Nonlinear Kalman Filter Parameter Estimation Algorithm for Low-Thrust Space Trajectory Optimization

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Abstract

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1. Introduction

The present paper studies the minimum-time and minimum-fuel optimal orbital transfer problems using low thrust.

Low-thrust propulsion systems including solar electric propulsion and solar sails, have been implemented in several deep space missions[1, 2, 3] and geosynchronous satellite station keeping for a long time[4]. Solar electric propulsions(SEP) were used to raise orbit to salvage missions from launcher or chemical engine failures like ARTEMIS[5] and AEHF-1. High specific impulse of the SEP may increase the mass fraction of the payload and expand the lifetime of satellite. Recently, dual all-electric-propelled communication satellites based on the Boeing 702SP platform were put into geosynchronous transfer orbit by Falcon 9 rocket. The satellites then spiraled to GEO orbit using SEP in about six months. The application of SEP reduces the launch cost significantly and increases the flexibility in choice of launch vehicles.

Preprint submitted to Mathematical Problems in Engineering *December 11, 2017*

2. The Optimal Control Problem in Space Flight Mechanics

2.1. General Form of the Optimal Control Problem

The general form for the optimal control problem is expressed as:

Find the control history of $\boldsymbol{u}(t)$ to minimize:

$$J = \varphi\left(t_f, \boldsymbol{x}(t_f)\right) + \int_{t_0}^{t_f} L\left(t, \boldsymbol{x}, \boldsymbol{u}\right) dt \tag{1}$$

subject to the first-order differential equations:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}\left(t, \boldsymbol{x}, \boldsymbol{u}\right) \tag{2}$$

where $\boldsymbol{x}$ is an n-vector of state variables and $\boldsymbol{u}$ is an m-vector of admissible control variables. t represents the independent time variable, $t \in [t_0, t_f]$. The initial conditions $\boldsymbol{x}\left(t_0\right) = \boldsymbol{x}_0$ are given at the fixed initial time t_0 and the final time t_f is free. The final boundary constraints are

$$\boldsymbol{\Psi}\left(t_f, \boldsymbol{x}\left(t_f\right)\right) = \boldsymbol{0} \tag{3}$$

The solution to this problem is derived by the calculus of variations, which leads to the derivation of the Euler-Lagrange equations. Let $\boldsymbol{\nu}$ be a constant vector of multipliers of the final boundary constraints, and let $\boldsymbol{\lambda}$ be the n-dimensional variable vector of adjoint or costate multipliers of the dynamics. The augmented performance index is defined as

$$\bar{J} = \varphi\left(t_f, \boldsymbol{x}\left(t_f\right)\right) + \boldsymbol{\nu}^T \boldsymbol{\psi}\left(t_f, \boldsymbol{x}\left(t_f\right)\right) + \int_{t_0}^{t_f} \left[L\left(t, \boldsymbol{x}, \boldsymbol{u}\right) + \boldsymbol{\lambda}^T\left(\boldsymbol{f}\left(t, \boldsymbol{x}, \boldsymbol{u}\right) - \dot{\boldsymbol{x}}\right)\right] dt \tag{4}$$

Write the Hamiltonian as

$$H\left(t, \boldsymbol{x}, \boldsymbol{u}, \boldsymbol{\lambda}\right) = L\left(t, \boldsymbol{x}, \boldsymbol{u}\right) + \boldsymbol{\lambda}^T \boldsymbol{f}\left(t, \boldsymbol{x}, \boldsymbol{u}\right) \tag{5}$$

The necessary conditions are as follows:

$$\dot{\boldsymbol{\lambda}} = -\frac{\partial H}{\partial \boldsymbol{x}}, \frac{\partial H}{\partial \boldsymbol{u}} = 0 \tag{6}$$

with boundary condition (3) and the following transversality conditions

$$\boldsymbol{\lambda}\left(t_f\right) = \left[\frac{\partial \boldsymbol{\varphi}}{\partial \boldsymbol{x}} + \boldsymbol{\nu}^T \frac{\partial \boldsymbol{\Psi}}{\partial \boldsymbol{x}}\right]_{t=t_f} \tag{7}$$

The first of (6) describes the dynamics of the costates, and the second is an application of the Pontryagin maximum principle. A more general expression is as follows:

$$\boldsymbol{u} = \arg\min_{\boldsymbol{u}\in U} H\left(t, \boldsymbol{x}, \boldsymbol{u}, \boldsymbol{\lambda}\right) \tag{8}$$

where U defines the domain of feasible controls. The maximum principle states that the control variables must be chosen to optimize the Hamiltonian at every instant of time.

