Lecture 1 Introduction to Data Structures

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Our Roadmap



- Programming basics
- How to solve a problem? by an algorithm?

• How to analyze the running time of an algorithm?

What are data structures?

Programming

This is not a programming course

- We will introduce some basic programming concepts (in the Java language)
 - so that you can do practices on computers

- Building blocks in a Java program:
 - \diamond Variables, constants, operators (e.g., x, y, 3, 5, +, =)
 - \rightarrow Expressions (e.g., x+y)
 - \rightarrow Statements (e.g., int x=3;)
 - → Methods (e.g., main)
 - → Classes (e.g., Test)
- A variable can be used to store a value
 - Primitive type (e.g., int, float, boolean) or object type (e.g., String)
- The execution starts at the main method

```
Write a program in the file "Test.java"

public class Test {
   public static void main(String args[]) {
     int x = 3;
     int y = 5;
     String msg = "hello";
     System.out.println(x+y);
     System.out.println(msg);
}
```

```
Compile a program

javac Test.java

Run a program

java Test

8
hello
```

Java: Array

- Array: for storing multiple variables (of the same type)
 - \diamond Create an array A with n items

$$int[] A = new int[n];$$

- A[i] denotes the variable at position i of array A
 - In Java, the range of *i* is from 0 to n-1

Examples

```
\Rightarrow int[] A = new int[7]; // create an array
\Rightarrow x = A[2]; // copy value A[2] to variable x
\Rightarrow A[3] = 5; // store value 5 to A[3]
```

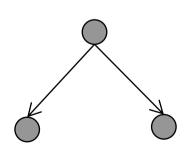
Position	0	1	2	3	4	5	6
Value	6	2	7	8	4	9	1
			1				

Control Flow

- Sequence (step-by-step)
 - A sequence of statements implies the execution ordering
 - Each statement is a 'step', e.g.,
 - Assignment statement: c = a * b;
 - Call statement:

```
System.out.println("hello");
```

- Selection (choose a branch)
 - if statement, switch statement
 - Example of if statement:
 - Evaluate the condition x>y
 - Condition is true \rightarrow execute v=x;
 - Condition is false \rightarrow execute v=y;



```
b = 5;

c = a * b;

d = a + b;
```

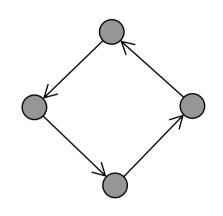
v=x;

else

int a,b,c,d;

Control Flow

Loop: repeat the execution of the loop body
 (a sequence of statements) for multiple times



Example: print "hello" 5 times

```
int i = 5;
while (i>0) {
    System.out.println("hello");
    i--;
}
```

System.out.print1n("hello");

for (int i=0; i<5; i++) {</pre>

- while-loop
 - Evaluate <cond> before the loop body; the loop stops when <cond> is false
- for-loop: counter-based
 - Execute <init> once before the loop starts '
 - Evaluate <cond> before the loop body;
 the loop stops when <cond> is false
 - Execute <update> after the loop body

Our Roadmap

Programming basics



• How to solve a problem? by an algorithm?

- How to analyze the running time of an algorithm?
- What are data structures?

How to solve a problem?

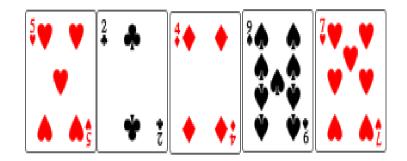
♠ A problem for human (in the real world):

Sort poker cards on hand

- Input:
 - A list of cards
- Procedure:

 - 2. **Pick** the smallest card from the left hand

 - 4. Repeat Steps 2-3 until the left hand is empty
- How to solve a problem by using a program?
 - Use variables, operators, statements,



Algorithms for Problem Solving

Input: a sequence *A* of *n* integers Output: a sequence A of n integers such that

Define a computational problem

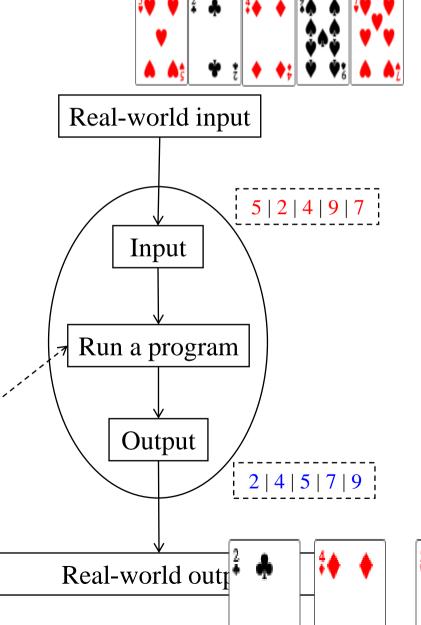
Selection-Sort (...)

