Discussion 1

Course website: `https://bu-cs332.github.io`
PS1 out yesterday, due next Tuesday before lecture... Start early!

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Problems

1. (**Review of basic logic and set notation.**)

   (a) Negate the following statement: "*The first DFA has an even number of states, but the second DFA does not.*"

   (b) Negate the following statement: "*Every CS major who is awesome is enrolled in CS 332.*"

   (c) Write the contrapositive of the following statement: "*Your homework assignment is not graded if you have not handed in a signed copy of the Collaboration policy.*"

   (d) If $A = \{1, \{2, 3\}, \varepsilon\}$, what is the power set of A?

   (e) Choose five distinct integers between 0 and 7, inclusive. Show that there exists a pair of them which add up to 7.

2. (**Induction.**) Suppose we are trying to divide a class of n students into groups of either 4 or 5 students.

   (a) Find an error in the following proof that a class with $n \geq 8$ students can be divided into groups of 4 or 5. That is, identify the first incorrect sentence and explain what went wrong.

   Proof. The proof is by strong induction. Let $P(n)$ be the proposition that a class with n students can be divided into teams of 4 or 5.

   Base case: We prove that $P(n)$ is true for $n = 8, 9$, and 10 by showing how to break classes of these sizes into groups of 4 or 5 students:

$$\begin{aligned} 8 &= 4+4; \\ 9 &= 4+5; \\ 10 &= 5+5. \end{aligned}$$

   Induction hypothesis: Next, we must show that $P(8), ..., P(n)$ imply $P(n+1)$ for all $n \geq 10$. That is, we assume that $P(8), ..., P(n)$ are all true and show how to divide up a class of $n+1$ students into groups of 4 or 5. We first form one group of 4 students. Then we can divide the remaining $n-3$ students into groups of 4 or 5 by the assumption $P(n-3)$. This proves $P(n+1)$, and so the claim holds by induction. □

   (b) Provide a correct strong induction proof that a class with $n \geq 12$ students can be divided into groups of 4 or 5.

3. We call string x is a *prefix* of a string y if a string z exists where $xz = y$, and that x is a *proper prefix* of y if in addition $x \neq y$. Show that if a language A is regular, then $NOPREFIX(A) = \{w \in A \mid \text{no proper prefix of } w \text{ is a member of } A\}$ is also a regular language.