

Lecture 1

Introduction to Data Structures

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Some parts has been revised and indicated.

Our Roadmap



- ◆ Programming basics
- ◆ How to solve a problem? by an algorithm?
- ◆ How to analyze the running time of an algorithm?
- ◆ What are data structures?

Programming

- ◆ This is not a programming course
- ◆ We will introduce some basic programming concepts (in the Java language)
 - ◆ so that you can do practices on computers

- ◆ Building blocks in a Java program:
 - ◆ Variables, constants, operators (e.g., `x`, `y`, `3`, `5`, `+`, `=`)
 - Expressions (e.g., `x+y`)
 - Statements (e.g., `int x=3;`)
 - Methods (e.g., `main`)
 - Classes (e.g., `Test`)
- ◆ A *variable* can be used to store a value
 - ◆ Primitive type (e.g., `int`, `float`, `boolean`) or object type (e.g., `String`)
- ◆ The execution starts at the *main* method

Write a program in the file "Test.java"

```
public class Test {  
    public static void main(String args[]) {  
        int x = 3;  
        int y = 5;  
        String msg = "hello";  
        System.out.println( x+y );  
        System.out.println( msg );  
    }  
}
```

Compile a program

```
javac Test.java
```

Run a program

```
java Test
```

```
8  
hello
```

Java: Array

◆ *Array*: for storing multiple variables (of the same type)

◆ Create an array A with n items

```
int[] A = new int[n];
```

◆ $A[i]$ denotes the variable at position i of array A

◆ In Java, the range of i is from 0 to $n - 1$


◆ Examples

◆ `int[] A = new int[7];` // create an array

◆ `x = A[2];` // copy value $A[2]$ to variable x

◆ `A[3] = 5;` // store value 5 to $A[3]$

Position	0	1	2	3	4	5	6
Value	6	2	7	8	4	9	1



x : 5

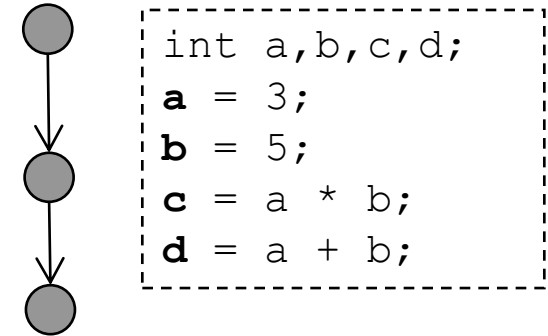
Control Flow

◆ Sequence (step-by-step)

- ◆ A sequence of statements implies the execution ordering
- ◆ Each statement is a ‘step’, e.g.,

- ◆ Assignment statement: `c = a * b;`

- ◆ Call statement: `System.out.println("hello");`

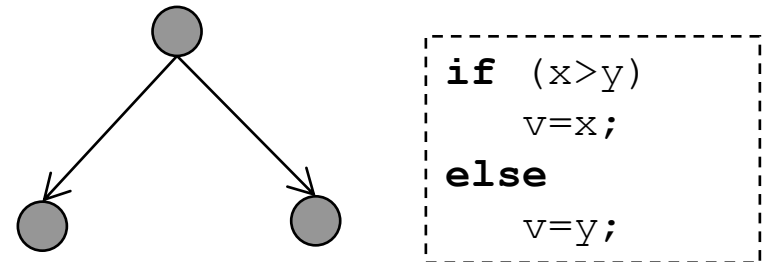


◆ Selection (choose a branch)

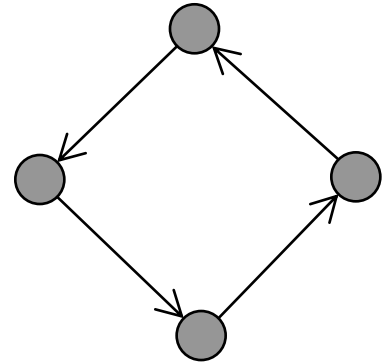
- ◆ *if* statement, *switch* statement

- ◆ *Example of if statement:*

- ◆ Evaluate the condition `x > y`
- ◆ Condition is true → execute `v = x;`
- ◆ Condition is false → execute `v = y;`