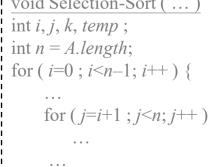
- 1. for integer $i \leftarrow 0$ to n-2
- $2. k \leftarrow i$
- for integer $i \leftarrow i+1$ to n-1
- if A[k] > A[j] then

Design an

algorithm

void Selection-Sort (...) Convert it into a program





Computational problem

- Example: the sorting problem
 - Input: a sequence A of n integers

- 5 | 2 | 4 | 9 | 7
- \diamond Output: a sequence A' of n integers such that
- 2 | 4 | 5 | 7 | 9

- A' is an *increasing* sequence, and
- A and A' contain the same integers.

Algorithms

- Algorithm: a well defined sequence of steps for solving a computational problem
 - It always produces the correct output
 - It uses well-defined steps or operations
 - It finishes in finite time
- It is written for human readers
 - Like programming, we can use variables and data structures, control flow, call statement
 - We can also use natural language provided that it is clear

```
Selection-Sort (Array A[0..n-1])
```

```
1<sub>₹</sub> for integer i \leftarrow 0 to n–2
```

- 2. $k \leftarrow i$
- 3. for integer $j \leftarrow i+1$ to n-1
- 4. if A[k] > A[j] then
- 5. $k \leftarrow j$
- -6:---> swap A[i] and A[k]

Algorithm vs. Program

- An algorithm can be implemented as a program in any language (e.g., C, C++, Java, Python)
- An algorithm is more readable than a program

<mark>Algorithm</mark>

```
Selection-Sort (Array A[0..n-1])
```

- 1. for integer $i \leftarrow 0$ to n-2
- 2. $k \leftarrow i$
- 3. for integer $j \leftarrow i+1$ to n-1
- 4. if A[k] > A[j] then
- 5. $k \leftarrow j$
- 6. swap A[i] and A[k]

Java program

```
// K.K.F. Yuen
void SelectionSort(int[] A){
int i, j, k, temp;
                        Or i <= n-2
int n = A.length;
    for (i = 0; i < n-1; i ++){
        k = i;
        for ( j=i+1 ; j<n; j++ ){
        if (A[k] > A[j] )
            k = i;
        temp = A[i];
        A[i] = A[k];
        A[k] = temp ;
                                       13
```

A trivial mistake is NOT trivial

Error result

```
void SelectionSort(int[] A){
    int i,j,k,temp;
    int n = A.length;
    for (i = 0; i < n-1; i++){
        k = i;
        for (j = i+1; j < n; j++){}
            if(A[k] > A[j])
                 k = j;
            temp = A[i];
            A[i] = A[k];
            A[k] = temp;
```

```
// K.K.F. Yuen
void SelectionSort(int[] A){
int i, j, k, temp;
int n = A.length;
   for (i = 0; i<n-1; i ++){
        k = i;
        for (j=i+1; j<n; j++){
        if (A[k] > A[j])
            k = j;

        temp = A[i];
        A[i] = A[k];
        A[k] = temp;
}
```

```
void SelectionSort(int[] A){
int i, j, k, temp;
int n = A.length;
   for (i = 0; i<n-1; i ++){
        k = i;

        for ( j=i+1 ; j<n; j++ )
        if ( A[k] > A[j] )
            k = j;|

        temp = A[i];
        A[i] = A[k];
        A[k] = temp;
}
```



```
Selection-Sort (Array A[0..n-1])
   1. for integer i \leftarrow 0 to n-2
                                                   Java
   2.
          k \leftarrow i
          for integer j \leftarrow i+1 to n-1
   3.
   4.
                if A[k] > A[j] then
   5.
                        k \leftarrow j
          swap A[i] and A[k]
   6.
                                             Python
                R
SelectionSort <- function(A)</pre>
    n = length(A)
    for (i in 1:(n-1)){
      k = i
         for (j in (i+1):(n))
              if (A[k] > A[j]){
                  k \leftarrow j
    # swap A[i] and A[k]
    temp = A[i]
    A[i] = A[k]
```

