Lecture 1
Introduction to Data Structures

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Some parts has been revised and indicated.

Our Roadmap

- Programming basics
- How to solve a problem? by an algorithm?
- How to analyze the running time of an algorithm?
- What are data structures?

Programming

- This is not a programming course
- We will introduce some basic programming concepts (in the Java language)
  - so that you can do practices on computers

- Building blocks in a Java program:
  - Variables, constants, operators (e.g., `x,y,3,5,+,=`)
  - → Expressions (e.g., `x+y`)
  - → Statements (e.g., `int x=3;`)
  - → Methods (e.g., `main`)
  - → Classes (e.g., `Test`)
- A *variable* can be used to store a value
  - Primitive type (e.g., `int,float,boolean`) or object type (e.g., `String`)
- The execution starts at the *main* method

Write a program in the file "Test.java"

```
public class Test {
   public static void main(String args[]) {
      int x = 3;
      int y = 5;
      String msg = "hello";
      System.out.println( x+y );
      System.out.println( msg );
   }
}
```

Compile a program

```
javac Test.java
```

Run a program

```
java Test

8
hello
```

Java: Array

- *Array*: for storing multiple variables (of the same type)
  - Create an array A with n items

```
int[] A = new int[n];
```

  - $A[i]$ denotes the variable at position i of array A
    - In Java, the range of i is from 0 to $n - 1$
- Examples
  - `int[] A = new int[7];` // create an array
  - `x = A[2];` // copy value $A[2]$ to variable x
  - `A[3] = 5;` // store value 5 to $A[3]$

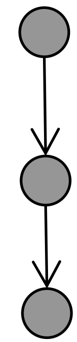
Position	0	1	2	3	4	5	6
Value	6	2	7	8	4	9	1

x: ___

5

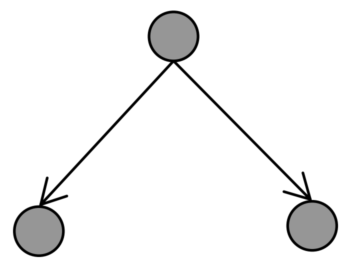
Control Flow

- **Sequence** (step-by-step)
  - A sequence of statements implies the execution ordering
  - Each statement is a ‘step’, e.g.,
    - Assignment statement: `c = a * b;`
    - Call statement: `System.out.println("hello");`



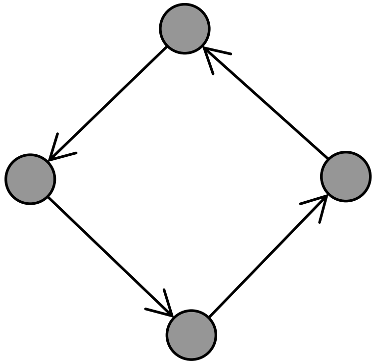
```
int a,b,c,d;
a = 3;
b = 5;
c = a * b;
d = a + b;
```

- **Selection** (choose a branch)
  - *if* statement, *switch* statement
  - *Example* of *if* statement:
    - Evaluate the condition `x>y`
    - Condition is true → execute `v=x;`
    - Condition is false → execute `v=y;`



```
if (x>y)
    v=x;
else
    v=y;
```

Control Flow



- **Loop**: repeat the execution of the loop body (a sequence of statements) for multiple times

- Example: print "hello" 5 times

```
int i = 5;
while (i>0) {
    System.out.println("hello");
    i--;
}
```

- *while*-loop
  - Evaluate `<cond>` **before** the loop body; the loop stops when `<cond>` is false

```
for (int i=0; i<5; i++) {
    System.out.println("hello");
}
```

- *for*-loop: counter-based
  - Execute `<init>` once before the loop starts
  - Evaluate `<cond>` **before** the loop body; the loop stops when `<cond>` is false
  - Execute `<update>` **after** the loop body

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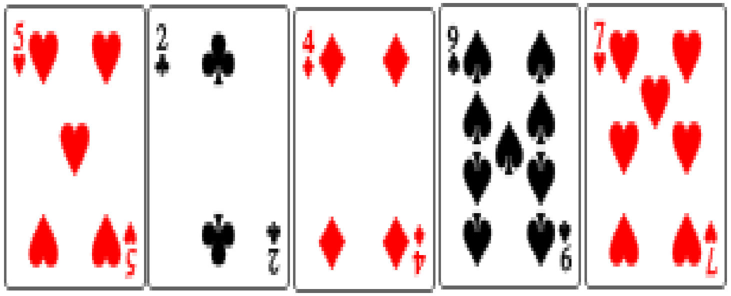
How to solve a problem?

