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Summary Sheet

Managing The Zambezi River

Summary

The Kariba Dam is located on the Zambezi River. As one of the largest dam in Africa, its current safety state is controversial. We assume that the Kariba Dam will be removed and replaced by a multi-dam system for the sake of safety. For a complete description of this dam system, we propose two models concerning the location of new dams and the strategy for modulating the water flow through the new dam system. In order to get the optimal solution of this problem, we take the whole Zambezi River basin as research object.

In Model A, our primary aim is to find the optimal locations of the new dams. By looking at the elevation data and making preliminary estimates, we firstly identify the general area suitable for the construction. However, the rough estimation is apparently not enough, so we narrow the candidate area by referring to other information such as geography conditions. We also established a benchmark formula by using analytic hierarchy process. This formula is a comprehensive analysis of the terrain condition, construction cost, income and safety condition of a specific location, with high reliability. Based on this formula, we tested a series of locations within the candidate areas and choose some candidate location with higher score as our solution.

In Model B, our primary aim is to design a modulating strategy for the multi-dam system established in the Model A.

Keywords: Kariba Dam; Multi-dams arrangement; AHP method

Managing The Zambezi River

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January 24, 2017

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1 Introduction

The Kariba Dam is one of the biggest dam in the world, which is constructed on the Zambezi River. It supplies 1626 megawatts of electricity to parts of both Zambia and Zimbabwe. However, the Kariba Dam is in a dangerous state now. In the past 50 years, the torrents from the spillway have eroded its bedrock, carving a vast crater that has undercut the dam's foundations.[1] A number of options are available to solve this problem. This paper focuses on the third option – Removing the Kariba Dam and replacing it with a series of small dams along the Zambezi River. To find the best location for new dams and do the best arrangement with the multi-dam system, we propose two mathematical models. This paper describe these two models minutely and give suggestions on where to build new small dams and how to dispatch all the dams.

2 Model A: Search for possible locations to build dams

2.1 Description

To build new dams, we need to find some proper locations at first. However, we cannot pick all the suitable locations manually since Zambezi River is rather long. So our underlying idea is fairly simple. Firstly, we find a serial of possible river reaches. Although only a rough estimate, it does help us to exclude many reaches which cannot meet the requirements. Then we can pick some suitable locations from the reaches left. In this step, we need to refine this problem. Generally, the choice of dam's location should be related to geology, terrain, economic, ecology, disaster and other factors. Among all the factors, the dominant factor should be terrain for it decides both the safety and the economy of the reservoir. We build a formula as a benchmark to quantify their impact to the selection and pick the locations with the highest grade as our result. The following is the detailed discussion.

2.2 Analysis and Assumptions

To be specific, we get two principle of searching for possible reaches to build dams:

1. The higher the vertical drop is, the more abundant hydropower resource is contained;
2. In consideration of reducing ecological impact and water evaporation, the surface area of reservoirs should be small under certain requirement of volume. To build reservoir with small surface area and certain volume, the average depth of reservoir should be deep, thus, dams should be built between deep ravines.

Hydropower station convert gravity potential of water into electrical energy. The gravity potential is calculated as $E_p = mgh$, thus, higher vertical drop means bigger electricity-generation capacity. In order to simplify the expression, all symbol used in this model are listed in the table below.

Symbols	Meanings
V	the volume of reservoir
S	the water surface area of reservoir
H	the water depth of reservoir
α	the angle of bank slope
g	the gravitational acceleration

Table 1: Symbol Table of Model A

To simplify model, we assume that the vertical section of a reservoir is an approximate trapezoid, then the submerged area can be expressed as below:

$$\int_A^B \frac{H(l)}{\sin[\alpha(l)]}dl + \int_C^D \frac{H(L)}{\sin[\alpha(L)]}dL + S_{bottom} \tag{1}$$

where l, L are respectively the lengths of left bank and right bank; A, B indicate the starting position and end position of l; similarly, C, D indicate the starting position and end position of L. The expression (1) is still hard to use because of the difficult estimation of α. In order to make our assessment feasible, we need to simplify the expression (1). Noticed that:

$$\int_A^B \frac{H(l)}{\sin[\alpha(l)]}dl = \frac{1}{sin[\alpha(\zeta)]}\int_A^B H(l)\,dl = KH_{average}l \tag{2}$$

where K is a coefficient of inclination. expressions (2) is a application of mean value theorem for integrals, it fits in with the physics intuition. Then, the submerged area can be estimated as:

$$(K_1 l + K_2 L)\,H_{average} + S_{bottom} \tag{3}$$

Using expression (3), we can qualitatively explain why small surface area is needed. Using equation $H_{average} = \frac{V}{S}$, we get:

$$S_{submerged} \approx (K_1 l + K_2 L)\frac{V}{S} + S_{bottom}$$

and using the assumption of vertical section, the area of the bottom of a reservoir can be estimated as:

$$S_{bottom} = \int(\beta dS) \propto S \tag{4}$$

where β is a coefficient of water surface area size and bottom area size, the expression (4) qualitatively explain that S_{bottom} is proportional to S, thus we get:

$$S_{submerged} \approx (K_1 l + K_2 L)\frac{V}{S} + CS \tag{5}$$

where C is a coefficient to indicate that S_{bottom} is proportional to S.

