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1. (a) For each of the following optimization problems, write a CVX code that solves it, if possible.

Also write down the optimal value returned by CVX (corrected to 4 decimal places).

i. (10 points)

$$\begin{aligned} \text{Minimize} \quad & \sqrt{x_1^2 + (x_2 - 5)^4 + (x_3 - 1)^6 + (x_4 + 1)^8 + 2} + |x_3 - x_4 + 7| \\ \text{Subject to} \quad & x_1^2 + x_2^2 + x_4^2 \leq 2. \end{aligned}$$

ii. (10 points)

$$\begin{aligned} \text{Minimize} \quad & x_1^2 + 8x_2^2 + 9x_3^2 + 5x_1x_2 - x_1x_3 + 8(|x_1 - 1| + |x_2 + 3| + |x_3 - 5|) \\ \text{Subject to} \quad & \begin{bmatrix} 5 & x_2^2 \\ x_3^2 & x_2 + 1 \end{bmatrix} \succeq 0. \end{aligned}$$

iii. (10 points)

$$\begin{aligned} \text{Minimize} \quad & 2x_1 + 3x_2 - x_3 + \sqrt{x_1^2 + (x_2 - 5)^2 + 6x_3^2 - 4x_1x_3 + 1} \\ \text{Subject to} \quad & \max\{x_1 + x_2, x_3 + x_2, x_1 + x_3\} \leq 2, \quad x_1 \geq 0, \quad x_2 \geq 0. \end{aligned}$$

1. (a) i) reformulate : Min $y + |x_3 - x_4 + 7|$
 s.t. $y^2 \geq x_1^2 + (x_2 - 5)^4 + (x_3 - 1)^6 + (x_4 + 1)^8 + 2, \quad y \geq 0$
 $x_1^2 + x_2^2 + x_4^2 \leq 2$

\Leftrightarrow Min $y + |x_3 - x_4 + 7|$
 s.t. $z_2 \geq (x_2 - 5)^2, \quad z_3 \geq |x_3 - 1|^3, \quad z_4 \geq (x_4 + 1)^2,$
 $z_5 \geq z_4^2, \quad y \geq 0$
 $y^2 \geq x_1^2 + z_2^2 + z_3^2 + z_5^2 + 2$
 $x_1^2 + x_2^2 + x_4^2 \leq 2$

\Leftrightarrow Min $y + |x_3 - x_4 + 7|$
 s.t. $z_2 \geq (x_2 - 5)^2, \quad p_1 \geq |x_3 - 1|, \quad p_1 \geq 0,$
 $z_3 \cdot p_1 \geq p_2^2, \quad p_2 \geq p_1^2, \quad z_3 \geq 0,$
 $z_4 \geq (x_4 + 1)^2, \quad z_5 \geq z_4^2, \quad y \geq 0, \quad y^2 \geq x_1^2 + z_2^2 + z_3^2 + z_5^2 + 2$
 $x_1^2 + x_2^2 + x_4^2 \leq 2.$

\Leftrightarrow Min $y + |x_3 - x_4 + 7|$
 s.t. $\begin{pmatrix} z_2 & x_2 - 5 \\ x_2 - 5 & 1 \end{pmatrix} \succeq 0, \quad p_1 \geq |x_3 - 1|, \quad p_1 \geq 0$
 $\begin{pmatrix} z_3 & p_2 \\ p_2 & p_1 \end{pmatrix} \succeq 0, \quad \begin{pmatrix} p_2 & p_1 \\ p_1 & 1 \end{pmatrix} \succeq 0, \quad \begin{pmatrix} z_4 & (x_4 + 1) \\ (x_4 + 1) & 1 \end{pmatrix} \succeq 0$
 $\begin{pmatrix} z_5 & z_4 \\ z_4 & 1 \end{pmatrix} \succeq 0, \quad y \geq 0, \quad \begin{pmatrix} y & x_1 & z_2 & z_3 & z_5 & \sqrt{2} \\ x_1 & z_2 & z_3 & z_5 & \sqrt{2} & y \cdot 15 \end{pmatrix} \succeq 0$
 $x_1^2 + x_2^2 + x_4^2 \leq 2. \quad \text{That is CVX format.}$

