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A vehicle routing problem with movement synchronization of drones, sidewalk robots, or foot-walkers

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Abstract

The vehicle routing problem (VRP) and its variants have many city logistics applications, such as goods delivery. The VRP extension with movement synchronization (VRPMS) has potential applications of drone and robot technologies to assist with the delivery of parcels. VRPMS seeks the optimal route for a set of composite resources, e.g. delivery van with drones, or delivery van with sidewalk robots. This paper proposes an exact formulation of the problem, and a metaheuristic approach to solve larger instances of the VRPMS in order to assess the economic benefits of the different technologies. It is shown that with the current physical constraints of drone technology, assisted delivery with drones has some challenges because of its capacity. Sidewalk robots and walkers, however, do contribute a cost savings compared to truck deliveries. As the technology matures, the presented metaheuristic approach can be used to evaluate improved economic benefits and cost benefit ratios.

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Keywords: freight; logistics; drone; sidewalk robot; vehicle routing problem; metaheuristic

1. Introduction

Increasing e-commerce demand has led to more parcel deliveries at high and irregular frequencies requiring courier companies to optimize capacity dynamically and be innovative with their assets and operations. Strategies include customer pick up locations; mobile depots; underground passageways for deliveries; and composite delivery resources. In the latter strategy, additional resources assist the delivery truck, for example, additional delivery persons, drones or delivery robots. The composite delivery concept beckons a VRP extension suited to schedule additional delivery resources. This paper presents an exact formation of the vehicle routing problem with movement

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synchronization (VRPMS) and a metaheuristic solution algorithm to solve for large instances.

Two types of vehicle in the VRPMS are heavy and light resources. Heavy is an independent vehicle that can move on its own through time and space. Light is a special vehicle (or person) that assists the heavy en route by servicing selected customers. The light can be transported by the heavy and move in time and space on its own, but has limited range and capacity compared to the heavy. Depending on the technology, parking and congestion, the average travel time could be faster for the light. Each time the light rejoins the heavy, it can be instantaneously resupplied with the demand of following customer(s). Both heavy and light must start and end at the depot as one unit. The objective is to determine the least expensive set of routes for the heavy and light to service all the customers.

2. Literature review

The VRP is usually formulated as an integer programming problem, and only small sized instances are solved with exact methods. For large problems, heuristic methods approximate the solution but with much faster computation time. Commonly used heuristic and metaheuristic algorithms include the Clarke and Wright's savings algorithm, sweep algorithm, Christofides-Mingozzi-Toth two-phase algorithm, and tabu search (Laporte, 1992).

There are several classic extensions of the VRP: capacity, time windows, multiple vehicle types, pickup and deliveries, backhauls, multiple depots, and dynamic information. Literature reviews such as Gendreau et al. (2008) and Kumar and Panneerselvam (2012) summarize the algorithms used to tackle these VRP variants. More recent research recommends incorporating metaheuristic techniques to further enhance heuristic solutions to tackle the more complex variants of the VRP (Lin, 2008; Kumar and Panneerselvam, 2012). Metaheuristics are techniques which use interactions between local improvements and high-level strategies to escape local optima (Glover, 2003). Common metaheuristics include ant colony optimization, genetic algorithm, simulated annealing, and tabu search.

The extension to the VRP that pertains to assisted delivery is synchronization, and Drexl (2012) identified five aspects of synchronization: task, operation, movement, load, and resource. This paper focuses on the synchronization of vehicle movements at the depot or en route. Two interrelated decisions with movement synchronization are where and when should the vehicles join and separate. The following section reviews existing research on such problems.

Lin (2008, 2011) formulated a pick-up and delivery problem with heavy and light resources with the ability to travel as a composite unit. Using two types of delivery resources is similar to the VRPMS problem, and enabling coordination of these two resources has benefits over independent operations. Lin's formulation contains assumptions that are relaxed in the VRPMS, including: an incapacitated problem; a maximum number of lights each heavy could carry; and clustering customers with similar pickup times.