A[k] = temp

Α

```
void SelectionSort(int[] A){
int i, j, k, temp;
int n = A.length;
    for (i = 0; i < n-1; i ++){
        k = i;
        for ( j=i+1 ; j<n; j++ ){</pre>
        if (A[k] > A[j])
            k = j;
        temp = A[i];
        A[i] = A[k];
        A[k] = temp ;
```

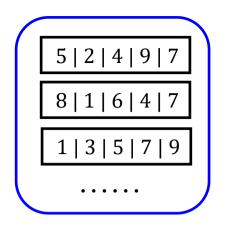
Algorithms: Running Steps

- Run an algorithm manually on a sample input, then draw running steps
 - E.g., show the content of important variables in each iteration
 - Useful for understanding how the algorithm works

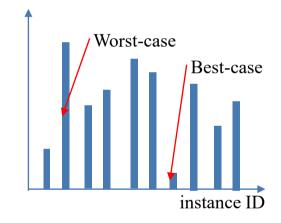
Selection-Sort (Array A[0..n-1]) 1. for integer $i \leftarrow 0$ to n-2 $k \leftarrow i$ 3. for integer $j \leftarrow i+1$ to n-1if A[k] > A[j] then 4. $k \leftarrow j$ 5. 6. swap A[i] and A[k]

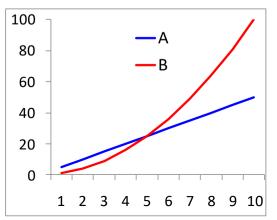
Algorithmic Analysis

- There are many possible input instances
- Is your algorithm always correct?
 - Not enough even if you have tested your algorithm on many instances
 - Will your algorithm fail on some other instance?
 - Need a correctness proof to show that:
 - "For every possible input instance, the algorithm produces the correct output"
- How fast is your algorithm?
 - Consider all instances of the same size n and their running time
 - Estimate the worst-case running time as a function of the input size n
 - What's the trend of the worst-case running time?



Running time





Our Roadmap

Programming basics

• How to solve a problem? by an algorithm?



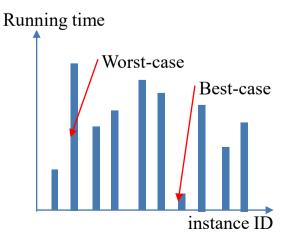
- How to analyze the running time of an algorithm?
- What are data structures?

Why do we analyze the running time of an algorithm?

- The same problem (e.g., sorting) can be solved by several different algorithms
- It is time consuming to convert all algorithms to programs
- We just want to choose the "fastest" algorithm and then convert it into a program
- Understand the scalability of a program.
 E.g., if the input size doubles, how much would its running time increase?

How to analyze the running time of an algorithm?

- Assumptions
 - #1. Each basic step takes constant time
 - * E.g., add two integers: c_{add} compare two numbers: $c_{compare}$ copy a value to a variable: c_{copy}
 - #2. The memory is large enough to store all input/intermediate/output data
- Find the (worst-case) frequency and cost of executing each line, then sum up the total cost



How to estimate running time?

Search (Array $A[0n-1]$, Key k)	<u>frequency</u>	
1. for integer $i \leftarrow 0$ to $n-1$	c_{I}	at most n
2. if $A[i] = k$	c_2	at most n
3. return <i>i</i>	c_3	at most 1
4. return −1		at most 1

- What is the frequency of executing Line 1?
 - \bullet It depends on which element A[i] is equal to k
 - Line 1 is executed at most n times
- What is the frequency of executing Line 2?
- Worst-case running time of the algorithm

$$= c_1 * n + c_2 * n + c_3 * 1 + c_4 * 1$$

= $(c_1 + c_2) n + c_3 + c_4$

- This expression contains many constants
- How to simplify it?

