日期: Solution 1:

2.(b) ① $h_1(x) = \|Ax\|_2^2 - 1$,

$g_i(x) = -x_i \le 0$

$g_i(x)$ is convex, set $\|Ae\|_2^2 \triangleq l$, $e = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$

since $\det A \ne 0$, $e \ne 0$, then we have $l \ne 0$

$\|A\frac{e}{\sqrt{l}}\|_2^2 = 1$, and $\frac{e}{\sqrt{l}}$ satisfy $\frac{e}{\sqrt{l}} < 0$, it is slater point.

MFCQ holds ——

$Ax = b$ only, $\|Ax\|_2^2 = 1$ does not satisfy.

② solution 2:

consider $\mu \cdot \nabla h_1(x) + \sum_{i \in I(x)} \lambda_i \cdot \nabla g_i(x) = 0$, $\lambda_i \ge 0$

$I(x) = \{ i \mid g_i(x) = 0 \}$,

$\Rightarrow \mu \cdot 2B^TBx + \sum_{i \in I(x)} \lambda_i(-e_i) = 0$, $\lambda_i \ge 0$

set $e_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$, ---- $e_n = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$

$\Rightarrow 2\mu B^TBx + \sum_{i \in I} \lambda_i(-e_i) = 0$, $\lambda_i \ge 0$, $\lambda_i \cdot g_i(x) = 0$, that is $\lambda_i \cdot x_i = 0$.

$\Rightarrow 2\mu x^TB^TBx + x^T\left(\sum_{i \in I} \lambda_i(-e_i)\right) = 2\mu x^TB^TBx + \sum_{i \in I} -\lambda_i x_i = 0$

$\Rightarrow 2\mu x^TB^TBx = 0$, $B^TB \succ 0$, $x^TB^TBx > 0$, $x \ne 0$ since $\|Ax\|_2^2 = 1$.

$\Rightarrow \mu = 0$

$\Rightarrow \sum_{i \in I} \lambda_i(-e_i) = 0$, $\lambda_i \ge 0$

$\Rightarrow \lambda_i = 0$, $\forall i \in I$.

hence, MFCQ holds. □

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Primal

5. (1)

$$\begin{aligned} \text{Min} \quad & tr(CX) \\ \text{s.t.} \quad & tr(A_1 X) = 5 \\ & tr(A_2 X) = 1 \\ & X \succeq 0 \end{aligned}$$

$$A_1 := I, \quad A_2 := \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 1/2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad b = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

Dual:

$$\begin{aligned} \text{Max} \quad & b^T y \\ \text{s.t.} \quad & C - y_1 \cdot A_1 - y_2 \cdot A_2 \succeq 0 \end{aligned}$$

(3) $tr(CX^*) = 6$

$$\begin{pmatrix} 2 & 1 & & \\ 1 & 2 & & \\ & & 2 & 0 \\ & & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & & & \\ & 0 & 1/2 & \\ & 1/2 & 1 & \\ & & & 0 \end{pmatrix}$$

we consider dual problem:

$$\begin{pmatrix} 2-y_1 & 1 & 0 & 0 \\ 1 & 2-y_1 & -1/2 y_2 & 0 \\ 0 & -1/2 y_2 & 2-y_1-y_2 & 0 \\ 0 & 0 & 0 & 1-y_1 \end{pmatrix} \succeq 0,$$

$\Rightarrow 2-y_1-y_2 \geq 0, \quad 1-y_1 \geq 0.$

$\Rightarrow y_1 \leq 1, \quad y_1 + y_2 \leq 2.$

$b^T y = 5y_1 + y_2 \leq 4y_1 + (y_1 + y_2) \leq 6.$

but if and only if $y_1 = 1, \; y_1 + y_2 = 2, \; y_2 = 1$, it ahieve the upper bound 6.

$$C - y_1 A_1 - y_2 A_2 = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & -1/2 & 0 \\ 0 & -1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$ is not semi positive definite.

and we know v_p, v_d are attainable.

hence, $y_1 = 1, y_2 = 1$ is not the optimal solution of dual problem.

we know that $v_d < 6$, that means $v_p < 6$. X^* is not optimal solution.

$\square$

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