More recently, researchers investigated movement synchronization involving drones. In these papers, drones make single deliveries within a limited service area. Murray and Chu (2015) examined two scenarios. The first arises when the depot is located within range to directly service customers from the depot. Their heuristic is capable of solving instances with 10 to 20 customers. The second scenario is an extension of the classical TSP, referred to as TSP-D (traveling salesman problem with drone). Ha *et al.* (2018) proposed two heuristics to solve a variant of this TSP-D and Bouman *et al.* (2018) presented an exact solution with dynamic programming capable of solving problems with 20 nodes. While Sacremento *et al.* (2019) relaxed the single vehicle constraint and proposed an Adaptive Large Neighborhood Search metaheuristic to solve this variant.

Di Puglia Pugliese and Guerriero (2017) relaxed the single vehicle and drone limitation. They formulated this extension with MIP and solved small instances of 5 to 10 customers. Completion time were found to be lower when drones are included as part of the fleet. They also confirmed that the vehicle route length decreases with drones. The framework presented in that paper is used as the starting point for the MIP model in this paper. Similarly, Wang and Sheu (2019) formulated a MIP with a branch-and-cut algorithm to solve problems with 8 and 13 customers.

3. Methodology

3.1. Mixed integer programming formulation

Nomenclature:

Constants

C_1, C_2	Cost to move one unit of length for heavy and light resources, respectively					
d_{ij} , $ar{d}_{ij}$	The distance between customer i, j for heavy and light resources, respectively					
t_{ij} , $ar{t}_{ij}$	The time between customer <i>i</i> , <i>j</i> for heavy and light resources, respectively					
s_i, \bar{s}_i	The time to service customer <i>i</i> for heavy and light resources, respectively					
T	The maximum time the light resource can wait for the heavy resource					
E	The maximum distance the light resource can continuously travel during one sub-route					
N	A set of customer nodes in the instance $[1, n]$, where n is the number of customers					
V_L , V_R	A set of nodes the light resource can launch from $[0, n]$, and return to $[1, n + 1]$					
0 and $n+1$	The index of the depot, and a copy of depot					
K,D	A set of heavy resources $[1, k]$, and light resources $[1, d]$					
S	A set of sub-routes [1, s]					
$v_i - w_i$	The service time window for customer <i>i</i>					
Decision Variables						
$x_{i,i}^k$	1, if resource k uses arc (i, j) and services customers i, j ; 0, otherwise					
y_{ij}^{kds}	1, if resource d of resource k on sub-route s uses arc (i,j) and services customers i,j ; 0,					
y_{ij}	otherwise					
l_{ij}^{kds}	1, if resource d of resource k on sub-route s launches from node i and services customer j ; 0,					
	otherwise					
r_{ij}^{kds}	1, if resource d of resource k on sub-route s services customer i and returnes to node j ; 0,					
	otherwise					
u_i^k	An integer variable, position of customer i in the route of resource k ; 0, if not in the route					
u_i^{kds}	An integer variable, position of customer i in the route of resource d of resource k in sub-route					
	s; 0, if it is not in the route					
a_i^k	A continuous variable representing the arrival time of resource k at customer i					
a_i^{kd}	A continuous variable representing arrival time of resource d of resource k at customer i					

Mixed integer programming model:

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Min. \ C_1 \sum_{i \in V_L} \sum_{j \in V_R} \sum_{k \in K} d_{ij} x_{ij}^k + C_2 \sum_{i \in V_L} \sum_{j \in V_R} \sum_{k \in K} \sum_{d \in D} \sum_{s \in S} \bar{d}_{ij} \left( y_{ij}^{kds} + l_{ij}^{kds} + r_{ij}^{kds} \right)
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         Subject to:
         \begin{array}{l} \overline{\sum_{j \in N} x_{0j}^k - \sum_{i \in N} x_{i,n+1}^k = 0}; \forall k \in K \\ \overline{\sum_{i \in V_L} x_{ih}^k - \sum_{j \in V_R} x_{hj}^k = 0}; \forall h \in N, \forall k \in K \\ \overline{\sum_{i \in V_L} y_{ih}^{kds} - \sum_{j \in V_R} y_{hj}^{kds} + \sum_{i \in V_L} l_{ih}^{kds} - \sum_{j \in V_R} r_{hj}^{kds} = 0}; \forall h \in N, \forall k \in K, \forall d \in D, \forall s \in S \end{array}
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         \sum_{j \in N} x_{0j}^{k} \leq 1; \forall k \in K
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      \sum_{i \in V_L} \sum_{k \in K} x_{ij}^k + \sum_{i \in V_L} \sum_{k \in K} \sum_{d \in D} \sum_{s \in S} y_{ij}^{kds} + \sum_{i \in V_L} \sum_{k \in K} \sum_{d \in D} \sum_{s \in S} l_{ij}^{kds} = 1; \forall j \in N
\sum_{j \in V_R} \sum_{k \in K} x_{ij}^k + \sum_{j \in V_R} \sum_{k \in K} \sum_{d \in D} \sum_{s \in S} y_{ij}^{kds} + \sum_{j \in V_R} \sum_{k \in K} \sum_{d \in D} \sum_{s \in S} r_{ij}^{kds} = 1; \forall i \in N
\sum_{i \in \overline{N}} r_{ih}^{kds} \leq \sum_{i \in N} x_{ih}^k; \forall h \in V_R, \forall k \in K, \forall d \in D, \forall s \in S
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   \begin{split} & \sum_{j \in \mathbb{N}} l_{hj}^{kds} \leq \sum_{j \in \mathbb{N}} x_{ih}, \forall h \in V_R, \forall k \in \mathcal{K}, \forall d \in \mathcal{D}, \forall s \in \mathcal{S} \\ & \sum_{j \in \mathbb{N}} l_{hj}^{kds} \leq \sum_{j \in \mathbb{N}} x_{hj}^{kds}; \forall h \in V_L, \forall k \in \mathcal{K}, \forall d \in \mathcal{D}, \forall s \in \mathcal{S} \\ & \sum_{j \in \mathbb{N}_R} l_{0j}^{kds} = \sum_{i \in \mathbb{N}_L} r_{i,n+1}^{kds} = 0; \forall k \in \mathcal{K}, \forall d \in \mathcal{D}, \forall s \in \mathcal{S} \\ & \sum_{i \in \mathbb{N}_L} l_{i,n+1}^{kds} = \sum_{j \in \mathbb{N}_R} r_{0j}^{kds} = 0; \forall k \in \mathcal{K}, \forall d \in \mathcal{D}, \forall s \in \mathcal{S} \\ & u_j^k \geq u_i^k + 1 - (n+1)(1-x_{ij}^k); \forall k \in \mathcal{K}, \forall i \in \mathbb{N}, \forall j \in \mathcal{N}_R \\ & \sum_{k \neq k} l_{i,k}^{kds} \leq l_{i,k}^{kds} \leq l_{i,k}^{kds} \langle \mathcal{N}_{i,k}^{kds} \rangle \langle \mathcal{N}_{i,k}^{kds} \rangle
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\begin{array}{l} u_{j}^{k} \geq u_{i}^{k} + 1 - (n+1)(1-x_{ij}^{k}); \forall k \epsilon K, \forall i \epsilon N, \forall j \epsilon V_{R} \\ u_{j}^{kds} \geq u_{i}^{kds} + 1 - (n)(1-y_{ij}^{kds}); \forall k \epsilon K, \forall d \epsilon D, \forall s \epsilon S, \forall i \epsilon N, \forall j \epsilon V_{R} \\ u_{j}^{kds} \geq u_{i}^{kds} + 1 - (n)(1-l_{ij}^{kds}); \forall k \epsilon K, \forall d \epsilon D, \forall s \epsilon S, \forall i \epsilon