Control Flow



- ◆ **Loop:** repeat the execution of the loop body (a sequence of statements) for multiple times

- ◆ Example: print “hello” 5 times

- ◆ *while*-loop

- ◆ Evaluate <cond> **before** the loop body;
the loop stops when <cond> is false

```
int i = 5;  
while (i>0) {  
    System.out.println("hello");  
    i--;  
}
```

- ◆ *for*-loop: counter-based

- ◆ Execute <init> once before the loop starts
- ◆ Evaluate <cond> **before** the loop body;
the loop stops when <cond> is false
- ◆ Execute <update> **after** the loop body

```
for (int i=0; i<5; i++) {  
    System.out.println("hello");  
}
```

Our Roadmap

- ◆ Programming basics



- ◆ How to solve a problem? by an algorithm?
- ◆ How to analyze the running time of an algorithm?
- ◆ What are data structures?

How to solve a problem?



- ◆ *A problem for human (in the real world):*

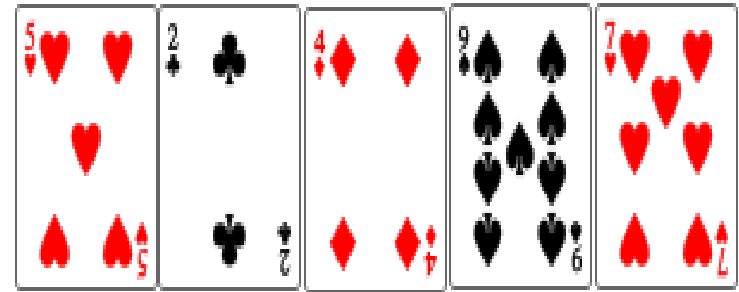
Sort poker cards on hand

- ◆ Input:

- ◆ A list of cards

- ◆ Procedure:

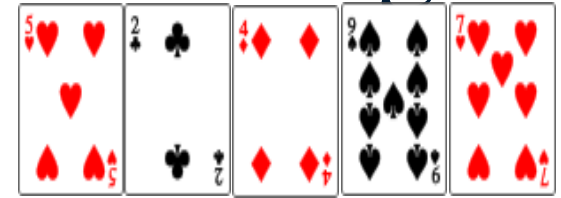
- ◆ 1. **Put** all cards on the left hand 🖐
- ◆ 2. **Pick** the smallest card from the left hand 🖐
- ◆ 3. **Move** that card to the right hand 🖐
- ◆ 4. Repeat Steps 2-3 until the left hand is empty 🖐



- ◆ How to solve a problem by using a program?

- ◆ Use variables, operators, statements,

Algorithms for Problem Solving



Input: a sequence A of n integers
Output: a sequence A of n integers
 such that

Define a
computational
problem

Selection-Sort (...)

```
1. for integer  $i \leftarrow 0$  to  $n-2$ 
2.    $k \leftarrow i$ 
3.   for integer  $j \leftarrow i+1$  to  $n-1$ 
4.     if  $A[k] > A[j]$  then
       ....
```

void Selection-Sort (...)

```
int  $i, j, k, temp$  ;
int  $n = A.length$ ;
for (  $i=0$  ;  $i<n-1$ ;  $i++$  ) {
    ...
    for (  $j=i+1$  ;  $j<n$ ;  $j++$  )
        ...
    ...
}
```

Design an
algorithm

Convert it into
a program

Real-world input

Input

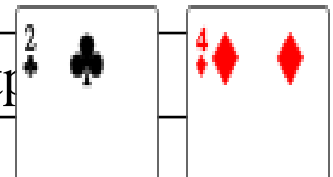
Run a program

Output

Real-world output

5 | 2 | 4 | 9 | 7

2 | 4 | 5 | 7 | 9



Computational problem

- ◆ *Example:* the **sorting** problem

- ◆ Input: a *sequence* A of n integers

5		2		4		9		7
---	--	---	--	---	--	---	--	---

- ◆ Output: a sequence A' of n integers such that

- ◆ A' is an **increasing** sequence, and
- ◆ A and A' contain the same integers.