Since the right side of the expression (5) is a hyperbolic function, it monotonically decrease when $S \geq \sqrt{\frac{(K_1 l+K_2 L)V}{C}}$. In the actual situation, $S \gg \sqrt{V}$, so we can qualitatively conclude that under certain requirement of volume small surface area of reservoir is more beneficial than the bigger surface area.

According to the discussion above, we should find the reaches with big throws on the Zambezi river based on the first principle. In accordance with the second principle, the possible reaches should between deep ravines, because a reservoir in deep ravines can have deeper water depth and thus smaller surface area.

2.3 Model Building

2.3.1 Search for candidate areas

In order to find suitable dam sites along the Zambezi River, we established a simple model based on the geographical conditions, costs, safety of the whole dam system and other fatal conditions.

The storage capacity of dams depends on the height of dams which is limited by the slope and height of the river bank. The cost of construction basically depends on the dam's height and length as well as the width of river. Thus, there are three major parameter to be taken into account in choosing candidate regions:

- The vertical drop of the river.
- The slope and height of the river bank.
- The width of river.

We download the geomorphological data of the Zambezi River Basin and generate a Digital Elevation Model (DEM). The Figure 1 is a general overview of the elevation in that region (the Zambezi River is marked in red line in the chart).

By using DEM, we obtain the elevation along the whole Zambezi River and plot the elevation figure as Figure 2. From Figure 2, We can obviously notice that the river is divided into 3 parts. There is a clear trend that the upstream is relatively plain, the water level decreases remarkably in the midstream and shoulders the most responsibility of storing water, the downstream have a rapid change of water level as well but there are few dams. Three prominent falls of the elevation are the Victoria Fall, the Kariba Dam and the Cahora Bassa Dam.

From the perspective of the vertical drop, we know the following areas are suitable for the establishment of dams: a few areas of upstream, reaches between Victoria Fall and Kariba Lake, reaches between Kariba Dam and Cahora Bassa Dam, some areas in the downstream. The exact areas are marked in red in Figure 3. In addition to the drop, we also need to analyze the slope and height as well as the breadth of the Zambezi River itself for they have great affect on the cost of dam construction. We plotted the contours