V_{L}, \forall j \epsilon N \\ u_{j}^{kds} \geq u_{i}^{kds} + 1 - (n)(1-r_{ij}^{kds}); \forall k \epsilon K, \forall d \epsilon D, \forall s \epsilon S, \forall i \epsilon N, \forall j \epsilon V_{R} \\ \sum_{i \epsilon V_{L}} \sum_{j \epsilon N} l_{ij}^{kds} = \sum_{i \epsilon N} \sum_{j \epsilon V_{R}} r_{ij}^{kds} \leq 1; \forall k \epsilon K, \forall d \epsilon D, \forall s \epsilon S \\ \sum_{j \epsilon V_{R}} y_{hj}^{kds} \leq \sum_{i \epsilon V_{L}} l_{ih}^{kds}; \forall h \epsilon N, \forall k \epsilon K, \forall d \epsilon D, \forall s \epsilon S \\ \sum_{i \epsilon V_{L}} y_{ih}^{kds} \leq \sum_{j \epsilon V_{R}} r_{hj}^{kds}; \forall h \epsilon N, \forall k \epsilon K, \forall d \epsilon D, \forall s \epsilon S \\ M(x_{ij}^{k} - 1) + a_{i}^{k} + s_{i} + t_{ij} \leq a_{j}^{k}; \forall k \epsilon K, \forall d \epsilon D, \forall i \epsilon V_{L}, \forall j \epsilon V_{R} \\ M(y_{ij}^{kds} - 1) + a_{i}^{k} + \bar{s}_{i} + \bar{t}_{ij} \leq a_{j}^{kd}; \forall k \epsilon K, \forall d \epsilon D, \forall s \epsilon S, \forall i \epsilon V_{L}, \forall j \epsilon V_{R} \\ M(l_{ij}^{kds} - 1) + a_{i}^{k} + \bar{t}_{ij} \leq a_{j}^{kd}; \forall k \epsilon K, \forall d \epsilon D, \forall s \epsilon S, \forall i \epsilon V_{L}, \forall j \epsilon V_{R} \\ M(r_{ij}^{kds} - 1) + a_{i}^{kd} + \bar{s}_{i} + \bar{t}_{ij} \leq a_{j}^{k} + s_{j}; \forall k \epsilon K, \forall d \epsilon D, \forall s \epsilon S, \forall i \epsilon V_{L}, \forall j \epsilon V_{R} \\ M(r_{ij}^{kds} - 1) + a_{i}^{kd} + \bar{s}_{i} + \bar{t}_{ij} \leq a_{j}^{k} + s_{j}; \forall k \epsilon K, \forall d \epsilon D, \forall s \epsilon S, \forall i \epsilon V_{L}, \forall j \epsilon V_{R} \\ M(r_{ij}^{kds} - 1) + a_{i}^{kd} + \bar{s}_{i} + \bar{t}_{ij} \leq a_{j}^{k} + s_{j}; \forall k \epsilon K, \forall d \epsilon D, \forall s \epsilon S, \forall i \epsilon V_{L}, \forall j \epsilon V_{R} \\ M(r_{ij}^{kds} - 1) + a_{i}^{kd} + \bar{s}_{i} + \bar{t}_{ij} \leq a_{j}^{k} + s_{j}; \forall k \epsilon K, \forall d \epsilon D, \forall s \epsilon S, \forall i \epsilon V_{L}, \forall j \epsilon V_{R} \\ M(r_{ij}^{kds} - 1) + a_{i}^{kd} + \bar{s}_{i} + \bar{t}_{ij} \leq a_{j}^{k} + s_{j}; \forall k \epsilon K, \forall \delta \epsilon D, \forall s \epsilon S, \forall i \epsilon V_{L}, \forall j \epsilon V_{R} \\ M(r_{ij}^{kds} - 1) + a_{i}^{kd} + \bar{s}_{i} + \bar{t}_{ij} \leq a_{j}^{k} + s_{j}; \forall k \epsilon K, \forall \delta \epsilon D, \forall s \epsilon S, \forall i \epsilon V_{L}, \forall j \epsilon V_{R} \\ M(r_{ij}^{kds} - 1) + a_{i}^{kd} + \bar{s}_{i} + \bar{t}_{ij} \leq a_{j}^{k} + s_{j}; \forall k \epsilon K, \forall \delta \epsilon D, \forall \epsilon \delta, \forall \epsilon \delta