The system of equations (1)-(8) constitutes a two-point-boundary-value-problem (TPBVP), which consists in finding the unknown initial costate vector. This is the essence of indirect methods.

2.2. An Application: Planar Low-Thrust Transfer Problem

A planar low-thrust transfer problem is considered in this paper. Polar coordinates are used for the planar two-body dynamics, as followed.

$$\begin{aligned}
\dot{r} &= v_r \\
\dot{\theta} &= \frac{v_t}{r} \\
\dot{v}_r &= \frac{v_t}{r} - \frac{\mu}{r^2} + u\sin\phi \\
\dot{v}_t &= -\frac{v_r v_t}{r} + u\cos\phi
\end{aligned} \tag{9}$$

r is the spacecraft orbit radius, θ is phase angle, v_r and v_t are the radial and transversal velocities, respectively. μ is the gravitational constant, u is the propulsive acceleration, and ϕ is thrust angle.

The initial boundary conditions for the spacecraft initial circular orbit is set as $[r_0, \theta_0, v_{r0}, v_{t0}]$, while the final boundary conditions place the spacecraft into a target circular orbit at t_f: $[r_f, v_{rf}, v_{tf}]$, where the final phase angle is free. The maximum of thrust acceleration is $u_{\max}$.

2.2.1. Time-Optimal Problem

The goal of time-optimal problem is to find the functions $u(t)$ and $\phi(t)$ that minimize the performance index:

$$J = t_f \tag{10}$$

From (5), the Hamiltonian is as follows:

$$H = 1 + \lambda_r v_r + \lambda_\theta \frac{v_t}{r} + \lambda_{vr}\left(\frac{v_t^2}{r} - \frac{\mu}{r^2} + u\sin\phi\right) + \lambda_{vt}\left(-\frac{v_r v_t}{r} + u\cos\phi\right) \tag{11}$$

where $\boldsymbol{\lambda} = [\lambda_r, \lambda_\theta, \lambda_{vr}, \lambda_{vt}]^{\mathrm{T}}$ is the costate vector.

From (6), we have

$$\begin{aligned}
\dot{\lambda}_r &= \frac{\lambda_\theta v_t}{r^2} + \frac{\lambda_{vr} v_t^2}{r^2} - \frac{2\mu\lambda_{vr}}{r^3} - \frac{\lambda_{vt} v_r v_t}{r^2} \\
\dot{\lambda}_\theta &= 0 \\
\dot{\lambda}_{vr} &= -\lambda_r + \frac{\lambda_{vr} v_t}{r} \\
\dot{\lambda}_{vt} &= -\frac{\lambda_\theta}{r} - \frac{2\lambda_{vr} v_t}{r} + \frac{\lambda_{vt} v_r}{r}
\end{aligned} \tag{12}$$

(9) and (12) complete the differential equations for the problem. The conditions at the initial time are

$$\boldsymbol{x}(t_0) = [r_0, \theta_0, v_{r0}, v_{t0}]^{\mathrm{T}} \tag{13}$$

whereas those at the final time are

$$\boldsymbol{\Psi}(t_f, \boldsymbol{x}(t_f)) = \boldsymbol{x}(t_f) - [r_f, v_{rf}, v_{tf}]^{\mathrm{T}} = \mathbf{0} \tag{14}$$

and from (7), we get

$$\lambda_\theta (t_f) = 0 \tag{15}$$

Rewrite (11) as the following

$$H = 1 + \lambda_r v_r + \lambda_\theta \frac{v_t}{r} + \lambda_{vr}\left(\frac{v_t^2}{r} - \frac{\mu}{r^2}\right) - \lambda_{vt}\frac{v_r v_t}{r} + u\sqrt{\lambda_{vr}^2 + \lambda_{vt}^2}\sin(\phi + \delta) \tag{16}$$

where $\delta = \arctan(\lambda_{vt}, \lambda_{vr})$. Since the Hamiltonian has to be minimized at any time, the following observations can be made.

$$u^* = u_{max}, \sin(\phi^* + \delta) = -1 \tag{17}$$

Thus, the Planar low-thrust optimal time transfer problem can be converted into a TPBVP, which consists in finding the unknown initial costate vector and the transfer time t_f.