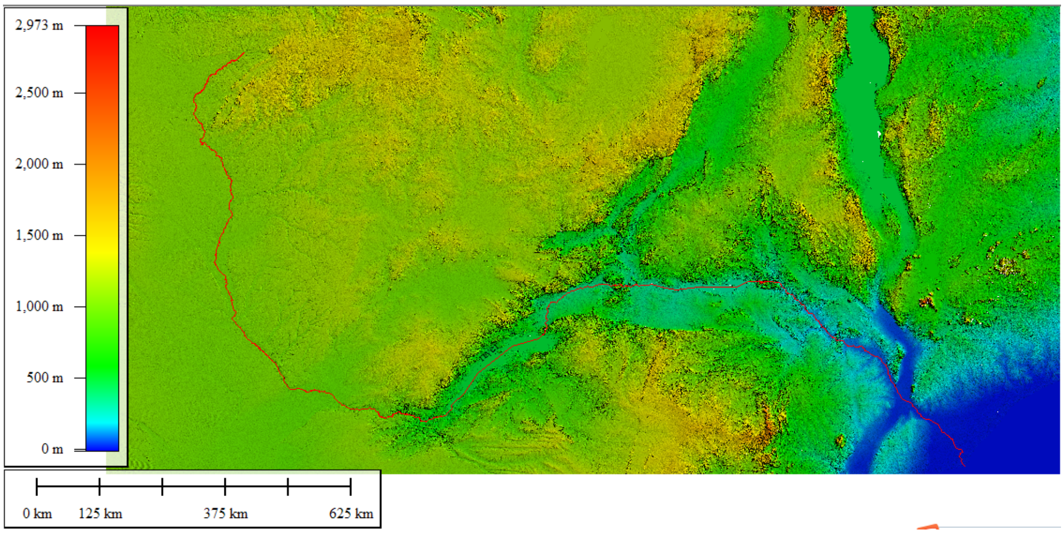


Figure 1: Overview Elevation Chart

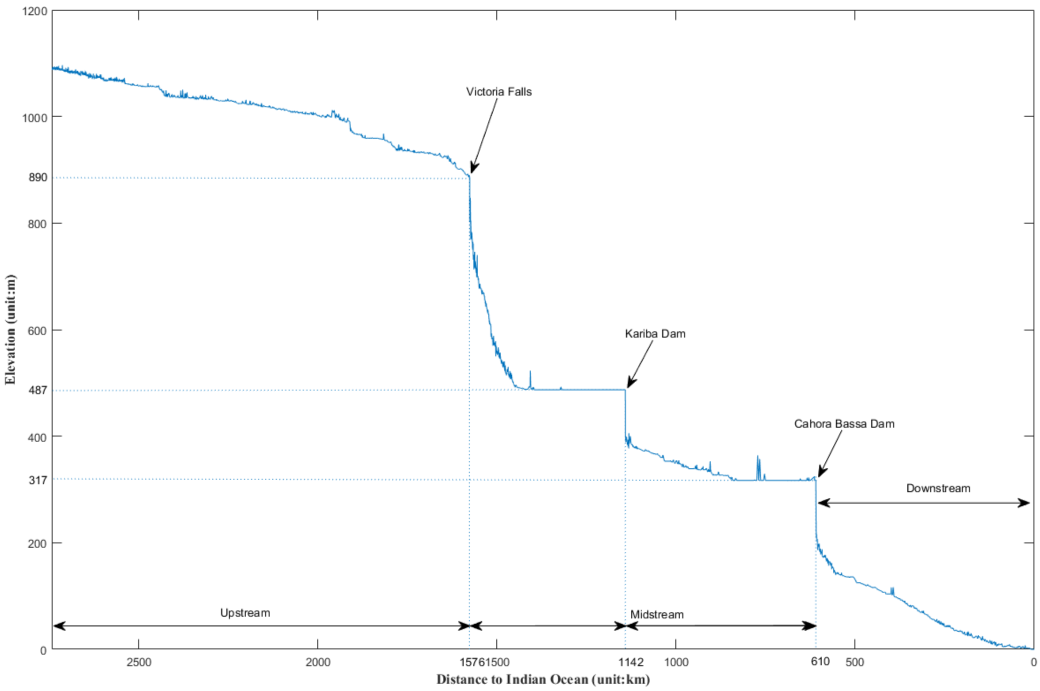


Figure 2: Elevation along the Zambezi River

of the Zambezi River basin based on the DEM model above. Generally, for the sake of storage capacity, safety and ecological impact, the bank of the reservoir should be steep as possible and have suitable height. The steepness of the bank means less flooded area and less impact on the surrounding environment. We also know that the dam should never be higher than the bank, so considerable height of the bank allows the reservoir to have a higher water level and proofs the robustness of the dam in extreme cases like flood. In addition, we don't expect to build our dam across a wide river. Not only because building dams in narrow valley can significantly reduce construction cost, but also since the too long dam body may have a negative impact on the safety of the dam.

We plot the contours of Zambezi River basin by using the DEM model above. The followings are 2 samples of the whole picture whose contour interval is 5 meters. The dense the contours means the river bank is steep which is good for us to build the dam.

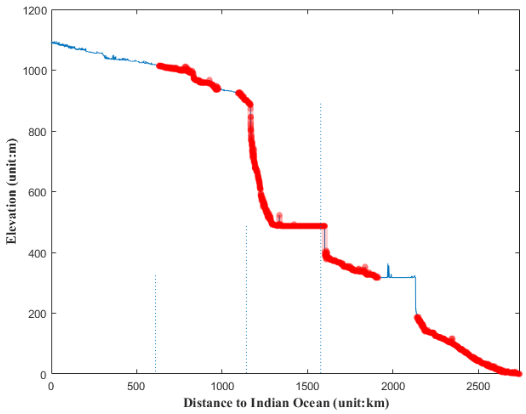
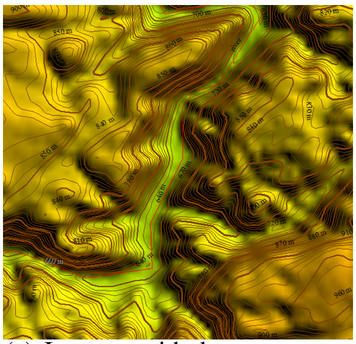
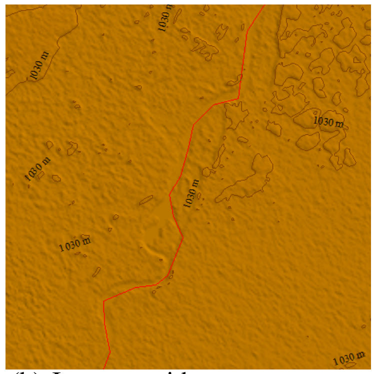


Figure 3: Candidate areas according to vertical drop



(a) Instance with dense contours



(b) Instance with sparse contours

Figure 4: Contour samples along Zambezi River

Taking these factors into account, we further reduced the candidate areas as Figure 5.

In addition, we need to consider the geological foundation conditions on both sides of the river. The rock layer of the candidate areas are rather solid and the vegetation along the river is lush which means it is less like to happen landslide in these areas. Base on the discussion above, we don't narrow the candidate areas any more since they are suitable for the construction of dams, the next step is build a function as benchmark and pick the exact locations from these candidate areas.

2.3.2 Building Benchmark function

From the discussion above, we have obtained the candidate dam construction area. However, evaluate from the continuous area is a heavy load work with great difficulty

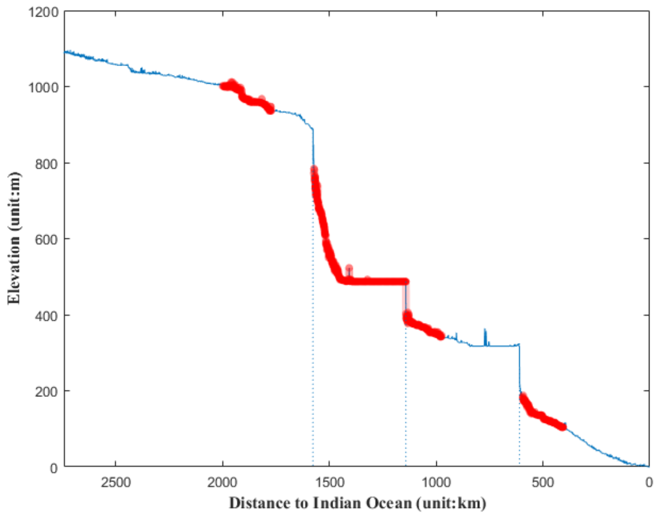


Figure 5: Candidate areas according to combined factors

and complexity, so we try to turn the continuous problem into discrete problem and reduce the complexity of the problem.