2		4		5		7		9
---	--	---	--	---	--	---	--	---

Algorithms

◆ *Algorithm*: a well defined sequence of steps for solving a computational problem

- ◆ It always produces the *correct output*
- ◆ It uses *well-defined* steps or operations
- ◆ It finishes in *finite time*

◆ It is written for human readers

- ◆ Like programming, we can use variables and data structures, control flow, call statement
- ◆ We can also use natural language provided that it is clear

Selection-Sort (Array $A[0..n-1]$)

1. for integer $i \leftarrow 0$ to $n-2$
2. $k \leftarrow i$
3. for integer $j \leftarrow i+1$ to $n-1$
4. if $A[k] > A[j]$ then
5. $k \leftarrow j$
6. swap $A[i]$ and $A[k]$

Algorithm vs. Program

- ◆ An algorithm can be implemented as a program in any language (e.g., C, C++, Java, Python)
- ◆ An algorithm is more *readable* than a program

Algorithm

Selection-Sort (Array $A[0..n-1]$)

1. for integer $i \leftarrow 0$ to $n-2$
2. $k \leftarrow i$
3. for integer $j \leftarrow i+1$ to $n-1$
4. if $A[k] > A[j]$ then
5. $k \leftarrow j$
6. swap $A[i]$ and $A[k]$

Java program


```
// K.K.F. Yuen
void SelectionSort(int[] A){
    int i, j, k, temp;
    int n = A.length;
    for (i = 0; i < n-1; i ++){
        k = i ;
        for ( j=i+1 ; j<n; j++ ){
            if ( A[k] > A[j] )
                k = j ;
        }
        temp = A[i] ;
        A[i] = A[k] ;
        A[k] = temp ;
    }
}
```

Or $i \leq n-2$


A trivial mistake is NOT trivial

Error result

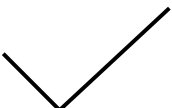
```
void SelectionSort(int[] A){
    int i,j,k,temp;
    int n = A.length;
    for (i = 0; i<n-1; i++){
        k = i;
        for (j = i+1; j<n; j++){
            if(A[k] > A[j])
                k = j;
            temp = A[i];
            A[i] = A[k];
            A[k] =temp;
        }
    }
```



```
// K.K.F. Yuen
void SelectionSort(int[] A){
    int i, j, k, temp;
    int n = A.length;
    for (i = 0; i<n-1; i++){
        k = i ;
        for ( j=i+1 ; j<n; j++ ){
            if ( A[k] > A[j] )
                k = j ;
        }
        temp = A[i] ;
        A[i] = A[k] ;
        A[k] = temp ;
    }
}
```



```
void SelectionSort(int[] A){
    int i, j, k, temp;
    int n = A.length;
    for (i = 0; i<n-1; i++){
        k = i ;
        for ( j=i+1 ; j<n; j++ )
            if ( A[k] > A[j] )
                k = j ;
        temp = A[i] ;
        A[i] = A[k] ;
        A[k] = temp ;
    }
}
```



Selection-Sort (Array $A[0..n-1]$)

1. for integer $i \leftarrow 0$ to $n-2$
2. $k \leftarrow i$
3. for integer $j \leftarrow i+1$ to $n-1$
4. if $A[k] > A[j]$ then
5. $k \leftarrow j$
6. swap $A[i]$ and $A[k]$

Java

```
void SelectionSort(int[] A){
    int i, j, k, temp;
    int n = A.length;
    for (i = 0; i < n-1; i ++){
        k = i ;
        for ( j=i+1 ; j<n; j++ ){
            if ( A[k] > A[j] )
                k = j ;
        }
        temp = A[i] ;
        A[i] = A[k] ;
        A[k] = temp ;
    }
}
```

R

Python

```
SelectionSort <- function(A)
{
    n = length(A)
    for (i in 1:(n-1)){
        k = i
        for (j in (i+1):(n))
        {
            if (A[k] > A[j]){
                k <- j
            }
        }
        # swap A[i] and A[k]
        temp = A[i]
        A[i] = A[k]
        A[k] = temp
    }
    A
}
```

```
def selectionSort2(A):
    for i in range(0, len(A)-1):
        k = i
        for j in range(i+1, len(A)):
            if(A[k] > A[j]):
                k = j;
        # swap A[i] and A[k]
        A[i], A[k] = A[k], A[i]
    return A
```

