

This article is related to [mathematical programming](#). For other uses see [complementarity](#).

A **complementarity problem** is a type of [mathematical optimization](#) problem. It is the problem of optimizing (minimizing or maximizing) a function of two [vector](#) variables subject to certain requirements (constraints) which include: that the [inner product](#) of the two vectors must equal zero, i.e. they are orthogonal.<sup>[1]</sup> In particular for finite-dimensional real vector spaces this means that, if one has vectors  $X$  and  $Y$  with all *nonnegative* components ( $x_i \geq 0$  and  $y_i \geq 0$  for all  $i$ : in the [first quadrant](#) if 2-dimensional, in the first [octant](#) if 3-dimensional), then for each pair of components  $x_i$  and  $y_i$  one of the pair must be zero, hence the name *complementarity*. e.g.  $X = (1, 0)$  and  $Y = (0, 2)$  are complementary, but  $X = (1, 1)$  and  $Y = (2, 0)$  are not. A complementarity problem is a special case of a [variational inequality](#).

## History

Complementarity problems were originally studied because the [Karush–Kuhn–Tucker conditions](#) in [linear programming](#) and [quadratic programming](#) constitute a [linear complementarity problem](#) (LCP) or a [mixed complementarity problem](#) (MCP). In 1963 [Lemke](#) and [Howson](#) showed that, for two person games, computing a [Nash equilibrium](#) point is equivalent to an LCP. In 1968 [Cottle and Dantzig](#) unified linear and quadratic programming and bimatrix games. Since then the study of complementarity problems and variational inequalities has expanded enormously.

Areas of [mathematics](#) and [science](#) that contributed to the development of complementarity theory include: [optimization](#), [equilibrium](#) problems, [variational inequality theory](#), [fixed point theory](#), [topological degree theory](#) and [nonlinear analysis](#).

## See also

- [Mathematical programming with equilibrium constraints](#)
- [nl format](#) for representing complementarity problems

## References

- Billups, Stephen; Murty, Katta (2000). "Complementarity Problems" . *Journal of Computational and Applied Mathematics*. **124**: 303. Bibcode:2000JCoAM.124..303B . doi:10.1016/S0377-0427(00)00432-5 .

## Further reading

- Richard W. Cottle; Jong-Shi Pang; Richard E. Stone (1992). *The Linear Complementarity Problem*. Academic Press. ISBN 978-0-12-192350-1.

- George Isac (1992). *Complementarity Problems*. Springer. ISBN 978-3-540-56251-1.
- George Isac (2000). *Topological Methods in Complementarity Theory*. Springer. ISBN 978-0-7923-6274-6.
- Francisco Facchinei; Jong-Shi Pang (2003). *Finite-Dimensional Variational Inequalities and Complementarity Problems: v.1 and v.2*. Springer. ISBN 978-0-387-95580-3.
- Murty, K. G. (1988). *Linear complementarity, linear and nonlinear programming* . Sigma Series in Applied Mathematics. **3**. Berlin: Heldermann Verlag. pp. xviii+629 pp. ISBN 3-88538-403-5. MR 0949214 . Archived from [the original](#) on 2010-04-01.

## Collections

- Richard Cottle; F. Giannessi; Jacques Louis Lions, eds. (1980). *Variational Inequalities and Complementarity Problems: Theory and Applications*. John Wiley & Sons. ISBN 978-0-471-27610-4.
- Michael C. Ferris; Jong-Shi Pang, eds. (1997). *Complementarity and Variational Problems: State of the Art*. SIAM. ISBN 978-0-89871-391-6.

## External links

- [CPNET:Complementarity Problem Net](#)

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