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(ii) \Leftrightarrow Min (x_1, x_2, x_3) $A := \begin{pmatrix} 1 & 5/2 & -1/2 \\ 5/2 & 8 & 0 \\ -1/2 & 0 & 9 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + 8(|x_1 - 1| + |x_2 + 3| + |x_3 - 5|)$

s.t. $5(x_2 + 1) - x_3^2 \geq 0, x_2 + 1 \geq 0$

\Leftrightarrow Min (x_1, x_2, x_3) $A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + 8(|x_1 - 1| + |x_2 + 3| + |x_3 - 5|)$

s.t.

$$\begin{pmatrix} 5 & y \\ y & x_2 + 1 \end{pmatrix} \geq 0, \begin{pmatrix} y & x_3 \\ x_3 & 1 \end{pmatrix} \geq 0$$

(iii) \Leftrightarrow Min $2x_1 + 3x_2 - x_3 + \sqrt{\frac{1}{3}x_1^2 + (x_2 - 5)^2 + 6(x_3 - \frac{1}{3}x_1)^2 + 1}$

s.t. $\max\{x_1 + x_2, x_3 + x_2, x_1 + x_3\} \leq 2, x_1 \geq 0, x_2 \geq 0,$
 $x_1 + x_2 \leq 2, x_3 + x_2 \leq 2, x_1 + x_3 \leq 2.$

result is on next page.

1.

```
1 cvx_solver sdpt3
2 % 1
3 cvx_begin
4     variables x(4) y z(5) p(2)
5     minimize y + abs(x(3)-x(4)+7)
6     subject to
7         [z(2), x(2)-5; x(2)-5, 1] == semidefinite(2);
8         p(1) >= abs(x(3) - 1);
9         p(1) >= 0;
10        [z(3), p(2); p(2), p(1)] == semidefinite(2);
11        [p(2), p(1); p(1), 1] == semidefinite(2);
12        [z(4), x(4)+1; x(4)+1, 1] == semidefinite(2);
13        [z(5), z(4); z(4), 1] == semidefinite(2);
14        y >= 0;
15        [[y, x(1), z(2), z(3), z(5), sqrt(2)]; [x(1); z(2); z(3);
16          z(5); sqrt(2)], eye(5)*y]] == semidefinite(6);
17        x(1)^2 + x(2)^2 + x(4)^2 <= 2;
18 cvx_end
```

Optimal value (cvx_optval): +19.8227

2.

```
1 % 2
2 A = [1, 5/2, -1/2; 5/2, 8, 0; -1/2, 0, 9];
3 cvx_begin
4     variables x(3) y
5     minimize quad_form(x, A) + ...
6         8*(abs(x(1)-1) + abs(x(2)+3) + abs(x(3)-5))
7     subject to
8         [5, y; y, x(2)+1] == semidefinite(2);
9         [y, x(3); x(3), 1] == semidefinite(2);
10    cvx_end
```

Optimal value (cvx_optval): +57.4688

3.

```
1 % 3
2 cvx_begin
3     variables x(3)
4     minimize 2*x(1) + 3*x(2) - x(3) + ...
5         norm([1/sqrt(3)*x(1), x(2)-5, sqrt(6)*(x(3)-1/3*x(1))), 1],
6
7         subject to
8             x(1)+x(2)<=2;
9             x(3)+x(2)<=2;
10            x(1)+x(3)<=2;
11            x(1)>=0;
12            x(2)>=0;
13 cvx_end
```

Optimal value (cvx_optval): +4.65475

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- (b) Explain whether the following optimization problems can be reformulated equivalently as an SDP problem.

i. (10 points)

$$\begin{aligned} & \text{Minimize } |x_1 + x_2 + x_3 - 8| + \sqrt{x_1^2 + x_2^2 + x_3^2} \\ & \text{Subject to } x_1^2 + 5x_2^4 + 4x_1x_2^2 + x_3^2 \leq 1, \quad x_1 \geq 1. \end{aligned}$$

ii. (10 points)

$$\begin{aligned} & \text{Minimize } \max\{x_1 + x_2^2 + 3, \sqrt{x_1^2 + x_2^2 + 3}\} + |x_1 + x_3 - 8| \\ & \text{Subject to } x_1^3 + x_2^4 + x_3 \leq 5, \quad x_1 \geq 0. \end{aligned}$$

