

Lecture 11

Database normalization

Subject Lecturer: Kevin K.F. YUEN, PhD.

Acknowledgement: Slides were offered from Prof. Ken Yiu.
Some parts might be revised and indicated.

Outline



- ◆ Motivation

- ◆ Concepts: functional dependency, closure, cover

- ◆ Properties of normalization

- ◆ Normal forms: BCNF vs. 3NF

First Normal Form

- ◆ An attribute is **atomic** if it cannot be divided into parts

Examples of non-atomic attributes

- ◆ Set of names, composite attributes
- ◆ Course code (CS101) that can be divided into “CS” and “101”
 - programmer may extract “CS” to find the department
 - bad idea: encode data in program rather than in DBMS
- ◆ **Drawback** of non-atomic attributes
 - ◆ Complicated to store them
 - ◆ Redundant storage of data → may cause data inconsistency
- ◆ In this lecture, we assume all attributes are atomic, i.e., each relation schema is in **first normal form**

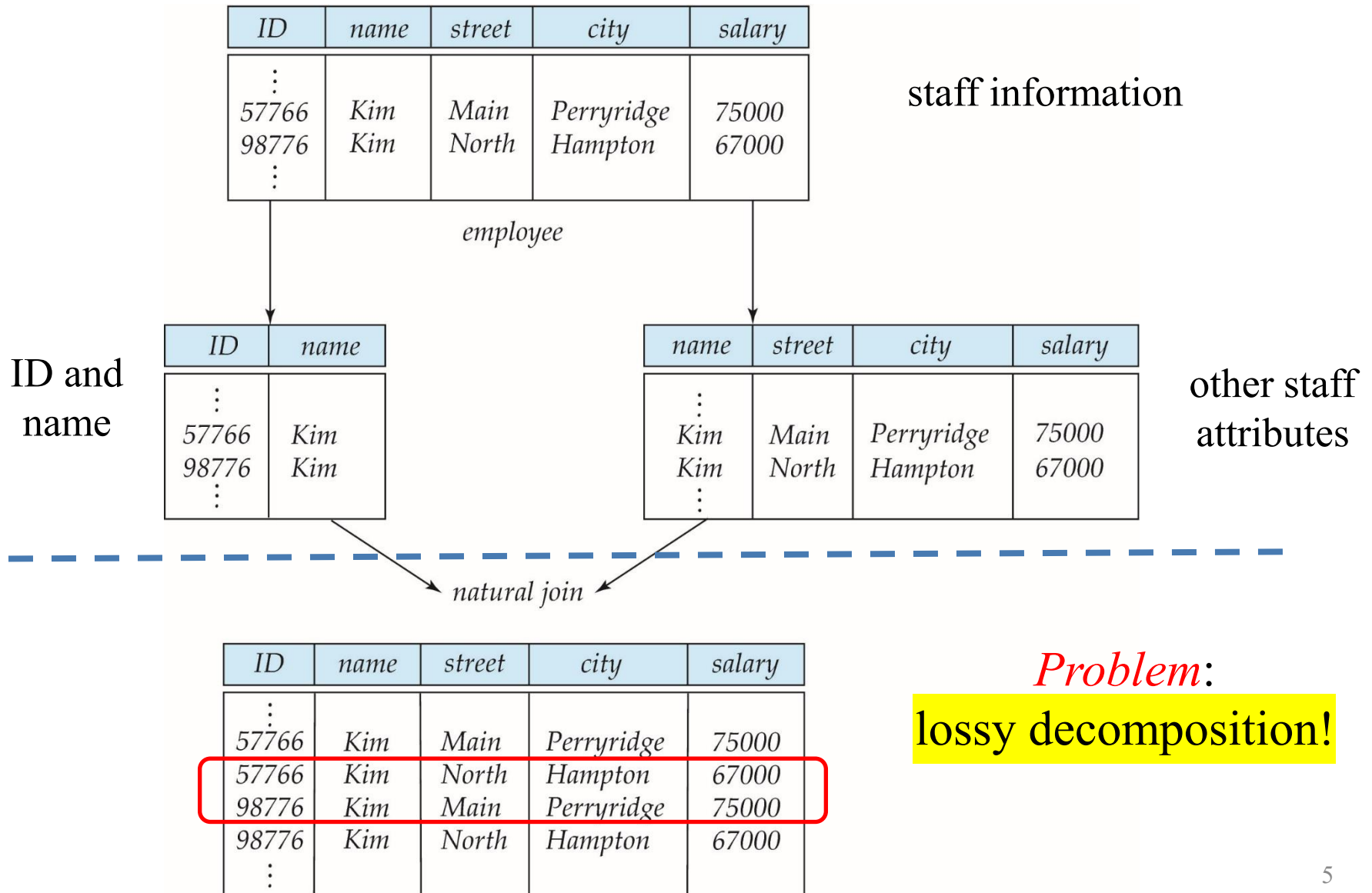


Example Application: University Information Management System

- What if we use one (large) table only?
- Problem:** redundant information

| staff name | | staff salary | dept. location | | dept. budget |
|------------|------------|--------------|----------------|----------|--------------|
| ID | name | salary | dept_name | building | budget |
| 22222 | Einstein | 95000 | Physics | Watson | 70000 |
| 12121 | Wu | 90000 | Finance | Painter | 120000 |
| 32343 | El Said | 60000 | History | Painter | 50000 |
| 45565 | Katz | 75000 | Comp. Sci. | Taylor | 100000 |
| 98345 | Kim | 80000 | Elec. Eng. | Taylor | 85000 |
| 76766 | Crick | 72000 | Biology | Watson | 90000 |
| 10101 | Srinivasan | 65000 | Comp. Sci. | Taylor | 100000 |
| 58583 | Califieri | 62000 | History | Painter | 50000 |
| 83821 | Brandt | 92000 | Comp. Sci. | Taylor | 100000 |
| 15151 | Mozart | 40000 | Music | Packard | 80000 |
| 33456 | Gold | 87000 | Physics | Watson | 70000 |
| 76543 | Singh | 80000 | Finance | Painter | 120000 |

What if we use many (small) tables?