Algorithms: Running Steps

- ◆ Run an algorithm manually on a sample input, then draw *running steps*
 - ◆ E.g., show the content of important variables in each iteration
 - ◆ Useful for understanding how the algorithm works

Selection-Sort (Array $A[0..n-1]$)

1. for integer $i \leftarrow 0$ to $n-2$
2. $k \leftarrow i$
3. for integer $j \leftarrow i+1$ to $n-1$
4. if $A[k] > A[j]$ then
5. $k \leftarrow j$
6. swap $A[i]$ and $A[k]$

input

5 | 2 | 4 | 9 | 7

$i = 1$

2 | 5 | 4 | 9 | 7

$i = 2$

2 | 4 | 5 | 9 | 7

$i = 3$

2 | 4 | 5 | 9 | 7

$i = 4$

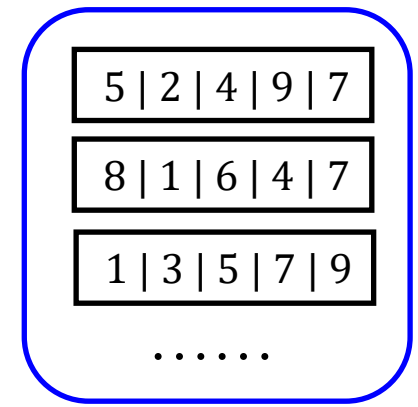
2 | 4 | 5 | 7 | 9

$i = 5$

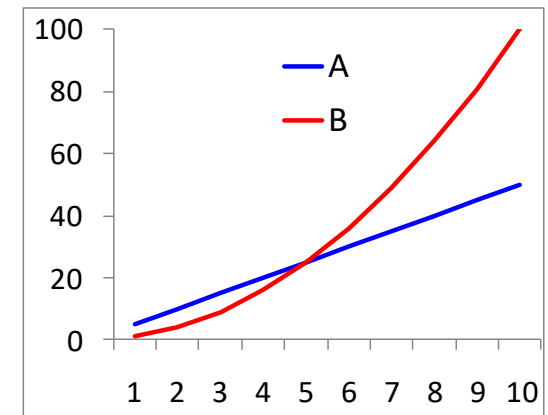
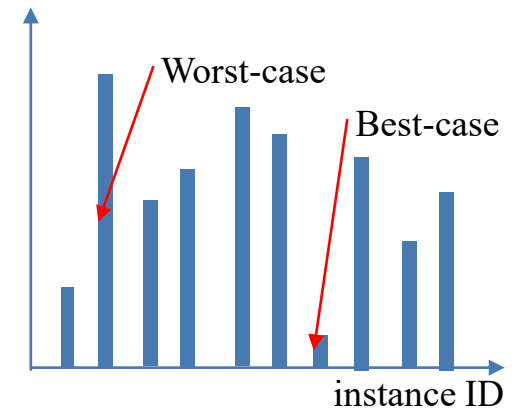
2 | 4 | 5 | 7 | 9

Algorithmic Analysis


- ◆ There are many possible **input instances**
- ◆ Is your algorithm always **correct**?
 - ◆ Not enough even if you have tested your algorithm on many instances
 - ◆ Will your algorithm fail on some other instance?
 - ◆ Need a **correctness proof** to show that:
 - ◆ “For every possible input instance, the algorithm produces the correct output”
- ◆ **How fast** is your algorithm?
 - ◆ Consider all instances of the **same size n** and their running time
 - ◆ Estimate the worst-case running time as a function of the input size n
 - ◆ What’s the trend of the worst-case running time?



Running time



Our Roadmap

- ◆ Programming basics
- ◆ How to solve a problem? by an algorithm?
-  ◆ How to analyze the running time of an algorithm?
- ◆ What are data structures?

Why do we analyze the running time of an algorithm?

- ◆ The same problem (e.g., sorting) can be solved by several different algorithms
- ◆ It is time consuming to convert all algorithms to programs
- ◆ We just want to choose the “fastest” algorithm and then convert it into a program
- ◆ Understand the scalability of a program.
E.g., if the input size doubles, how much would its running time increase?

How to analyze the running time of an algorithm?