$$(b) \quad \text{(i)} \iff \begin{aligned} & \text{Min } z_1 + z_2 \\ & \text{s.t. } y \geq x_1^2, \quad y \geq 0 \end{aligned}$$

$$x_1^2 + 5y^2 + 4x_1y + x_3^2 \leq 1, \quad x_1 \geq 1$$

$$z_1 \geq |x_1 + x_2 + x_3 - 8|$$

$$z_2 \geq \sqrt{x_1^2 + x_2^2 + x_3^2}$$

$$\iff \begin{aligned} & \text{Min } z_1 + z_2 \\ & \text{s.t. } y \geq x_1^2, \quad y \geq 0 \end{aligned}$$

$$(x_1 \quad y \quad x_3) \begin{pmatrix} 1 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ y \\ x_3 \end{pmatrix} \leq 1, \quad x_1 \geq 1$$

$$z_1 \geq |x_1 + x_2 + x_3 - 8|, \quad z_1 \geq 0$$

$$z_2 \geq x_1^2 + x_2^2 + x_3^2, \quad z_2 \geq 0$$

$$\iff \begin{aligned} & \text{Min } z_1 + z_2 \\ & \text{s.t. } \end{aligned}$$

$$\left(\begin{array}{ccc} 5 & -2 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right) \begin{pmatrix} x_1 \\ y \\ x_3 \end{pmatrix} \geq 0, \quad \begin{pmatrix} x_1 & y & x_3 & 1 \end{pmatrix} \geq 0, \quad x_1 \geq 1$$

$$z_1 \geq x_1 + x_2 + x_3 - 8 \geq -z_1$$

$$\begin{pmatrix} z_2 & x_1 & x_2 & x_3 \\ x_1 & & & \\ x_2 & & z_2 \cdot I_3 & \\ x_3 & & & \end{pmatrix} \geq 0.$$

$$(ii) \iff \begin{aligned} & \text{Min } \max\{y, z\} + p \\ & \text{s.t. } \end{aligned}$$

$$y \geq x_1 + x_2^2 + 3, \quad x_1 \geq 0, \quad y - x_1 \geq 0$$

$$z \geq \sqrt{x_1^2 + x_2^2 + 3}, \quad z \geq 0$$

$$p \geq |x_1 + x_3 - 8|$$

$$x_1^3 + x_2^4 + x_3 \leq 5$$

$$\iff \begin{aligned} & \text{Min } \frac{1}{2}(y + z + |y - z|) + p \\ & \text{s.t. } \end{aligned}$$

$$\begin{pmatrix} y - x_1 & x_2 & \sqrt{3} \\ x_2 & I_2 & \end{pmatrix} \geq 0, \quad x_1 \geq 0,$$

日期: / $\begin{pmatrix} z & x_1 & x_2 & 3 \\ x_1 & x_2 & x_3 & 0 \\ x_2 & x_3 & z \cdot I_3 & 0 \\ 3 & & & \end{pmatrix} \geq 0$

$$p \geq x_1 + x_3 - 8 \geq -p$$

$$x_1^3 + x_2^4 + x_3 \leq 5$$

from the discussion of (i), we know objective function
can be reformulated to SDP,

and now we discuss constraints

$$\begin{cases} x_1^3 + x_2^4 + x_3 \leq 5 \\ x_1 \geq 0 \end{cases}$$

it's equivalent to: $\Leftrightarrow: \begin{cases} x_2^2 \leq p_1, p_1^2 \leq p_2 \\ x_1^3 + p_2 + x_3 \leq 5, x_1 \geq 0 \end{cases}$

$$\Leftrightarrow: \begin{cases} x_2^2 \leq p_1, p_1^2 \leq p_2 \\ x_1^2 \leq q_1, \\ q_1^2 \leq x_1 \cdot (5 - p_2 - x_3), x_1 \geq 0, (5 - p_2 - x_3) \geq 0 \end{cases}$$

$$\Leftrightarrow: \begin{cases} \begin{pmatrix} p_1 & x_2 \\ x_2 & 1 \end{pmatrix} \geq 0, \begin{pmatrix} p_2 & p_1 \\ p_1 & 1 \end{pmatrix} \geq 0, \begin{pmatrix} q_1 & x_1 \\ x_1 & 1 \end{pmatrix} \geq 0 \\ \begin{pmatrix} x_1 & q_1 \\ q_1 & (5 - p_2 - x_3) \end{pmatrix} \geq 0 \end{cases}$$