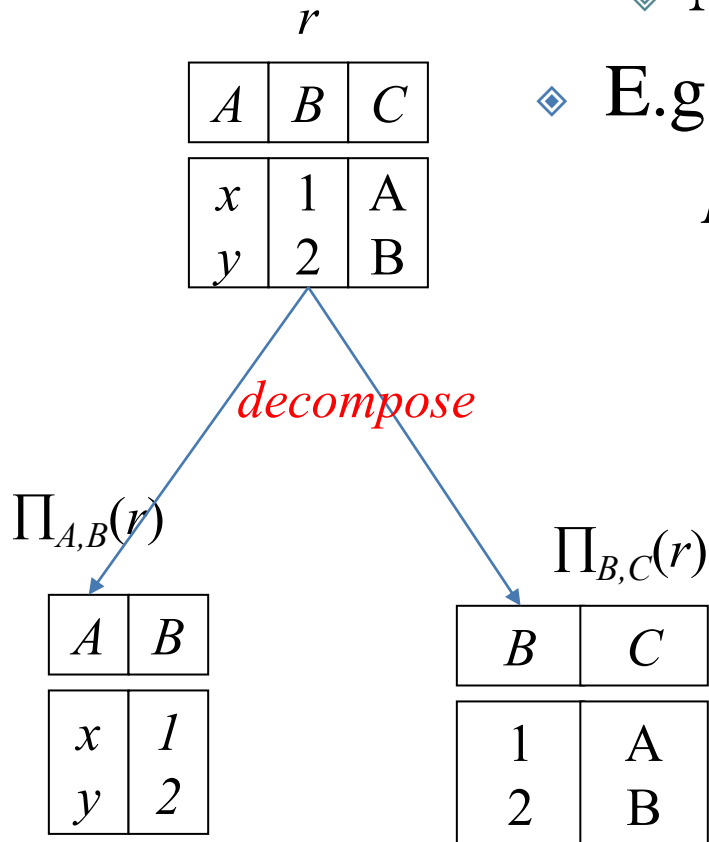


Example of Lossless-Join Decomposition



◆ Lossless-join decomposition

- ◆ No information lost
- ◆ E.g., decompose $R = (A, B, C)$ into $R_1 = (A, B)$ and $R_2 = (B, C)$



$$\Pi_{A,B}(r) \bowtie \Pi_{B,C}(r)$$

| A | B | C |
|---|---|---|
| x | 1 | A |
| y | 2 | B |

How to design a good schema?

- ◆ **Decide** whether a particular relation R is in “good” form
 - ◆ E.g., BCNF, 3NF
- ◆ If a relation R is not in “good” form, **decompose** it into a set of relations $\{R_1, R_2, \dots, R_n\}$ such that
 - ◆ each relation is in good form
 - ◆ the decomposition is a lossless-join decomposition
- ◆ Our theory is based *functional dependencies*

Outline

- ◆ Motivation



- ◆ Concepts: functional dependency, closure, cover

- ◆ Properties of normalization

- ◆ Normal forms: BCNF vs. 3NF

Functional dependency: $\alpha \rightarrow \beta$



generalize the concept of **key**

- ◆ $\alpha \rightarrow \beta$ is like a **rule**: the values of attributes in α **determines uniquely** the values of attributes in β

- ◆ *Example:* consider this **instance** of relation $r(A,B)$

- ◆ $A \rightarrow B$ doesn't hold on r

- ◆ $B \rightarrow A$ holds on r



| | |
|---|---|
| 1 | 4 |
| 1 | 5 |
| 3 | 7 |

- ◆ *Definition:* Given a **relation schema** R , the **functional dependency** $\alpha \rightarrow \beta$ holds on R means that:
for any **legal relation** $r(R)$, whenever **any two tuples** t_1 and t_2 of r agree on attributes in α , they also agree on attributes in β . That is,

$$t_1[\alpha] = t_2[\alpha] \Rightarrow t_1[\beta] = t_2[\beta]$$

Express the concept of key by $\alpha \rightarrow \beta$

- ◆ K is a **superkey** for relation schema R if and only if $K \rightarrow R$
- ◆ K is a **candidate key** for R if and only if
 - ◆ $K \rightarrow R$, and
 - ◆ there exists no $\alpha \subset K$ such that $\alpha \rightarrow R$
- ◆ Functional dependencies allow us to express constraints that cannot be expressed using superkeys.