2.2.2. Fuel-Optimal Problem

In the fuel-optimal problem, the performance index is following:

$$J = \int_{t_0}^{t_f} u\,dt \tag{18}$$

The final time t_f is fixed. The Hamiltonian for the fuel-optimal problem is following:

$$H = \lambda_r v_r + \lambda_\theta \frac{v_t}{r} + \lambda_{vr}\left(\frac{v_t^2}{r} - \frac{\mu}{r^2}\right) - \lambda_{vt}\frac{v_r v_t}{r} + u\left[1 + \sqrt{\lambda_{vr}^2 + \lambda_{vt}^2}\sin\left(\phi + \delta\right)\right] \tag{19}$$

The boundary conditions, the costate differential equations, and the transversality conditions are the same as the time-optimal problem. The optimal control $[u^*, \phi^*]$ is therefore given by the well-known bang-bang law as follows:

$$\begin{cases} u^* = u_{\max}, \sin(\phi^* + \delta) = -1 \quad if\ \psi > 0 \\ u^* = 0, \phi^* free \quad if\ \psi \leq 0 \end{cases} \tag{20}$$

ψ is the switching function defined as:

$$\psi = \sqrt{\lambda_{vr}^2 + \lambda_{vt}^2} - 1 \tag{21}$$

The solution of optimal control problems is commonly obtained through multiple shooting or single shooting methods. We define the shooting function:

$$\boldsymbol{S}\left(\boldsymbol{\lambda}_0\right) = \boldsymbol{x}\left(t_f; \boldsymbol{x}\left(t_0\right), \boldsymbol{\lambda}_0\right) - \boldsymbol{x}\left(t_f\right) \tag{22}$$

where $\boldsymbol{x}\left(t_f; \boldsymbol{x}\left(t_0\right), \boldsymbol{\lambda}_0\right)$ is the solution of the initial-value problem. Solving the TPBVP by the shooting method thus consists of solving the equations $\boldsymbol{S}\left(\boldsymbol{\lambda}_0\right) = \mathbf{0}$.

For orbital transfer problems, the shooting functions tend to be numerically unstable and very sensitive to the initial guesses. In particular, the value of $\boldsymbol{S}$ is computed through the integration of a problem with a discontinuous righthand side, which becomes quite difficult as the number of switches grows (typically for low-thrust transfers). The Jacobian of $\boldsymbol{S}$ has to be approximated by finite differences method, whose step-length must be set according to the precision with respect to $\boldsymbol{S}$. For instance, taking a too small step leads to an erroneous approximation of the Jacobian, which severely impairs the convergence. These difficulties make it nearly impossible to find a solution of $\boldsymbol{S} = \mathbf{0}$ without a very close initial guess.

3. The Nonlinear Kalman Filter Parameter Estimation Method

The shooting function solution for the optimal control problem is tricky to obtain, because of the numerical problems due to the nonsmoothness and discontinuities in the differential equations defining the optimal problem. For this reason, current numerical solvers typically present a narrow convergence radius for the shooting function. To overcome this problem, a nonlinear Kalman filter parameter estimation method is used to solve the TPBVP shooting functions.

3.1. Parameter Estimation

Parameter estimation, sometimes referred to as system identification or machine learning, involves determining a nonlinear transformation

$$\boldsymbol{y}_k = \boldsymbol{G}(\boldsymbol{z}_k, \boldsymbol{w}) \tag{23}$$

where $\boldsymbol{z}_k$ is the input, $\boldsymbol{y}_k$ is the output, and the nonlinear map $\boldsymbol{G}(\cdot)$ is parameterized by the vector $\boldsymbol{w}$. Typically, a training set is provided with sample pairs consisting of known input and desired outputs, $\{\boldsymbol{x}_k, \boldsymbol{d}_k\}$. The error of the machine is defined as $\boldsymbol{e}_k = \boldsymbol{d}_k - \boldsymbol{G}(\boldsymbol{z}_k, \boldsymbol{w})$, and the goal

1. Numbered list item one
2. Numbered list item two

3.2. Subsection One

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Treatments	**Response 1**	**Response 2**
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Table 1: Table caption

3.3. Subsection Two

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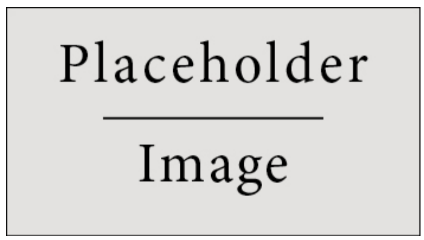

Figure 1: Figure caption

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$$e = mc^2 \tag{24}$$

4. The Second Section

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