

# TopoAna: A generic tool for the topology analysis of inclusive Monte-Carlo samples in high energy physics experiments

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## Abstract

1 Inclusive Monte-Carlo samples are indispensable for signal selection and background suppression in many high energy physics experiments. A clear knowledge of the topology of the samples, including the types of physics processes and the number of processes in each type, is a great help to investigating signals and backgrounds. To help analysts obtain the topology knowledge from the truth information of the samples, we develop a topology analysis program, TopoAna, with C++, ROOT, and LaTeX. The program implements the functionalities of component analysis and signal identification with many kinds of fine, customizable clustering and matching algorithms. It tags physics processes in individual events accurately in the output root files, and exports the obtained topology information at the sample level clearly to the output plain text, tex source, and pdf files. Independent of specific software frameworks, the program is applicable to many experiments. At present, it has come into use in three  $e^+e^-$  colliding experiments: the BESIII, Belle, and Belle II experiments. The use of the program in other experiments is also prospective.

13 **Keywords:** event topology; component analysis; signal identification; inclusive Monte-Carlo samples; high energy physics experiments

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## 1. Introduction

16 One of the most important tasks in the data analysis of high energy physics experiments is to select signals, or in other words, to suppress backgrounds. As for the task, inclusive/generic Monte-Carlo (MC) samples are extremely useful, in that they provide basic, though not perfect, descriptions of the signals and/or backgrounds involved. However, due to the similarities between signals and some backgrounds, it usually takes efforts to establish a set of selection criteria that retain a high signal efficiency and meanwhile keep a low background level. Further optimization of preliminary criteria is often needed in the process. Under the circumstances, a

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The program is now available at <https://github.com/buaazhouxingyu/topoana>.

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comprehensive understanding of the samples is required. In particular, a clear knowledge of the topology of the samples is quite helpful. To be specific, the topology information includes the types of physics processes and the number of processes in each type, involved both in the entire samples and in the individual events. Here, the physics process could be a complete production and decay process involved in an event, or merely a part of it, such as the decay of an intermediate resonance. With the information, one can figure out the main backgrounds (especially the peaking ones), and optimize the selection criteria further by analyzing the differences between the main backgrounds and the signals. Even if it is difficult to further suppress these backgrounds, the knowledge of their topology is beneficial to estimate the systematic uncertainties associated with them.

The analysis of the topology information described above is a sort of component analysis, or in technical words, cluster analysis. It is complex since it has to classify physics processes actively and finely. Another sort of topology analysis often required in practice is signal identification, which only aims to search for certain processes of interests. It is relatively simple because its core technique is merely pattern matching. Mostly, signal and background events coexist in the inclusive MC samples. It is meaningful to differentiate them in such cases. The identified signal events can be used to make up a signal sample (removed to avoid repetition) in the absence (presence) of specialized signal samples. Occasionally, we have to pick out some decay branches in order to re-weight them according to new theoretical predictions or updated experimental measurements. Signal identification also plays a part in this occasion.

However, since the raw topology truth information of inclusive MC samples is counter-intuitive, diverse, and overwhelming, it is difficult for analysts to check the topology of the samples directly. To help them do the checks quickly and easily, a topology analysis program called TopoAna is developed with C++, ROOT [1], and LaTeX. Here, C++ is the programming language, ROOT is the C++ based data analysis software universally used in modern high energy physics experiments, and LaTeX is used for generating pdf documents containing the obtained topology information. The program implements the functionalities of component analysis and signal identification based on accurate pattern recognition. To meet a variety of practical requirements, many kinds of fine, customizable clustering and matching algorithms are implemented in the program. Generally, the program recognizes, categorizes, and counts physics processes in each event of the samples, and tags them in the corresponding entry of the output root (TFile [2]) files. After processing the events, the program exports the obtained topology information at the sample level to three files in the plain text, tex source, and pdf formats. Here, the output of the highly readable pdf files is a key feature of the program.

The program is applicable to inclusive MC samples at any data analysis stage of high energy physics experiments. In the overwhelming majority of situations, it is run over the samples which have undergone some selections, in order to examine the signals and backgrounds in the selected samples as well as the effect of the imposed selections. In such situations, the results of topology analysis are usually used together with other quantities for physics analysis. In spite of this, applying the program to the samples without undergoing any selection facilitates us to validate the generators and decay cards that produce the samples and helps novices get familiar with the topology of the samples.

Not relying on any specific software frameworks, the program applies to many high energy physics experiments. At first, the program was developed for the BESIII experiment, an experiment in the  $\tau$ -Charm energy region with abundant research topics under study [3, 4]. Then, it was extended substantially for the Belle II experiment, which is primarily dedicated to search for physics beyond the Standard Model in the flavor sector and has already started data taking in the

recent two years [5]. Besides, the program has also been tried and used in the Belle experiment, the predecessor of the Belle II experiment, where some physics studies are still ongoing [6].

This user guide gives a detailed description of TopoAna. It proceeds as follows: Section 2 introduces the basics of the program; Sections 3 and 4 expatiate the two categories of functionalities of the program — component analysis and signal identification, respectively; Sections 5 and 6 present some common settings and auxiliary facilities for the executing of the program, respectively; Section 7 summarizes the user guide. It is worth mentioning here that, aside from the detailed description in the user guide, an essential description of the program can be found in the file “paper\_draft\_v\*.pdf” under the directory “share” of the package.

## 2. Basics of the program

This section introduces the basics of the program, including the package, input, execution, and output of the program. The package implements the program via a C++ class called “topoana” and a main function invoking the class. Compiling the package creates the executable file of the program, that is, “topoana.exe”. To execute the program, we have to first obtain the input data of the program, namely the raw topology truth information of the inclusive MC samples, with some interfaces to the program in the software systems of the corresponding experiments. Normally, the input data contain all the topology information of the samples. With the data, all kinds of the topology analysis presented in the user guide can be performed.

To carry out the topology analysis desired in our work, we have to provide some necessary input, functionality, and output information to the program. The information is required to be filled in the setting items designed and implemented in the program, and the items have to be put in a plain text file named with a suffix “.card”. With the card file, one can execute the program with the command line: “topoana.exe cardFileName”, where the argument “cardFileName” is optional and its default value is “topoana.card”. After the execution of the program, we can examine the results of topology analysis in the output files and use them to analyze other experimental quantities. The results help us gain a better understanding of the signals and backgrounds and are conducive to carrying our work forward. In the next four subsections, we will present the package, input, execution, and output of the program in detail, with each part in one subsection.

### 2.1. Package of the program

The package consists of six directories — “include”, “src”, “bin”, “share”, “examples”, and “utilities” — and five files — “LICENSE”, “README.md”, “Configure”, “Makefile”, and “Setup”. While the directory “include” only includes one header file “topoana.h”, the directory “src” contains sixty source files “\*.cpp” as well as a script file “topoana.C”. At present, only one class, namely “topoana”, is defined in the program for all of its functionalities. The class is declared in “topoana.h”, implemented in “\*.cpp” files, and invoked in “topoana.C”.

The file “template\_topoana.card” under the directory “share” saves all the items which are developed for users to specify information for the execution of the program. One can refer to the file when filling in the cards for their own needs. Some plain text files “pid\_3pchg\_txtprnm\_texpnm\_iccp.dat.\*” are also included in the directory “share”. They store the basic information of the particles used in the program. The suffixes of their names indicate the experiments they apply to. One of them will be copied to “pid\_3pchg\_txtprnm\_texpnm\_iccp.dat” when we set up the program. Besides, the directory “share” also contains three LaTeX style files “geometry.sty”, “ifxetex.sty”, and “makecell.sty”, which are invoked by the program for generating pdf files.

113 The directory “examples” includes plenty of detailed examples. Particularly, all the examples in-  
 114 volved in this user guide are under its sub-directory “in\_the\_user\_guide”. The directory “utilities”  
 115 contains some useful bash scripts.

116 The program is released under MIT license [7]. The file “README.md” briefly introduces  
 117 how to install and use the program. To set up the program, one should first set the package path  
 118 with the command “./Configure”. Standard outputs of the command are the guidelines for man-  
 119 ually adding the absolute path of “topoana.exe” to the environment variable “PATH”, in order to  
 120 execute it without any path. The second step is executing the command “make”. This command  
 121 compiles the header, source, and script files into the executable file “topoana.exe” under the di-  
 122 rectory “bin”, according to the rules specified in the “Makefile”. The last step is specifying the  
 123 experiment name with the command line “./Setup experimentName”. Currently, the supported  
 124 experiment names are “BESIII”, “Belle”, and “Belle.II”. Besides, “./Setup Example” is required  
 125 for the execution of the examples in the user guide.

## 126 2.2. Input of the program

The input of the program is one or more root files including a TTree [8] object which con-  
 tains raw topology truth information of the inclusive MC samples under study. To be specific,  
 the information in each entry of the TTree object consists of the following three ingredients as-  
 sociated with the particles produced in an event of the samples: the number of particles, PDG [9]  
 codes of particles, and mother indices of particles. Notably, the particles do not include the initial  
 state particles ( $e^+$  and  $e^-$  in  $e^+e^-$  colliding experiments), which are default and thus omitted. Be-  
 sides, the indices of particles are integers starting from zero (included) to the number of particles  
 (excluded); they are obvious and hence not taken as an input ingredient for topology analysis.  
 Equation (1) shows an example of the input data.

$$\begin{array}{ll}
 \text{Number of particles} & : \quad 63 \\
 \text{PDG codes of particles} & : \quad 300553, \\
 & \quad -511, 511, -433, 421, 211, 22, -413, 111, 111, 113, \\
 & \quad 211, -431, 22, -323, 213, -421, -211, 22, 22, 22, \\
 & \quad 22, 211, -211, 333, 11, -12, 22, -311, -211, 211, \\
 & \quad 111, 221, 331, 321, -321, 310, 22, 22, 111, 111, \\
 & \quad 111, 111, 111, 221, 111, 111, 22, 22, 22, 22, \\
 & \quad 22, 22, 22, 22, 22, 22, 22, 22, 22, \\
 & \quad 22, 22 \\
 \text{Mother indices of particles} & : \quad -1, \\
 & \quad 0, 0, 1, 1, 1, 1, 2, 2, 2, 2, \\
 & \quad 2, 3, 3, 4, 4, 7, 7, 8, 8, 9, \\
 & \quad 9, 10, 10, 12, 12, 12, 12, 14, 14, 15, \\
 & \quad 15, 16, 16, 24, 24, 28, 31, 31, 32, 32, \\
 & \quad 32, 33, 33, 33, 36, 36, 39, 39, 40, 40, \\
 & \quad 41, 41, 42, 42, 43, 43, 44, 44, 45, 45, \\
 & \quad 46, 46
 \end{array} \tag{1}$$