Consider the schema:

inst_dept (ID, *name*, *salary*, dept_name, *building*, *budget*)

We expect these **functional dependencies** to hold:

dept_name \rightarrow *building*

and *ID* \rightarrow *building*

but would not expect the following to hold:

dept_name \rightarrow *salary*

Use of Functional Dependencies



- ◆ We use functional dependencies to:
 - ◆ **test a relation** r to see if it is legal under a given set \mathcal{F} of functional dependencies
 - ◆ If yes, then we say that: **r satisfies \mathcal{F}**
 - ◆ **specify constraints** on the set of legal relations
 - ◆ We say that **\mathcal{F} holds on R** if all legal relations on R satisfy \mathcal{F}
- ◆ **Note:** we ignore a functional dependency if it cannot hold on all legal instances
 - ◆ E.g., an instance of *instructor* may satisfy, **by chance**,
$$name \rightarrow ID$$
but we should **ignore** this functional dependency

Functional Dependencies (Cont.)

- ◆ A functional dependency is **trivial** if it is satisfied by all instances of a relation
- ◆ $\alpha \rightarrow \beta$ is trivial if $\beta \subseteq \alpha$
- ◆ Example:
 - ◆ $ID, name \rightarrow ID$
 - ◆ $name \rightarrow name$

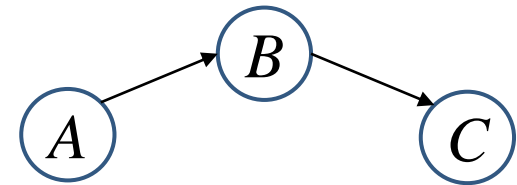
Closure of Functional Dependencies

◆ Armstrong's axioms

- ◆ if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$ (reflexivity)
- ◆ if $\alpha \rightarrow \beta$, then $\gamma \alpha \rightarrow \gamma \beta$ (augmentation)
- ◆ if $\alpha \rightarrow \beta$, and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$ (transitivity)

◆ Example:

- ◆ given $A \rightarrow B$ and $B \rightarrow C$,
we obtain: $A \rightarrow C$



◆ The closure of \mathcal{F} , denoted by \mathcal{F}^+

- ◆ The set of all functional dependencies implied by a set \mathcal{F} of functional dependencies
- ◆ We compute \mathcal{F}^+ by repeatedly applying Armstrong's axioms

Example

$$\diamond R = (A, B, C, G, H, I)$$

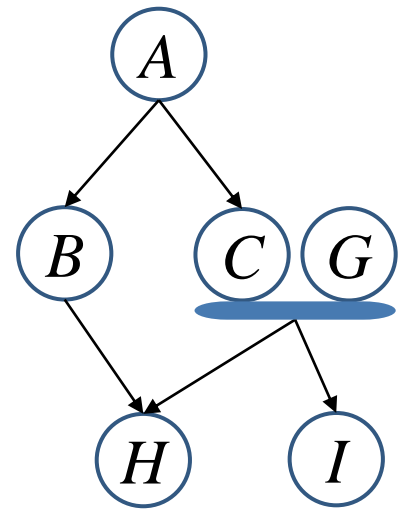
$$\diamond \mathcal{F} = \{ A \rightarrow B$$

$$A \rightarrow C$$

$$CG \rightarrow H$$

$$CG \rightarrow I$$

$$B \rightarrow H\}$$



\mathcal{F}^+ contains:

$$\diamond A \rightarrow H$$

\diamond by transitivity from $A \rightarrow B$ and $B \rightarrow H$

$$\diamond AG \rightarrow I$$

\diamond by augmenting $A \rightarrow C$ with G , to get $AG \rightarrow CG$
 and then transitivity with $CG \rightarrow I$

$$\diamond CG \rightarrow HI$$

\diamond by augmenting $CG \rightarrow I$ to infer $CG \rightarrow CGI$,
 and augmenting of $CG \rightarrow H$ to infer $CGI \rightarrow HI$,
 and then transitivity

$$\diamond \dots \rightarrow \dots$$

\diamond until you cannot find a new functional dependency


We have a systematic way to compute \mathcal{F}^+ (see page 18)

Closure of Functional Dependencies (Cont.)

