on  $R$**  if all legal relations on  $R$  satisfy  $\mathcal{F}$
- ◆ **Note:** we ignore a functional dependency if it cannot hold on all legal instances
  - ◆ E.g., an instance of *instructor* may satisfy, **by chance**,  
$$name \rightarrow ID$$
but we should **ignore** this functional dependency

# Functional Dependencies (Cont.)

- ◆ A functional dependency is **trivial** if it is satisfied by all instances of a relation
- ◆  $\alpha \rightarrow \beta$  is trivial if  $\beta \subseteq \alpha$
- ◆ Example:
  - ◆  $ID, name \rightarrow ID$
  - ◆  $name \rightarrow name$

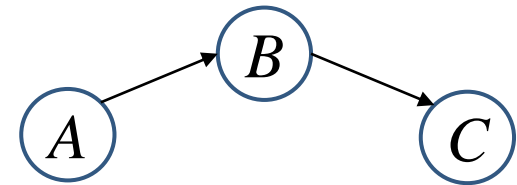
# Closure of Functional Dependencies

## ◆ Armstrong's axioms

- ◆ if  $\beta \subseteq \alpha$ , then  $\alpha \rightarrow \beta$  (reflexivity)
- ◆ if  $\alpha \rightarrow \beta$ , then  $\gamma \alpha \rightarrow \gamma \beta$  (augmentation)
- ◆ if  $\alpha \rightarrow \beta$ , and  $\beta \rightarrow \gamma$ , then  $\alpha \rightarrow \gamma$  (transitivity)

## ◆ Example:

- ◆ given  $A \rightarrow B$  and  $B \rightarrow C$ ,  
we obtain:  $A \rightarrow C$



## ◆ The closure of $\mathcal{F}$ , denoted by $\mathcal{F}^+$

- ◆ The set of all functional dependencies implied by a set  $\mathcal{F}$  of functional dependencies
- ◆ We compute  $\mathcal{F}^+$  by repeatedly applying Armstrong's axioms

# Example

$$\diamond R = (A, B, C, G, H, I)$$

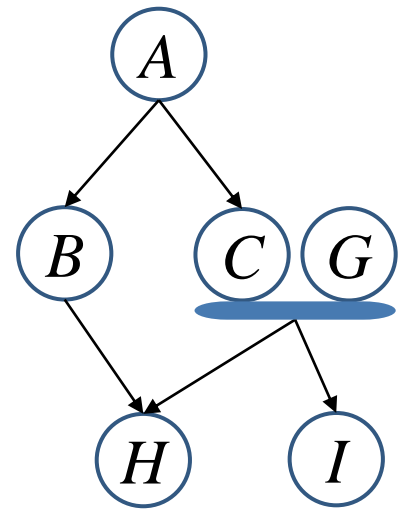
$$\diamond \mathcal{F} = \{ A \rightarrow B$$

$$A \rightarrow C$$

$$CG \rightarrow H$$

$$CG \rightarrow I$$

$$B \rightarrow H\}$$



$\mathcal{F}^+$  contains:

$$\diamond A \rightarrow H$$

$\diamond$  by transitivity from  $A \rightarrow B$  and  $B \rightarrow H$

$$\diamond AG \rightarrow I$$

$\diamond$  by augmenting  $A \rightarrow C$  with  $G$ , to get  $AG \rightarrow CG$   
        and then transitivity with  $CG \rightarrow I$

$$\diamond CG \rightarrow HI$$

$\diamond$  by augmenting  $CG \rightarrow I$  to infer  $CG \rightarrow CGI$ ,  
        and augmenting of  $CG \rightarrow H$  to infer  $CGI \rightarrow HI$ ,  
        and then transitivity

$$\diamond \dots \rightarrow \dots$$

$\diamond$  until you cannot find a new functional dependency

We have a systematic way to compute  $\mathcal{F}^+$  (see page 18)