Geography Analysis

The choose of the dam should based on various geographical factors, such as vertical drop, rock and soil condition, the slope and height of the bank and the width of the river etc. With so many factors to be considered, we decided to use analytic hierarchy process(AHP) to do this job.

Firstly, we need to establish the natural coordinate along the river. We assume l to be the distance from the upstream source to the the candidate point.

Intensity of Importance	meaning
1	Equal importance
3	Moderate importance
5	Strong importance
7	Very strong or demonstrated importance
9	Extreme importance
$2,4,6,8$	Intensity between two hierarchies
$1,1/2,...,1/9$	The opposite numbers

Table 2: Intensity of Importance

We combined these factors into a hierarchical factor vector $\mathbf{P}_1$, and get $\mathbf{P}_1 = (F_l, \alpha_l, H_l, C_l, W_l)$. Then we can get the comparison matrix $\mathbf{A}_1$ which is satisfied with the following equation:

$$\mathbf{A}_1 = (a_{i,j})_{5\times 5}, \qquad a_{ij} > 0, \quad a_{ij} = \frac{1}{a_{ji}} \tag{6}$$

Now we need to get the importance of each factor, that is , the weight vector $\mathbf{w}$. The C_l and W_l decides whether the construction is feasible to a great extent, so they are regarded as the most important factors. Between W_l and C_l, W_l has more flexibility, so we decide to choose W_l as a more effective judge factor. In the other 3 elements, F_l and H_l decide they storage capacity of the dam, so they are also with a certain importance. However, α_l mainly influence the submerged area, so we think it a less significant factor. So we get the comparison matrix as below:

$$\mathbf{A}_1 = \begin{bmatrix} 1 & 5 & 1 & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{5} & 1 & \frac{1}{5} & \frac{1}{7} & \frac{1}{8} \\ 1 & 5 & 1 & \frac{1}{3} & \frac{1}{4} \\ 3 & 7 & 3 & 1 & \frac{1}{2} \\ 4 & 8 & 4 & 2 & 1 \end{bmatrix}$$

Calculating the maximum eigenvalues of matrix A, we get $\lambda = 5.1415$. The consistency indicators of $\mathbf{A}_1$ is

$$CI = \frac{\lambda - n}{n-1} = 0.0354$$

where n is the order of the matrix. When $n = 5$, the random consistency index RI is 1.12. Then the consistency ratio of matrix A is

$$CR = \frac{CI}{RI} = 0.0316 < 0.1$$

The error of $\mathbf{A}_1$ is acceptable, and the eigenvector of λ is $[0.2185, 0.063, 0.2185, 0.5190, 0.7945]$ Based on the calculation above, we get the weight vector $\mathbf{w}_1$.

$$\mathbf{w}_1 = \lambda = [0.2185, 0.063, 0.2185, 0.5190, 0.7945] \tag{7}$$

and our initial benchmark function is:

$$E(l) = \mathbf{P}\mathbf{w}_1^T = 0.2185F_l + 0.063\alpha_l + 0.2185H_l + 0.519C_l + 0.7945W_l \tag{8}$$

There are something need to note when using this function:

- This function can only determine **one** point each time because every selected point will change H_l.
- When establishing the function of these five factors, it is necessary to set a maximum value of this function, since the dam establishment only needs to meet the appropriate conditions. The function can be expressed as

$$F(x) = \begin{cases} f(x) & 0 < x < \theta \\ F_{max} & x \geq \theta \end{cases} \tag{9}$$

Our location selecting algorithm are as follow:

Dam Location Selecting Algorithm

1. Create a natural coordinate system of the candidate area, and take the upstream source as the origin. The evaluation threshold is set as T_0.
2. Based on the coordinate and DEM data we get, create the function of 5 factors including $F(l), \alpha(l), H(l), C(l)$ and $W(l)$, then set the maximum value of it.
3. Bring l into evaluation function $E(l)$, and select the point that satisfies the condition best as candidate point.
4. According to the obtained candidate point, divide this area into 2 segment. Repeat step 2 and step 3 to each of these segments until no new candidate points are generated.
5. Put all candidate points into candidate set.

Cost Analysis

Besides the geography conditions, we also needs to consider the cost of dam construction, including the design cost of the whole project, the overall construction cost of the project, the feasibility analysis report of the later stage of the project etc.
Firstly, we need to calculate the construction costs. According to the cost formula given by Hydro Review Maga, the construction cost is directly related to the size of the dam.

$$C_{build} = K(\frac{V_{cap}}{(H/\Delta)^{\eta}})^{\gamma} \tag{10}$$

where V_{cap} is the installed capacity, H is the height of the water head, Δ and η are two coefficient to adjust the relationship between V_{cap} and H, γ is the cost index, K is the cost coefficient. The value of K often depends on the size of the dam. For example, the K of the big dam is 7.7×10^4, while the small dam could be 5.0×10^4. We can calculate the installed capacity V_{cap} based on the annual flow of the river, the detailed data can be found from the Internet. The formula to calculate V_{total} is:

$$V_{total} = V_{flow} - V_{evap} + V_{rain} = V_{flow} - (1-\alpha)v_0 S(\mu T_{aver})^{\theta} + \beta v_1 S \tag{11}$$

where v_0 is the evaporation of the water per day per unit area, α is the evaporation reflux coefficient, μ is the temperature regulation coefficient, β is the proportion of precipitation into the reservoir, v_1 is the annual average rainfall per unit area, S is the area of river, t is the time.