Hence, can be reformulated to

$$\text{Min } \frac{1}{2}(y+z+c) + p$$

$$\text{s.t. } c \geq y - z \geq -c$$

$$\begin{pmatrix} y-x_1 & x_2 & \sqrt{3} \\ x_2 & I_2 & 0 \\ \sqrt{3} & 0 & \end{pmatrix} \geq 0, \quad \begin{pmatrix} z & x_1 & x_2 & 3 \\ x_1 & x_2 & x_3 & 0 \\ x_2 & x_3 & z \cdot I_3 & 0 \\ 3 & & & \end{pmatrix} \geq 0, \quad p \geq x_1 + x_3 - 8 \geq -p$$

$$\begin{pmatrix} p_1 & x_2 \\ x_2 & 1 \end{pmatrix} \geq 0, \quad \begin{pmatrix} p_2 & p_1 \\ p_1 & 1 \end{pmatrix} \geq 0, \quad \begin{pmatrix} q_1 & x_1 \\ x_1 & 1 \end{pmatrix} \geq 0$$

$$\begin{pmatrix} x_1 & q_1 \\ q_1 & (5 - p_2 - x_3) \end{pmatrix} \geq 0 \quad \square$$

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2. Consider the following optimization problem.

$$\begin{array}{ll}\text{Minimize}_{x \in \mathbb{R}^2} & -x_1^3 + 2x_2^2 \\ \text{Subject to} & x_1^2 + x_2^2 \leq 5, \\ & x_2 \geq x_1^2 + 2.\end{array}$$

- (a) (5 points) Show that the MFCQ holds at every feasible point.
(b) (25 points) Write down the KKT conditions and find all global minimizers.

assume

$$2. \quad (a) \quad g_1 := x_1^2 + x_2^2 - 5 \leq 0,$$

$$g_2 := x_1^2 - x_2 + 2 \leq 0,$$

$$\nabla^2 g_1 = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \geq 0, \quad \nabla^2 g_2 = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} \geq 0$$

we find a slater point, $x = \begin{pmatrix} 0 \\ 2.1 \end{pmatrix}$ satisfy $g_1(x) < 0, g_2(x) < 0$.

Hence MFCQ holds on feasible region.

$$(b) \quad \text{Min } f(x) := -x_1^3 + 2x_2^2$$

$$\text{s.t. } g_1 := x_1^2 + x_2^2 - 5 \leq 0$$

$$g_2 := x_1^2 - x_2 + 2 \leq 0$$

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KKT conditions: ① $g_1(x) \leq 0, g_2(x) \leq 0$

② $\nabla f(x) + \lambda_1 \cdot \nabla g_1(x) + \lambda_2 \cdot \nabla g_2(x) = 0, \lambda_1 \geq 0, \lambda_2 \geq 0$

③ $\lambda_1 \cdot g_1(x) = 0, \lambda_2 \cdot g_2(x) = 0$

$$\textcircled{2} \Rightarrow \begin{pmatrix} -3x_1^2 \\ 4x_2 \end{pmatrix} + \lambda_1 \begin{pmatrix} 2x_1 \\ 2x_2 \end{pmatrix} + \lambda_2 \begin{pmatrix} 2x_1 \\ -1 \end{pmatrix} = 0, \lambda \geq 0$$

if $\lambda_1 = 0, \lambda_2 = 0$, we have $(x_1, x_2) = (0, 0)$

$g_2(x) \leq 0$ is not satisfied.

if $\lambda_1 = 0, \lambda_2 \neq 0$, we have $\begin{pmatrix} -3x_1^2 \\ 4x_2 \end{pmatrix} + \lambda_2 \begin{pmatrix} 2x_1 \\ -1 \end{pmatrix} = 0, x_1^2 - x_2 + 2 = 0$

we get solution $(x_1, x_2) = (0, 2), \lambda_2 = 8, g_1 \leq 0, g_2 \leq 0$ satisfy.