127 The complete physics process contained in the data is displayed as follows.

|   |    |  |    |
|---|----|--|----|
| 0 $e^+e^- \rightarrow \Upsilon(4S)$                 | -1 | 9 $\rho^+ \rightarrow \pi^0\pi^+$                  | 6  |
| 1 $\Upsilon(4S) \rightarrow B^0\bar{B}^0$           | 0  | 10 $K^{*-} \rightarrow \pi^- \bar{K}^0$            | 6  |
| 2 $B^0 \rightarrow \pi^0\pi^0\rho^+\pi^+D^{*-}$     | 1  | 11 $D_s^- \rightarrow e^- \bar{\nu}_e \phi \gamma$ | 7  |
| 3 $\bar{B}^0 \rightarrow \pi^+ D^0 D_s^{*-} \gamma$ | 1  | 12 $\eta \rightarrow \pi^0\pi^0\pi^0$              | 8  |
| 4 $\rho^0 \rightarrow \pi^+\pi^-$                   | 2  | 13 $\eta' \rightarrow \pi^0\pi^0\eta$              | 8  |
| 5 $D^{*-} \rightarrow \pi^- \bar{D}^0$              | 2  | 14 $\bar{K}^0 \rightarrow K_S^0$                   | 10 |
| 6 $D^0 \rightarrow \rho^+ K^{*-}$                   | 3  | 15 $\phi \rightarrow K^+ K^-$                      | 11 |
| 7 $D_s^{*-} \rightarrow D_s^- \gamma$               | 3  | 16 $\eta \rightarrow \gamma\gamma$                 | 13 |
| 8 $\bar{D}^0 \rightarrow \eta\eta'$                 | 5  | 17 $K_S^0 \rightarrow \pi^0\pi^0$                  | 14 |

(2)

Here, the decay branches in the process are placed into two blocks in order to make full use of the page space. In both blocks, the first, second, and third columns are the indices, textual expressions, and mother indices of the decay branches. Notably, all the decay branches of  $\pi^0 \rightarrow \gamma\gamma$  are omitted in Eq. (2) in order to make the process look more concise. Since the topology diagram of such a process looks like a tree, we refer to the complete processes as decay trees. Obviously, the input data do not show the structure automatically. Thus, we need the program to do the topology analysis work.

From the first branch in Eq. (2), only one particle  $\Upsilon(4S)$  is produced after the  $e^+e^-$  annihilation. Thus,  $\Upsilon(4S)$  can be referred to as the root particle of the decay tree. Similarly, many other resonances with the quantum numbers  $J^{PC} = 1^{--}$ , such as  $J/\psi$ , can be solely produced at other proper energy points. Besides the cases with only one root particle, the program can deal with the cases with multiple root particles. For example, the program can recognize the following raw topology truth information

|                             |   |   |
|-----------------------------|---|---|
| Number of particles         | : | 25  |
| PDG codes of particles      | : | 433,  |
|                             |   | -321, 223, 211, -413, 431, 111, 211, -211, 111, -411, |
|                             |   | 111, 321, 113, 22, 22, 22, 321, -211, -211,           |
|                             |   | 22, 22, 211, -211                                     |
| Mother indices of particles | : | -1,   |
|                             |   | -1, -1, -1, -1, 0, 0, 2, 2, 2, 4,                     |
|                             |   | 4, 5, 5, 6, 6, 9, 9, 10, 10, 10,                      |
|                             |   | 11, 11, 13, 13  |

(3)

as the following process

|  |    |                                     |   |
|--|----|-------------------------------------|---|
| 0 $e^+e^- \rightarrow \pi^+\omega K^- D^{*-} D_s^{*+}$ | -1 | 4 $D^- \rightarrow \pi^- \pi^- K^+$ | 2 |
| 1 $\omega \rightarrow \pi^0\pi^+\pi^-$                 | 0  | 5 $D_s^+ \rightarrow \rho^0 K^+$    | 3 |
| 2 $D^{*-} \rightarrow \pi^0 D^-$                       | 0  | 6 $\rho^0 \rightarrow \pi^+\pi^-$   | 5 |
| 3 $D_s^{*+} \rightarrow \pi^0 D_s^+$                   | 0  |                                     |   |

(4)

Here, the particles  $\pi^+\omega K^- D^{*-} D_s^{*+}$  in the first branch arise from hadronization processes, in which quark pairs produced from initial state particles turn into hadrons. The processes with hadronization ignored have a tree structure and thus are easy to resolve. On the other hand, some hadronization processes, particularly those in high energy regions, contain complicated loop structures that are difficult to resolve without sophisticated algorithms. Resolving these intricate hadronization processes is not involved in the program at present.

It is recommended to save the input data in the TTree object together with other quantities for physics analyses, in order to facilitate the examination of the distributions of these quantities with the topology information. The input data can be stored in several types. Normally, the number of particles can be simply stored in a TBranch [10] object as a scalar integer, while the PDG

codes of particles, as well as the mother indices of particles, can be stored in a TBranch object as an array of integers, in a TBranch object as a vector of integers, or in a group of TBranch objects as multiple scalar integers. In the analysis software of the Belle II experiment, double-precision variables are used uniformly to store all the quantities involved in the experiment, and TBranch objects are not recommended to store arrays and vectors in order to use other tools such as NumPy [11] and pandas [12]. In such a situation, we have to store the number of particles in a TBranch object as a scalar double-precision number, and store the PDG codes of particles, as well as the mother indices of particles, in a group of TBranch objects as multiple scalar double-precision numbers. Summing up the above, we have mentioned four storage types of the input information. For the sake of simplification, we refer to them with the following acronyms: AOI, VOI, MSI, and MSD, which are short for array of integers, vector of integers, multiple scalar integers, and multiple scalar double-precision numbers, respectively. All of the storage types are supported by the program, and their acronyms will be used in the related item of the card file (see next subsection for details).

It is easy to get the input of the program within the software framework of high energy physics experiments. To facilitate its use, we have developed the interfaces of the program to the software systems of the BESIII, Belle, and Belle II experiments. Similar interfaces for other experiments can also be implemented with ease. Beyond the scope of the user guide, we will not discuss the details of the interfaces here.

Considering the diversity of the raw topology truth information, the program does not simply translate the counter-intuitive data into the intuitive processes. Here, the diversity means that a definite process can be represented with multiple permutations of the same set of data. For example, both the two permutations (113  $\rightarrow$  211, -211) and (113  $\rightarrow$  -211, 211) stand for the decay branch  $\rho^0 \rightarrow \pi^+\pi^-$ . A decay tree can consist of many decay branches. As a consequence, the diversity issue is complex. To avoid the same set of data in different permutations are classified as different processes, the program first sorts the input data to adjust the possible permutations to a unique order, according to the PDG codes and charges of the involved particles, and the numbers of daughter particles in the case of identical particles present in the same decay branch. The sorting algorithm is implemented in the source file “sortPs.cpp”, where some common settings are also involved. One can see the reference file “sortPs.cpp\_core” for the core of the sorting algorithm. This is the foundation of accurate pattern recognition in the program.

### 2.3. Execution of the program

To execute the program, we have to first configure some necessary setting items in a card file, and then run the program with the command line: “topoana.exe cardFileName”. This subsection introduces the essential items for the input, basic functionality, and output of the program. More items that can be set in the card file will be described in the following three sections. Sections 3 and 4 expatiate the available items for the functionalities of the program, and Section 5 presents the optional items for the common settings to control the execution of the program.

An example of the card file containing the essential items is shown as follows.

```
# The following six items set the input of the program.

% Names of input root files
{
  ../input/jpsi_1.root
  ../input/jpsi_2.root
```

```

192     }
193
194     % TTree name
195     {
196         evt
197     }
198
199     % Storage type of input raw topology truth information (Four options: AOI, VOI, MSI, and MSD. Default: AOI)
200     {
201         AOI
202     }
203
204     % TBranch name of the number of particles (Default: nMCGen)
205     {
206         Nmcps
207     }
208
209     % TBranch name of the PDG codes of particles (Default: MCGenPDG)
210     {
211         Pid
212     }
213
214     % TBranch name of the mother indices of particles (Default: MCGenMothIndex)
215     {
216         Midx
217     }
218
219     # The following item sets the basic functionality of the program.
220
221     % Component analysis — decay trees
222     {
223         Y
224     }
225
226     # The following item sets the output of the program.
227
228     % Main name of output files (Default: Main name of the card file)
229     {
230         jpsi.ta
231     }
232

```

In the card file, “#”, “%”, and the pair of “{” and “}”, are used for commenting, prompting, and grouping, respectively. The first six, seventh, and last items are set for the input, basic functionality, and output of the program, respectively.

The first item sets the names of the input root files. The names ought to be input one per line without trailing characters, such as comma, semicolon, and period. In the names, both the absolute and relative paths are allowed and wildcards “[ ]?\*” are supported, just like those in the root file names input to the method Add() of the class TChain [13]. The second item specifies the TTree name. The third item tells the program the storage type of the input raw topology truth information, and the input should be one of the following four acronyms: AOI, VOI, MSI, and MSD, as we introduce in the previous subsection. The following three items set the TBranch names of the three ingredients of the input raw topology truth information. Of the first six items, the former two are indispensable, whereas the latter four can be removed or left empty if the input values are identical to the default values indicated in their prompts. Besides, the latter four items can be moved to the underlying card file, which is developed for frequently used items and

will be introduced in Section 6.1, because the input values are usually fixed for a user or a group of users, though they might be different from the default values.

The seventh item sets the basic functionality of the program, namely the component analysis over decay trees. The item can be replaced or co-exist with other functionality items expatiated in Sections 3 and 4. Here, we note that at least one functionality item has to be specified explicitly in the card file, otherwise the program will terminate soon after its start because no topology analysis to be performed is set up.

The last item specifies the main name of the output files. Though in different formats, the files are denominated with the same main name for the sake of uniformity. They will be introduced at length in the next subsection. This item is also optional, with the main name of the card file as its default input value. It is a good practice to first denominate the card file with the desired main name of the output files and then remove this item or leave it empty.

To provide a complete description, we list and explain all the essential items in the paragraphs above. However, in practical uses, we suggest removing the optional items if the input values are identical to the default ones, or moving them to the underlying card file if the input values are fixed for most of your use cases. In this way, the contents of the card file will become much more concise, making the use of the program easier and quicker. For example, unless otherwise stated, only the following two items are used to set the essential information in Sections 3, 4, and 5.

```

266
267 % Names of input root files
268 {
269     ../input/mixed.1.root
270     ../input/mixed.2.root
271 }
272
273 % TTree name
274 {
275     evt
276 }
277

```

Besides, all the items in the program, also including those to be introduced in the following sections, are not required to be filled in the card files in a certain order. Nonetheless, we recommend filling them in a logical order for clearness.