# Closure of Functional Dependencies (Cont.)

- ◆ Additional *short-cut* rules:
  - ◆ If  $\alpha \rightarrow \beta$  holds and  $\alpha \rightarrow \gamma$  holds, then  $\alpha \rightarrow \beta\gamma$  holds  
**(union)**
  - ◆ If  $\alpha \rightarrow \beta\gamma$  holds, then  $\alpha \rightarrow \beta$  holds and  $\alpha \rightarrow \gamma$  holds  
**(decomposition)**
  - ◆ If  $\alpha \rightarrow \beta$  holds and  $\gamma\beta \rightarrow \delta$  holds, then  $\alpha\gamma \rightarrow \delta$  holds  
**(pseudotransitivity)**

The above rules can be derived from Armstrong's axioms

# Attribute Set Closure

- ◆ Let  $\alpha$  be a set of attributes
- ◆ The *closure* of  $\alpha$  **under**  $\mathcal{F}$  (denoted by  $\alpha^+$ ) is the set of attributes that are determined by  $\alpha$  under  $\mathcal{F}$
- ◆ Algorithm to compute  $\alpha^+$ :  
 $result := \alpha$   
**while** ( $result$  has changed) **do**  
    **for each**  $\beta \rightarrow \gamma$  **in**  $\mathcal{F}$  **do**  
        **if**  $\beta \subseteq result$  **then**  
             $result := result \cup \gamma$





# Example on $\alpha^+$

◆  $R = (A, B, C, G, H, I)$

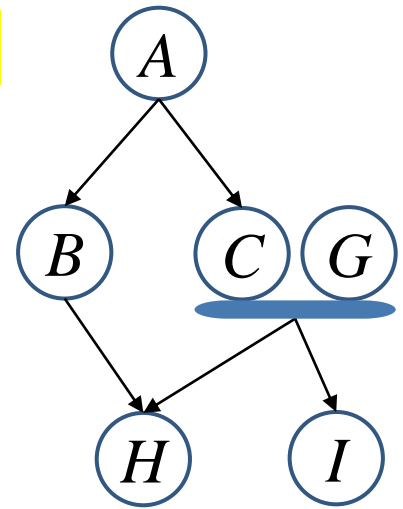
◆  $\mathcal{F} = \{A \rightarrow B$

$A \rightarrow C$

$CG \rightarrow H$

$CG \rightarrow I$

$B \rightarrow H\}$



◆  $(AG)^+$

1. It contains  $AG$

2. It contains  $ABCG$  ( $A \rightarrow C$  and  $A \rightarrow B$ )

3. It contains  $ABCGH$  ( $CG \rightarrow H$ )

4. It contains  $ABCGHI$  ( $CG \rightarrow I$ )

Therefore,  $(AG)^+ = ABCGHI$

◆ Is  $AG$  a candidate key?

□ Is  $AG$  a super key?

□ Does  $(AG)^+$  contain  $R$ ?

□ Is any subset of  $AG$  a superkey?

□ Does  $(A)^+$  contain  $R$ ?

□ Does  $(G)^+$  contain  $R$ ?

The above figure is for reference only.  
In general, it can be complicated  
visualize  $\mathcal{F}$  by using a figure.

# Uses of Attribute Closure

- ◆ How to test if  $\alpha$  is a superkey?
  - ◆ Compute  $\alpha^+$ , then check if  $\alpha^+$  contains all attributes of  $R$
- ◆ How to test if a functional dependency  $\alpha \rightarrow \beta$  holds?
  - ◆ (in other words, we want to check whether  $\alpha \rightarrow \beta$  is in  $\mathcal{F}^+$ )
  - ◆ Compute  $\alpha^+$  by using attribute closure, and then check if  $\alpha^+$  contains  $\beta$
- ◆ How to compute  $\mathcal{F}^+$  (i.e., the closure of  $\mathcal{F}$ )?
  - ◆ for each  $\gamma \subseteq R$ 
    - find the closure  $\gamma^+$
    - for each  $S \subseteq \gamma^+$ 
      - output a functional dependency  $\gamma \rightarrow S$

