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1. Let $X_1, \ldots, X_n$ be a random sample from a geometric distribution that has pmf $f(x|\theta) = (1-\theta)^x\theta$, $x = 0, 1, 2 \ldots$, $0 < \theta < 1$, zero elsewhere. Show that $\sum_{i=1}^n X_i$ is a sufficient statistic for θ.

1. $f(x|\theta) = \exp\{x \cdot \log(1-\theta) + \log\theta\}$ is exponential class

hence Σx_i is sufficient statistics. $\square$

2. Let $X_1, \ldots, X_n$ be a random sample from a Beta$(\theta, 5)$. Show that the product $X_1 \times \cdots \times X_n$ is a sufficient statistic for θ.

2. $f(x|\theta,5) = \dfrac{x^{\theta-1}(1-x)^4}{B(\theta,5)}$

$L(\theta,5) = (\prod x_i)^{\theta-1} \cdot \prod \dfrac{(1-x_i)^4}{B(\theta,5)}$

$\prod \dfrac{(1-x_i)^4}{B(\theta,5)}$ is irrelevant with θ,

hence $\prod x_i$ is sufficient statistics. $\square$

3. Write the pdf

$$f(x|\theta) = \frac{1}{6\theta^4}x^3e^{-x/\theta}, \quad 0 < x < \infty,\ 0 < \theta < \infty,$$

zero elsewhere, in the exponential form. If $X_1, X_2, \ldots, X_n$ is a random sample from this distribution, find a complete sufficient statistic Y_1 for θ and the unique function $\phi(Y_1)$ of this statistic that is the MVUE of θ. Is $\phi(Y_1)$ itself a complete sufficient statistic?

3. ① X is exponential form,

$f(x|\theta) = \exp\{-\log(6\theta^4) + 3\log x - \frac{x}{\theta}\}$

$\Rightarrow \sum_{i=1}^n x_i$ is complete sufficient statistics, $Y_1 = \sum_{i=1}^n x_i$.

② $E(x) = \int_0^{+\infty} \frac{1}{6\theta^4} x^4 \cdot e^{-x/\theta} dx$

$= \int_0^{+\infty} -\frac{1}{6\theta^3} x^4 d e^{-x/\theta} = \int_0^{+\infty} \frac{4}{6\theta^3} x^3 e^{-x/\theta} dx$

$= \int_0^{+\infty} \frac{4 \times 3 \times 2 \times 1}{6} e^{-x/\theta} dx = 4\theta$

$E(\frac{1}{4n}\sum_{i=1}^{n} X_i) = \theta$, is unbiased estimator,

$\phi(T_1) = E(\frac{1}{4n}\sum_{i=1}^{n} X_i \mid \sum_{i=1}^{n} X_i) = \frac{1}{4}\bar{X}$,

$\frac{1}{4}\bar{X}$ is unique MVUE.

③ since $T_1 = \sum_{i=1}^{n} X_i$ is complete sufficient statistics,

$\frac{1}{4}\bar{X}$ is complete sufficient statistics as well. □

4. Let $X_1, \ldots, X_n$ be a random sample from the density function $f(x|\theta) = \frac{1}{\theta}e^{x/\theta}$, $x < 0$, $\theta > 0$. Determine the MVUE of θ.

4.

$f(x|\theta) = \exp\{-\log\theta + \frac{x}{\theta}\}$

so $\sum_{i=1}^{n} x_i$ is complete sufficient statistics.

$E(X) = -\theta$, $E(\bar{X}) = -\theta$.

$-\bar{X}$ is unbiased estimator of θ,

$\phi(\bar{X}) = E(-\bar{X} \mid \bar{X}) = -\bar{X}$ is the unique MVUE for θ. □

5. Let $X_1, \ldots, X_n$ be a random sample from a uniform distribution $[0, \theta]$, for some unknown parameter $\theta > 0$. Is $X_{(n)}$, the largest order statistic among all samples, a sufficient statistic for θ?

5. $X_{(n)} = \max\{X_1, \cdots, X_n\}$

$$P(X_{(n)} \le t) = P(X_1 \le t) \cdot P(X_2 \le t) \cdots P(X_n \le t)$$
$$= (\frac{t}{\theta})^n \quad , \text{for } 0 \le t \le \theta.$$

$$f_{X_{(n)}}(t|\theta) = n \cdot \frac{t^{n-1}}{\theta^n}, \quad 0 \le t \le \theta.$$

$$\frac{f(x_1|\theta) \cdots f(x_n|\theta)}{f_{X_{(n)}}(t|\theta)} = \frac{(\frac{1}{\theta})^n}{n \cdot \frac{t^{n-1}}{\theta^n}} = \frac{1}{n \cdot t^{n-1}}$$ is independent with θ,

hence $X_{(n)}$ is sufficient. □

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