During the execution of the program, some standard output and error messages are printed to the screen to provide some information on the input, progress, and output of the program, as well as the possible problems and proposed solutions to them. The standard output messages include the following four parts: (1) the values of the items with active inputs; (2) the total number of entries contained in the input root files and the progress of the program to process these entries; (3) the information output by the pdflatex command when it compiles the tex source file to get the pdf file; (4) and the hints on the output of the program. The standard error messages are prompted with “Error:” and “Infor:” in order to differentiate themselves from the standard output messages. The messages started with “Error:” point out the problems encountered by the program directly, while those started with “Infor:” give more information on the problems as well as some guidelines on the solutions to the problems.

The processing rate of the program is partly related to the performance of the detailed computing systems. Figure 1 shows the typical trends of the progress of the program, where the blue line displays the trend of running the example in this subsection. From the figure, the number of elapsed seconds grows linearly with the number of processed entries. This linear pattern, rather than a quadratic pattern, is a nice feature. It guarantees the program has a high rate even



in the case of processing huge samples. In this example, the program can process one hundred thousand events within five seconds.

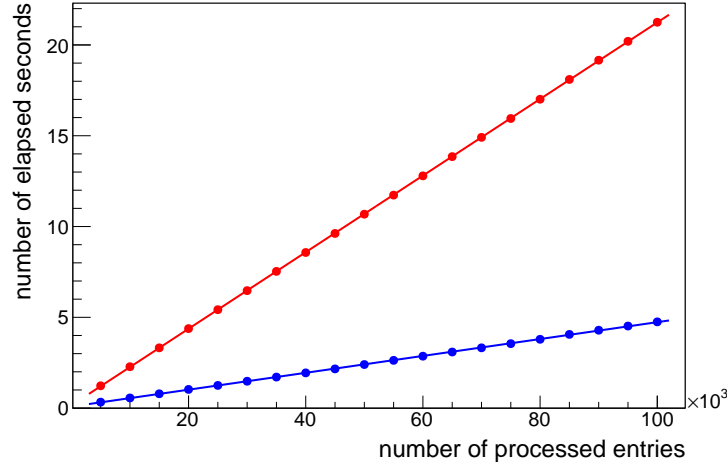


Figure 1: Typical trends of the progress of the program. The dots show the timing data from the standard output of the program, and the lines display the results of fitting linear functions to the data. The blue and red lines illustrate the trends of running the  $J/\psi$  example in Section 2.3 and the  $\Upsilon(4S)$  example in Section 3.1, respectively. It takes more time in the  $\Upsilon(4S)$  example than in the  $J/\psi$  example, because the decay of the  $\Upsilon(4S)$  resonance is more complex than that of the  $J/\psi$  resonance.

#### 2.4. Output of the program

The program gains the topology information from input data and saves it to output files. As mentioned in Section 1, the information includes the types of physics processes and the number of processes in each type, involved both in entire samples and in individual events. We refer to the information at the sample level as topology maps. In the topology maps, we assign an integer to each type of physics processes as its index. We term the indices of processes as well as the numbers of processes involved in each type in the individual events as topology tags.

The program outputs topology maps to three different files: one plain text file, one tex source file, and one pdf file, with the same main name specified in the card file. For instance, the three files are “jpsi.ta.txt”, “jpsi.ta.tex”, and “jpsi.ta.pdf” in the example. Although in different formats, the three files have the same information. The pdf file is the easiest to read. It is converted from the tex source file with the command `pdflatex`. The tex source file is convenient to us if we want to change the style of the pdf file to our taste and when we need to copy and paste (parts of) the topology maps to our slides, papers, and so on. For example, all of the tables displaying topology maps in this user guide are taken from associated tex source files. The plain text file has its own advantage, because the topology maps in it can be checked with text processing commands as well as text editors, and can be used on some occasions as input to the functionality items (see Sections 3 and 4 for details) of another card file.

In addition to the three files for topology maps, one or more root files are output to save topology tags. The root files only include one TTree object, which is entirely the same as that in the input root files, except for the topology tags inserted in all of its entries. The number of root

files depends on the size of output data. The program switches to one new root file whenever the size of the TTree object in memory exceeds 3 GB. In the case of the size less than 3 GB, only one root file is output. While the sole or first root file has the same main name as the three files above, more possible root files are denominated with the suffix “\_n” (n=1, 2, 3, and so on) appended to the main name. In the example, the first root file is “jpsi.ta.root”, and more possible root files would be “jpsi.ta.1.root”, “jpsi.ta.2.root”, “jpsi.ta.3.root”, and so on.

In the example of the previous subsection, the program conducts its basic functionality, namely the component analysis over decay trees. From the 100000 events of the input sample, the program recognizes 17424 decay trees and outputs all of them to the plain text, tex source, and pdf files. Table 1 only shows the top ten decay trees and their respective final states listed in the output pdf file. With the help of the symbolic expressions, the components of the sample are clearly displayed in the table, which brings great convenience to us in examining the signals and backgrounds involved in the sample. In the table, “rowNo”, “iDcyTr”, “nEtr”, and “nCEtr” are abbreviations for the row number, index of decay tree, number of entries of decay tree, and number of the cumulative entries from the first to the current decay trees, respectively. The values of “iDcyTr” are assigned from small to large in the program but listed according to the values of “nEtr” from large to small in the table. This is the reason why they are not in natural order like the values of “rowNo”. Since  $J/\psi$  is the only root particle for the  $J/\psi$  sample, the production branch  $e^+e^- \rightarrow J/\psi$  is omitted to save page space. Similar rules also apply to other samples with only one root particle. Considering  $\pi^0$  has a very large production rate and approximately 99% of it decays to  $\gamma\gamma$ , the program is designed to discard the decay  $\pi^0 \rightarrow \gamma\gamma$  by default at the early phase of processing the input data (see Section 5.1.2 for the setting item to alter the behavior). As a result,  $\pi^0 \rightarrow \gamma\gamma$  does not show itself in the table. Besides, the superscripts “ $f$ ” and “ $F$ ” in  $\gamma^f$  and  $\gamma^F$  indicate the final state radiation effect (see Section 5.1.3 for their difference).

Table 1: Top ten decay trees and their respective final states.

| rowNo | decay tree  | decay final state                     | iDcyTr | nEtr | nCEtr |
|-------|---|---------------------------------------|--------|------|-------|
| 1     | $J/\psi \rightarrow \mu^+\mu^-$   | $\mu^+\mu^-$                          | 6      | 5269 | 5269  |
| 2     | $J/\psi \rightarrow e^+e^-$   | $e^+e^-$                              | 4      | 4513 | 9782  |
| 3     | $J/\psi \rightarrow \pi^0\pi^+\pi^-\pi^-$   | $\pi^0\pi^+\pi^-\pi^-$                | 0      | 2850 | 12632 |
| 4     | $J/\psi \rightarrow \pi^0\pi^+\pi^+\pi^-\pi^-$  | $\pi^0\pi^+\pi^+\pi^-\pi^-$           | 2      | 1895 | 14527 |
| 5     | $J/\psi \rightarrow \pi^0\pi^+\pi^-K^+K^-$  | $\pi^0\pi^+\pi^-K^+K^-$               | 20     | 1698 | 16225 |
| 6     | $J/\psi \rightarrow \rho^+\rho^-\omega, \rho^+ \rightarrow \pi^0\pi^+, \rho^- \rightarrow \pi^0\pi^-, \omega \rightarrow \pi^0\pi^+\pi^-$ | $\pi^0\pi^0\pi^0\pi^+\pi^-\pi^-$      | 19     | 1453 | 17678 |
| 7     | $J/\psi \rightarrow e^+e^-\gamma^f$   | $e^+e^-\gamma^f$                      | 70     | 1222 | 18900 |
| 8     | $J/\psi \rightarrow \pi^0\pi^0\pi^+\pi^-\pi^-$  | $\pi^0\pi^0\pi^+\pi^-\pi^-$           | 127    | 1161 | 20061 |
| 9     | $J/\psi \rightarrow \pi^0\pi^+\pi^+\pi^+\pi^-\pi^-\pi^-$  | $\pi^0\pi^+\pi^+\pi^+\pi^-\pi^-\pi^-$ | 234    | 836  | 20897 |
| 10    | $J/\psi \rightarrow \pi^0\pi^0\pi^+\pi^-\gamma^F$   | $\pi^0\pi^0\pi^+\pi^-\gamma^F$        | 43     | 792  | 21689 |

In the table, “iDcyTr” is the topology tag for decay trees. Thus, it is also saved in the TTree objects of the output root file, together with other quantities for physics analysis. Therefore, it can be used to pick out the entries of specific decay trees and then examine the distributions of the other quantities over the decay trees. In the example, besides the raw topology truth information, only a random variable following the standardized normal distribution, namely X, is stored in the input root files and thus copied by default to the output root file. Though not a genuine variable for physics analysis, X is quite good to illustrate the usage of the topology tag. Figure 2

351 shows the distribution of X accumulated over the top ten decay trees. The figure is drawn with  
 352 the root script

353

354 `examples/in_the_user_guide/ex_for_tb_01/draw_X/v2/draw_X.C,`

355

356 where, for example, a statement equivalent to

357

358 `chain->Draw("X >>h0", "iDcyTr==6")`

359

360 is used to import X over the decay tree  $J/\psi \rightarrow \mu^+ \mu^-$  from the output root file to the histogram  
 361 named h0. With such a figure, we can clearly see the contribution of each decay tree. Particu-  
 362 larly, we can get to know whether a decay tree has a peak contribution or a contribution mainly  
 363 distributed in a different region. Based on these distributions, we can get a better understand-  
 364 ing of our signals and backgrounds, and thus optimize event selection criteria by applying new  
 365 requirements on the displayed quantities.

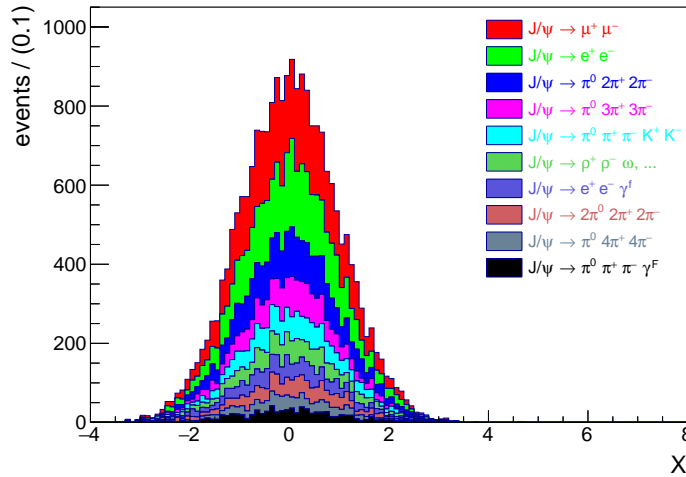


Figure 2: Distribution of X accumulated over the top ten decay trees. In the legend entry “ $J/\psi \rightarrow \rho^+ \rho^- \omega, \dots$ ”, the dots “ $\dots$ ” represent the secondary decay branches:  $\rho^+ \rightarrow \pi^0 \pi^+$ ,  $\rho^- \rightarrow \pi^0 \pi^-$ ,  $\omega \rightarrow \pi^0 \pi^+ \pi^-$ .