# Example on finding all candidate keys

◆  $R = (A, B, C, G, H, I)$

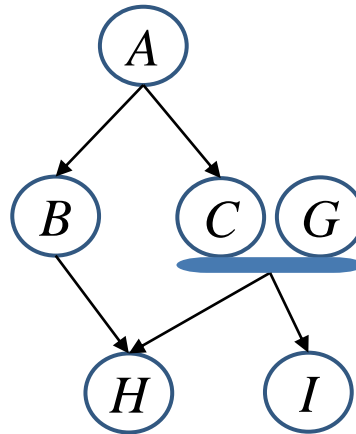
◆  $\mathcal{F} = \{A \rightarrow B$

$A \rightarrow C$

$CG \rightarrow H$

$CG \rightarrow I$

$B \rightarrow H\}$



◆ Size-3 attribute sets  
except supersets of candidate keys

◆  $(ABC)^+ = ABCH$

◆  ~~$(ABG)^+$~~

◆  $(ABH)^+ = ABCH$

◆  $(ABI)^+ = ABCHI$

◆  $(ACH)^+ = ABCH$

◆ .....

◆ Size-2 attribute sets

◆  $(AB)^+ = ABCH$

◆  $(AC)^+ = ABCH$

◆  $(AG)^+ = ABCGHI$

◆ Candidate key!

◆  $(AH)^+ = ABCH$

◆  $(AI)^+ = ABCHI$

◆  $(BC)^+ = BCH$

◆ .....

◆ Size-1 attribute sets

◆  $A^+ = ABCH$

◆  $B^+ = BH$

◆  $C^+ = C$

◆  $G^+ = G$

◆  $H^+ = H$

◆  $I^+ = I$

# Example on finding $\mathcal{F}^+$ (the closure of $\mathcal{F}$ )

◆  $R = (A, B, C, G, H, I)$

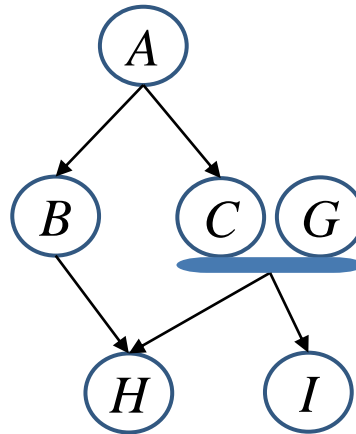
◆  $\mathcal{F} = \{A \rightarrow B$

$A \rightarrow C$

$CG \rightarrow H$

$CG \rightarrow I$

$B \rightarrow H\}$



◆ Size-3 attribute sets

◆  $ABC \rightarrow ABCH$

◆  $ABC \rightarrow \dots$

◆  $\dots$

◆  $ABG \rightarrow ABCGHI$

◆  $\dots$

◆  $ABH \rightarrow ABCH$

◆  $\dots$

◆  $ABI \rightarrow ABCHI$

◆  $\dots$

◆  $ACH \rightarrow ABCH$

◆  $\dots$

◆  $\dots$

◆ Size-1 attribute sets

◆  $A \rightarrow ABCH$

◆  $A \rightarrow ABC$

◆  $A \rightarrow ABH$

◆  $A \rightarrow ACH$

◆  $A \rightarrow BCH$

◆  $A \rightarrow AB$

◆  $A \rightarrow AC$

◆  $\dots$

◆  $\dots$

◆ Size-2 attribute sets

◆  $AB \rightarrow ABCH$

◆  $AB \rightarrow \dots$

◆  $\dots$

◆  $AC \rightarrow ABCH$

◆  $\dots$

◆  $AG \rightarrow ABCGHI$

◆  $\dots$

◆  $\dots$

# Canonical Cover

- ◆ A set of functional dependencies may have redundant dependencies that can be inferred from the others
  - ◆ *Example #1*:  $A \rightarrow C$  is redundant in:
$$\{A \rightarrow B, B \rightarrow C, A \twoheadrightarrow C\}$$
  - ◆ *Example #2*: parts of a functional dependency redundant  
E.g., RHS:  $\{A \rightarrow B, B \rightarrow C, A \rightarrow \textcolor{red}{C}D\}$  can be simplified to
$$\{A \rightarrow B, B \rightarrow C, A \rightarrow D\}$$
  