- *A problem for human (in the real world)*:
  - Sort poker cards on hand



- Input:
  - A list of cards
- Procedure:
  - 1. **Put** all cards on the left hand
  - 2. **Pick** the smallest card from the left hand
  - 3. **Move** that card to the right hand
  - 4. Repeat Steps 2-3 until the left hand is empty
- How to solve a problem by using a program?
  - Use variables, operators, statements, ……

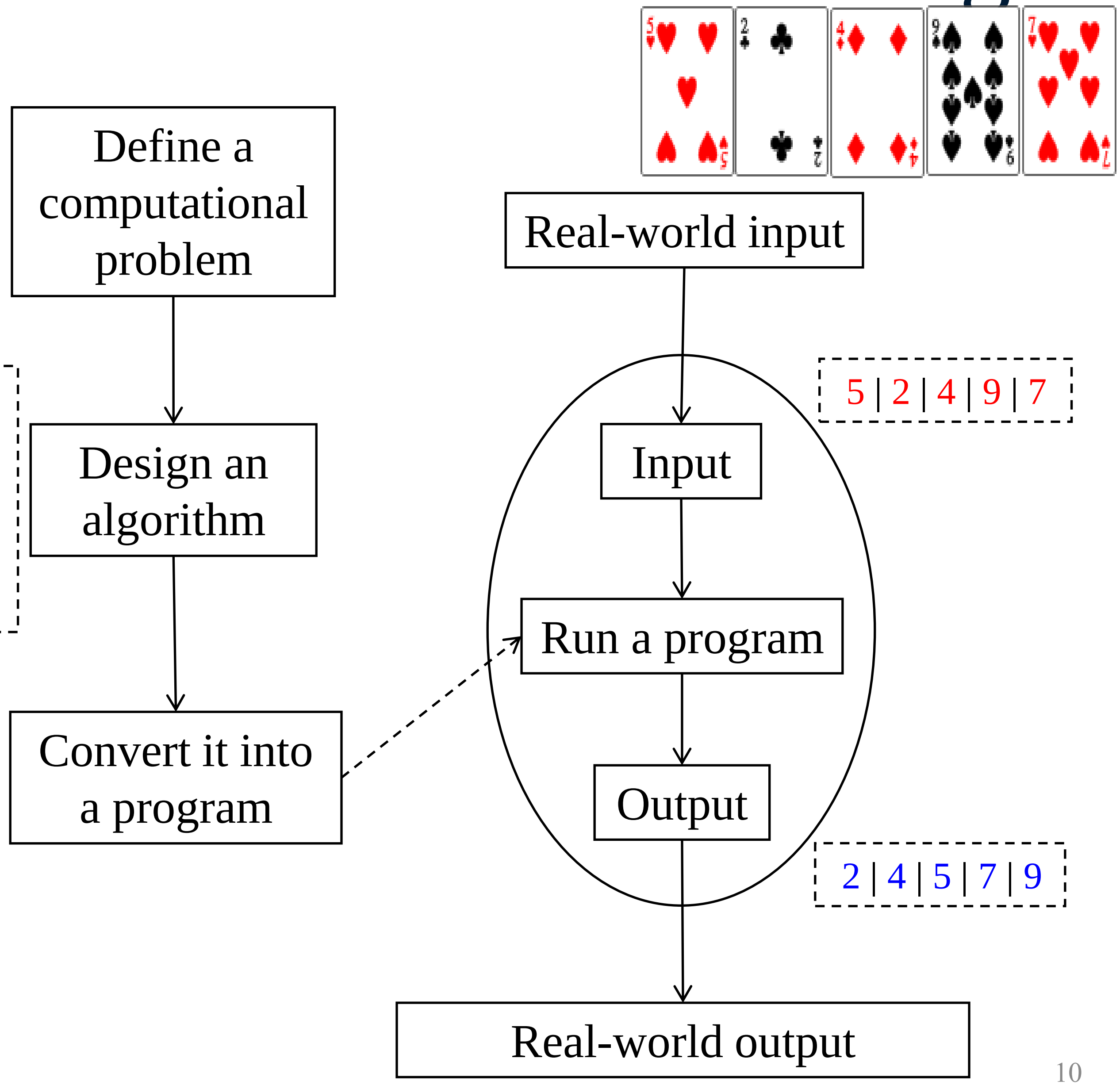
Algorithms for Problem Solving

Input: a sequence A of n integers
Output: a sequence A of n integers such that ……

<ins>Selection-Sort (…)</ins>

1. for integer $i \leftarrow 0$ to $n{-}2$
2. $k \leftarrow i$
3. for integer $j \leftarrow i{+}1$ to $n{-}1$
4.   if $A[k] > A[j]$ then
   ……

```
void Selection-Sort ( … )
int i, j, k, temp ;
int n = A.length;
for ( i=0 ; i<n–1; i++ ) {
    …
    for ( j=i+1 ; j<n; j++ )
        …
    …
}
```



Computational problem

- *Example*: the **sorting** problem
  - Input: a *sequence* A of n integers
  - Output: a sequence A' of n integers such that
    - A' is an ***increasing*** sequence, and
    - A and A' contain the same integers.

5 | 2 | 4 | 9 | 7

2 | 4 | 5 | 7 | 9

Algorithms

- *Algorithm*: a well defined sequence of steps for solving a computational problem
  - It always produces the *correct output*
  - It uses *well-defined* steps or operations