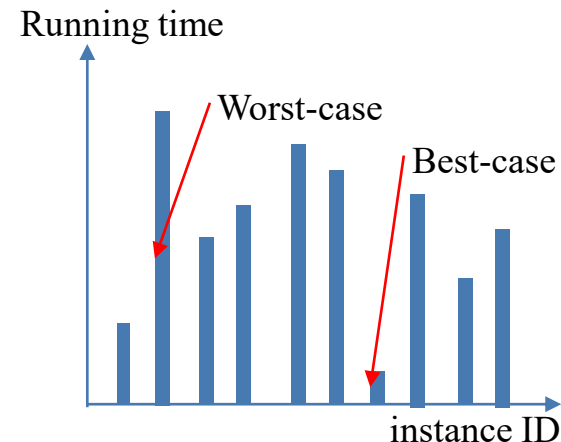
◆ Assumptions

◆ #1. Each basic step takes constant time

- ◆ E.g., add two integers: c_{add}
compare two numbers: $c_{compare}$
copy a value to a variable: c_{copy}

◆ #2. The memory is large enough to store all input/intermediate/output data

◆ Find the (worst-case) frequency and cost of executing each line, then sum up the total cost



How to estimate running time?

<u>Search (Array $A[0..n-1]$, Key k)</u>	<u>cost</u>	<u>frequency</u>
1. for integer $i \leftarrow 0$ to $n-1$	c_1	at most n
2. if $A[i] = k$	c_2	at most n
3. return i	c_3	at most 1
4. return -1	c_4	at most 1

◆ What is the frequency of executing Line 1?

- ◆ It depends on which element $A[i]$ is equal to k
- ◆ Line 1 is executed **at most** n times

◆ What is the frequency of executing Line 2?

◆

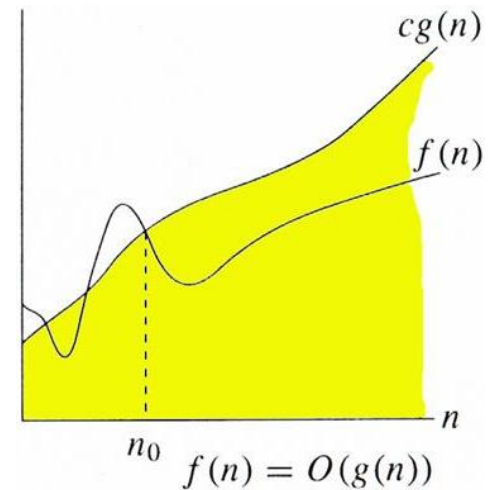
◆ Worst-case running time of the algorithm

$$\begin{aligned} &= c_1 * n + c_2 * n + c_3 * 1 + c_4 * 1 \\ &= (c_1 + c_2) n + c_3 + c_4 \end{aligned}$$

- ◆ This expression contains many constants
- ◆ How to simplify it?

Asymptotic Notation

- ◆ How to describe the trend of running time in a concise way?
- ◆ Asymptotic **upper-bound**, **O**-notation
 - ◆ We can write $f(n) = O(g(n))$ if
 - we can find positive constants c, n_0 such that
$$0 \leq f(n) \leq c g(n) \text{ for all } n \geq n_0$$
- ◆ Is $(5n^2 + 3n) = O(n^2)$?
 - ◆ YES. choose $c=6$ and $n_0=10$
[note: other choices also possible]
- ◆ Is $(5n^2 + 3n) = O(n^3)$?
 - ◆ YES, *but we prefer to use a tighter expression*



Simplification Rules of O-Notation

*These rules provide
shortcuts for showing that
 $f(n) = O(g(n))$*

- ◆ Ignore constant factors

- ◆ E.g., $3 = O(1)$

- ◆ E.g., $3n = O(n)$

- ◆ Combine fixed number of terms with same complexity

- ◆ E.g., $O(n) + O(n) = O(n)$

$\leftarrow \text{dashed arrow} \rightarrow c_1 n + c_2 n = (c_1 + c_2) n$

- ◆ E.g., $O(n) + O(n) + O(n) = O(n)$

- ◆ Polynomial

- ◆ Just use the highest-degree term.

Why?