E.g., LHS:  $\{A \rightarrow B, B \rightarrow C, A\textcolor{red}{C} \rightarrow D\}$  can be simplified to
$$\{A \rightarrow B, B \rightarrow C, A \rightarrow D\}$$
- ◆ A canonical cover of  $\mathcal{F}$  is a “**minimal**” set of functional dependencies equivalent to  $\mathcal{F}$ 
  - ◆ no redundant (parts of) dependencies

# Testing if an Attribute is Extraneous

- ◆ Consider a functional dependency  $\alpha \rightarrow \beta$  in a given set  $\mathcal{F}$  of functional dependencies
- ◆ To test if attribute  $A$  is **extraneous** in  $\alpha$ 
  - compute  $(\alpha - A)^+$  using dependencies in  $\mathcal{F}$
  - if  $(\alpha - A)^+$  contains  $\beta$ , then  $A$  is extraneous in  $\alpha$

◆ *Example:* Given  $\mathcal{F} = \{A \rightarrow C, AB \rightarrow C\}$

- ◆ Is  **$B$**  extraneous in  **$AB \rightarrow C$** ?

$$\begin{aligned}\text{Compute } (AB - B)^+ &= A^+ \\ &= AC\end{aligned}$$

- ◆ Yes, because the above result contains  $C$

# Testing if an Attribute is Extraneous

- ◆ Consider a functional dependency  $\alpha \rightarrow \beta$  in a given set  $\mathcal{F}$  of functional dependencies
- ◆ To test if attribute  $B \in \beta$  is **extraneous** in  $\beta$ 
  - compute  $\alpha^+$  using only dependencies in  $\mathcal{F}' = (\mathcal{F} - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - B)\}$
  - if  $\alpha^+$  contains  $B$ , then  $B$  is extraneous in  $\beta$



- ◆ *Example:* Given  $\mathcal{F} = \{A \rightarrow C, AB \rightarrow CD\}$

- ◆ Is **C** extraneous in  $AB \rightarrow CD$  ?

Use only dependencies in  $\mathcal{F}' = \{A \rightarrow C, AB \rightarrow D\}$

Compute  $(AB)^+ = ABCD$

- ◆ Yes, because the above result contains  $C$

# Canonical Cover

- ◆ A **canonical cover** for  $\mathcal{F}$  is a set of dependencies  $\mathcal{F}_c$  such that
  - ◆  $\mathcal{F}$  implies all dependencies in  $\mathcal{F}_c$ , and
  - ◆  $\mathcal{F}_c$  implies all dependencies in  $\mathcal{F}$ , and
  - ◆ No functional dependency in  $\mathcal{F}_c$  contains an **extraneous** attribute, and
  - ◆ Each left side of functional dependency in  $\mathcal{F}_c$  is unique
- ◆ To compute a canonical cover for  $\mathcal{F}$ :  
**repeat**
  - Use the union rule to replace any dependencies in  $\mathcal{F}$   
 $\alpha_1 \rightarrow \beta_1$  and  $\alpha_1 \rightarrow \beta_2$  with  $\alpha_1 \rightarrow \beta_1 \beta_2$
  - Find a functional dependency  $\alpha \rightarrow \beta$  with an  
extraneous attribute either in  $\alpha$  or in  $\beta$
  - If an extraneous attribute is found, delete it from  $\alpha \rightarrow \beta$**until**  $\mathcal{F}$  does not change