  - It finishes in *finite time*
- It is written for human readers
  - Like programming, we can use variables and data structures, control flow, call statement
  - We can also use natural language provided that it is clear

Selection-Sort (Array $A[0..n-1]$)

```
for integer i ← 0 to n–2
    k ← i
    for integer j ← i+1 to n–1
        if  A[k] > A[j]  then
            k ← j
    swap A[i] and A[k]
```

Algorithm vs. Program

- An algorithm can be implemented as a program in any language (e.g., C, C++, Java, Python)
- An algorithm is more *readable* than a program

Algorithm

Selection-Sort (Array $A[0..n-1]$)
1. for integer $i \leftarrow 0$ to $n-2$
2. $k \leftarrow i$
3. for integer $j \leftarrow i+1$ to $n-1$
4. if $A[k] > A[j]$ then
5. $k \leftarrow j$
6. swap $A[i]$ and $A[k]$

Java program

```
// K.K.F. Yuen
void SelectionSort(int[] A){
int i, j, k, temp;
int n = A.length;
    for (i = 0; i<n-1; i ++){
        k = i ;
        for ( j=i+1 ; j<n; j++ )
        if  ( A[k] > A[j] )
            k = j ;
        temp = A[i] ;
        A[i] = A[k] ;
        A[k] = temp ;

    }
}
```

Algorithms: Running Steps

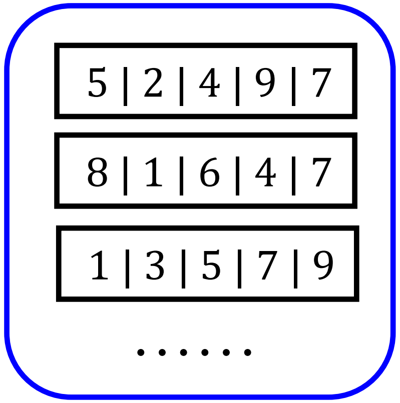
- Run an algorithm manually on a sample input, then draw *running steps*
  - E.g., show the content of important variables in each iteration
  - Useful for understanding how the algorithm works