### 366 3. Component analysis

367 Component analysis is the primary functionality of the program. It is developed mainly for  
 368 the background analysis involved in our physics studies. We perform it over decay trees in the  
 369 previous example. Also, it can be carried out as follows: over decay initial-final states; with  
 370 specified particles to check their decay branches, production branches, mothers, cascade decay  
 371 branches, and decay final states; with specified inclusive decay branches to examine their exclu-  
 372 sive components; and with specified intermediate-resonance-allowed (IRA) decay branches to  
 373 investigate their inner structures. This section introduces the nine (five for specified particles)  
 374 kinds of component analysis, with each in a subsection. For each kind of component analysis,

one item is designed and implemented in the program to set related parameters. In each subsection, we take an example to demonstrate the corresponding setting item and show the resulting topology map. For easy exposition, all of the essential topology tags involved in the component analysis functionalities are presented in another separate subsection, namely the last subsection.

Similar to the case over decay trees, to perform the component analysis over decay initial-final states, we only need to input a positive option “Y” to the corresponding item. Different from the former two kinds, to carry out the latter seven kinds of component analysis, we have to explicitly specify one or more desired particles, inclusive decay branches, or IRA decay branches in the associated items. In the following examples, two particles or decay branches are set to illustrate the use of these items, but only the topology map related to one of them is shown to save space in the paper.

In addition to the indispensable parameters, two sorts of common optional parameters can be set in the items. The first sort is designed for all the nine kinds of component analysis to restrict the maximum number of components output to the plain text, tex source, and pdf files. Without the optional parameters, all components will be output. This is fine if the number of components is not massive. In cases of too many (around ten thousand or more) components, it takes a long time for the program to output the components to the plain text and tex source files as well as to get the pdf file from the tex source file. In such cases, it also takes up a large disk space to save these components in the output files. Considering further that the posterior components are generally unimportant and our time and energy to examine them are limited, it is better to set a maximum to the number of output components. To save space in the paper, we set the maximum number to five in the following examples.

The second sort of optional parameters are developed for the latter seven kinds of component analysis to assign meaningful aliases to the specified particles, inclusive decay branches, and IRA decay branches. By default, the indices 0, 1, 2, and so on are used to tag the particles and decay branches in the names of the TBranch objects appended in the TTree object of the output root files. This is fine, but it is significative to replace the indices with meaningful aliases, particularly in cases of many specified particles or decay branches.

### 3.1. Decay trees

Component analysis over decay trees is the basic kind of topology analysis. It is quite useful to study the backgrounds involved in our research works where the signals are the complete decay trees fully reconstructed from final state particles. It has already been widely performed in the BESIII experiment, as illustrated in the previous section with the  $J/\psi$  example. This subsection introduces it further with the available optional settings using the  $\Upsilon(4S)$  sample. The following example shows the associated item with the maximum number of output components set to five. In the item, a third parameter is also filled and set to “Y”. With the setting, the decay final states in the output pdf file are put under their respective decay trees, rather than in a column next to that for decay trees. It is recommended to use this optional parameter in cases there are too many (about ten or more) particles in some final states. Here, we note that the symbol “-” can be used as a placeholder for the maximum number of output components, if only the third parameter is desired.

```
% Component analysis — decay trees
{
  Y 5 Y
}
```

422

423 Component analysis over decay trees is one kind of the most time-consuming topology anal-  
 424 ysis tasks. To check further the efficiency of the program, the progress of running this example,  
 425 in addition to the example in Section 2.3, is illustrated in Fig. 1 as well. Clearly, a similar linear  
 426 pattern is also observed. However, since the decay of the  $\Upsilon(4S)$  resonance is more complex  
 427 than that of the  $J/\psi$  resonance, it takes more than twenty seconds for the program to process one  
 428 hundred thousand events in this example. Nonetheless, the program still has a high processing  
 429 rate.

430 Table 2 shows the decay trees. In the table, while the first five decay trees are listed exclu-  
 431 sively in the main part, the rest decay trees are only summarized inclusively at the bottom row.  
 432 Here, we note that the events are not densely populated over the first five decay trees because the  
 433 inclusive  $\Upsilon(4S)$  sample used here is not selected beforehand with any requirements. In the sym-  
 434 bolic expressions of decay initial-final states, the dashed right arrow ( $--\rightarrow$ ) instead of the plain  
 435 right arrow ( $\rightarrow$ ) is used, in order to reflect that the initial states do not necessarily decay to the  
 436 final states in a direct way. Similarly, it is also used in the symbolic expressions of IRA decay  
 437 branches, which will be introduced in Section 3.9.

Table 2: Decay trees and their respective initial-final states.

| rowNo | decay tree<br>(decay initial-final states)  | iDcyTr | nEtr  | nCEtr  |
|-------|---|--------|-------|--------|
| 1     | $\Upsilon(4S) \rightarrow B^0 \bar{B}^0, B^0 \rightarrow e^+ \nu_e D^{*-} \gamma^F, \bar{B}^0 \rightarrow \mu^- \bar{\nu}_\mu D^{*+}, D^{*-} \rightarrow \pi^- \bar{D}^0,$<br>$D^{*+} \rightarrow \pi^+ D^0, \bar{D}^0 \rightarrow \pi^0 \pi^- K^+, D^0 \rightarrow \pi^0 \pi^+ K^-$<br>$(\Upsilon(4S) --\rightarrow e^+ \nu_e \mu^- \bar{\nu}_\mu \pi^0 \pi^+ \pi^- \pi^- K^+ K^- \gamma^F)$   | 20870  | 3     | 3      |
| 2     | $\Upsilon(4S) \rightarrow B^0 \bar{B}^0, B^0 \rightarrow \pi^0 \pi^+ \pi^- \rho^- D^-, \bar{B}^0 \rightarrow \mu^- \bar{\nu}_\mu D^{*+}, \rho^- \rightarrow \pi^0 \pi^-,$<br>$D^- \rightarrow \pi^- \pi^- K^+, D^{*+} \rightarrow \pi^+ D^0, D^0 \rightarrow K_L^0 \pi^+ \pi^-$<br>$(\Upsilon(4S) --\rightarrow \mu^- \bar{\nu}_\mu \pi^0 \pi^+ \pi^- \pi^+ \pi^- \pi^- \pi^- \pi^- K^+)$   | 5295   | 2     | 5      |
| 3     | $\Upsilon(4S) \rightarrow B^0 \bar{B}^0, B^0 \rightarrow \mu^+ \nu_\mu D^{*-}, \bar{B}^0 \rightarrow e^- \bar{\nu}_e D^+, D^{*-} \rightarrow \pi^- \bar{D}^0,$<br>$D^+ \rightarrow e^+ \nu_e \bar{K}^*, \bar{D}^0 \rightarrow \pi^0 \pi^+ \pi^- K_S^0, \bar{K}^* \rightarrow \pi^0 \bar{K}^0, K_S^0 \rightarrow \pi^+ \pi^-, \bar{K}^0 \rightarrow K_L^0$<br>$(\Upsilon(4S) --\rightarrow e^+ e^- \nu_e \bar{\nu}_e \mu^+ \nu_\mu \pi^0 \pi^+ \pi^- \pi^+ \pi^- \pi^- \pi^- K^+)$ | 11954  | 2     | 7      |
| 4     | $\Upsilon(4S) \rightarrow B^0 \bar{B}^0, B^0 \rightarrow e^+ \nu_e D^{*-}, \bar{B}^0 \rightarrow \pi^0 \pi^- \omega D^+, D^{*-} \rightarrow \pi^- \bar{D}^0,$<br>$\omega \rightarrow \pi^0 \pi^+ \pi^-, D^+ \rightarrow e^+ \nu_e \pi^+ K^-, \bar{D}^0 \rightarrow \pi^0 \pi^- K^+$<br>$(\Upsilon(4S) --\rightarrow e^+ e^+ \nu_e \nu_e \pi^0 \pi^+ \pi^- \pi^- \pi^- \pi^- K^+ K^-)$   | 14345  | 2     | 9      |
| 5     | $\Upsilon(4S) \rightarrow B^0 \bar{B}^0, B^0 \rightarrow \mu^+ \nu_\mu D^{*-}, \bar{B}^0 \rightarrow e^- \bar{\nu}_e D^{*+} \gamma^F, D^{*-} \rightarrow \pi^- \bar{D}^0,$<br>$D^{*+} \rightarrow \pi^0 D^+, \bar{D}^0 \rightarrow \pi^- K^+, D^+ \rightarrow e^+ \nu_e \bar{K}^*, \bar{K}^* \rightarrow \pi^+ K^-$<br>$(\Upsilon(4S) --\rightarrow e^+ e^- \nu_e \bar{\nu}_e \mu^+ \nu_\mu \pi^0 \pi^+ \pi^- \pi^- K^+ K^- \gamma^F)$  | 15332  | 2     | 11     |
| rest  | $\Upsilon(4S) \rightarrow \text{others (99980 in total)}$<br>$(\Upsilon(4S) --\rightarrow \text{corresponding to others})$  | —      | 99989 | 100000 |

### 438 3.2. Decay initial-final states

439 On some occasions, we need to investigate the decay initial-final states of backgrounds for  
 440 some sophisticated physics analyses. Particularly, it is necessary to differentiate the following  
 441 two fundamental types of backgrounds: the one with the same initial-final states as the signal,  
 442 and the other with different initial-final states from the signal. While the latter type of back-  
 443 grounds needs to be suppressed as much as possible, the former type usually needs to be kept to  
 444 study more physical effects, for example, the interference effect. Besides, examining the decay  
 445 initial-final states of backgrounds sheds light on the misjudgment of final state particles at the  
 446 level of signal candidates. Below is an example demonstrating the related item with the maxi-  
 447 mum number of output components set to five.

448

449 % Component analysis — decay initial-final states

```

450     {
451       Y 5
452     }
453

```

454 The decay initial-final states are displayed in Table 3. The layout of the table is similar to that of  
455 Table 2, which shows the decay trees.