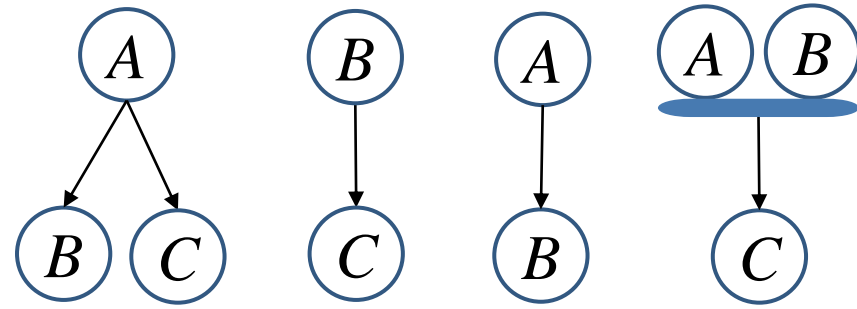


# Example

- ◆  $R = (A, B, C)$

- $\mathcal{F} = \{A \rightarrow BC, B \rightarrow C$

- $A \rightarrow B, AB \rightarrow C\}$



- ◆ Combine  $A \rightarrow BC$  and  $A \rightarrow B$  into  $A \rightarrow BC$

- ◆  $\mathcal{F}$  becomes  $\{A \rightarrow BC, B \rightarrow C, AB \rightarrow C\}$  now

- ◆  $A$  is extraneous in  $AB \rightarrow C$

- ◆ Check if the result of deleting  $A$  from  $AB \rightarrow C$  is implied by other dependencies

- ◆ Yes: in fact,  $B \rightarrow C$  is already present!

- ◆  $\mathcal{F}$  becomes  $\{A \rightarrow BC, B \rightarrow C\}$  now

- ◆  $C$  is extraneous in  $A \rightarrow BC$


- ◆ Check if  $A \rightarrow C$  is implied by  $A \rightarrow B$  and other dependencies

- ◆ Yes: using transitivity on  $A \rightarrow B$  and  $B \rightarrow C$

- ◆ Can use attribute closure of  $A$  in more complex cases

- ◆ The **canonical cover** is:  $\{A \rightarrow B, B \rightarrow C\}$

# Outline

- ◆ Motivation
- ◆ Concepts: functional dependency, closure, cover
-  ◆ Properties of normalization
- ◆ Normal forms: BCNF vs. 3NF

# Property 1: Lossless-join Decomposition

- For the case of  $R = (R_1, R_2)$ , we require that for all possible relations  $r$  on schema  $R$ :

$$r = \Pi_{R_1}(r) \bowtie \Pi_{R_2}(r)$$

- A decomposition of  $R$  into  $R_1$  and  $R_2$  is **lossless join** if at least one of the following dependencies is in  $F^+$ :

$$R_1 \cap R_2 \rightarrow R_1 \quad \text{or} \quad R_1 \cap R_2 \rightarrow R_2$$

## Example

- $R = (A, B, C), \quad \mathcal{F} = \{A \rightarrow B, B \rightarrow C\}$
- Decompose into  $R_1 = (A, B), \quad R_2 = (B, C)$ 
  - Lossless because:  $R_1 \cap R_2 = \{B\}$  and  $B \rightarrow BC$
- Decompose into  $R_1 = (A, B), \quad R_2 = (A, C)$ 
  - Lossless because:  $R_1 \cap R_2 = \{A\}$  and  $A \rightarrow AB$

# Property 2: Dependency Preservation

- Let  $\mathcal{F}_i$  be the set of dependencies  $\mathcal{F}^+$  that include only attributes in  $R_i$

- A decomposition is **dependency preserving** if

$$(\mathcal{F}_1 \cup \mathcal{F}_2 \cup \dots \cup \mathcal{F}_n)^+ = \mathcal{F}^+$$

- If it is not, then checking updates for violation of functional dependencies may require computing joins, which is expensive

## Example

- $R = (A, B, C), \quad \mathcal{F} = \{A \rightarrow B, B \rightarrow C\}$

- Decompose into  $R_1 = (A, B), \quad R_2 = (B, C)$



- Dependency preserving**

- Decompose into  $R_1 = (A, B), \quad R_2 = (A, C)$

- Not dependency preserving**

(cannot check  $B \rightarrow C$  without computing  $R_1 \bowtie R_2$ )