<u>Selection-Sort (Array $A[0..n-1]$)</u>

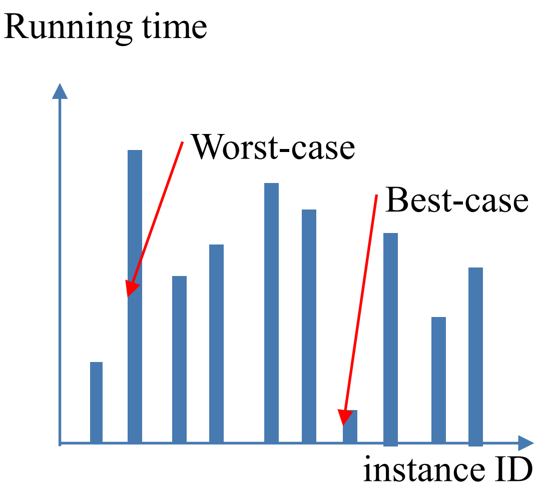
```
for integer i ← 0 to n–2
    k ← i
    for integer j ← i+1 to n–1
        if  A[k] > A[j]  then
            k ← j
    swap A[i] and A[k]
```

input	5	2	4	9	7
$i = 1$	2	5	4	9	7
$i = 2$	2	4	5	9	7
$i = 3$	2	4	5	9	7
$i = 4$	2	4	5	7	9
$i = 5$	2	4	5	7	9

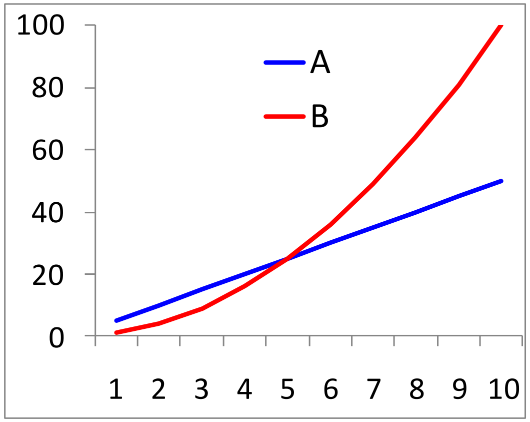
Algorithmic Analysis



- There are many possible **input instances**
- Is your algorithm always **correct**?
  - Not enough even if you have tested your algorithm on many instances
    - Will your algorithm fail on some other instance?
  - Need a **correctness proof** to show that:
    - "For every possible input instance, the algorithm produces the correct output"



- **How fast** is your algorithm?
  - Consider all instances of the same size n and their running time
  - Estimate the worst-case running time as a function of the input size n
  - What's the trend of the worst-case running time?



Our Roadmap

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- How to analyze the running time of an algorithm?
- What are data structures?

Why do we analyze the running time of an algorithm?

- The same problem (e.g., sorting) can be solved by several different algorithms

- It is time consuming to convert all algorithms to programs

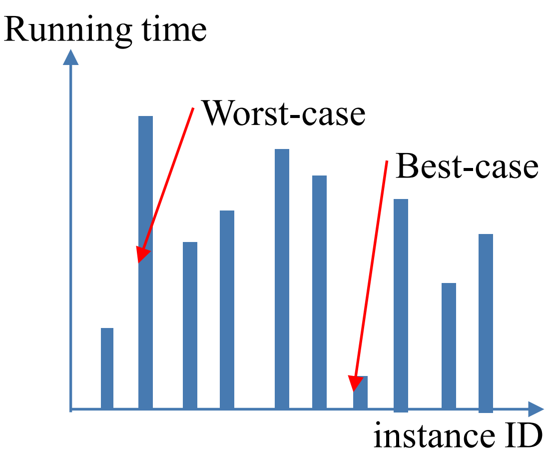
- We just want to choose the “fastest” algorithm and then convert it into a program

- Understand the scalability of a program.

  E.g., if the input size doubles, how much would its running time increase?

How to analyze the running time of an algorithm?

- Assumptions
  - #1. Each basic step takes constant time
    - E.g., add two integers: c_{add}
      compare two numbers: $c_{compare}$
      copy a value to a variable: c_{copy}
  - #2. The memory is large enough to store all input/intermediate/output data
- Find the (worst-case) frequency and cost of executing each line, then sum up the total cost



How to estimate running time?