Asymptotic Notation

- How to describe the trend of running time in a concise way?
- Asymptotic upper-bound, O-notation
 - We can write f(n) = O(g(n)) if we can find positive constants c, n_0 such that

$$0 \le f(n) \le c \ g(n)$$
 for all $n \ge n_0$

• Is
$$(5n^2 + 3n) = O(n^2)$$
?

$$5n^2 + 3n$$

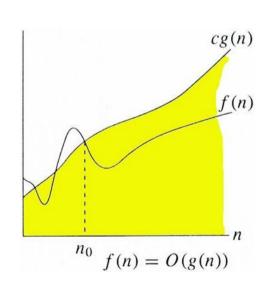
$$\leq 5n^2 + 3n^2 \quad \text{[true for all } n \geq 1\text{]}$$

$$\leq 8n^2$$

- \diamond YES, we choose c=8 and $n_0=1$.
- [*Note*: other proofs/choices are also possible]

• Is
$$(5n^2 + 3n) = O(n^3)$$
?

YES, but we prefer to use a tighter expression



Proof

- $f(n) = (5n^2 + 3n)$ and $g(n) = n^2$.
- To find $(5n^2 + 3n) = O(n^2)$, show $f(n) \le c g(n)$ for all $n \ge n_0$.
- We need to find c and n_0 .
- Since the coefficient of the highest order of n is 5, the minimal c is 6 if there is a valid n_0 . > n = n0 = 3
- Find the tight n, e.g., n_0 , such that $(5n^2 + 3n) \le 6n^2$, i.e., $3n \le n^2$.
- For $n \ge n_0 = 3$, we have $3n \le n^2$.

So, the order of $(5n^2 + 3n)$ is $O(n^2)$

```
> n = n0 = 3
> c = 6
> 5*n^2 + 3*n
[1] 54
> 6*n^2
[1] 54
> 5*n^2 + 3*n <= 6*n^2
[1] TRUE
> n = n0 = 2
> 5*n^2 + 3*n <= 6*n^2
[1] FALSE
> n = n0 = 4
> 5*n^2 + 3*n <= 6*n^2
[1] TRUE</pre>
```

Simplification Rules of O-Notation

Ignore constant factors

$$\bullet$$
 E.g., $3 = O(1)$

♦ E.g.,
$$3n = O(n)$$

These rules provide shortcuts for showing that f(n) = O(g(n))

Why?

 Combine fixed number of terms with same complexity $c_1 n + c_2 n = (c_1 + c_2) n$

♦ E.g.,
$$O(n) + O(n) = O(n)$$

♦ E.g.,
$$O(n) + O(n) + O(n) = O(n)$$

- Polynomial
 - Just use the highest-degree term.

$$\bullet$$
 E.g., $5n^2 + 3n = O(n^2)$

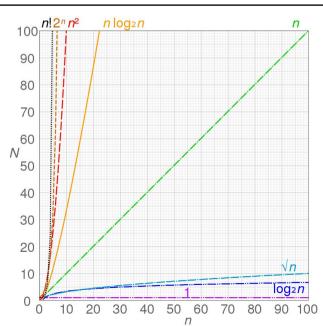
$$\bullet$$
 E.g., $2n^3 + 5n^2 + 3n = O(n^3)$

Asymptotic running time of some algorithms

Con	nplexity	Algorithm		
O(1)	Constant time	Compare two numbers		
$O(\log n)$	Logarithmic	Binary search (on a sorted array)		
O(n)	Linear time	Linear search (on an unsorted array)		
$O(n \log n)$		Merge-sort		
$O(n^2)$	Quadratic	Selection-sort		

The trend matters

- We care about the performance at large input size
- Constant factors (e.g., c) do not affect the order of growth



Asymptotic running time: Example 1

Search (Array A[0..n-1], Key k)

- 1. for integer $i \leftarrow 0$ to n-1
- 2. if A[i] = k
- 3. return i
- 4. return -1

5 | 2 | 4 | 9 | 7

- Let's analyze the asymptotic running time of this algorithm
 - \diamond Line 1: at most *n* basic steps = O(n)
 - Line 2: at most 1*n basic steps = O(n)
 - Lines 3 and 4: O(1)
- \bullet Running time of algorithm T(n)

$$= O(n) + O(n) + O(1)$$

$$= O(n)$$

Asymptotic running time: Example 2

Selection-Sort (Array A[0..n-1])

- 1. for integer $i \leftarrow 0$ to n-2
- 2. $k \leftarrow i$
- 3. for integer $j \leftarrow i+1$ to n-1
- 4. if A[k] > A[j] then
- 5. $k \leftarrow j$
- 6. swap A[i] and A[k]
- Line 1: n-1 basic steps = O(n)
- Line 2: 1*(n-1) basic steps = O(n)
- Lines 3 to 5:
 - Now many times can we execute Line 3 at most?
 - \bullet For each given value of *i*, what is the possible range of *j*?



