Sequential linear-quadratic programming (**SLQP**) is an iterative method for nonlinear optimization problems where objective function and constraints are twice continuously differentiable. Similarly to sequential quadratic programming (SQP), SLQP proceeds by solving a sequence of optimization subproblems. The difference between the two approaches is that:

- in SQP, each subproblem is a quadratic program, with a quadratic model of the objective subject to a linearization of the constraints
- in SLQP, two subproblems are solved at each step: a linear program (LP) used to determine an active set, followed by an equality-constrained quadratic program (EQP) used to compute the total step

This decomposition makes SLQP suitable to large-scale optimization problems, for which efficient LP and EQP solvers are available, these problems being easier to scale than full-fledge quadratic programs.

Algorithm basics

Consider a nonlinear programming problem of the form:

$$\begin{array}{rl}\min\limits_{x} & f(x)\\ \text{s.t.} & b(x)\geq 0\\ & c(x)=0.\end{array}$$

The Lagrangian for this problem is[1]

$$\mathcal{L}(x,\lambda,\sigma)=f(x)-\lambda^T b(x)-\sigma^T c(x),$$

where λ and σ are Lagrange multipliers.

LP phase

In the LP phase of SLQP, the following linear program is solved:

$$\begin{array}{rl}\min\limits_{d} & f(x_k)+\nabla f(x_k)^T d\\ \mathrm{s.\,t.} & b(x_k)+\nabla b(x_k)^T d\geq 0\\ & c(x_k)+\nabla c(x_k)^T d=0.\end{array}$$

Let $\mathcal{A}_k$ denote the *active set* at the optimum d^*_{LP} of this problem, that is to say, the set of constraints that are equal to zero at d^*_{LP}. Denote by $b_{\mathcal{A}_k}$ and $c_{\mathcal{A}_k}$ the sub-vectors of b and c corresponding to elements of $\mathcal{A}_k$.

EQP phase

In the EQP phase of SLQP, the search direction d_k of the step is obtained by solving the following quadratic program:

$$\begin{array}{rl} \min\limits_{d} & f(x_k) + \nabla f(x_k)^T d + \tfrac{1}{2} d^T \nabla^2_{xx} \mathcal{L}(x_k, \lambda_k, \sigma_k) d \\ \mathrm{s.t.} & b_{\mathcal{A}_k}(x_k) + \nabla b_{\mathcal{A}_k}(x_k)^T d = 0 \\ & c_{\mathcal{A}_k}(x_k) + \nabla c_{\mathcal{A}_k}(x_k)^T d = 0. \end{array}$$

Note that the term $f(x_k)$ in the objective functions above may be left out for the minimization problems, since it is constant.

See also

- Newton's method
- Secant method
- Sequential linear programming
- Sequential quadratic programming

Notes

1. Jorge Nocedal and Stephen J. Wright (2006). *Numerical Optimization* . Springer. ISBN 0-387-30303-0.

References

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