Search (Array $A[0..n-1]$, Key k)	*cost*	*frequency*
1. for integer $i \leftarrow 0$ to $n-1$	c_1	at most n
2. if $A[i] = k$	c_2	at most n
3. return i	c_3	at most 1
4. return -1	c_4	at most 1

- What is the frequency of executing Line 1?
  - It depends on which element $A[i]$ is equal to k
  - Line 1 is executed **at most** n times
- What is the frequency of executing Line 2?
- ……
- Worst-case running time of the algorithm

$$= c_1 * n + c_2 * n + c_3 * 1 + c_4 * 1$$
$$= (c_1 + c_2) n + c_3 + c_4$$

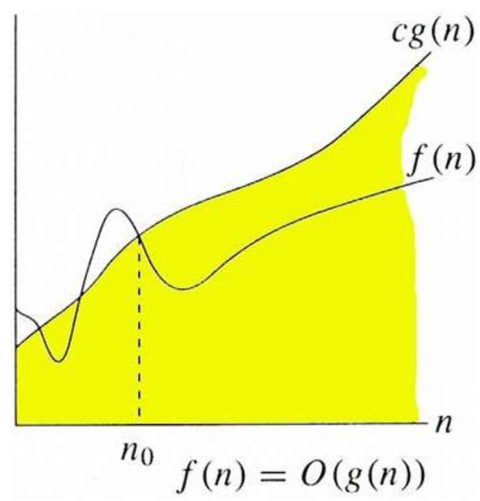
- This expression contains many constants
- How to simplify it?

Asymptotic Notation

- How to describe the trend of running time in a concise way?
- Asymptotic **upper-bound**, **O**-notation
  - We can write $\boldsymbol{f(n)} = \mathbf{O}(\boldsymbol{g(n)})$ if

    we can find positive constants c, n_0 such that

    $0 \leq f(n) \leq c\, g(n)$ for all $n \geq n_0$



- Is $(5n^2 + 3n) = O(n^2)$?
  - YES. choose c=6 and n_0=10

    [note: other choices also possible]
- Is $(5n^2 + 3n) = O(n^3)$?
  - YES, *but we prefer to use a tighter expression*

Simplification Rules of O-Notation

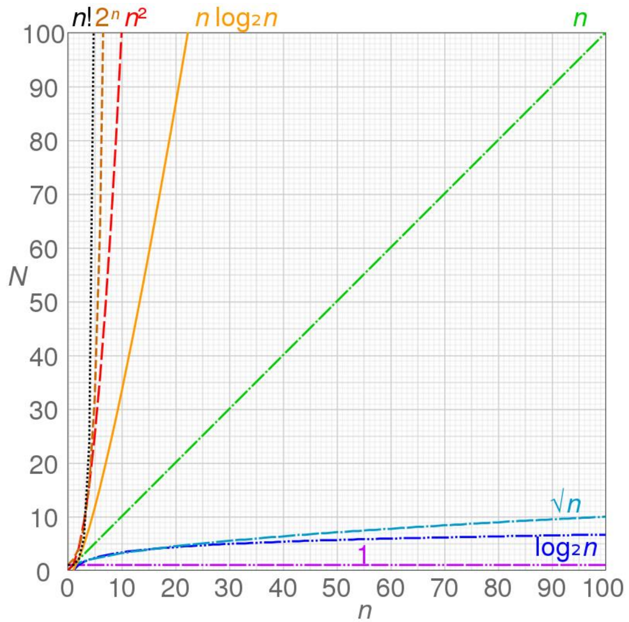
These rules provide shortcuts for showing that $f(n) = O(g(n))$

- Ignore constant factors
  - E.g., $3 = O(1)$
  - E.g., $3n = O(n)$
- Combine fixed number of terms with same complexity
  - E.g., $O(n) + O(n) = O(n)$ $\quad c_1 n + c_2 n = (c_1+c_2)\, n$
  - E.g., $O(n) + O(n) + O(n) = O(n)$
- Polynomial
  - Just use the highest-degree term. Why?
  - E.g., $5n^2 + 3n = O(n^2)$
  - E.g., $2n^3 + 5n^2 + 3n = O(n^3)$