S can be calculated by using following formula:

$$S \approx (K_1 l + K_2 L)\frac{V}{S} + CS \tag{12}$$

Now we get the formula to calculate the installed capacity:

$$V_{cap} = \rho g V_{total} H = \rho g(V_{flow} - (1-\alpha)v_0 S(\mu T_{aver})^{\theta} t + \beta v_1 S t)H \tag{13}$$

Then we consider the cost of the construction of the dam. There are some relations between the design costs and the construction cost, so we can directly get

$$C_{design} = \delta(C_{build})^{\tau} \tag{14}$$

where δ is the proportional coefficient, τ is the proportional index. According to the data we get, τ generally go for 0.9 and δ generally goes for 0.34. Then we start to consider other costs when building a dam. According to the statistic given by the World Bank, the cost of feasibility report before and after the construction account for about 12% of total design cost.

$$C_{prev} + C_{later} = (\alpha_1 + \alpha_2)C_{design} = 0.12C_{design} \tag{15}$$

Finally, we get the total cost of the project:

$$C_{project} = C_{build} + C_{design} + C_{prev} + C_{later} \tag{16}$$

$$C_{project} = K(\frac{V_{cap}}{(H/\Delta)^{\eta}})^{\gamma} + (1 + \alpha_1 + \alpha_2)\delta(K(\frac{V_{cap}}{(H/\Delta)^{\eta}})^{\gamma})^{\tau} \tag{17}$$

Income Analysis

After the cost analysis, let's come to the income analysis. We can calculate the profit gained from annual power generation by using the installed capacity of the dam.

$$B_{elec} = V_{cap}T_{use}p_{elec} \tag{18}$$

where T_{use} is the power-generation time in a year(The hydropower is stopped during the low water period.), p_{elec} is the local electricity price. Usually, T_{use} is between $3700-4200$h and local electricity price is $0.384 \sim 0.468$Cent/KWh. In addition, building new dams will bring some extra income such as Fishery income, tourism income, irrigation income and so on.Based on the model built by Welcomme[2], we can calculate the fishery income by using the equation below:

$$B_{fish} = \gamma S^{\theta} P_{fish} \tag{19}$$

where γ is the relation coefficient between watershed area and fishery output, θ is the relation index between watershed area and fishery output, p_{fish} represent the local fish price on average. In this paper, γ is set to 0.03 and θ is set to 0.97. According to the fishery data in 2016, we set p_{fish} to 3.5 dollar per kilogram. There are also some benefit that cannot be quantized like the irrigation benefit and emergency response capability.
The final dam income formula is:

$$B_{total} = B_{elec} + B_{fish} + B_{other} \tag{20}$$

where B_{other} is a basic estimate of other benefits including the irrigation benefit and emergency response capability. Specific values can refer to other dams on the downstream of the Zambezi River.

Safety Analysis

Since we are analyzing the impact on safety made by local terrain and geological. The evaluation function obtained above is used directly as a measure of security here.

$$S_i = E_{l,i} \tag{21}$$

Other Factors

At the same time, the influencing factors of people and environment should be put into out evaluation function as a measurement factor. Some of them are more prominent such as the impact of the residents living around(cost of immigration) and the animals and plants living in the submerged areas. We use the formula below to quantify the impact.

$$F_{other} = P_i(\xi S^{\sigma})m_0 + S_{submerge}(\beta_1 m_1 + \beta_2 m_2) + F_{balance} \tag{22}$$

where P_i means the density of the population around the ith dam, ξS^{σ} is a formula used to estimate how many people will be influenced by the new dam, S is the area of reservoir, m_0 is the migration costs per person, $S_{submerge}$ is the size of submerged area, $\beta_1 and \beta_2$ are two relation coefficients between dam and the creatures living around, m_1, m_2 is the value of the living creature living in this location. Besides, $F_{balance}$ is a constant used to balance F_{other}.

Combined Analysis

We have got some quantified formula from above. In the final benchmark function, we need to put all these factors together and set proper weight to them. Here, we still use the AHP method to calculate the proportion of each factor. Combined the 4 factors above into the influencing factor vector $\mathbf{P}_2$. $\mathbf{P}_2 = (C_{project}, S_i, B_{total}, F_{other})$, then we get the comparison matrix $\mathbf{A}_2$ which is satisfied with the equation below.

$$\mathbf{A}_2 = (a_{i,j})_{4\times 4}, \qquad a_{i,j} > 0, \qquad a_{i,j} = \frac{1}{a_{j,i}}$$

Similarly, using the standard above as a measure. The safety and geography condition are two most important factors, however the income is of less importance and its not our goal to maximum the income of the dam. The influence from other factors should be bigger than the income, but smaller than the cost and safety. So we created the comparison matrix as below.

$$\mathbf{A}_2 = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 1 & 1 & 3 & 2 \\ \frac{1}{2} & \frac{1}{3} & 1 & 1 \\ \frac{1}{2} & \frac{1}{2} & 1 & 1 \end{bmatrix}$$