$(x_1, x_2) = (0, 2)$ is stationary point.

if $\lambda_1 \neq 0, \lambda_2 = 0$, we have $\begin{pmatrix} -3x_1^2 \\ 4x_2 \end{pmatrix} + \lambda_1 \begin{pmatrix} 2x_1 \\ 2x_2 \end{pmatrix} = 0, x_1^2 + x_2^2 = 5$

$4x_2 + 2\lambda_1 x_2 = 0$, we get $x_2 = 0$ or $\lambda_1 = -2$.

if $x_2 = 0$, we have $x_1 = \pm\sqrt{5}$, but $g_2(x) \leq 0$ is not satisfied.

if $\lambda_1 = -2$, it's contradictory with $\lambda \geq 0$.

if $\lambda_1 \neq 0, \lambda_2 \neq 0$, we have $x_1^2 + x_2^2 = 5, x_2 = x_1^2 + 2$

we have solution $x_1^2 = \frac{-5 + \sqrt{29}}{2}, x_2 = \frac{-1 + \sqrt{29}}{2}$

$\begin{pmatrix} 2x_1 & 2x_1 \\ 2x_2 & -1 \end{pmatrix}$ is full rank, so $\left(\pm \sqrt{\frac{-5 + \sqrt{29}}{2}}, \frac{-1 + \sqrt{29}}{2} \right)$ is stationary points.

and $\nabla f(x) + \lambda_1 \nabla g_1(x) + \lambda_2 \nabla g_2(x) = \begin{pmatrix} -3x_1^2 + \lambda_1 2x_1 + \lambda_2 \cdot 2x_1 \\ 4x_1 + \lambda_1 2x_2 - \lambda_2 \end{pmatrix} = 0$.

日期 and we don't need to test if it satisfies $\lambda_1, \lambda_2 \geq 0$,
 since if $x = (\pm \sqrt{\frac{-5+\sqrt{89}}{2}}, \frac{-1+\sqrt{89}}{2})$, $f(x)$ is greater than $f(0, 2)$, they cannot be global minimizer.
 we find the minimal point is $(0, 2)$, $\min f(x) = 8$.
 $(0, 2)$ is global minimizer. \square

3. Consider the following optimization problem, where n is an integer greater than 2024:

$$\underset{x \in \mathbb{R}^n}{\text{Minimize}} \quad \frac{1}{2} \sum_{i=1}^n (x_i - i)^2$$

$$\text{Subject to } 1 + \sum_{i=1}^n ix_i \leq 0.$$

You may leave the term S_n in your answer, where $S_n := \sum_{i=1}^n i^2$.

(a) (10 points) For each $c > 0$, define

$$q_c(x) := \frac{1}{2} \sum_{i=1}^n (x_i - i)^2 + \frac{c}{2} \left(1 + \sum_{i=1}^n ix_i \right)_+^2$$

Argue that q_c is convex and find the global minimizer of q_c .

3. (a) ① $\nabla q_c(x) = \begin{pmatrix} x_1 - 1 \\ \vdots \\ x_n - n \end{pmatrix} + c \cdot \left(1 + \sum_{i=1}^n ix_i \right)_+ \cdot \begin{pmatrix} 1 \\ 2 \\ \vdots \\ n \end{pmatrix}$

since $\frac{\partial^2}{\partial x_i^2} (x_i - i)^2 = 2 > 0$, we know $(x_i - i)^2$ is convex,

so $\frac{1}{2} \sum_{i=1}^n (x_i - i)^2$ is convex.

assume that $y := 1 + \sum_{i=1}^n ix_i$, y is linear affine of x .

and y^2 is convex, \Rightarrow we have $\sum_{i=1}^n (1 + \sum_{i=1}^n ix_i)_+^2$, $c > 0$ is convex

Hence, $q_c(x)$ is convex.

Lemma: $g(y) = y^2$ is convex.

Proof: $\forall \lambda \in (0, 1)$, if $y_1, y_2 < 0$, we have

$$\lambda \cdot g(y_1) + (1-\lambda) g(y_2) = g(\lambda y_1 + (1-\lambda) y_2) = 0$$

if $y_1, y_2 \geq 0$, since y^2 is convex, the formula above still holds.

if $y_1 > 0$, and $y_2 < 0$, $\lambda g(y_1) + (1-\lambda) g(y_2) = \lambda g(y_1) = \lambda y_1^2$

$$g(\lambda y_1 + (1-\lambda) y_2) \leq g(\lambda y_1) = \lambda^2 y_1^2 \leq \lambda y_1^2.$$

Hence $g(y) = y^2$ is convex.