Table 3: Decay initial-final states.

| rowNo | decay initial-final states   | iDcyIFSts | nEtr  | nCEtr  |
|-------|--|-----------|-------|--------|
| 1     | $\Upsilon(4S) \rightarrow \mu^+ \nu_\mu \pi^0 \pi^0 \pi^+ \pi^+ \pi^- \pi^- \pi^- K^+ K^-$             | 41        | 18    | 18     |
| 2     | $\Upsilon(4S) \rightarrow \pi^0 \pi^0 \pi^0 \pi^0 \pi^+ \pi^+ \pi^+ \pi^- \pi^- \pi^- K^+ K^-$         | 887       | 18    | 36     |
| 3     | $\Upsilon(4S) \rightarrow \mu^- \bar{\nu}_\mu \pi^0 \pi^0 \pi^+ \pi^+ \pi^+ \pi^- \pi^- \pi^- K^+ K^-$ | 3350      | 18    | 54     |
| 4     | $\Upsilon(4S) \rightarrow \pi^0 \pi^0 \pi^0 \pi^0 \pi^+ \pi^+ \pi^+ \pi^- \pi^- \pi^- K^+ K^-$         | 1215      | 17    | 71     |
| 5     | $\Upsilon(4S) \rightarrow \pi^0 \pi^0 \pi^0 \pi^0 K_L^0 \pi^+ \pi^+ \pi^+ \pi^- \pi^- \pi^- K^-$       | 1207      | 17    | 88     |
| rest  | $\Upsilon(4S) \rightarrow \text{others (78208 in total)}$  | —         | 99912 | 100000 |

### 456 3.3. Decay branches of particles

457 The invariant mass constraint is one of the most frequently used event selection requirements  
458 in high energy physics experiments. With the requirement applied to certain particle, the main  
459 backgrounds (especially the peaking ones) to its signal decay mode are very likely to be its other  
460 decay modes. In this case, it is significant to examine the decay branches of the particle. The  
461 following example shows the associated item with the two particles  $D^{*+}$  and  $J/\psi$  set as research  
462 objects. In the item, each row holds the information of a specified particle, and the first, sec-  
463 ond and third columns are the textual expressions, aliases, and maximum numbers of output  
464 components, respectively. As we introduce at the beginning part of this section, the aliases and  
465 maximum numbers of output components are both optional. Here, we note that the symbol “—”  
466 can be used as a placeholder for an unassigned alias, if only the maximum number of output  
467 components is desired.

```

468 % Component analysis — decay branches of particles
469 {
470   D*+  Dsp  5
471   J/psi Jpsi 5
472 }
473

```

474  
475 Table 4 shows the decay branches of  $D^{*+}$ . From the table, only four decay branches of  $D^{*+}$  are  
476 found in the input inclusive MC sample. Since there is likely one or more cases of  $D^{*+}$  decays in  
477 one input entry, “nCase” and “nCCase”, instead of “nEtr” and “nCEtr”, are used in the table in  
478 order to accurately indicate what we are counting are the numbers of  $D^{*+}$  decays, rather than the  
479 numbers of entries involving the  $D^{*+}$  decays.

Table 4: Decay branches of  $D^{*+}$ .

| rowNo | decay branch of $D^{*+}$                | iDcyBrP | nCase | nCCase |
|-------|---|---------|-------|--------|
| 1     | $D^{*+} \rightarrow \pi^+ D^0$          | 0       | 31180 | 31180  |
| 2     | $D^{*+} \rightarrow \pi^0 D^+$          | 1       | 13978 | 45158  |
| 3     | $D^{*+} \rightarrow D^+ \gamma$         | 2       | 700   | 45858  |
| 4     | $D^{*+} \rightarrow \pi^+ D^0 \gamma^F$ | 3       | 28    | 45886  |

### 3.4. Production branches of particles

In some cases, we have interest in the production branches of certain particles. Below is an example demonstrating the related item also by taking the two particles  $D^{*+}$  and  $J/\psi$  as objects of study. The input to this item is the same as that to the above item.

```
% Component analysis — production branches of particles
{
  D*+   Dsp   5
  J/psi  Jpsi   5
}
```

The production branches of  $D^{*+}$  are displayed in Table 5. In the production branches,  $D^{*+}$  is marked in blue so as to make it noticeable. From the table, the number of production branches of  $D^{*+}$  found in the input sample is 3277, much bigger than 4, which is the number of its decay branches.

Table 5: Production branches of  $D^{*+}$ .

| rowNo | production branch of $D^{*+}$                           | iProdBrP | nCase | nCCase |
|-------|---|----------|-------|--------|
| 1     | $\bar{B}^0 \rightarrow \mu^- \bar{\nu}_\mu D^{*+}$      | 9        | 4154  | 4154   |
| 2     | $\bar{B}^0 \rightarrow e^- \bar{\nu}_e D^{*+}$          | 7        | 2886  | 7040   |
| 3     | $\bar{B}^0 \rightarrow D^{*+} D_s^{*-}$                 | 4        | 1691  | 8731   |
| 4     | $\bar{B}^0 \rightarrow e^- \bar{\nu}_e D^{*+} \gamma^F$ | 10       | 1623  | 10354  |
| 5     | $\bar{B}^0 \rightarrow \pi^0 \pi^+ \pi^- \pi^- D^{*+}$  | 40       | 1429  | 11783  |
| rest  | others (3272 in total)                                  | —        | 34103 | 45886  |

### 3.5. Mothers of particles

Occasionally, we may want to check the mothers of certain particles. The following example shows the associated item also with the two particles  $D^{*+}$  and  $J/\psi$  set as research objects. The input to this item is identical to those to the two items above.

```
% Component analysis — mothers of particles
{
  D*+   Dsp   5
  J/psi  Jpsi   5
}
```

Table 6 shows the mothers of  $D^{*+}$ . Notably, the PDG codes of the mother particles, instead of additional indices, are listed in the table, since they are sufficient to tag the mother particles. From the table, six sources of  $D^{*+}$  are found in the input sample and the dominant one is the  $\bar{B}^0$  decay.

Table 6: Mothers of  $D^{*+}$ .

| rowNo | mother of $D^{*+}$  | PDGMoth | nCase | nCCase |
|-------|---------------------|---------|-------|--------|
| 1     | $\bar{B}^0$         | -511    | 41751 | 41751  |
| 2     | $B^0$               | 511     | 2983  | 44734  |
| 3     | $D_1'^+$            | 20413   | 455   | 45189  |
| 4     | $D_1^+$             | 10413   | 368   | 45557  |
| 5     | $D_2^{*+}$          | 415     | 247   | 45804  |
| rest  | others (1 in total) | —       | 82    | 45886  |

### 3.6. Cascade decay branches of particles

Sometimes, the invariant mass constraint is applied to certain particle and the signal process is its cascade decay branch. In this case, it is necessary to investigate the cascade decay branches of the particle, rather than its first decay branches, so as to analyze the backgrounds effectively. Below is an example demonstrating the related item by taking the two particles  $B^0$  and  $D^0$  as objects of study. While the first three columns of the input to this item have the same meanings as those to the three items above, the additional fourth column sets the maximum hierarchy of decay branches to be examined. Here, the hierarchy reflects the rank of a decay branch in a cascade decay branch of one specific particle. For instance, in the following cascade decay branch of  $B^0$ :  $B^0 \rightarrow \pi^0 \pi^0 \rho^0 \pi^+ D^{*-}$ ,  $\rho^0 \rightarrow \pi^+ \pi^-$ ,  $D^{*-} \rightarrow \pi^- \bar{D}^0$ ,  $\bar{D}^0 \rightarrow \eta \eta'$ ,  $\eta \rightarrow \pi^0 \pi^0 \pi^0$ ,  $\eta' \rightarrow \pi^0 \pi^0 \eta$ ,  $\eta \rightarrow \gamma \gamma$ , the hierarchies of the seven individual decay branches are 1, 2, 2, 3, 4, 4, and 5, respectively. In the example, the maximum hierarchy of decay branches is set to two for both  $B^0$  and  $D^0$ , and hence only the first two hierarchies of branches in their cascade decays will be investigated. Without such settings, all the branches in their cascade decays will be examined.

```
% Component analysis — cascade decay branches of particles
{
  B0  B0  5  2
  D0  D0  5  2
}
```

The cascade decay branches of  $B^0$  are displayed in Table 7.

Table 7: Cascade decay branches of  $B^0$  (only the first two hierarchies are involved).

| rowNo | cascade decay branch of $B^0$  | iCascDcyBrSP | nCase | nCCase |
|-------|--|--------------|-------|--------|
| 1     | $B^0 \rightarrow \mu^+ \nu_\mu D^{*-}, D^{*-} \rightarrow \pi^- \bar{D}^0$                               | 12           | 2912  | 2912   |
| 2     | $B^0 \rightarrow e^+ \nu_e D^{*-}, D^{*-} \rightarrow \pi^- \bar{D}^0$                                   | 6            | 1991  | 4903   |
| 3     | $B^0 \rightarrow \mu^+ \nu_\mu D^{*-}, D^{*-} \rightarrow \pi^0 D^-$                                     | 70           | 1283  | 6186   |
| 4     | $B^0 \rightarrow e^+ \nu_e D^{*-} \gamma^F, D^{*-} \rightarrow \pi^- \bar{D}^0$                          | 18           | 1132  | 7318   |
| 5     | $B^0 \rightarrow D^{*-} D_s^{*+}, D^{*-} \rightarrow \pi^- \bar{D}^0, D_s^{*+} \rightarrow D_s^+ \gamma$ | 20           | 1119  | 8437   |
| rest  | $B^0 \rightarrow \text{others (42074 in total)}$   | —            | 91594 | 100031 |

### 3.7. Decay final states of particles

When the invariant mass constraint is applied to certain particle reconstructed directly from a specific final state, it is significant to examine the decay final states of the particle, rather than its first or cascade decay branches, in order to study the backgrounds effectively. The following example shows the associated item also with the two particles  $B^0$  and  $D^0$  set as research objects. The format of the input to the item is the same as that to the above item, but the fourth parameters here are designed to restrict the numbers of final state particles. Without the fourth parameters, all the decay final states of the specified particles will be investigated. In the example, the parameters are set to three for both  $B^0$  and  $D^0$ , and thus only the three-body decay final states of them will be examined.

```
% Component analysis — decay final states of particles
{
  B0  B0  5  3
  D0  D0  5  3
}
```

Table 8 shows the three-body decay final states of  $D^0$ . In the table,  $\pi^0$  only decays to  $\gamma \gamma$ ; otherwise, it will be replaced with its decay products, resulting in different decay final states of  $D^0$ .