The maximum eigenvalue of A_2 is $\lambda = 4.0206$. The consistency indicator of $\mathbf{A}_2$ is

$$CI = \frac{\lambda - n}{n-1} = 0.0069$$

where n means the order of the matrix. When $n = 4$, we get the random consistency indicator RI to be 0.90. So the consistency ratio of A_2 is

$$CR = \frac{CI}{RI} = 0.0076 < 0.1$$

So the inconsistency of $\mathbf{A}_2$ is within the allowable range, the feature vector of λ is $[0.6092, 0.6780, 0.2765, 0.3046]$. Using the result from above, we can calculate the value of weight vector $\mathbf{w}_2$:

$$\mathbf{w}_2 = \lambda = [0.6092, 0.6780, 0.2765, 0.3046] \tag{23}$$

Finally, our benchmark function $V(i)$ is:

$$V(i) = 0.2185C(i) - 0.678S(i) - 0.2675B(i) + 0.3046F(i) \tag{24}$$

2.4 Model Validation

In order to validate the model, we firstly select the dam Batoka Gorge Dam which is being prepared to construction near the Victoria Fall. Taking the geography factor at this point to $E(l)$, we can get 63.82 which is far below the threshold 25. Then we can take other important factors into the benchmark formula $V(i)$ and get 2.132×10^9 which is far below the limit of the benchmark 4.5×10^9. In conclusion, this point is a good location to build new dams. The result shows that the candidate point P_7 is very close to the location of Batoka Gorge Dam, which means that our model has a certain sensitivity.

2.5 Result

From model A, we firstly get the benchmark formula to evaluate candidate point. Then after the normalizing the parameters, we set the threshold of $E(l)$ to 25. Based on the safety evaluation function, we can get the candidate points as below(fig.7(a)). Since the limit $10 \leq N_{dam} \leq 20$, we changed the threshold of the benchmark to 4.5×10^9. According to the threshold, we can get the candidate point as below(fig.7(b)). Using the effect diagram rendered by Google Earth, we can easily get the area size of the reservoir and estimate the capacity. Our final solution meets the requirement that the new multi-dam system can have the same water storage capacity as the Kariba Dam.

Figure 6: Effect diagram rendered by Google Earth

3 Model B : The Best Arrangement

3.1 Description

For a given dam system, a series of dam site selections and corresponding reservoir capacities, installed capacity, local precipitation limits, and so on, can already be determined. However, there are still many analyzes that can be carried out, among which

ID	longitude	latitude
P1	25.56074815	-17.84625225
P2	25.84942235	-17.92704119
P3	25.85055478	-17.93714677
P4	25.85494022	-17.9439995
P5	25.86157239	-17.97757717
P6	25.89778936	-17.98044942
P7	26.12832017	-17.92124915
P8	26.47975523	-17.97062722
P9	26.89451467	-17.98845261
P10	27.70843304	-16.92358133
P11	28.76169553	-16.52234338
P12	28.83013483	-16.47991064
P13	28.85485817	-16.38075771
P14	30.0333817	-15.6409043
P15	30.06686636	-15.62874014
P16	30.18478024	-15.63716229
P17	30.24615856	-15.65907206
P18	32.66641749	-15.58712386
P19	32.7881512	-15.59768824
P20	33.13497446	-15.7846197
P21	33.39813759	-16.00484265
P22	33.98177208	-16.60008856

(a) Data of candidate locations

ID	area(km)	height(m)	capacity(m^3)
p7	10.8	150	1.62*10^9
p23	73	100	7.3*10^9
p8	342	100	3.42*10^10
p9	3600	20	7.2*10^10
p11	13.4	50	6.7*10^8
p12	1343	30	4.029*10^10
p14	28.5	150	4.275*10^9
p15	22.1	20	4.42*10^8
p18	134	50	6.7*10^9
p19	93.2	30	2.796*10^9
p20	237	50	1.185*10^10
total			1.821*10^11

(b) Location of candidate points

Figure 7: Basic information on candidate points

the most interesting is the modulating of water resources between dams. The modulating scheme can protect the water resources and the benefits derived therefrom as much as possible on the basis of ensuring safety and responding to emergencies. It can be expected that the modulating scheme will be different in different situations (different periods in the water cycle, different water capacity of reservoirs, etc.). In order to cope with the different situations reasonably, a model of generating water resources modulating scheme is needed.

3.2 Symbols

Symbols	Meanings
V	Water Resources Volume
V_D	The volume of water lost to the dam system as a result of flood discharge
$V_{D,i}$	The volume of water discharged from the reservoir i during the unit time period.
$\triangle V_i$	Variation of Water Quantity per Unit Time in Reservoir i
$V_{n,i}$	The natural water increase in reservoir i per unit time
$V_{e,i}$	The amount of water used per unit time for power generation of reservoir i
$V_{u,i}$	The amount of water in reservoir i used for other purposes per unit of time
n	The number of dams in the Dam System, so the system also has n reservoirs

Table 3: Symbol Table of Model B

3.3 Assumptions

1. The use of water resources is divided into varieties, such as hydropower, agricultural irrigation, etc. In order to simplify the assessment of water resources value, we assume that the value of water resources is proportional to the volume of water resources V.

2. The amount of water lost to a dam system due to flood discharge can not be reused.

3. The power generation water is transferred into the downstream reservoir, but the water for other uses is not directly transferred into the dam system.

3.4 Model Building

According to our first two assumptions, the value of water lost per unit time of a dam system is proportional to the amount of water lost due to flood discharge. And water is lost from the entire dam system only if it is drained as flood discharge from the last dam of the dam system, so we get

$$V_D = V_{D,n} \tag{25}$$

In order to maximize the benefits of the dam system, it is necessary to make the $V_{D,i}$ as small as possible, and the restrictive factors in different situations must be taken into account when proposing the modulating scheme. Therefore, we consider the use of optimization model to obtain water resources modulating scheme under certain case.