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\begin{split} &M\left(y_{ij}^{kds}-1\right)+a_{i}^{kd}-\bar{s}_{i}-\bar{t}_{ij}-a_{i}^{kd}\leq \mathsf{T};\ \forall k\epsilon K, \forall d\epsilon \mathsf{D}, \forall s\epsilon \mathsf{S}, \forall i\epsilon V_{L}, \forall j\epsilon V_{R}\right.\\ &M\left(l_{ij}^{kds}-1\right)+a_{j}^{kd}-\bar{t}_{ij}-a_{i}^{k}\leq \mathsf{T};\ \forall k\epsilon K, \forall d\epsilon \mathsf{D}, \forall s\epsilon \mathsf{S}, \forall i\epsilon V_{L}, \forall j\epsilon V_{R}\right.\\ &M\left(r_{i}^{kds}-1\right)+a_{j}^{kd}-\bar{s}_{i}-\bar{t}_{ij}-a_{i}^{kd}\leq \mathsf{T};\ \forall k\epsilon K, \forall d\epsilon \mathsf{D}, \forall s\epsilon \mathsf{S}, \forall i\epsilon V_{L}, \forall j\epsilon V_{R}\right.\\ &M\left(r_{ij}^{kds}-1\right)+a_{i}^{kd}\leq a_{i}^{k};\ \forall k\epsilon K, \forall d\epsilon \mathsf{D}, \forall s\epsilon \mathsf{S}, \forall i\epsilon V_{L}, \forall j\epsilon V_{R}\right.\\ &M\left(r_{ij}^{kds}-1\right)+a_{j}^{kd}\leq a_{j}^{k}+s_{j}^{k};\ \forall k\epsilon K, \forall d\epsilon \mathsf{D}, \forall s\epsilon \mathsf{S}, \forall i\epsilon V_{L}, \forall j\epsilon V_{R}\right.\\ &\left.\sum_{i\epsilon V_{L}}\sum_{j\in \mathsf{V}_{R}}\bar{d}_{ij}\left(l_{i}^{kds}+y_{ij}^{kds}+r_{ij}^{kds}\right)\leq \mathsf{E};\forall k\epsilon K, \forall d\epsilon \mathsf{D}, \forall s\epsilon \mathsf{S}\right.\\ &\sum_{i\epsilon N}\sum_{j\in \mathsf{V}_{R}}q_{i}x_{j}^{k}+\sum_{i\epsilon N}\sum_{j\in \mathsf{V}_{R}}\sum_{d\epsilon \mathsf{D}}\sum_{s\epsilon \mathsf{S}}q_{i}\left(y_{ij}^{kds}+r_{ij}^{kds}\right)\leq \mathsf{C};\forall k\epsilon K\\ &\sum_{i\epsilon N}\sum_{j\in \mathsf{V}_{R}}q_{i}\left(y_{ij}^{kds}+r_{ij}^{kds}\right)\leq \bar{C};\forall k\epsilon K, \forall d\epsilon \mathsf{D}, \forall s\epsilon \mathsf{S}\right.\\ &\sum_{s\epsilon S}\sum_{h\epsilon N}l_{ih}^{kds}\leq \mathsf{D};\forall k\epsilon K, \forall d\epsilon \mathsf{D}, \forall i\epsilon V_{L}\\ &\sum_{s\epsilon S}\sum_{h\epsilon N}r_{h}^{kds}\leq \mathsf{D};\forall k\epsilon K, \forall d\epsilon \mathsf{D}, \forall j\epsilon V_{R}\\ &\sum_{i\epsilon V_{L}}\sum_{j\epsilon N}\sum_{s\epsilon \mathsf{S}}l_{ij}^{kds}\leq \mathsf{S};\forall k\epsilon K, \forall d\epsilon \mathsf{D}\\ &\sum_{i\epsilon V_{L}}\sum_{j\epsilon N}\sum_{s\epsilon \mathsf{S}}r_{ij}^{kds}\leq \mathsf{S};\forall k\epsilon K, \forall d\epsilon \mathsf{D}\\ &v_{j}\leq a_{i}^{k}\leq w_{j};\forall k\epsilon K, \forall j\epsilon N\\ &v_{j}\leq a_{i}^{kd}\leq w_{j};\forall d\epsilon \mathsf{D}, \forall j\epsilon N\\ \end{split}
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          v_i \leq a_i^{kd} \leq w_i; \forall d \in D, \forall j \in N
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Objective (1) minimizes total travel cost for all resources. Constraint (2) ensures the conservation of heavy, and constraints (3) and (4) are flow balance. Constraint (5) limits each heavy resource to only one tour; (6) and (7) ensure each customer is serviced only once by either a heavy or light; (8) and (9) ensure lights launch and return to the same heavy; and (10) and (11) restrict the lights to directly launch from or return to the depot (i.e. lights must be carried by the heavy resources in and out of the depot). Constraints (12) - (15) are sub-tour elimination constraints. Constraints (16) - (18) ensure the structure of each light resource sub-tour follows the pattern of launch, deliveries and return or launch and return as illustrated in Figure 1.