Table 8: Decay final states of  $D^0$  (only three-body final states are involved).

| rowNo | decay final state of $D^0$                    | iDcyFStP | nCase | nCCase |
|-------|---|----------|-------|--------|
| 1     | $D^0 \rightarrow \pi^0 \pi^+ K^-$             | 2        | 6258  | 6258   |
| 2     | $D^0 \rightarrow \mu^+ \nu_\mu K^-$           | 5        | 1487  | 7745   |
| 3     | $D^0 \rightarrow \pi^0 \pi^+ \pi^-$           | 1        | 1162  | 8907   |
| 4     | $D^0 \rightarrow K_L^0 \pi^+ \pi^-$           | 3        | 1158  | 10065  |
| 5     | $D^0 \rightarrow e^+ \nu_e K^-$               | 11       | 1148  | 11213  |
| rest  | $D^0 \rightarrow \text{others (24 in total)}$ | —        | 2407  | 13620  |

### 3.8. Inclusive decay branches

In a few physics studies, we take inclusive decay branches as signals. In such cases, it is essential to have a basic knowledge of the exclusive components of these inclusive decay branches. Below is an example demonstrating the related item by investigating the exclusive components of the two inclusive decay branches  $\bar{B}^0 \rightarrow D^{*+} + \text{anything}$  and  $B^0 \rightarrow K_S^0 + \text{anything}$ . In the item, each row holds the information of an inclusive decay branch, and the first, second, and third columns separated with the symbol “&” are the textual expressions, aliases, and maximum numbers of output components, respectively. As we introduce at the beginning part of this section, the aliases and maximum numbers of output components are both optional. Here, we note that the symbol “—” can be used as a placeholder for an unassigned alias, if only the maximum number of output components is desired.

```
% Component analysis — inclusive decay branches
{
  B0 --> D*+ & B2Dsp & 5
  B0 --> K_S0 & B2Ks & 5
}
```

The exclusive components of  $B^0 \rightarrow K_S^0 + \text{anything}$  are displayed in Table 9. From the table, ten exclusive components of the inclusive decay branch are found in the input sample, and the particles denoted with *anything* are mainly the traditional charmonium states.

Table 9: Exclusive components of  $B^0 \rightarrow K_S^0 + \text{anything}$ .

| rowNo | exclusive component of $B^0 \rightarrow K_S^0 + \text{anything}$ | iDcyBrIncDcyBr | nCase | nCCase |
|-------|--|----------------|-------|--------|
| 1     | $B^0 \rightarrow K_S^0 J/\psi$                                   | 0              | 45    | 45     |
| 2     | $B^0 \rightarrow K_S^0 \eta_c$                                   | 1              | 40    | 85     |
| 3     | $B^0 \rightarrow K_S^0 \psi'$                                    | 3              | 33    | 118    |
| 4     | $B^0 \rightarrow K_S^0 \chi_{c1}$                                | 2              | 20    | 138    |
| 5     | $B^0 \rightarrow K_S^0 \chi_{c0}$                                | 4              | 6     | 144    |
| rest  | $B^0 \rightarrow K_S^0 + \text{others (5 in total)}$             | —              | 9     | 153    |

### 3.9. Intermediate-resonance-allowed decay branches

In many research works, we take multi-body decay branches as signals. On such occasions, it is fundamental to investigate the intermediate resonances involved in these decay branches. In other words, we need to examine the exclusive components of these IRA decay branches. The following example shows the associated item with the two IRA decay branches  $D^{*+} \rightarrow \pi^0 \pi^+ \pi^- K^-$  and  $J/\psi \rightarrow \pi^0 \pi^+ \pi^-$  set as objects of study. Since IRA decay branches look like inclusive decay branches, the format of the input to the item for IRA decay branches is identical to that for inclusive decay branches, which is introduced in the previous subsection.

```

582 % Component analysis — intermediate-resonance-allowed decay branches
583 {
584   D*+ --> K- pi+ pi+ pi0 & Dsp2K3Pi & 5
585   J/psi --> pi+ pi- pi0 & Jpsi23Pi & 5
586 }

```

Table 10 shows the exclusive components of  $D^{*+} \rightarrow \pi^0 \pi^+ \pi^+ K^-$ . From the table, two intermediate particles  $D^0$  and  $D^+$  are found in the IRA decay branch, and they decay to  $\pi^0 \pi^+ K^-$  and  $\pi^+ \pi^+ K^-$ , respectively.

Table 10: Exclusive components of  $D^{*+} \rightarrow \pi^0 \pi^+ \pi^+ K^-$ .

| rowNo | exclusive component of $D^{*+} \rightarrow \pi^0 \pi^+ \pi^+ K^-$ | iDcyBrIRADcyBr | nCase | nCCase |
|-------|---|----------------|-------|--------|
| 1     | $D^{*+} \rightarrow \pi^+ D^0, D^0 \rightarrow \pi^0 \pi^+ K^-$   | 0              | 3869  | 3869   |
| 2     | $D^{*+} \rightarrow \pi^0 D^+, D^+ \rightarrow \pi^+ \pi^+ K^-$   | 1              | 1102  | 4971   |

### 3.10. Essential topology tags

Table 11: Essential topology tags involved in each kind of component analysis.

| Component type                      | Topology tag       | Interpretation  |
|-------------------------------------|--------------------|---|
| Decay trees                         | iDcyTr             | index of decay tree   |
| Decay initial-final states          | iDcyIFSts          | index of decay initial-final states   |
| Decay branches of particles         | nPDcyBr.i          | number of particle;s (or its decay branches)                                      |
|                                     | iDcyBrP.i.j        | index of decay branch of the $j^{\text{th}}$ particle; <sub>i</sub>               |
| Production branches of particles    | nPProdBr.i         | number of particle;s (or its production branches)                                 |
|                                     | iProdBrP.i.j       | index of production branch of the $j^{\text{th}}$ particle; <sub>i</sub>          |
| Mothers of particles                | nPMoth.i           | number of particle;s (or its mothers)   |
|                                     | PDGMothP.i.j       | PDG code of mother of the $j^{\text{th}}$ particle; <sub>i</sub>                  |
| Cascade decay branches of particles | nPCascDcyBr.i      | number of particle;s (or its cascade decay branches)                              |
|                                     | iCascDcyBrP.i.j    | index of cascade decay branch of the $j^{\text{th}}$ particle; <sub>i</sub>       |
| Decay final states of particles     | nPDcyFSt.i         | number of particle;s (or its decay final states)                                  |
|                                     | iDcyFStP.i.j       | index of decay final state of the $j^{\text{th}}$ particle; <sub>i</sub>          |
| Inclusive decay branches            | nIncDcyBr.i        | number of inclusive decay branch;es   |
|                                     | iDcyBrIncDcyBr.i.j | index of decay branch of the $j^{\text{th}}$ inclusive decay branch; <sub>i</sub> |
| IRA decay branches                  | nIRADcyBr.i        | number of IRA decay branch;es   |
|                                     | iDcyBrIRADcyBr.i.j | index of decay branch of the $j^{\text{th}}$ IRA decay branch; <sub>i</sub>       |

Table 11 lists and interprets all of the essential topology tags involved in the component analysis functionalities. The topology tag for the component analysis over decay initial-final states is iDcyIFSts. It has a similar interpretation as iDcyTr and is shown in the third column of Table 3. For the latter seven kinds of component analysis, there are two sorts of topology tags. The first sort, such as nPDcyBr.i, records the number of instances of the  $i^{\text{th}}$  specified particle or decay branch found in each event. The second sort, for example, iDcyBrP.i.j, keeps the associated index of the  $j^{\text{th}}$  found instance of the  $i^{\text{th}}$  specified particle or decay branch. The indices and the decays they stand for can be found in Tables 4 – 10.

In the topology tags, “i” in “i” is the default index of the specified particle or decay branch, and it ranges from 0 (included) to the number of specified particles or decay branches (excluded). If the alias of the particle or decay branch is also specified, the index “i” will be replaced with

the alias. For example, since “Dsp” and “Jpsi” are set as the aliases of  $D^{*+}$  and  $J/\psi$  in the component analysis over their decay branches, the specialized topology tags nPDcyBr.Dsp and nPDcyBr.Jpsi, instead of the default ones nPDcyBr.0 and nPDcyBr.1, are used to store the numbers of  $D^{*+}$  and  $J/\psi$  found in each event.

In addition, “j” in “\_j” is the default index of the found instance of certain particle or decay branch in an event, and it ranges from 0 (included) to the sample-level maximum of the number of the particles or decay branches found in each event (excluded). For example, the maximum of the number of  $D^{*+}$  found in each event is two for the whole sample, and thus two topology tags iDcyBrP.Dsp\_0 and iDcyBrP.Dsp\_1 are employed to store the indices of  $D^{*+}$  decay branches. These indices range from 0 (included) to the number of the types of  $D^{*+}$  decay branches found in the samples (excluded). In the events with only one  $D^{*+}$ , iDcyBrP.Dsp\_1 is assigned with the default value  $-1$ ; in the events that have no  $D^{*+}$ , the default value  $-1$  is assigned to both iDcyBrP.Dsp\_0 and iDcyBrP.Dsp\_1. We note that different from all other indices, PDGMoth.i.j has the default value 0, instead of  $-1$ .

#### 4. Signal identification

Signal identification is the other functionality of the program. Though relatively simple, it can help us identify the “signals” we desire directly, quickly, and easily. Here, the “signals” are not confined to the authentic signals in our research works but can be any physics processes of interests, particularly some important backgrounds we concern. At present, the following eight kinds of signals can be identified with the program: (1) decay trees, (2) decay initial-final states, (3) particles, (4) (regular) decay branches, (5) cascade decay branches, (6) inclusive decay branches, (7) inclusive cascade decay branches, and (8) IRA decay branches. For each kind of signals, one item is developed to specify related parameters. This section introduces the eight kinds of signal identification, with each in a subsection. In each subsection, we take an example to demonstrate the related setting item and show the obtained topology map. For easy exposition, all of the essential topology tags involved in the signal identification functionalities are presented in another separate subsection, that is, the last subsection.

Similar to the cases of the latter seven kinds of component analysis, one or more signals can be specified in each of the signal identification items, and two signals are set in the following examples to illustrate the use of the items. Besides, meaning aliases can also be optionally assigned to the specified signals so as to better tag them in the names of the TBranch objects appended in the TTree object of the output root files.

##### 4.1. Decay trees

Sometimes, we need to identify certain decay trees. The following example shows the associated item with the first two decay trees listed in Table 2 set as signals. In the item, each row holds a decay branch in the decay trees, and the first, second, and third columns separated with the symbol “&” are the indices, textual expressions, and mother indices of the decay branches, respectively. The decay branches with index 0 indicate the beginning of new decay trees, and their mother indices are equal to  $-1$ , suggesting they have no mother branches because they are the first decay branches of the decay trees. Besides, the name of each decay tree can be optionally filled in the fourth column of its first decay branch. Similar to the third parameter in the item for the component analysis over decay trees (see Section 3.1), a “Y” can be optionally filled in the fifth column of the first decay branch of the first decay tree, to adjust the positions of decay final states in the output pdf file.