- ◆ Additional *short-cut* rules:
 - ◆ If $\alpha \rightarrow \beta$ holds and $\alpha \rightarrow \gamma$ holds, then $\alpha \rightarrow \beta\gamma$ holds
(union)
 - ◆ If $\alpha \rightarrow \beta\gamma$ holds, then $\alpha \rightarrow \beta$ holds and $\alpha \rightarrow \gamma$ holds
(decomposition)
 - ◆ If $\alpha \rightarrow \beta$ holds and $\gamma\beta \rightarrow \delta$ holds, then $\alpha\gamma \rightarrow \delta$ holds
(pseudotransitivity)

The above rules can be derived from Armstrong's axioms

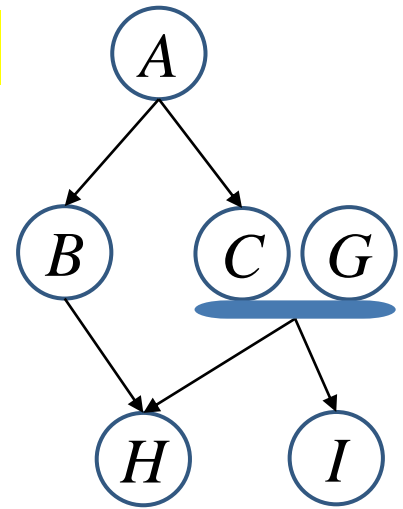
Attribute Set Closure

- ◆ Let α be a set of attributes
- ◆ The **closure** of α **under** \mathcal{F} (denoted by α^+) is the set of attributes that are determined by α under \mathcal{F}
- ◆ Algorithm to compute α^+ :
 $result := \alpha$
 while ($result$ has changed) **do**
 for each $\beta \rightarrow \gamma$ **in** \mathcal{F} **do** 
 if $\beta \subseteq result$ **then**
 $result := result \cup \gamma$

Example on α^+

◆ $R = (A, B, C, G, H, I)$

◆ $\mathcal{F} = \{A \rightarrow B$
 $A \rightarrow C$
 $CG \rightarrow H$
 $CG \rightarrow I$
 $B \rightarrow H\}$



◆ $(AG)^+$

1. It contains AG
2. It contains $ABCG$ ($A \rightarrow C$ and $A \rightarrow B$)
3. It contains $ABCGH$ ($CG \rightarrow H$)
4. It contains $ABCGHI$ ($CG \rightarrow I$)

Therefore, $(AG)^+ = ABCGHI$

◆ Is AG a candidate key?

- ❑ Is AG a super key?
 - ❑ Does $(AG)^+$ contain R ?
- ❑ Is any subset of AG a superkey?
 - ❑ Does $(A)^+$ contain R ?
 - ❑ Does $(G)^+$ contain R ?

The above figure is for reference only.
In general, it can be complicated
visualize \mathcal{F} by using a figure.

Uses of Attribute Closure

- ◆ How to test if α is a superkey?
 - ◆ Compute α^+ , then check if α^+ contains all attributes of R
- ◆ How to test if a functional dependency $\alpha \rightarrow \beta$ holds?
 - ◆ (in other words, we want to check whether $\alpha \rightarrow \beta$ is in \mathcal{F}^+)
 - ◆ Compute α^+ by using attribute closure, and then check if α^+ contains β
- ◆ How to compute \mathcal{F}^+ (i.e., the closure of \mathcal{F})?
 - ◆ for each $\gamma \subseteq R$
 - find the closure γ^+
 - for each $S \subseteq \gamma^+$
 - output a functional dependency $\gamma \rightarrow S$

Example on finding all candidate keys

◆ $R = (A, B, C, G, H, I)$

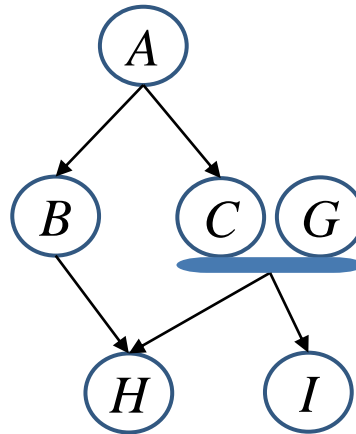
◆ $\mathcal{F} = \{A \rightarrow B$

$A \rightarrow C$

$CG \rightarrow H$

$CG \rightarrow I$

$B \rightarrow H\}$



◆ Size-3 attribute sets
except supersets of
candidate keys

◆ $(ABC)^+ = ABCH$

◆ ~~$(ABG)^+$~~

◆ $(ABH)^+ = ABCH$

◆ $(ABI)^+ = ABCHI$

◆ $(ACH)^+ = ABCH$

◆

◆ Size-2 attribute sets

◆ $(AB)^+ = ABCH$

◆ $(AC)^+ = ABCH$

◆ $(AG)^+ = ABCGHI$

◆ Candidate key!