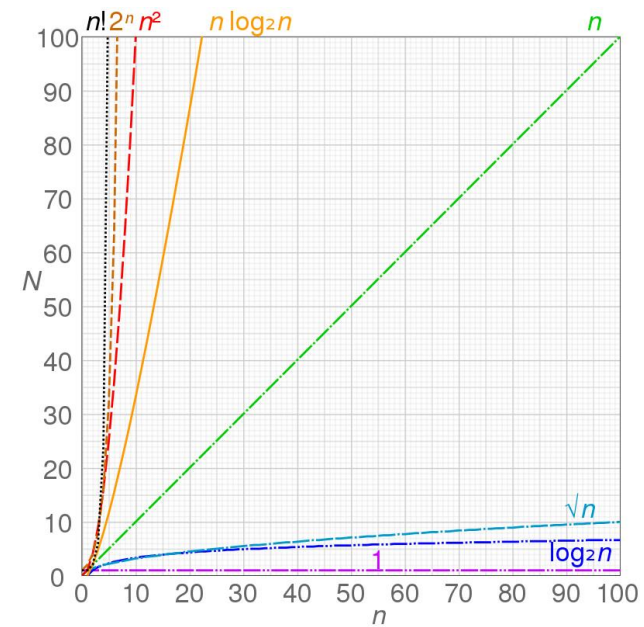
- ◆ E.g., $5n^2 + 3n = O(n^2)$

- ◆ E.g., $2n^3 + 5n^2 + 3n = O(n^3)$

Asymptotic running time of some algorithms

<i>Complexity</i>		<i>Algorithm</i>
$O(1)$	Constant time	Compare two numbers
$O(\log n)$	Logarithmic	Binary search (on a sorted array)
$O(n)$	Linear time	Linear search (on an unsorted array)
$O(n \log n)$		Merge-sort
$O(n^2)$	Quadratic	Selection-sort

- ◆ The trend matters
 - ◆ We care about the performance at *large input size*
 - ◆ Constant factors (e.g., c) do not affect *the order of growth*



Asymptotic running time: Example 1

Search (Array $A[0..n-1]$, Key k)

1. for integer $i \leftarrow 0$ to $n-1$

2. if $A[i] = k$

3. return i

4. return -1

5		2		4		9		7
---	--	---	--	---	--	---	--	---

◆ Let's analyze the asymptotic running time of this algorithm

◆ Line 1: at most n basic steps = $O(n)$

◆ Line 2: at most $1 * n$ basic steps = $O(n)$

◆ Lines 3 and 4: $O(1)$

◆ Running time of algorithm $T(n)$

$$= O(n) + O(n) + O(1)$$

$$= O(n)$$

Asymptotic running time: Example 2

Selection-Sort (Array $A[0..n-1]$)

1. for integer $i \leftarrow 0$ to $n-2$
2. $k \leftarrow i$
3. for integer $j \leftarrow i+1$ to $n-1$
4. if $A[k] > A[j]$ then
5. $k \leftarrow j$
6. swap $A[i]$ and $A[k]$

5		2		4		9		7
---	--	---	--	---	--	---	--	---

- ◆ Line 1: $n - 1$ basic steps $= O(n)$
- ◆ Line 2: $1 * (n - 1)$ basic steps $= O(n)$
- ◆ Lines 3 to 5:
- ◆ How many times can we execute Line 3 at most?

- ◆ For each given value of i , what is the possible range of j ?



Asymptotic running time: Example 2

- ◆ Worst-case analysis of Line 3:

- ◆ Values of j in that loop: 1 to $n-1$, 2 to $n-1$, ..., $n-1$ to $n-1$
- ◆ Number of iterations: $n-1, n-2, n-3, \dots, 1$

$$\begin{aligned}\sum_{i=1..n-1} (n-i) &= (n-1) + (n-2) + \dots + 1 \\ &= (n-1)*n/2 = O(n^2)\end{aligned}$$

- ◆ Running time of algorithm $T(n)$:

- ◆ Line 1. $O(n)$
- ◆ Lines 2, 6. same as Line 1
- ◆ Line 3. $O(n^2)$
- ◆ Lines 4, 5. same as Line 3

- ◆ Thus, we have: $\mathbf{T(n)} = 3*O(n) + 3*O(n^2) = O(n^2)$

Formula:

$$\sum_{i=1}^n i = 1 + 2 + \dots + n = \frac{n(n+1)}{2}$$

Our Roadmap

- ◆ Programming basics
- ◆ How to solve a problem? by an algorithm?
- ◆ How to analyze the running time of an algorithm?
- ◆ What are data structures?