According to our discussion above, the objective function of optimization is very simple, i.e.

$$\min V_D = V_{D,n} \tag{26}$$

In the following, we will focus on finding constraints. First, the amount of water in the reservoir after the unit time is equal to the sum of original amount of water, natural increment of the reservoir and drainage of upstream reservoirs (including power generation and flood discharge) then subtract the sum of water for other purposes, power generation and flood discharge, i.e.

$$V_i^{'} = \begin{cases} V_i + V_{n,i} + V_{D,i-1} + V_{e,i-1} - V_{u,i} - V_{e,i} - V_{D,i} & i > 1 \\ V_i + V_{n,i} - V_{u,i} - V_{e,i} - V_{D,i} & i = 1 \end{cases} \tag{27}$$

where $V_i^{'}$ is the amount of water in reservoir i at the end of the period. $\triangle V = V^{'} - V$, then there have:

$$V_{n,i} = \begin{cases} \triangle V_i - V_{D,i-1} - V_{e,i-1} + V_{u,i} + V_{e,i} + V_{D,i} & i > 1 \\ \triangle V_i + V_{u,i} + V_{e,i} + V_{D,i} & i = 1 \end{cases} \tag{28}$$

This is the equality constraint between variables

We consider more constraints on the variables The Flood discharge capacity of the dam is clearly greater than or equal to zero and has an upper limit, ie

$$0 \le V_{D,i} \le V_{D,i}^{max} \tag{29}$$

$V_{D,i}^{max}$ is the maximum flood discharge capacity of dam i

Dam electricity generation capacity is limited, accordingly, the amount of water used for electricity generation is also limited. In addition, the amount of water used for electricity generation can not be negative, thus we get

$$0 \le V_{e,i} \le V_{e,i}^{\max} \tag{30}$$

Similarly, for $V_{u,i}$ we have:

$$V_{u,i}^{\min} \leq V_{u,i} \leq V_{u,i}^{\max} \tag{31}$$

The lower limit of $V_{u,i}$ is not 0, considering that the rigid demand for agricultural water use around the reservoir.

The constraint on $\triangle V_i$ should depend on the reservoir's existing water volume, future plans for reservoir volumes, and natural replenishment of future reservoirs. But intuitively, the water level of the reservoir should not be violent fluctuations; and our model should allow professionals to give limits of the reservoir water level fluctuations where used in specific case in the relevant areas, so we express the constraints of $\triangle V_i$ as:

$$\alpha_i \leq \frac{\triangle V_i}{V_i^{\max}} \leq \beta_i \tag{32}$$

where ${V_i}^{max}$ is the maximum volume of the reservoir.

Although some dams in the system can stop generating electricity at a certain time due to the regulation of the power grid, considering the need to meet the electricity needs of the local and other areas, the total generation of the dam system should have a lower limit of more than 0, thus we get:

$$\sum_{i=1}^{n} V_{e,i} \geq E_{min} \tag{33}$$

E_{min} is the amount of water required to produce the minimum electricity demand.

3.5 Macro Plan of Water Resources modulating

Based on the characteristics of the dam system obtained in Model A and the experience of dam management, we have proposed the following macro scenario for water resources modulating. The scenario can guide the staff of the River Authority to choose different values of α_i and β_i under different circumstances. Then, the optimal scheduling scheme of water resources is obtained by solving the above model. It is observed that in the dam system we get, the reservoirs with large volume have smaller drops than that with small volume and the reservoirs with small volume usually have larger drops. That is, the smaller reservoirs have more potential in energy generation, so we need them to run in a water level as high as possible. However, the big reservoir should drain some water ahead of the rainy season to make sure it safety when the flood is coming. Based on the factors above, we think the macro plan should have several principles as follow:

- In the dry season, the reservoir in upstream should reduce the power generation to ensure other water need.
- In the rainy season, the reservoir in the midstream should catch the water from upstream and reduce the flood discharge pressure of downstream. Meanwhile, the the reservoir in upstream and downstream should keep in safety water level.

- To the small dams in midstream, do the same activity as small reservoirs in downstream.

Change the principles above into the choose of α_i and β_i:

- To the small reservoir along the river, we should choose bigger α_i in the rainy season and smaller β_i in rainy season.
- To the big reservoir in the midstream, choose smaller β_i in the dry season.

The choose above makes the water level of small dams remains high in the dry season and make it possible to storage the water that cannot be used in the rainy season, which improves the utilization of water resources. Meanwhile, the schedule above performs well in emergency.

3.6 Model Validation

3.7 Result

4 Strengths and weaknesses

4.1 Model A: Search for possible locations to build dams

Model A has the following weaknesses.

- When building the benchmark function, some factors can not be quantified which will bring some errors.
- In the rough selection of the candidate areas, some of the valuable points may be ignored, resulting in the non-optimal solution.

Despite of the weakness, it has more strengths.

- The factors considered in this model are rather comprehensive. The benchmark function can be a good reflection of the impact of various factors.
- Using the analytic hierarchy process(AHP) to find the final solution, the result is accurate.
- The established benchmark function has a good guiding role for address selection.
- The specific dam construction plan is obtained, and the requirement of water storage capacity can be satisfied. Meanwhile, the multi-dam system have great advantage in adjusting the flow of Zambezi River basin.