% Signal identification — decay trees

```

649 {
650   0 & Upsilon(4S) --> B0 anti-B0 & -1 & 1stDcyTrInTb2 & Y
651   1 & B0 --> e+ nu_e D*- gamma & 0
652   2 & anti-B0 --> mu- anti-nu_mu D*+ & 0
653   3 & D*- --> pi- anti-D0 & 1
654   4 & D*+ --> pi+ D0 & 2
655   5 & anti-D0 --> pi0 pi- K+ & 3
656   6 & D0 --> pi0 pi+ K- & 4
657
658   0 & Upsilon(4S) --> B0 anti-B0 & -1 & 2ndDcyTrInTb2
659   1 & B0 --> pi0 pi+ pi- rho- D- & 0
660   2 & anti-B0 --> mu- anti-nu_mu D*+ & 0
661   3 & rho- --> pi0 pi- & 1
662   4 & D- --> pi- pi- K+ & 1
663   5 & D*+ --> pi+ D0 & 2
664   6 & D0 --> K_L0 pi+ pi- & 5
665 }
666

```

Table 12 shows the resulting topology map. The results are the same as those displayed in the first two rows of Table 2.

Table 12: Signal decay trees and their respective initial-final states.

| rowNo | signal decay tree<br>(signal decay initial-final states)  | iSigDcyTr | nEtr | nCEtr |
|-------|---|-----------|------|-------|
| 1     | $\Upsilon(4S) \rightarrow B^0 \bar{B}^0, B^0 \rightarrow e^+ \nu_e D^{*-} \gamma^F, \bar{B}^0 \rightarrow \mu^- \bar{\nu}_\mu D^{*+}, D^{*-} \rightarrow \pi^- \bar{D}^0,$<br>$D^{*+} \rightarrow \pi^+ D^0, \bar{D}^0 \rightarrow \pi^0 \pi^- K^+, D^0 \rightarrow \pi^0 \pi^+ K^-$<br>$(\Upsilon(4S) \rightarrow e^+ \nu_e \mu^- \bar{\nu}_\mu \pi^0 \pi^+ \pi^- \pi^- K^+ K^- \gamma^F)$       | 0         | 3    | 3     |
| 2     | $\Upsilon(4S) \rightarrow B^0 \bar{B}^0, B^0 \rightarrow \pi^0 \pi^+ \pi^+ \rho^- D^-, \bar{B}^0 \rightarrow \mu^- \bar{\nu}_\mu D^{*+}, \rho^- \rightarrow \pi^0 \pi^-,$<br>$D^- \rightarrow \pi^- \pi^- K^+, D^{*+} \rightarrow \pi^+ D^0, D^0 \rightarrow K_L^0 \pi^+ \pi^-$<br>$(\Upsilon(4S) \rightarrow \mu^- \bar{\nu}_\mu \pi^0 \pi^0 K_L^0 \pi^+ \pi^+ \pi^+ \pi^- \pi^- \pi^- K^+ K^-)$ | 1         | 2    | 5     |

#### 4.2. Decay initial-final states

In a few cases, we have an interest in some decay initial-final states. Below is an example demonstrating the related item by taking the first two decay initial-final states listed in Table 3 as signals. Similar to IRA decay branches, decay initial-final states look like inclusive decay branches. Hence, except that only two columns are involved in the item, the format of the input to the item for decay initial-final states is identical to that for the component analysis over inclusive decay branches, which is introduced in Section 3.8. As we can see from the example, the numbers of identical particles are supported to be written in front of their textual names in order to simplify the textual expressions of the final states. The obtained topology map is displayed in Table 13. The results are identical to those shown in the first two rows of Table 3.

```

679 % Signal identification — decay initial-final states
680 {
681   Y(4S) --> mu+ nu_mu 3 pi0 3 pi+ 4 pi- K+ K- & 2ndDcyIFStsInTb3
682   Y(4S) --> 5 pi0 5 pi+ 5 pi- K+ K- & 2ndDcyIFStsInTb3
683 }
684

```

Table 13: Signal decay initial-final states.

| rowNo | signal decay initial-final states  | iSigDcyIFSts2 | nEtr | nCEtr |
|-------|--|---------------|------|-------|
| 1     | $\Upsilon(4S) \rightarrow \mu^+ \nu_\mu \pi^0 \pi^0 \pi^0 \pi^+ \pi^+ \pi^- \pi^- \pi^- K^+ K^-$ | 0             | 18   | 18    |
| 2     | $\Upsilon(4S) \rightarrow \pi^0 \pi^0 \pi^0 \pi^0 \pi^+ \pi^+ \pi^+ \pi^- \pi^- \pi^- K^+ K^-$   | 1             | 18   | 36    |

#### 4.3. Particles

Occasionally, we may want to identify some particles. The following example shows the associated item with the two particles  $D^{*+}$  and  $J/\psi$  set as signals. Except that only two columns are involved in the item, the format of the input to the item is identical to that for the component analysis over decay branches of particles, which is introduced in Section 3.3.

```

691 % Signal identification — particles
692 {
693     D*+   Dsp
694     J/psi  Jpsi
695 }

```

Table 14 shows the resulting topology map. As a cross-check, the number of  $D^{*+}$ s in the table equals those in Tables 4, 5, and 6.

Table 14: Signal particles.

| rowNo | signal particle | iSigP | nCase | nCCase |
|-------|-----------------|-------|-------|--------|
| 1     | $D^{*+}$        | 0     | 45886 | 45886  |
| 2     | $J/\psi$        | 1     | 2654  | 48540  |

#### 4.4. Decay branches

On some occasions, we have to identify certain regular decay branches. Below is an example demonstrating the related item by taking the two decay branches  $\bar{B}^0 \rightarrow \mu^- \bar{\nu}_\mu D^{*+}$  and  $B^0 \rightarrow K_S^0 J/\psi$  as signals. Since regular decay branches also look like inclusive decay branches, except that only two columns are involved in the item, the format of the input to the item for regular decay branches is identical to that for the component analysis over inclusive decay branches, which is introduced in Section 3.8.

```

707 % Signal identification — decay branches
708 {
709     B0 --> mu- anti-nu_mu D*+ & B2munuDsp
710     B0 --> K_S0 J/psi & B2KsJpsi
711 }

```

The obtained topology map is displayed Table 15. For cross-checks, we note that the number of  $\bar{B}^0 \rightarrow \mu^- \bar{\nu}_\mu D^{*+}$  ( $B^0 \rightarrow K_S^0 J/\psi$ ) in the table is equal to that in the first row of Table 5 (9).

Table 15: Signal decay branches.

| rowNo | signal decay branch                                | iSigDcyBr | nCase | nCCase |
|-------|--|-----------|-------|--------|
| 1     | $\bar{B}^0 \rightarrow \mu^- \bar{\nu}_\mu D^{*+}$ | 0         | 4154  | 4154   |
| 2     | $B^0 \rightarrow K_S^0 J/\psi$                     | 1         | 45    | 4199   |

#### 4.5. Cascade decay branches

Sometimes, we are interested in certain cascade decay branches. The following example shows the associated item with the two cascade decay branches  $B^0 \rightarrow D^{*-} D_s^{*+}$ ,  $D^{*-} \rightarrow \pi^- \bar{D}^0$ ,  $D_s^{*+} \rightarrow D_s^+ \gamma$  and  $B^0 \rightarrow D^{*-} D_s^{*+}$ ,  $D^{*-} \rightarrow \pi^- \bar{D}^0$  set as signals. While the first cascade decay branch is identical to the fifth one in Table 7, the second is only part of it, which demonstrates that the cascade decay branches supported in the item are not necessarily fully specified at the level of certain hierarchy. Similar to decay trees, cascade decay branches are made up of regular

decay branches. Hence, the format of the input to the item for cascade decay branches is identical to that for decay trees, which is introduced in Section 4.1.

```

725 % Signal identification — cascade decay branches
726 {
727   0 & B0 --> D*- D_s*+ & -1
728   1 & D*- --> pi- anti-D0 & 0
729   2 & D_s*+ --> D_s+ gamma & 0
730
731   0 & B0 --> D*- D_s*+ & -1
732   1 & D*- --> pi- anti-D0 & 0
733 }

```

Table 16 shows the resulting topology map. As a cross-check, the number of cases of the first cascade decay branch in the table equals that of the fifth cascade decay branch in Table 7.

Table 16: Signal cascade decay branches.

| rowNo | signal cascade decay branch  | iSigCascDcyBr | nCase | nCCase |
|-------|--|---------------|-------|--------|
| 1     | $B^0 \rightarrow D^{*-} D_s^{*+}, D^{*-} \rightarrow \pi^- \bar{D}^0, D_s^{*+} \rightarrow D_s^+ \gamma$ | 0             | 1119  | 1119   |
| 2     | $B^0 \rightarrow D^{*-} D_s^{*+}, D^{*-} \rightarrow \pi^- \bar{D}^0$                                    | 1             | 1180  | 2299   |

#### 4.6. Inclusive decay branches

In a few cases, we have to identify some inclusive decay branches. Below is an example demonstrating the related item by taking the two inclusive decay branches  $\bar{B}^0 \rightarrow D^{*+} + anything$  and  $B^0 \rightarrow K_S^0 + anything$  as signals. Except that only two columns are involved in the item, the format of the input to the item is identical to that for the component analysis over inclusive decay branches, which is introduced in Section 3.8.

```

744 % Signal identification — inclusive decay branches
745 {
746   anti-B0 --> D*+ & B2Dsp
747   B0 --> K_S0 & B2Ks
748 }

```

The obtained topology map is displayed in Table 17. As a cross-check, the number of  $B^0 \rightarrow K_S^0 + anything$  in the table equals that in Table 9.

Table 17: Signal inclusive decay branches.

| rowNo | signal inclusive decay branch             | iSigIncDcyBr | nCase | nCCase |
|-------|---|--------------|-------|--------|
| 1     | $\bar{B}^0 \rightarrow D^{*+} + anything$ | 0            | 41751 | 41751  |
| 2     | $B^0 \rightarrow K_S^0 + anything$        | 1            | 153   | 41904  |

#### 4.7. Inclusive cascade decay branches

Occasionally, we may have an interest in certain inclusive cascade decay branches. The following example shows the associated item with the two inclusive cascade decay branches  $\bar{B}^0 \rightarrow D^{*+} + anything$ ,  $D^{*+} \rightarrow \pi^+ D^0$  and  $B^0 \rightarrow K_S^0 J/\psi$ ,  $K_S^0 \rightarrow \pi^+ \pi^-$ ,  $J/\psi \rightarrow \mu^+ + anything$  set as signals. Similar to decay trees and cascade decay branches, inclusive cascade decay branches are made up of regular decay branches. Hence, the format of the input to the item for inclusive cascade decay branches is also identical to that for decay trees, which is introduced in Section

4.1. and the independent textual name “\*” denotes anything.

```

% Signal identification — inclusive cascade decay branches
{
  0 & anti-B0 --> D*+ * & -1
  1 & D*+ --> pi+ D0 & 0

  0 & B0 --> K_S0 J/psi & -1
  1 & K_S0 --> pi+ pi- & 0
  2 & J/psi --> mu+ * & 0
}

```

Table 18 shows the resulting topology map.