5 | 2 | 4 | 9 | 7

Asymptotic running time: Example 2

- Worst-case analysis of Line 3:
 - \diamond Values of j in that loop: 1 to n-1, 2 to n-1, ..., n-1 to n-1
 - \diamond Number of iterations: n-1, n-2, n-3, ..., 1

$$\Sigma_{i=1..n-1} (n-i) = (n-1) + (n-2) + ... + 1$$
$$= (n-1)*n/2 = O(n^2)$$

- \diamond Running time of algorithm T(n):
 - ♦ Line 1.

- O(n)
- ♦ Lines 2, 6.
- same as Line 1

 $O(n^2)$

- \bullet Thus, we have: $T(n) = 3*O(n) + 3*O(n^2) = O(n^2)$

Formula:

$$\sum_{i=1}^{n} i = 1 + 2 + \ldots + n = \frac{n(n+1)}{2}$$

Our Roadmap

Programming basics

• How to solve a problem? by an algorithm?

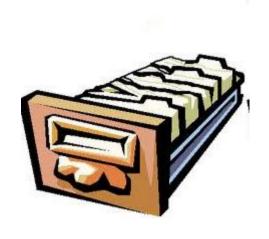
• How to analyze the running time of an algorithm?



What are data structures?

Data Structures

- What are the data structures for human?
 - Used in libraries, books, clinics, companies,



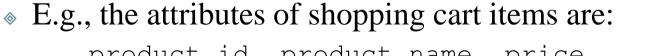
Oriental lampshades, 105-107 Patterns, how to make, 87, 135, 137 Piping, 120 Pleating, 99-104 Pricing your work, 152 Relining lampshades, 118 Rewiring lamps, 80-82 Roses, 126 Ruffles, how to make, 122-123 Scallops, 33, 85 Shampooing lampshades, 151 Shapes of lampshades, 31-41 Silhouettes of lamps, 21-30 Slipcovers for lamps, 108 Smocking, 96, 99 Spiders, different kinds, 31-32 Sunburst pleating, 100-102

How about the data structures for computers?

Data Objects

- Examples of data objects
 - Employees in payroll application
 - Tasks in the scheduler of OS
 - Shopping cart items in online shopping
 - In any application that you can think of





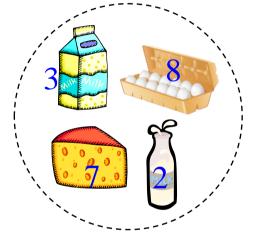
- product_id, product_name, price,
- The search key depends on the application, e.g.,
 - product_id for a shopping cart application





Data Structures

- Let S be a set of items, and x be a search key
 - A key is a number, e.g., product id
- Useful operations on a set S
 - Search(S, x): search whether x appears in S
 - Insert(S, x): insert item x into S
 - \bigcirc Delete(S, x): remove item x from S



set

Data structure:

- A way of organizing data objects for efficient usage
- Building blocks for designing algorithms

search key

The problems in Lectures 2-5

- How to solve these problems quickly?
 - Sort a set of items
 - Search an item from a set of items
 - Insert / Delete an item from a set of items
- Question 1
 - \diamond Selection-Sort takes $O(n^2)$ time
 - Is there a faster sorting algorithm?
- Question 2
 - Is an array good enough for searching, insertion and deletion?

Is Array Good Enough?



- Search a key in an unsorted array (of length n)
- Linear search algorithm
 - Running time: O(n)
- Example #1: search k=21

 - Return 0
- Example #2: search k=17
 - Check the cases i=0,1,2,3,4,5,6
 - Return 6
- How to search faster?

```
21 | 12 | 8 | 3 | 35 | 1 | 17
```

Search (Array A[0..n-1], Key k)

- 1. for integer $i \leftarrow 0$ to n-1
- 2. if A[i] = k
- 3. return i
- 4. return -1

Is Array Good Enough?