Data Structures

- ◆ What are the data structures for human?
 - ◆ Used in libraries, books, clinics, companies,



Oriental lampshades, 105-107
Patterns, how to make, 87, 135, 137
Piping, 120
Pleating, 99-104
Pricing your work, 152
Relining lampshades, 118
Rewiring lamps, 80-82
Roses, 126
Ruffles, how to make, 122-123
Scallops, 33, 85
Shampooing lampshades, 151
Shapes of lampshades, 31-41
Silhouettes of lamps, 21-30
Slipcovers for lamps, 108
Smocking, 96, 99
Spiders, different kinds, 31-32
Sunburst pleating, 100-102

- ◆ How about the data structures for computers?

Data Objects



- ◆ Examples of data objects

- ◆ Employees in payroll application
- ◆ Tasks in the scheduler of OS
- ◆ Shopping cart items in online shopping
- ◆ In any application that you can think of



- ◆ A data object has attributes

- ◆ E.g., the attributes of shopping cart items are:

`product_id, product_name, price,`



- ◆ The search key depends on the application, e.g.,

- ◆ `product_id` for a shopping cart application

Data Structures

- ◆ Let S be a set of items, and x be a search key

- ◆ A key is a number, e.g., product id

- ◆ Useful operations on a set S



Search(S, x): search whether x appears in S



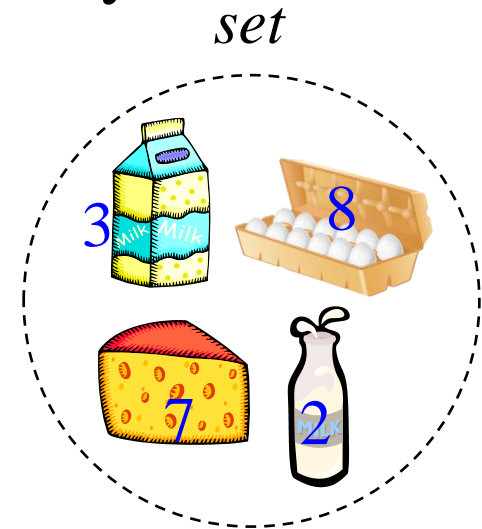
Insert(S, x): insert item x into S



Delete(S, x): remove item x from S

- ◆ *Data structure:*

- ◆ A way of organizing data objects for efficient usage
- ◆ Building blocks for designing algorithms



search key



The problems in Lectures 2-5

- ◆ How to solve these problems quickly?
 - ◆ Sort a set of items
 - ◆ Search an item from a set of items
 - ◆ Insert / Delete an item from a set of items
- ◆ Question 1
 - ◆ Selection-Sort takes $O(n^2)$ time
 - ◆ Is there a faster sorting algorithm?
- ◆ Question 2
 - ◆ Is an array good enough for searching, insertion and deletion?

Is Array Good Enough?



- ◆ Search a key in an unsorted array (of length n)
- ◆ Linear search algorithm
 - ◆ Running time: $O(n)$
- ◆ Example #1: search $k=21$
 - ◆ Check the case $i=0$
 - ◆ Return **0**
- ◆ Example #2: search $k=17$
 - ◆ Check the cases $i=0,1,2,3,4,5,6$
 - ◆ Return **6**
- ◆ How to search faster?

21 12 8 3 35 1 17

Search (Array $A[0..n-1]$, Key k)

1. for integer $i \leftarrow 0$ to $n-1$
2. if $A[i] = k$
3. return i
4. return -1

Is Array Good Enough?



- ◆ Search in a **sorted** array (of length n)
- ◆ Binary search algorithm
 - ◆ Running time: $O(\log n)$

1		3		8		12		17		23		35
---	--	---	--	---	--	----	--	----	--	----	--	----

- ◆ *Idea:*

- ◆ Compare k with the middle item $A[mid]$
- ◆ Is k in the left sub-array or in the right sub-array?