Constraints (19) – (26) deal with the movement synchronization between the resources. Specifically, (19) – (22) assign the arrival time at each customer for both resources. The maximum waiting time for light is imposed by constraints (23) – (25). Constraints (26) and (27) guarantee the light's arrival time is before the heavy. Constraint (28) ensures the light resource's maximum travel distance. Constraint (29) and (30) represent the capacity constraint for the heavy and light. It is assumed the light can instantaneously transfer the load onto the heavy upon returning, the capacity constraint for the light resource is restricted to each sub-tour. Constraints (31) and (32) ensure the number of light resources launched or returned simultaneously does not exceed the preassigned value. Constraints (33) and (34) ensure the number of sub-routes per light resource does not exceed the preassigned value. Constraints (35) and (36) define the time window requirement which the resource can service the customer.

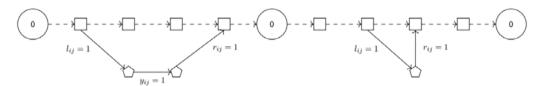


Fig. 1. Light resource movement pattern (heavy resource is dotted; light resource is solid)

3.2. Genetic algorithm procedure for VRPMS

To solve the VRPMS, the metaheuristic solves two sub-problems (Figure 2a). A reason to split the problem into two sub-problems is that the size of the solution pool for VRPMS is overwhelming even for the GA procedure. The first step is to use a GA to obtain the VRP solution with an assumed extended capacity for the heavy resource. This assumption simulates the effects of increased capacity when a composite unit of heavy and light is used. In practice, the true capacity of the heavy is usually constrained by the allowable working hours, so assistance from the light can extend the heavy resource's capacity. Then, each tour in the VRP solution is re-solved using a GA as separate TSP-D problems. The output from this step informs the user how many light resources are required to optimally service each tour in the VRP solution. Then the final solution to the VRPMS is obtained.

Figure 2b depicts the process used to solve both the general VRP and the extension with light resources. To begin, an initialization function randomly generates individuals as the first generation of population (i.e. potential solutions). Individuals' fitness are then evaluated by a fitness function working in conjunction with the split function to decode each individual into viable solutions. The best individual (elite offspring) is preserved for the next generation. Meanwhile, a pool of potential offspring is selected using a roulette wheel. Potential offspring are randomly mutated with the single point mutation function and randomly mated with the partially mapped crossover function.

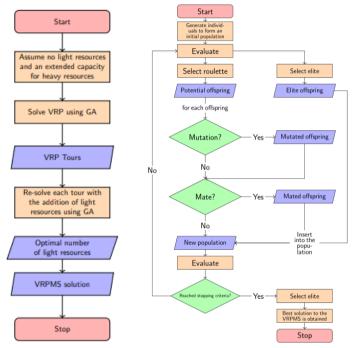


Fig. 2. (a) Overview of metaheuristic; (b) Overview of genetic algorithm.

New generations are thus created and evaluated by the evaluation function. If the fitness threshold or the number of generations criteria are met then the GA procedure ends with the best individual in the latest generation.

Since the GA encodes the solution as one giant tour, a split mechanism is required to decode the giant tour back into distinct tours for each resource. The first split algorithm, used to split the giant TSP tour into a VRP with only heavy resource, is a procedural capacity and range constraint check. The second split algorithm is used to split the VRP tours into VRPMS tours with heavy and light sub-routes. In each heavy tour, the algorithm starts a light resource cluster candidate with the closest pair of customers and adds neighbouring customers until the capacity or range constraint is reached. The evaluation function calculates the fitness and includes two functions to consider the capital cost, unit travel cost, wait cost and delay cost of the heavy and light resources for the TSP and VRPMS tours respectively.