# Goals of Normalization

- ◆ If a relation scheme  $R$  is not in “good” form, decompose it into  $\{R_1, R_2, \dots, R_n\}$  such that
  - ◆ each relation scheme is in “good” form, and
  - ◆ the decomposition is a **lossless-join decomposition**
  - ◆ [*preferably*] the decomposition should be **dependency preserving**

# Outline

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- ◆ Normal forms: BCNF vs. 3NF



# Boyce-Codd Normal Form (BCNF)

A relation schema  $R$  is in **BCNF** with respect to a set  $\mathcal{F}$  of functional dependencies if for **all** functional dependencies in  $\mathcal{F}^+$  of the form

$$\alpha \rightarrow \beta$$

where  $\alpha \subseteq R$  and  $\beta \subseteq R$ , **at least one** of the following holds:

- ◆  $\alpha \rightarrow \beta$  is trivial (i.e.,  $\beta \subseteq \alpha$ )
- ◆  $\alpha$  is a superkey for  $R$

Example schema *not* in BCNF:

*instr\_dept* (ID, name, salary, dept\_name, building, budget )

because  $dept\_name \rightarrow building, budget$  holds on *instr\_dept*,  
but  $dept\_name$  is not a superkey

# BCNF Decomposition Algorithm

$result := \{R\}$

compute  $\mathcal{F}^+$

**while** (some schema  $R_i$  in  $result$  is not in BCNF) **do**

    let  $\alpha \rightarrow \beta$  be a nontrivial functional dependency that  
    holds on  $R_i$  such that  $\alpha \rightarrow R_i$  is not in  $\mathcal{F}^+$ ,

    and  $\alpha \cap \beta = \emptyset$

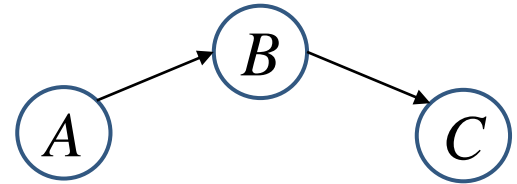
$result := (result - R_i) \cup (R_i - \beta) \cup (\alpha, \beta)$

Note: each  $R_i$  is in BCNF, and  
decomposition is lossless-join



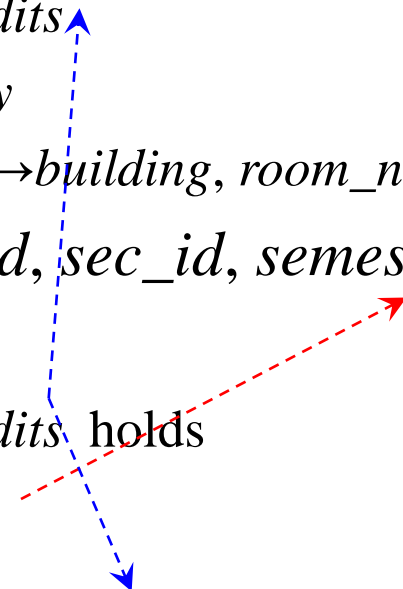
# Example of BCNF Decomposition

- ◆  $R = (A, B, C)$   
 $\mathcal{F} = \{A \rightarrow B$   
 $B \rightarrow C\}$   
Key =  $\{A\}$



- ◆  $R$  is not in BCNF  
( $B \rightarrow C$  but  $B$  is not superkey)
- ◆ Decompose  $R$  into:  $R_1 = (A, B)$ ,  $R_2 = (B, C)$ 
  - ◆  $R_1$  and  $R_2$  are in BCNF
  - ◆ Lossless-join decomposition