BinarySearch (Array A , Integers low , $high$, Key k)

1. $mid \leftarrow \lfloor (low+high)/2 \rfloor$
2. if $A[mid] = k$
3. return mid
4. else if $low = high$
5. return -1
6. if $k < A[mid]$
7. return BinarySearch(A , low , $mid-1$, k)
8. else
9. return BinarySearch(A , $mid+1$, $high$, k)

Initial call: BinarySearch (A , 0 , $n-1$, k)

Example: find the key “35” from this array A

1 | 3 | 8 | 12 | 17 | 23 | 35

BinarySearch (Array A, Integers *low*, *high*, Key *k*)

1. $mid \leftarrow \lfloor (low+high)/2 \rfloor$
2. if $A[mid] = k$
3. return mid
4. else if $low = high$
5. return -1
6. if $k < A[mid]$
7. return BinarySearch($A, low, mid-1, k$)
8. else
9. return BinarySearch($A, mid+1, high, k$)

Initial call: BinarySearch (A, 0, $n-1$, k)

BinarySearch (A, 0, 6, **35**),
 $mid = 3$



1 | 3 | 8 | 12 | 17 | 23 | 35

BinarySearch (A, 4, 6, **35**),
 $mid = 5$



1 | 3 | 8 | 12 | 17 | 23 | 35

BinarySearch (A, 6, 6, **35**),
 $mid = 6$



1 | 3 | 8 | 12 | 17 | 23 | 35

Key found, return 6

```

int i = 0; // for understanding recursive function only
public int binarySearch(int[] A, int low, int high, int k){
    //int mid = (int) Math.floor( ((double) low + (double) high) / 2.0);
    i = i + 1;
    System.out.println("call " + i); // tracking
    int mid = (low + high) / 2;
    System.out.println("mid position is " + mid); // tracking
    System.out.println("value is " + A[mid]); // tracking
    if(A[mid] == k) return mid;
    else if (low == high) return -1;
    if(k < A[mid]) return binarySearch(A, low, mid-1, k);
    else return binarySearch(A, mid+1, high, k);
}

```

```

int[] A = {1,3,8,12,17,23,35};
binarySearch(A, 0, A.length - 1, 35);

```

```

call 1
mid position is 3
value is 12
call 2
mid position is 5
value is 23
call 3
mid position is 6
value is 35

```

Is Array Good Enough?

- ◆ How about insertion/deletion?



- ◆ Can an unsorted array support fast insertion/deletion?

21		12		8		3		35		1		17
----	--	----	--	---	--	---	--	----	--	---	--	----

- ◆ Can a sorted array support fast insertion/deletion?

1		3		8		12		17		23		35
---	--	---	--	---	--	----	--	----	--	----	--	----

Data Structures in Lectures 2-5

- ◆ Learn the **properties** of data structures
 - ◆ E.g., stack's LIFO property, queue's FIFO property, growable structure like linked list
- ◆ Learn the **operations** of data structures
 - ◆ They support different operations, and with different running time
- ◆ You may play with them in <https://visualgo.net/en>
- ◆ What's the most suitable data structure to solve a particular problem?
 - ◆ It depends on the properties and operations most frequently used in your algorithm

Stack (<i>Lecture 2</i>)
Queue (<i>Lecture 2</i>)
Linked List (<i>Lecture 2</i>)
Tree (<i>Lecture 3</i>)
Heap (<i>Lecture 4</i>)
.....

Java Programmers

- ◆ At least know how to use the data structures and algorithms in the `java.util` library
(<https://docs.oracle.com/javase/8/docs/api/java/util/package-summary.html>)
 - ◆ `Stack`, `Queue`, `LinkedList`, `PriorityQueue`,
 - ◆ `Arrays.sort`, `Arrays.binarySearch`,
- ◆ We hope you can also understand the “magic” behind them

Summary

- ◆ Basic concepts in Java, algorithm, asymptotic running time, data structure
- ◆ Please read Chapters 1, 4.2.1-4.2.6 in the book “*Data Structures and Algorithms in Java*”
- ◆ *Next lecture:*
linear data structures (stack, queue, linked list)