When using the GA to solve the VRP, the ordering of the genes in the chromosome becomes very important. As noted by Goldberg and Lingle (1985) a new type of crossover operator known as the partially-mapped crossover (PMX), the normal GA procedure assumes values of the chromosomes independent of their position, the GA for the VRP will take the order of the chromosome into consideration.

4. Results

The hardware setup used to obtain the empirical results is a 2.5 GHz Intel Core i5 CPU with 4GB 1600 MHz DDR3 of memory. The MIP was configured to timeout at 9999s or stop at a tolerance of 1.00e-04. Fixed configurations for the GA are: mating ratio (90%), mutating ratio (5%) and population size (20n), n is the number of customers. Other configurations varied depending on the problem to minimize runtime while maintaining performance.

4.1. Comparison between exact and metaheuristic methods

A total of 30 random instances were solved with the exact MIP model and the GA metaheuristic to evaluate the difference in their performance. Each instance has either 10, 15 or 20 customers distributed over a 100 by 100 unit length square region. Each customer requires either 2 or 5 pieces and a service time of either 10 or 5 units of time for the heavy and light, respectively. The depot is located either at the origin or the centre of the region. Detailed specifications for the heavy and light resource are summarized in Table 1.

As shown in Figure 3, the solution cost gap for the VRPMS is not as tight as the TSP because of the rudimentary split method employed. The use of a greedy algorithm to sequentially group customers may not consistently produce good solutions for the GA to iterate on. Average solution gaps for the VRPMS are 31%, 32% and 27% for the 10, 15 and 20 customer size instances respectively, while computation times for running the MIP and the GA are drastically different. The average runtime for the MIP to solve the 10 customer instances is 4,639 s, compared to 22.9 s for the GA.

The MIP was also tested against VRPs and solutions by Augerat *et al.* (1995), to validate the MIP for solving regular VRPs. Eight instances with less than 30 customers were selected from Set P, and 16 from Set A and B which contained customers ranging from 30 to 40. The larger instances proved to be computationally expensive for the MIP to solve. This resulted in almost all the runs from Set A and B reaching the time out limit of 9999 seconds with higher solution bounds, thus creating the larger variance in performance compared to Set P.

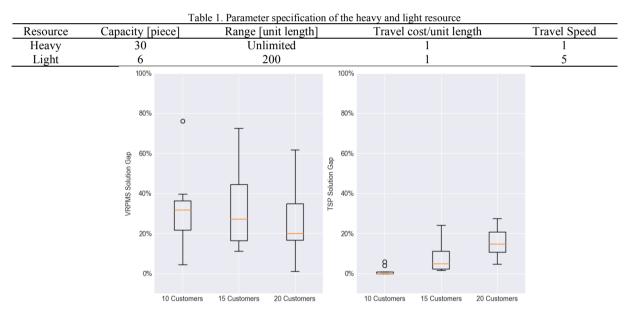


Figure 3. Comparison between GA and MIP solution quality for VRPMS and TSP problems

4.2. Empirical results for the three technologies from the metaheuristic method

Initial pilot tests and economic estimates indicate the cost per stop is cheaper for drones compared to traditional delivery vans (Kim, 2016). USPS averages \$2.50 for a typical shoebox delivery and drone/robot technology could cost around \$1.74 per delivery (Jenkins *et al.*, 2017). This is estimated from the following costs: capital, operation and maintenance, motor, battery, and electricity. Sidewalk robots are at an even earlier stage of development. Thus, capital cost estimates for this technology range from \$2,000 to \$7,000 (Pettitt, 2015; Huang, 2017). The projected cost per stop for the sidewalk robot ranges are low, like the drone, but robots can carry much higher loads. The most established light resource is the foot walker with a handcart. Advantages of foot walkers are reliability, range and capacity while the higher cost is a disadvantage. Table 2 presents assumptions for heavy and various light resource technologies. A capacity of 100 pieces is assumed for the delivery van. Other parameters of the heavy resource are:

capital cost of \$80,000, travel cost of \$0.1/unit length, travel speed of 1 unit length/unit time, wait cost of \$0.05/unit time and delay cost of \$0.01/unit time. A heavy can carry up to two walkers; two robots or three drones.