◆ $(AH)^+ = ABCH$

◆ $(AI)^+ = ABCHI$

◆ $(BC)^+ = BCH$

◆

◆ Size-1 attribute sets

◆ $A^+ = ABCH$

◆ $B^+ = BH$

◆ $C^+ = C$

◆ $G^+ = G$

◆ $H^+ = H$

◆ $I^+ = I$

Example on finding \mathcal{F}^+ (the closure of \mathcal{F})

◆ $R = (A, B, C, G, H, I)$

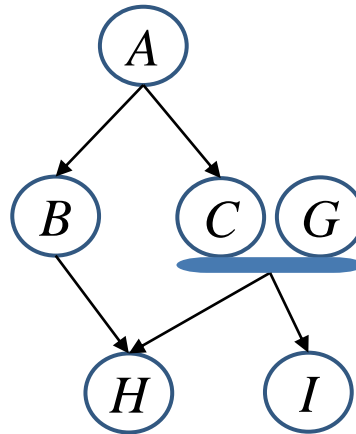
◆ $\mathcal{F} = \{A \rightarrow B$

$A \rightarrow C$

$CG \rightarrow H$

$CG \rightarrow I$

$B \rightarrow H\}$



◆ Size-3 attribute sets

◆ $ABC \rightarrow ABCH$

◆ $ABC \rightarrow \dots$

◆ $\dots\dots$

◆ $ABG \rightarrow ABCGHI$

◆ $\dots\dots$

◆ $ABH \rightarrow ABCH$

◆ $\dots\dots$

◆ $ABI \rightarrow ABCHI$

◆ $\dots\dots$

◆ $ACH \rightarrow ABCH$

◆ $\dots\dots$

◆ $\dots\dots$

◆ Size-1 attribute sets

◆ $A \rightarrow ABCH$

◆ $A \rightarrow ABC$

◆ $A \rightarrow ABH$

◆ $A \rightarrow ACH$

◆ $A \rightarrow BCH$

◆ $A \rightarrow AB$

◆ $A \rightarrow AC$

◆ $\dots\dots$

◆ $\dots\dots$

◆ Size-2 attribute sets

◆ $AB \rightarrow ABCH$

◆ $AB \rightarrow \dots$

◆ $\dots\dots$

◆ $AC \rightarrow ABCH$

◆ $\dots\dots$

◆ $AG \rightarrow ABCGHI$

◆ $\dots\dots$

◆ $\dots\dots$

Canonical Cover

- ◆ A set of functional dependencies may have redundant dependencies that can be inferred from the others
 - ◆ *Example #1*: $A \rightarrow C$ is redundant in:
$$\{A \rightarrow B, B \rightarrow C, A \rightarrow C\}$$
 - ◆ *Example #2*: parts of a functional dependency redundant
E.g., RHS: $\{A \rightarrow B, B \rightarrow C, A \rightarrow CD\}$ can be simplified to
$$\{A \rightarrow B, B \rightarrow C, A \rightarrow D\}$$

E.g., LHS: $\{A \rightarrow B, B \rightarrow C, AC \rightarrow D\}$ can be simplified to
$$\{A \rightarrow B, B \rightarrow C, A \rightarrow D\}$$
- ◆ A canonical cover of \mathcal{F} is a “minimal” set of functional dependencies equivalent to \mathcal{F}
 - ◆ no redundant (parts of) dependencies

Testing if an Attribute is Extraneous

- ◆ Consider a functional dependency $\alpha \rightarrow \beta$ in a given set \mathcal{F} of functional dependencies
- ◆ To test if attribute A is **extraneous** in α
 - compute $(\alpha - A)^+$ using dependencies in \mathcal{F}
 - if $(\alpha - A)^+$ contains β , then A is extraneous in α

- ◆ *Example:* Given $\mathcal{F} = \{A \rightarrow C, AB \rightarrow C\}$

- ◆ Is **B** extraneous in **$AB \rightarrow C$** ?

$$\begin{aligned}\text{Compute } (AB - B)^+ &= A^+ \\ &= AC\end{aligned}$$

- ◆ Yes, because the above result contains C

Testing if an Attribute is Extraneous

- ◆ Consider a functional dependency $\alpha \rightarrow \beta$ in a given set \mathcal{F} of functional dependencies
- ◆ To test if attribute $B \in \beta$ is **extraneous** in β
 - compute α^+ using only dependencies in $\mathcal{F}' = (\mathcal{F} - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - B)\}$
 - if α^+ contains B , then B is extraneous in β



- ◆ *Example:* Given $\mathcal{F} = \{A \rightarrow C, AB \rightarrow CD\}$

- ◆ Is **C** extraneous in $AB \rightarrow CD$?