4.2 Model B: The Best Arrangement

In Model B, we consider that the objective function of the optimization model is simplified to a relatively large extent and the given reference parameters may not be optimal for a particular case, which are weaknesses. However we have more strengths as below:

- The model can be adjusted by a professional.
- The model gives the global optimal solution under certain constraints.
- The reference parameters for controlling the reservoir water volume in usual, drought and flood years are given.

5 Conclusion

In model A, we firstly analyzed the elevation along Zambezi River. Combined with other geography factors, we found some candidate areas to build new small dams. According to our research result, the new dams should be concentrated in the middle and lower reaches of the Zambezi River, which is the basis of our next step. After that, we established a benchmark function to evaluate the candidate locations. This function is built by using AHP, which combine various important factors such as terrain conditions, construction cost, income and safety. We evaluated a series of specific locations by using this function and selected the candidate points with higher scores to build our new dam system.

Having chosen the specific locations, we further discussed the modulating strategy of our multi-dam system in face of complex conditions. Since the water level of Zambezi River changes intensely in a year, we anticipate that it is very difficult to obtain a unified scheduling scheme directly. So, we consider whether we can get a model to generate a specific scheduling strategy. This model should generate the corresponding optimal scheduling scheme according to the specific situation. We first establish a linear model with a global optimal solution under certain constraints. Moreover, we considered the action of people and take it as a parameter. After that, we use the historical data to simulate and get the value of parameters in normal year, in the dry season and rainy season.

References

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6 Brief Assessment of the options

The solution to the Kariba Dam problem can simply be divided into three options: repairing it, rebuilding it or removing it then replacing it with other dams. To the third method, ZRA suggests to build $10 \sim 20$ small dams to replace the huge Kariba Dam.

Evaluating the options from the perspective of cost and benefit is a complex task, since it can be influence by a number of factors. Only considering the cost of building dams, although it can be estimated accurately by using the cost formula below

$$C_p = K \left(\frac{V}{\left(\frac{H}{0.3}\right)^{0.3}} \right)^{0.82}$$

where C_p is the cost of building the hydropower station, V is the installed capacity, H is the design head, K is the proportional coefficient. However, the ecological costs of dam construction need to be considered more cautiously because damage to the ecological environment may be irreversible.

Option 1. Repairing the existing Kariba Dam. This is the option with the lowest cost of construction. Meanwhile, it won't change the submerged area, so there is no extra ecological cost. From the aspect of revenue, the reconstruction and expansion of Kariba Dam hydropower station can be carried out at the same time, which can effectively increase the total installed capacity of hydropower station, and thus improve the income of hydropower station. In fact, the expansion of the Kariba Dam hydropower station is underway. Since the reconstruction will not affect the Kariba Lake, the benefits from the use of water from the lake won't be reduced. The analysis above is based on the assumption that the climate will not change drastically in the future and no rare disasters which is outside the historical statistics will occur.

Option 2. Rebuilding the existing Kariba Dam. Because rebuilding the Kariba Dam need to remove the existing the dam and rebuild it at the origin site, it is an option with high risk and cost. What's more, the reconstruction of the dam will inevitably lead to the result that the hydropower station can't generate electricity in quite a long period, so this part of loss should also be included in the cost of reconstruction. However, rebuilding dams do have benefits. It helps to expand the installed capacity of hydropower station (benefit from re-designing the internal structure and using more advanced equipment). The new designed Dam would have better flood protection capacity, which allows river management to handle emergency with more flexibility. Stronger water storage capacity means we can raise the water level of Kariba Lake. It will increase the energy generation as well as bring the risk of ecologic damage which needs to be treated with caution.

Option 3. Removing the Kariba Dam and replacing it with a series of $10 \sim 20$ smaller dams along the Zambezi River. This is quite an ambitious plan. Even if the sum of installed capacity of all these small dams is the same as that of Kariba Dam, the total construction cost is still expected to be higher than rebuilding Kariba Dam according to the cost formula above. With the same problem as option 2, removing Kariba Dam will definitely lead to the loss of energy generation, furthermore even the construction of a smaller dam in the original position of Kariba Dam may result in loss of water storage capacity, as the water level in Kariba Lake will decrease. Fortunately, these losses can be minimized through rational planning. Specifically, we can give priority to the construction of small dams, and then gradually replace the Kariba Dam with their power generation capacity. New dams built in the down stream would store the water from Kariba Dam when it is removed, which can reduce the loss of water resources. Different from the previous two options, economic compensation of the new reservoirs' reserved area also needs to be include in the cost.(Here we can make an estimate by calculating the unit area GDP of the catchment)From the ecological point of view, the third option is also accompanied by greater risk. It will not only flood new areas, but also affect the ecology of Lake Kariba (the water level drops and the lake is divided into several parts). In terms of revenue, the scheduling of water resources between dams will reduce the loss of water resources caused by flooding discharge, which will actually help to increase the power generation capacity of hydropower stations. Moreover, the rational allocation of flood storage between dams will increase the safety of the dam system, the reduced reservoir area will reduce the evaporation loss of water and, in the face of emergencies, river management can also adopt a more flexible approach. Because of the high cost of the third option, a long-term analysis is of great significance. In the future, the flow of Zambezi River may reduce by $40\% \sim 50\%$ due to the climate change. Although the climate predictions nowadays are with a large degree of uncertainty, but we should never be blindly optimistic about the benefits of the new dam system.