Sensitivity analysis of each light resource is performed over an instance set with 50 and 100 customers with technology parameters specified in Table 2. The instances are generated following the same properties presented before except the size of the instance is a 500 by 500 unit square and every customer has a fixed demand of 1. Each instance is solved 10 times with a randomized starting population for the GA. The number of generations used in the GA process varied from 300 to 500 depending on the size of the problem and how long it took to converge.

The solution results are summarized in Figure 4. The biggest takeaway is that drone technology did not significantly decrease the overall route cost. This could be because it can only make one delivery at a time within a small range. As such, if the capacity of the drone does not increase, it is not a competitive alternative. With a moderately sized capacity, range and lower travel cost, the sidewalk robot option performed better with a 10% decrease in travel cost despite its slower speed. Lastly, the high capacity and range walker option produced the largest decrease in travel cost with a 16% reduction, despite its higher unit travel cost and waiting cost.

Table 2. Technology cost ratios with respect to heavy resource

Technology	Capital cost ratio	Capacity ratio	Range [unit length]	Travel cost ratio [\$/unit length]	Travel Speed	Wait Cost [\$/unit time]	Delay Cost [\$/unit time]
Drone	25:1	100:1	200	5:1	1:5	1:0	1:1
Robot	15:1	25:1	500	10:1	1:1	1:0	1:1
Walker	8:1	10:1	1000	5:3	1:2	5:2	1:1

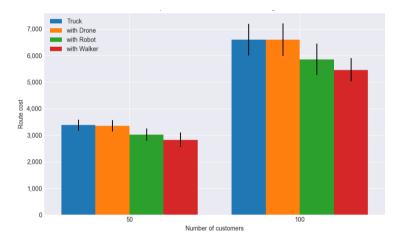


Fig. 4. Route costs for different technologies (standard deviation shown with black line).

The relative savings in capital costs for different customer sizes are summarized in Figure 5 showing that as the number of customers increases, there is more potential to save on capital costs. This makes intuitive sense because larger instances require more heavy resources, so adding the light can eliminate heavies which creates larger savings. The limited capacity of the drone means that many drones are required to replace the capacity of one heavy, and the relatively lower capital cost does not offset the total number of drones required. Figure 5 shows that the drone alternative consistently produces the smallest capital cost savings. Compared to the walker, the robot is more favourable for smaller customer sizes. Once the instance size reaches 200, walker is the cheaper option. The walker also incurs the highest route cost savings. The comparison demonstrates that the relationship between the parameters of heavy and light need to be closer to the walker alternative in order to be cost effective.

5. Conclusion

This paper introduces a model for cooperative delivery schemes through movement synchronization between delivery resources. This paper provides a method to optimize this new delivery mode and assess their cost implications. The insights gained from the analysis are valuable for express courier companies to rationalize

committing research and development efforts into promising light resource technologies.

Further research should focus on improving the greedy clustering component of the metaheuristic algorithm. Second, the time window constraint considered in this paper is limited to a static range. A better reflection of reality could be a dynamic time window for each customer and each technology. Third, drone and sidewalk robot technologies are rapidly changing, causing their attributes to be uncertain. The cost of delivery truck and foot-walker operations also vary between companies. So updates to the attribute assumptions made in this paper are required in any application. Lastly, there are other metaheuristic algorithms available for solving routing problems that may have potential for computation gains and better solution bounds.

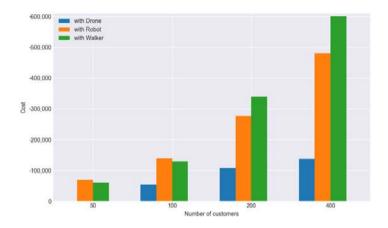


Fig. 5. Capital cost saved from replacing heavy resources with light resources.

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