Use only dependencies in $\mathcal{F}' = \{A \rightarrow C, AB \rightarrow D\}$

Compute $(AB)^+ = ABCD$

- ◆ Yes, because the above result contains C

Canonical Cover

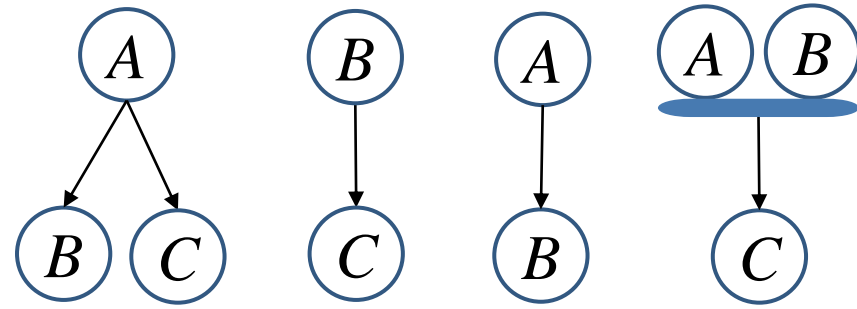
- ◆ A **canonical cover** for \mathcal{F} is a set of dependencies \mathcal{F}_c such that
 - ◆ \mathcal{F} implies all dependencies in \mathcal{F}_c , and
 - ◆ \mathcal{F}_c implies all dependencies in \mathcal{F} , and
 - ◆ No functional dependency in \mathcal{F}_c contains an **extraneous** attribute, and
 - ◆ Each left side of functional dependency in \mathcal{F}_c is unique
- ◆ To compute a canonical cover for \mathcal{F} :
repeat
 - Use the union rule to replace any dependencies in \mathcal{F}
 $\alpha_1 \rightarrow \beta_1$ and $\alpha_1 \rightarrow \beta_2$ with $\alpha_1 \rightarrow \beta_1 \beta_2$
 - Find a functional dependency $\alpha \rightarrow \beta$ with an
extraneous attribute either in α or in β
 - If an extraneous attribute is found, delete it from $\alpha \rightarrow \beta$**until** \mathcal{F} does not change

Example

- ◆ $R = (A, B, C)$

- $\mathcal{F} = \{A \rightarrow BC, B \rightarrow C$

- $A \rightarrow B, AB \rightarrow C\}$



- ◆ Combine $A \rightarrow BC$ and $A \rightarrow B$ into $A \rightarrow BC$

- ◆ \mathcal{F} becomes $\{A \rightarrow BC, B \rightarrow C, AB \rightarrow C\}$ now

- ◆ A is extraneous in $AB \rightarrow C$

- ◆ Check if the result of deleting A from $AB \rightarrow C$ is implied by other dependencies

- ◆ Yes: in fact, $B \rightarrow C$ is already present!

- ◆ \mathcal{F} becomes $\{A \rightarrow BC, B \rightarrow C\}$ now

- ◆ C is extraneous in $A \rightarrow BC$


- ◆ Check if $A \rightarrow C$ is implied by $A \rightarrow B$ and other dependencies

- ◆ Yes: using transitivity on $A \rightarrow B$ and $B \rightarrow C$

- ◆ Can use attribute closure of A in more complex cases

- ◆ The **canonical cover** is: $\{A \rightarrow B, B \rightarrow C\}$

Outline

- ◆ Motivation
- ◆ Concepts: functional dependency, closure, cover
-  ◆ Properties of normalization
- ◆ Normal forms: BCNF vs. 3NF

Property 1: Lossless-join Decomposition

- For the case of $R = (R_1, R_2)$, we require that for all possible relations r on schema R :

$$r = \Pi_{R_1}(r) \bowtie \Pi_{R_2}(r)$$

- A decomposition of R into R_1 and R_2 is **lossless join** if at least one of the following dependencies is in F^+ :

$$R_1 \cap R_2 \rightarrow R_1$$

$$\text{or } R_1 \cap R_2 \rightarrow R_2$$

Example

- $R = (A, B, C), \quad \mathcal{F} = \{A \rightarrow B, B \rightarrow C\}$
- Decompose into $R_1 = (A, B), \quad R_2 = (B, C)$
 - Lossless because: $R_1 \cap R_2 = \{B\}$ and $B \rightarrow BC$
- Decompose into $R_1 = (A, B), \quad R_2 = (A, C)$
 - Lossless because: $R_1 \cap R_2 = \{A\}$ and $A \rightarrow AB$

Property 2: Dependency Preservation

- Let \mathcal{F}_i be the set of dependencies \mathcal{F}^+ that include only attributes in R_i

- A decomposition is **dependency preserving** if

$$(\mathcal{F}_1 \cup \mathcal{F}_2 \cup \dots \cup \mathcal{F}_n)^+ = \mathcal{F}^+$$

- If it is not, then checking updates for violation of functional dependencies may require computing joins, which is expensive

Example

- $R = (A, B, C), \quad \mathcal{F} = \{A \rightarrow B, B \rightarrow C\}$

- Decompose into $R_1 = (A, B), \quad R_2 = (B, C)$



- Dependency preserving**

- Decompose into $R_1 = (A, B), \quad R_2 = (A, C)$

- Not dependency preserving**

(cannot check $B \rightarrow C$ without computing $R_1 \bowtie R_2$)