Asymptotic running time of some algorithms

Complexity		*Algorithm*
O(1)	Constant time	Compare two numbers
O(log n)	Logarithmic	Binary search (on a sorted array)
O(n)	Linear time	Linear search (on an unsorted array)
O(n log n)		Merge-sort
O(n^2)	Quadratic	Selection-sort

- The trend matters
  - We care about the performance at *large input size*
  - Constant factors (e.g., c) do not affect *the order of growth*



Asymptotic running time: Example 1

Search (Array $A[0..n-1]$, Key k)

1. for integer $i \leftarrow 0$ to $n-1$
2. if $A[i] = k$
3. return i
4. return –1

5 | 2 | 4 | 9 | 7

- Let's analyze the asymptotic running time of this algorithm
  - Line 1: at most n basic steps = $O(n)$
  - Line 2: at most $1*n$ basic steps = $O(n)$
  - Lines 3 and 4: $O(1)$
- Running time of algorithm $T(n)$

$= O(n) + O(n) + O(1)$

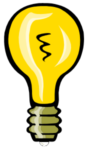
$= O(n)$

Asymptotic running time: Example 2

```
Selection-Sort ( Array A[0..n–1])
1. for integer i ← 0 to n–2
2.       k ← i
3.       for integer j ← i+1 to n–1
4.             if   A[k] > A[j]  then
5.                   k ← j
6.       swap A[i] and A[k]
```

5 | 2 | 4 | 9 | 7

- Line 1: $n - 1$ basic steps $= O(n)$
- Line 2: $1*(n - 1)$ basic steps $= O(n)$
- Lines 3 to 5:
  - How many times can we execute Line 3 at most?
  - For each given value of i, what is the possible range of j?



Asymptotic running time: Example 2

- Worst-case analysis of Line 3:
  - Values of j in that loop: 1 to n–1, 2 to n–1, …, n–1 to n–1
  - Number of iterations: $n-1, n-2, n-3, \ldots, 1$

$$\Sigma_{i=1..n-1} (n-i) = (n-1) + (n-2) + \ldots + 1 = (n-1)*n/2 = O(n^2)$$

- Running time of algorithm $T(n)$:
  - Line 1. $O(n)$
  - Lines 2, 6. same as Line 1
  - Line 3. $O(n^2)$
  - Lines 4, 5. same as Line 3
  - Thus, we have: $\mathbf{T(n)} = 3*O(n) + 3*O(n^2) = O(n^2)$

Formula:

$$\sum_{i=1}^{n} i = 1 + 2 + \ldots + n = \frac{n(n+1)}{2}$$

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Data Structures

- What are the data structures for human?
  - Used in libraries, books, clinics, companies, ……

- How about the data structures for computers?

Data Objects

- Examples of data objects
  - Employees in payroll application
  - Tasks in the scheduler of OS
  - Shopping cart items in online shopping
  - In any application that you can think of ……

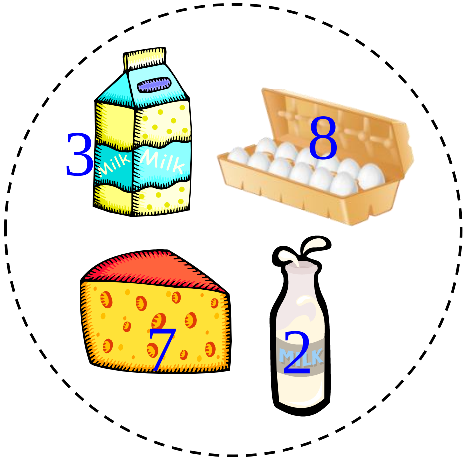
- A data object has attributes
  - E.g., the attributes of shopping cart items are:

    `product_id, product_name, price,` ……
- The search key depends on the application, e.g.,
  - `product_id` for a shopping cart application

Data Structures

- Let S be a set of items, and x be a search key
  - A key is a number, e.g., product id

set



- Useful operations on a set S
  - Search(S, x): search whether x appears in S
  - Insert(S, x): insert item x into S
  - Delete(S, x): remove item x from S

search key

- *Data structure*:
  - A way of organizing data objects for efficient usage
  - Building blocks for designing algorithms

The problems in Lectures 2-5

- How to solve these problems quickly?
  - *Sort* a set of items
  - *Search* an item from a set of items
  - *Insert / Delete* an item from a set of items
- Question 1
  - Selection-Sort takes $O(n^2)$ time
  - Is there a faster sorting algorithm?
- Question 2
  - Is an array good enough for searching, insertion and deletion?