# Example of BCNF Decomposition

- ◆ *class* (*course\_id*, *title*, *dept\_name*, *credits*, *sec\_id*, *semester*, *year*, *building*, *room\_number*, *capacity*, *time\_slot\_id*)
  - ◆ Given functional dependencies:
    - ◆ *course\_id* → *title*, *dept\_name*, *credits*
    - ◆ *building*, *room\_number* → *capacity*
    - ◆ *course\_id*, *sec\_id*, *semester*, *year* → *building*, *room\_number*, *time\_slot\_id*
  - ◆ Find a candidate key: (*course\_id*, *sec\_id*, *semester*, *year*)
  - ◆ BCNF Decomposition:
    - ◆ *course\_id* → *title*, *dept\_name*, *credits* holds
      - ◆ but *course\_id* is not a superkey
    - ◆ We replace *class* by:
      - ◆ **course**(*course\_id*, *title*, *dept\_name*, *credits*)
      - ◆ *class-1* (*course\_id*, *sec\_id*, *semester*, *year*, *building*, *room\_number*, *capacity*, *time\_slot\_id*)
- 

# BCNF Decomposition (Cont.)

- ◇ *course* is in BCNF
  - ◇ How do we know this?
- ◇ *building, room\_number* → *capacity* holds on *class-1*
  - ◇ but {*building, room\_number*} is not a superkey for *class-1*
  - ◇ We replace *class-1* by:
    - ◆ *classroom* (*building, room\_number, capacity*)
    - ◆ *section* (*course\_id, sec\_id, semester, year, building, room\_number, time\_slot\_id*)
- ◇ *classroom* and *section* are in BCNF

# Third Normal Form (3NF)

- ◆ A relation schema  $R$  is in **3NF** if for all:

$$\alpha \rightarrow \beta \text{ in } \mathcal{F}^+$$

at **least one** of the following holds:

- ◆  $\alpha \rightarrow \beta$  is trivial (i.e.,  $\beta \in \alpha$ )
- ◆  $\alpha$  is a superkey for  $R$
- ◆ Each attribute  $A$  in  $\beta - \alpha$  is contained in a candidate key for  $R$   
(**NOTE:** each attribute may be in a different candidate key)

- ◆ A relation is in BCNF  $\rightarrow$  it is in 3NF
  - ◆ Because BCNF requires one of the first two conditions
- ◆ The third condition is a relaxation of BCNF
  - ◆ Ensure dependency preservation

# 3NF Example

- ◆ Relation *dept\_advisor*:
  - ◆ *dept\_advisor* (*s\_ID*, *i\_ID*, *dept\_name*)  
 $\mathcal{F} = \{s\_ID, dept\_name \rightarrow i\_ID, i\_ID \rightarrow dept\_name\}$
  - ◆ Two candidate keys: (*s\_ID*, *dept\_name*), and (*i\_ID*, *s\_ID*)
  - ◆ *R* is in 3NF
    - ◆ *s\_ID*, *dept\_name*  $\rightarrow$  *i\_ID* *s\_ID*
      - ◇ *s\_ID*, *dept\_name* is a superkey
    - ◆ *i\_ID*  $\rightarrow$  *dept\_name*
      - ◇ *dept\_name* is contained in a candidate key

# 3NF Decomposition Algorithm

let  $\mathcal{F}_c$  be a canonical cover for  $\mathcal{F}$

$i := 0$

**for each** functional dependency  $\alpha \rightarrow \beta$  in  $\mathcal{F}_c$  **do**

**if** no schema  $R_j$ ,  $1 \leq j \leq i$  contains  $\alpha \beta$  **then**

$i := i + 1$

$R_i := \alpha \beta$

**if** no schema  $R_j$ ,  $1 \leq j \leq i$  contains a candidate key for  $R$  **then**

$i := i + 1$

$R_i :=$  any candidate key for  $R$

/\* remove redundant relations \*/

**while** (some schema  $R_j$  is contained in another schema  $R_k$ )

$R_j := R_i$

$i := i - 1$

**return** ( $R_1, R_2, \dots, R_i$ )

It ensures:

- ◆ each schema  $R_i$  is in 3NF
- ◆ decomposition is dependency preserving and lossless-join

# 3NF Decomposition: An Example

- ◆ Relation schema:

$\text{cust\_banker\_branch} = (\underline{\text{customer\_id}}, \underline{\text{employee\_id}}, \text{branch\_name}, \text{type})$

- ◆ Given functional dependencies:

- $\text{customer\_id}, \text{employee\_id} \rightarrow \text{branch\_name}, \text{type}$
- $\text{employee\_id} \rightarrow \text{branch\_name}$
- $\text{customer\_id}, \text{branch\_name} \rightarrow \text{employee\_id}$

- ◆ We first compute a **canonical cover**

- ◆  $\text{branch\_name}$  is extraneous in the r.h.s. of the 1<sup>st</sup> dependency
- ◆ No other attribute is extraneous, so we get  $\mathcal{F}_C =$

$\text{customer\_id}, \text{employee\_id} \rightarrow \text{type}$   
 $\text{employee\_id} \rightarrow \text{branch\_name}$   
 $\text{customer\_id}, \text{branch\_name} \rightarrow \text{employee\_id}$

# 3NF Decompsition Example (Cont.)

- ◆ The **for** loop generates following 3NF schema:
  - $(customer\_id, employee\_id, type)$
  - $(\underline{employee\_id}, branch\_name)$
  - $(customer\_id, branch\_name, employee\_id)$
- ◆ Observe that  $(customer\_id, employee\_id, type)$  contains a candidate key of the original schema, so no further relation schema needs to be added
- ◆ At end of for loop, detect and delete schemas, such as  $(\underline{employee\_id}, branch\_name)$ , which are subsets of other schemas
  - ◆ result will not depend on the order in which FDs are considered
- ◆ The resultant simplified 3NF schema is:
  - $(customer\_id, employee\_id, type)$
  - $(customer\_id, branch\_name, employee\_id)$



# BCNF vs. 3NF

- ◆ We can **always** decompose a relation into a set of relations that are in **3NF** such that:
  - ◆ the decomposition is **lossless**
  - ◆ **the dependencies are preserved**
- ◆ We can always decompose a relation into a set of relations that are in **BCNF** such that:
  - ◆ the decomposition is **lossless**
  - ◆ but it may **not preserve dependencies**

# BCNF vs. 3NF

- ◆ First, we try to satisfy all requirements:
  - ◆ BCNF
  - ◆ Lossless join
  - ◆ Dependency preservation
- ◆ If we cannot achieve this, we accept one of:
  - ◆ Lack of dependency preservation, or
  - ◆ Redundancy due to the use of 3NF

# Appendix: BCNF and Dependency Preservation

It is not always possible to get a BCNF decomposition that is dependency preserving

- ◆  $R = (J, K, L)$   
 $\mathcal{F} = \{ JK \rightarrow L$   
 $L \rightarrow K \}$

Two candidate keys =  $JK$  and  $JL$

- ◆  $R$  is not in BCNF
- ◆ Any decomposition of  $R$  will fail to preserve

$$JK \rightarrow L$$

This implies that testing for  $JK \rightarrow L$  requires a join

# Appendix: Redundancy in 3NF

- ◆ There is some redundancy in this schema
- ◆ Example of problems due to redundancy in 3NF

- ◆  $R = (J, K, L)$   
 $\mathcal{F} = \{JK \rightarrow L, L \rightarrow K\}$

$J$	$L$	$K$
$j_1$	$l_1$	$k_1$
$j_2$	$l_1$	$k_1$
$j_3$	$l_1$	$k_1$
$null$	$l_2$	$k_2$

- repetition of information (e.g., the relationship  $l_1, k_1$ )
  - $(i\_ID, dept\_name)$
- need to use null values (e.g., to represent the relationship  $l_2, k_2$  where there is no corresponding value for  $J$ ).
  - $(i\_ID, dept\_name)$  if there is no separate relation mapping instructors to departments