Goals of Normalization

- ◆ If a relation scheme R is not in “good” form, decompose it into $\{R_1, R_2, \dots, R_n\}$ such that
 - ◆ each relation scheme is in “good” form, and
 - ◆ the decomposition is a **lossless-join decomposition**
 - ◆ [*preferably*] the decomposition should be **dependency preserving**

Outline

- ◆ Motivation
- ◆ Concepts: functional dependency, closure, cover
- ◆ Properties of normalization
- ◆ Normal forms: BCNF vs. 3NF



Boyce-Codd Normal Form (BCNF)

A relation schema R is in **BCNF** with respect to a set \mathcal{F} of functional dependencies if for all functional dependencies in \mathcal{F}^+ of the form

$$\alpha \rightarrow \beta$$

where $\alpha \subseteq R$ and $\beta \subseteq R$, at least one of the following holds:

- ◇ $\alpha \rightarrow \beta$ is trivial (i.e., $\beta \subseteq \alpha$)
- ◇ α is a superkey for R

Example schema *not* in BCNF:

instr_dept (ID, name, salary, dept_name, building, budget)

because $dept_name \rightarrow building, budget$ holds on *instr_dept*,
but $dept_name$ is not a superkey

BCNF Decomposition Algorithm

$result := \{R\}$

compute \mathcal{F}^+

while (some schema R_i in $result$ is not in BCNF) **do**

let $\alpha \rightarrow \beta$ be a nontrivial functional dependency that
holds on R_i such that $\alpha \rightarrow R_i$ is not in \mathcal{F}^+ ,

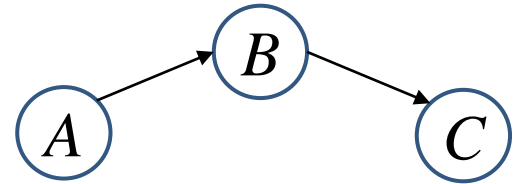
and $\alpha \cap \beta = \emptyset$

$result := (result - R_i) \cup (R_i - \beta) \cup (\alpha, \beta)$

Note: each R_i is in BCNF, and
decomposition is lossless-join

Example of BCNF Decomposition

- ◆ $R = (A, B, C)$
 $\mathcal{F} = \{A \rightarrow B$
 $B \rightarrow C\}$
Key = $\{A\}$



- ◆ R is not in BCNF
($B \rightarrow C$ but B is not superkey)
- ◆ Decompose R into: $R_1 = (A, B), R_2 = (B, C)$
 - ◆ R_1 and R_2 are in BCNF
 - ◆ Lossless-join decomposition

Example of BCNF Decomposition

- ◆ *class* (*course_id*, *title*, *dept_name*, *credits*, *sec_id*, *semester*, *year*, *building*, *room_number*, *capacity*, *time_slot_id*)
- ◆ Given functional dependencies:
 - ◆ *course_id* → *title*, *dept_name*, *credits*
 - ◆ *building*, *room_number* → *capacity*
 - ◆ *course_id*, *sec_id*, *semester*, *year* → *building*, *room_number*, *time_slot_id*
- ◆ Find a candidate key: (*course_id*, *sec_id*, *semester*, *year*)
- ◆ BCNF Decomposition:
 - ◆ *course_id* → *title*, *dept_name*, *credits* holds
 - ◆ but *course_id* is not a superkey
 - ◆ We replace *class* by:
 - ◆ **course**(*course_id*, *title*, *dept_name*, *credits*)
 - ◆ *class-1* (*course_id*, *sec_id*, *semester*, *year*, *building*, *room_number*, *capacity*, *time_slot_id*)

BCNF Decomposition (Cont.)

- ◇ *course* is in BCNF
 - ◇ How do we know this?
- ◇ *building, room_number* → *capacity* holds on *class-1*
 - ◇ but {*building, room_number*} is not a superkey for *class-1*
 - ◇ We replace *class-1* by:
 - ◆ *classroom* (*building, room_number, capacity*)
 - ◆ *section* (*course_id, sec_id, semester, year, building, room_number, time_slot_id*)
- ◇ *classroom* and *section* are in BCNF

Third Normal Form (3NF)

- ◆ A relation schema R is in **3NF** if for all:

$$\alpha \rightarrow \beta \text{ in } \mathcal{F}^+$$

at **least one** of the following holds:

- ◆ $\alpha \rightarrow \beta$ is trivial (i.e., $\beta \in \alpha$)
- ◆ α is a superkey for R
- ◆ Each attribute A in $\beta - \alpha$ is contained in a candidate key for R
(**NOTE:** each attribute may be in a different candidate key)