Is Array Good Enough?

- Search a key in an unsorted array (of length n)
- Linear search algorithm
  - Running time: O(n)
- Example #1: search k=21
  - Check the case i=0
  - Return **0**
- Example #2: search k=17
  - Check the cases i=0,1,2,3,4,5,6
  - Return **6**
- How to search faster?

21	12	8	3	35	1	17

Search (Array $A[0..n\text{-}1]$, Key k)

1. for integer $i \leftarrow 0$ to n–1
2. if $A[i] = k$
3. return i
4. return –1

Is Array Good Enough?

- Search in a **sorted** array (of length n)
- Binary search algorithm
  - Running time: O(log n)
- *Idea*:
  - Compare k with the middle item $A[mid]$
  - Is k in the left sub-array or in the right sub-array?

1	3	8	12	17	23	35

BinarySearch (Array *A*, Integers *low*, *high*, Key *k*)

```
mid ← ⌊ (low+high)/2 ⌋
if   A[mid] = k
      return mid
else if  low = high
      return –1
if  k < A[mid]
      return BinarySearch(A, low, mid–1, k)
else
      return BinarySearch(A, mid+1, high, k)
```

Initial call: BinarySearch (*A*, 0, *n*–1, *k*)

Example: find the key "35" from this array *A*

1 | 3 | 8 | 12 | 17 | 23 | 35

BinarySearch (Array *A*, Integers *low*, *high*, Key *k*)

1. $mid \leftarrow \lfloor (low+high)/2 \rfloor$
2. if $A[mid] = k$
3. return *mid*
4. else if $low = high$
5. return –1
6. if $k < A[mid]$
7. return BinarySearch(*A*, *low*, *mid*–1, *k*)
8. else
9. return BinarySearch(*A*, *mid*+1, *high*, *k*)

Initial call: BinarySearch (*A*, 0, *n-1*, *k*)

BinarySearch (*A*, 0, 6, **35**), *mid* = 3

1 | 3 | 8 | 12 | 17 | 23 | 35

BinarySearch (*A*, 4, 6, **35**), *mid* = 5

1 | 3 | 8 | 12 | 17 | 23 | 35

BinarySearch (*A*, 6, 6, **35**), *mid* = 6

1 | 3 | 8 | 12 | 17 | 23 | 35

Key found, return 6

Is Array Good Enough?

- How about insertion/deletion?

- Can an unsorted array support fast insertion/deletion?

  21 | 12 | 8 | 3 | 35 | 1 | 17

- Can a sorted array support fast insertion/deletion?

  1 | 3 | 8 | 12 | 17 | 23 | 35

Data Structures in Lectures 2-5

- Learn the **properties** of data structures
  - E.g., stack's LIFO property, queue's FIFO property, growable structure like linked list
- Learn the **operations** of data structures
  - They support different operations, and with different running time
- You may play with them in https://visualgo.net/en
- What's the most suitable data structure to solve a particular problem?
  - It depends on the properties and operations most frequently used in your algorithm

Stack (*Lecture 2*)
Queue (*Lecture 2*)
Linked List (*Lecture 2*)
Tree (*Lecture 3*)
Heap (*Lecture 4*)
……

Java Programmers

- At least know how to use the data structures and algorithms in the `java.util` library (*https://docs.oracle.com/javase/8/docs/api/java/util/package-summary.html*)
  - `Stack, Queue, LinkedList, PriorityQueue,` ……
  - `Arrays.sort,  Arrays.binarySearch,` ……

- We hope you can also understand the "magic" behind them

Summary

- Basic concepts in Java, algorithm, asymptotic running time, data structure

- Please read Chapters 1, 4.2.1-4.2.6 in the book “*Data Structures and Algorithms in Java*”

- *Next lecture*:

  linear data structures (stack, queue, linked list)