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$$\textcircled{2} \quad \nabla q_c(x) = \begin{pmatrix} x_1 - 1 \\ \vdots \\ x_i - i \\ \vdots \\ x_n - n \end{pmatrix} + c \cdot \left(1 + \sum_{i=1}^n i x_i \right)_+ \cdot \begin{pmatrix} 1 \\ 2 \\ \vdots \\ i \\ \vdots \\ n \end{pmatrix} = 0$$

we have $x_i - i + c \cdot \left(1 + \sum_{i=1}^n i x_i \right)_+ \cdot i = 0$, for $\forall i = 1, 2, \dots, n$

$$x_i = i \left[1 - c \cdot \left(1 + \sum_{i=1}^n i x_i \right)_+ \right]$$

if $1 + \sum_{i=1}^n i x_i < 0$, we have $x_i = i$,

$$1 + \sum_{i=1}^n i x_i = 1 + S_n > 0, \text{ it's contradiction.}$$

if $1 + \sum_{i=1}^n i x_i \geq 0$, define $z := 1 - c \cdot \left(1 + \sum_{i=1}^n i x_i \right)_+$, $y = \sum_{i=1}^n i x_i$

$$\sum_{i=1}^n i x_i = \sum_{i=1}^n i^2 \cdot z, \text{ that } \exists y = S_n \cdot (1 - c(1+y))$$

$$y = \frac{S_n(1-c)}{1+cS_n}, \quad z = \frac{1-c}{1+cS_n}$$

$$1 + \sum_{i=1}^n i x_i = \frac{1+S_n}{1+cS_n} \geq 0, \text{ satisfy the assumption.}$$

we get $x_i = i \cdot \frac{1-c}{1+cS_n}$, for $i = 1, 2, \dots, n$,

is the global minimizer, since convexity.

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(b) (10 points) For each $\mu > 0$, define

$$f_\mu(x) := \frac{1}{2} \sum_{i=1}^n (x_i - i)^2 - \mu \ln \left(-1 - \sum_{i=1}^n i x_i \right).$$

Argue that f_μ is convex and find the global minimizer of f_μ . You may use without proof the fact that the function $t \mapsto -\ln(-1-t)$ is convex (as an extended real-valued function).

(b) ① $(x_i - i)^2$ is convex, since $\frac{\partial^2}{\partial x_i^2} (x_i - i)^2 = 2 > 0$.

so $\frac{1}{2} \sum_{i=1}^n (x_i - i)^2$ is convex.

assume $t := \sum_{i=1}^n i x_i$, t is affine projection of x .

and $\mu > 0$, $-\mu \ln(-1-t)$ is convex.

it implies $-\mu \ln(-1 - \sum_{i=1}^n i x_i)$ is convex.

Hence, $f_\mu(x)$ is convex.

$$\textcircled{2} \quad \nabla f_\mu(x) = \begin{pmatrix} x_1 - 1 \\ \vdots \\ x_i - i \\ \vdots \\ x_n - n \end{pmatrix} - \frac{\mu}{1 + \sum_{i=1}^n i x_i} \cdot \begin{pmatrix} 1 \\ 2 \\ \vdots \\ i \\ \vdots \\ n \end{pmatrix} = 0$$

we have $x_i - i - \frac{\mu}{1 + \sum_{i=1}^n i x_i} \cdot i = 0$ for $i = 1, 2, \dots, n$.

assume that $y := \sum_{i=1}^n i x_i$,

$$x_i = i + \frac{\mu}{1+y} \cdot i = i \left(1 + \frac{\mu}{1+y} \right)$$

$$y = \sum_{i=1}^n i x_i = s_n \cdot \left(1 + \frac{\mu}{1+y} \right), \Rightarrow y = \frac{s_n - 1 \pm \sqrt{(s_n - 1)^2 + 4(1+\mu)s_n}}{2}$$

$$x_i = i \left(1 + \frac{\mu}{1+y} \right), \forall i = 1, 2, \dots, n, \text{ and } y < -1$$

we get $\boxed{y = \frac{s_n - 1 - \sqrt{(s_n - 1)^2 + 4(1+\mu)s_n}}{2} < -1}$ holds for $\forall \mu > 0$.

the global minimizer is $x_i = i + \frac{\mu}{1+y} \cdot i$, for $i = 1, 2, \dots, n$. \square

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