- ◆ A relation is in **BCNF** \rightarrow it is in 3NF

- ◆ Because BCNF requires one of the first two conditions

- ◆ The third condition is a relaxation of BCNF

- ◆ Ensure dependency preservation

3NF Example

- ◆ Relation *dept_advisor*:
 - ◆ *dept_advisor* (*s_ID*, *i_ID*, *dept_name*)
 $\mathcal{F} = \{s_ID, dept_name \rightarrow i_ID, i_ID \rightarrow dept_name\}$
 - ◆ Two candidate keys: (*s_ID*, *dept_name*), and (*i_ID*, *s_ID*)
 - ◆ *R* is in 3NF
 - ◆ *s_ID*, *dept_name* \rightarrow *i_ID* *s_ID*
 - ◇ *s_ID*, *dept_name* is a superkey
 - ◆ *i_ID* \rightarrow *dept_name*
 - ◇ *dept_name* is contained in a candidate key

3NF Decomposition Algorithm

let \mathcal{F}_c be a canonical cover for \mathcal{F}

$i := 0$

for each functional dependency $\alpha \rightarrow \beta$ in \mathcal{F}_c **do**

if no schema R_j , $1 \leq j \leq i$ contains $\alpha \beta$ **then**

$i := i + 1$

$R_i := \alpha \beta$

if no schema R_j , $1 \leq j \leq i$ contains a candidate key for R **then**

$i := i + 1$

$R_i :=$ any candidate key for R

/* remove redundant relations */

while (some schema R_j is contained in another schema R_k)

$R_j := R_i$



$i := i - 1$

return (R_1, R_2, \dots, R_i)

It ensures:

- ◆ each schema R_i is in 3NF
- ◆ decomposition is dependency preserving and lossless-join

3NF Decomposition: An Example

- ◆ Relation schema:

$\text{cust_banker_branch} = (\underline{\text{customer_id}}, \underline{\text{employee_id}}, \text{branch_name}, \text{type})$

- ◆ Given functional dependencies:

- $\text{customer_id}, \text{employee_id} \rightarrow \text{branch_name}, \text{type}$
- $\text{employee_id} \rightarrow \text{branch_name}$
- $\text{customer_id}, \text{branch_name} \rightarrow \text{employee_id}$

- ◆ We first compute a **canonical cover**

- ◆ branch_name is extraneous in the r.h.s. of the 1st dependency
- ◆ No other attribute is extraneous, so we get $\mathcal{F}_C =$

$\text{customer_id}, \text{employee_id} \rightarrow \text{type}$
 $\text{employee_id} \rightarrow \text{branch_name}$
 $\text{customer_id}, \text{branch_name} \rightarrow \text{employee_id}$

3NF Decompsition Example (Cont.)

- ◆ The **for** loop generates following 3NF schema:
 - $(customer_id, employee_id, type)$
 - $(\underline{employee_id}, branch_name)$
 - $(customer_id, branch_name, employee_id)$
- ◆ Observe that $(customer_id, employee_id, type)$ contains a candidate key of the original schema, so no further relation schema needs to be added
- ◆ At end of for loop, detect and delete schemas, such as $(\underline{employee_id}, branch_name)$, which are subsets of other schemas
 - ◆ result will not depend on the order in which FDs are considered
- ◆ The resultant **simplified 3NF schema is:**
 - $(customer_id, employee_id, type)$
 - $(customer_id, branch_name, employee_id)$

BCNF vs. 3NF

- ◆ We can **always** decompose a relation into a set of relations that are in **3NF** such that:
 - ◆ the decomposition is **lossless**
 - ◆ **the dependencies are preserved**
- ◆ We can always decompose a relation into a set of relations that are in **BCNF** such that:
 - ◆ the decomposition is **lossless**
 - ◆ but it may **not preserve dependencies**

BCNF vs. 3NF

- ◆ First, we try to satisfy all requirements:
 - ◆ BCNF
 - ◆ Lossless join
 - ◆ Dependency preservation
- ◆ If we cannot achieve this, we accept one of:
 - ◆ Lack of dependency preservation, or
 - ◆ Redundancy due to the use of 3NF

Appendix: BCNF and Dependency Preservation

It is not always possible to get a BCNF decomposition that is dependency preserving

- ◆ $R = (J, K, L)$
 $\mathcal{F} = \{ JK \rightarrow L$
 $L \rightarrow K \}$

Two candidate keys = JK and JL

- ◆ R is not in BCNF
- ◆ Any decomposition of R will fail to preserve

$$JK \rightarrow L$$

This implies that testing for $JK \rightarrow L$ requires a join

Appendix: Redundancy in 3NF

- ◆ There is some redundancy in this schema
- ◆ Example of problems due to redundancy in 3NF

- ◆ $R = (J, K, L)$

- $\mathcal{F} = \{JK \rightarrow L, L \rightarrow K\}$

| J | L | K |
|--------|-------|-------|
| j_1 | l_1 | k_1 |
| j_2 | l_1 | k_1 |
| j_3 | l_1 | k_1 |
| $null$ | l_2 | k_2 |

- repetition of information (e.g., the relationship l_1, k_1)
 - $(i_ID, dept_name)$
- need to use null values (e.g., to represent the relationship l_2, k_2 where there is no corresponding value for J).
 - $(i_ID, dept_name)$ if there is no separate relation mapping instructors to departments