- Search in a sorted array (of length n)
- Binary search algorithm
 - Running time: O(log n)
- Idea:
 - Compare k with the middle item A[mid]
 - Is k in the left sub-array or in the right sub-array?

```
1 | 3 | 8 | 12 | 17 | 23 | 35
```

BinarySearch (Array A, Integers low, high, Key k)

- 1. $mid \leftarrow \lfloor (low + high)/2 \rfloor$
- 2. if A[mid] = k
- 3. return *mid*
- 4. else if low = high
- 5. return -1
- 6. if k < A[mid]
- 7. return BinarySearch(A, low, mid-1, k)
- 8. else
- 9. return BinarySearch(*A*, *mid*+1, *high*, *k*)

Initial call: BinarySearch (A, 0, n-1, k)

Example: find the key "35" from this array A

BinarySearch (Array A, Integers low, high, Key k)

1.
$$mid \leftarrow \lfloor (low+high)/2 \rfloor$$

2. if
$$A[mid] = k$$

- 3. return *mid*
- 4. else if low = high
- 5. return -1
- 6. if k < A[mid]
- 7. return BinarySearch(A, low, mid-1, k)
- 8. else
- 9. return BinarySearch(A, mid+1, high, k)

Initial call: BinarySearch $(A, 0, \frac{n-1}{k}, k)$

BinarySearch (
$$A$$
, 4, 6, 35), $mid = 5$ $\frac{1}{\sqrt{3}}$ 1 | 3 | 8 | 12 | 17 | 23 | 35

BinarySearch (
$$A$$
, 6 , 6 , 35), $mid = 6$ $1 \mid 3 \mid 8 \mid 12 \mid 17 \mid 23 \mid 35$

Key found, return 6

```
int i = 0; // for understanding recursive function only
public int binarySearch(int[] A, int low, int high, int k){
  //int mid = (int) Math.floor( ((double) low + (double) high) / 2.0);
    i = i + 1:
    System.out.println("call " + i); // tracking
    int mid = (low + high) / 2;
    System.out.println("mid position is " + mid); // tracking
    System.out.println("value is " + A[mid]); // tracking
    if(A[mid] == k) return mid;
    else if (low == high) return -1;
    if(k < A[mid]) return binarySearch(A, low, mid-1, k);</pre>
    else return binarySearch(A, mid+1, high, k);
    }
                      int[] A = \{1,3,8,12,17,23,35\};
                      binarySearch(A, 0, A.length - 1, 35);
                      call 1
                      mid position is 3
                      value is 12
                      call 2
                      mid position is 5
                      value is 23
                      call 3
                      mid position is 6
                      value is 35
                      6
```

Is Array Good Enough?

How about insertion/deletion?





Can an unsorted array support fast insertion/deletion?

21 | 12 | 8 | 3 | 35 | 1 | 17

Can a sorted array support fast insertion/deletion?

1 | 3 | 8 | 12 | 17 | 23 | 35

Data Structures in Lectures 2-5

- Learn the **properties** of data structures
 - E.g., stack's LIFO property, queue's FIFO property, growable structure like linked list
- Learn the operations of data structures
 - They support different operations, and with different running time
- You may play with them in https://visualgo.net/en
- What's the most suitable data structure to solve a particular problem?
 - It depends on the properties and operations most frequently used in your algorithm

Stack (Lecture 2) Queue (Lecture 2) Linked List (Lecture 2) Tree (Lecture 3) Heap (Lecture 4)

Java Programmers

At least know how to use the data structures and algorithms in the java.util library

(https://docs.oracle.com/javase/8/docs/api/java/util/package-summary.html)

- Stack, Queue, LinkedList, PriorityQueue,
- Arrays.sort, Arrays.binarySearch,

• We hope you can also understand the "magic" behind them

Summary

 Basic concepts in Java, algorithm, asymptotic running time, data structure

Please read Chapters 1, 4.2.1-4.2.6 in the book "Data Structures and Algorithms in Java"

Next lecture:

linear data structures (stack, queue, linked list)