Table 18: Signal inclusive cascade decay branches.

| rowNo | signal inclusive cascade decay branch   | iSigIncCascDcyBrs | nCase | nCCase |
|-------|---|-------------------|-------|--------|
| 1     | $\bar{B}^0 \rightarrow D^{*+} + \text{anything}, D^{*+} \rightarrow \pi^+ D^0$                            | 0                 | 28367 | 28367  |
| 2     | $B^0 \rightarrow K_S^0 J/\psi, K_S^0 \rightarrow \pi^+ \pi^-, J/\psi \rightarrow \mu^+ + \text{anything}$ | 1                 | 1     | 28368  |

#### 4.8. Intermediate-resonance-allowed decay branches

On some occasions, we need to identify certain IRA decay branches. Below is an example demonstrating the related item by taking the two IRA decay branches  $D^{*+} \rightarrow \pi^0 \pi^+ \pi^+ K^-$  and  $J/\psi \rightarrow \pi^0 \pi^+ \pi^-$  as signals. Except that only two columns are involved in the item, the format of the input to the item is identical to that for the component analysis over IRA decay branches, which is introduced in Section 3.9.

```

% Signal identification — intermediate-resonance-allowed decay branches
{
  D*+ --> K- pi+ pi+ pi0 & Dsp2K3Pi
  J/psi --> pi+ pi- pi0 & Jpsi23Pi
}

```

The obtained topology map is displayed in Table 19. For the purpose of cross-checks, we note that the number of  $D^{*+} \rightarrow \pi^0 \pi^+ \pi^+ K^-$  in the table is equal to that in Table 10.

Table 19: Signal IRA decay branches.

| rowNo | signal IRA decay branch                    | iSigIRADcyBr | nCase | nCCase |
|-------|--|--------------|-------|--------|
| 1     | $D^{*+} \rightarrow \pi^0 \pi^+ \pi^+ K^-$ | 0            | 4971  | 4971   |
| 2     | $J/\psi \rightarrow \pi^0 \pi^+ \pi^-$     | 1            | 59    | 5030   |

#### 4.9. Essential topology tags

Table 20 summarizes and explains all of the essential topology tags involved in the signal identification functionalities. For signal decay trees and signal decay initial-final states, there are two sorts of topology tags. The first sort of tags, iSigDcyTr and iSigDcyIFSts, record the default indices of the specified signal decay trees and signal decay initial-final states. They have similar interpretations as iDcyTr and iDcyIFSts, and are shown in the third columns of Tables 12 and 13. The second sort of tags, nameSigDcyTr and nameSigDcyIFSts, save the specified aliases of the signal decay trees and signal decay initial-final states. In cases the aliases are not specified,

empty strings will be stored.

For the latter six kinds of signal identification, there is only one sort of topology tags, which records the number of instances of certain specified particle or decay branch found in each event. Similar to the cases in the latter seven kinds of component analysis, in the topology tags, “i” in “\_i” is the default index of the specified particle or decay branch, and it ranges from 0 (included) to the number of specified particles or decay branches (excluded). If the alias of the particle or decay branch is also specified, the index “i” will be replaced with the alias.

Table 20: Essential topology tags involved in each kind of signal identification.

| Signal type                      | Topology tag       | Interpretation   |
|----------------------------------|--------------------|--|
| Decay trees                      | iSigDcyTr          | index of signal decay tree                                     |
|                                  | nameSigDcyTr       | name of signal decay tree                                      |
| Decay initial-final states       | iSigDcyIFSts       | index of signal decay initial-final states                     |
|                                  | nameSigDcyIFSts    | name of signal decay initial-final states                      |
| Particles                        | nSigP_i            | number of signal particle <sub>s</sub>                         |
| Decay branches                   | nSigDcyBr_i        | number of signal decay branch <sub>ies</sub>                   |
| Cascade decay branches           | nSigCascDcyBr_i    | number of signal cascade decay branch <sub>ies</sub>           |
| Inclusive decay branches         | nSigIncDcyBr_i     | number of signal inclusive decay branch <sub>ies</sub>         |
| Inclusive cascade decay branches | nSigIncCascDcyBr_i | number of signal inclusive cascade decay branch <sub>ies</sub> |
| IRA decay branches               | nSigIRADcyBr_i     | number of signal IRA decay branch <sub>ies</sub>               |

## 5. Common settings

From Sections 3 and 4, the optional parameters of the functionality items give us more choices and thus help us do our jobs quicker and better. In addition to these parameters, many optional items are designed and implemented to control the execution of the program in order to meet practical needs. Unlike the optional parameters, which only affect the individual functionalities to which they belong, the optional items have an impact on all of the functionalities, or at least most of the functionalities. The current version of the program contains 24 commonly used items, which can be divided into the following three groups: items on the input of the program, items on the functionalities of the program, and items on the output of the program. This section introduces these items in the three groups, with each group in one subsection.

### 5.1. Settings on the input of the program

#### 5.1.1. Input entries

The program normally processes all of the entries in the input samples, but sometimes only a part of the entries are needed to be (first) processed. Running the program over a big sample usually takes a long time. In such a case, it is a good habit to run the program first over a small part of the sample to check possible exceptions, and then over the whole sample if no exceptions are found or after the found exceptions are handled. Besides, a small number of entries is usually sufficient to do tests in the development of the program. For these reasons, an item is developed to set up the maximum number of entries to be processed. Below is an example showing the item with the maximum number set at two thousand.

```
% Maximum number of entries to be processed
{
    2000
}
```



On some occasions, especially in the course of optimizing selection criteria, we need to run the program only over entries satisfying certain requirements. For this purpose, an item is developed to select entries. The following example shows the item with X set in the range (-1, 1).

```

832 % Cut to select entries
833 {
834     (X > -1) && (X < 1)
835 }

```

Notably, only a single-line selection requirement is supported in the item, like the cases in the methods Draw() [15] and GetEntries() [16] of the class TTree. In spite of this, such a requirement is able to express any requirement with the help of the parentheses “()” as well as the logical symbols “&&”, “||”, and “!”.

Occasionally, array variables are involved in the requirement. Under the circumstances, users have to tell the program how to determine the total logical value with the individual logical values. At present, two criteria are provided: (1) the total result is true as long as the result for one instance is true; (2) the total result is false as long as the result for one instance is false. By default, the second criterion is used in the program. One can alter it to the first one with the following item.

```

847 % Method to apply cut to array variables (Two options: T and F. Default: F)
848 {
849     T
850 }

```

In the item, “T” and “F” stand for the first and second criteria, respectively.

### 5.1.2. Input decay branches

Table 21: Decay trees and their respective initial-final states.

| rowNo | decay tree<br>(decay initial-final states)   | iDcyTr | nEtr  | nCEtr  |
|-------|--|--------|-------|--------|
| 1     | $\Upsilon(4S) \rightarrow B^0 \bar{B}^0$<br>( $\Upsilon(4S) \rightarrow B^0 \bar{B}^0$ )             | 0      | 81057 | 81057  |
| 2     | $\Upsilon(4S) \rightarrow B^0 B^0$<br>( $\Upsilon(4S) \rightarrow B^0 B^0$ )                         | 1      | 9487  | 90544  |
| 3     | $\Upsilon(4S) \rightarrow \bar{B}^0 \bar{B}^0$<br>( $\Upsilon(4S) \rightarrow \bar{B}^0 \bar{B}^0$ ) | 2      | 9456  | 100000 |

Normally, the program deals with all of the decay branches in every decay tree. However, examining all the branches is not always required in practice. Sometimes, we only concern the first  $n$  hierarchies of the branches. Similar to that in cascade decay branches of particles (as we introduce in Section 3.6), the hierarchy here reflects the rank of a decay branch in a decay tree. For example, in the decay tree  $\Upsilon(4S) \rightarrow B^0 \bar{B}^0$ ,  $B^0 \rightarrow e^+ \nu_e D^{*-} \gamma^F$ ,  $\bar{B}^0 \rightarrow \mu^- \bar{\nu}_\mu D^{*+}$ ,  $D^{*-} \rightarrow \pi^- \bar{D}^0$ ,  $D^{*+} \rightarrow \pi^+ D^0$ ,  $\bar{D}^0 \rightarrow \pi^0 \pi^- K^+$ ,  $D^0 \rightarrow \pi^0 \pi^+ K^-$ , the hierarchies of the seven individual branches are 1, 2, 2, 3, 3, 4, and 4, respectively. The program provides an item to set the maximum hierarchy. Below is an example showing the item with the maximum hierarchy set at one.

```

864 % Maximum hierarchy of heading decay branches to be processed in each event
865 {
866     1
867 }

```

868  
869  
870  
871  
872  
873  
874  
875  
876

With the setting, the decay branches with hierarchy larger than one will be ignored by the program. For the component analysis over the decay trees of the  $\Upsilon(4S)$  sample, only the first hierarchy of  $\Upsilon(4S)$  decay branches are analyzed, and the result is shown in Table 21. From the table, not only  $\Upsilon(4S) \rightarrow B^0 \bar{B}^0$  but also  $\Upsilon(4S) \rightarrow B^0 B^0$  and  $\Upsilon(4S) \rightarrow \bar{B}^0 \bar{B}^0$  are seen because of  $B^0$ - $\bar{B}^0$  mixing.

Similarly, in the case of the maximum hierarchy set at two, we could get the result of the component analysis over the first two hierarchies of  $\Upsilon(4S)$  decay branches, as displayed in Table 22.

Table 22: Decay trees and their respective initial-final states.

| rowNo | decay tree<br>(decay initial-final states)   | iDcyTr | nEtr  | nCEtr  |
|-------|--|--------|-------|--------|
| 1     | $\Upsilon(4S) \rightarrow B^0 \bar{B}^0, B^0 \rightarrow \mu^+ \nu_\mu D^{*-}, \bar{B}^0 \rightarrow \mu^- \bar{\nu}_\mu D^{*+}$<br>( $\Upsilon(4S) \rightarrow \mu^+ \mu^- \nu_\mu \bar{\nu}_\mu D^{*+} D^{*-}$ ) | 936    | 136   | 136    |
| 2     | $\Upsilon(4S) \rightarrow B^0 \bar{B}^0, B^0 \rightarrow e^+ \nu_e D^{*-}, \bar{B}^0 \rightarrow \mu^- \bar{\nu}_\mu D^{*+}$<br>( $\Upsilon(4S) \rightarrow e^+ \nu_e \mu^- \bar{\nu}_\mu D^{*+} D^{*-}$ )         | 1188   | 112   | 248    |
| 3     | $\Upsilon(4S) \rightarrow B^0 \bar{B}^0, B^0 \rightarrow \mu^+ \nu_\mu D^{*-}, \bar{B}^0 \rightarrow e^- \bar{\nu}_e D^{*+}$<br>( $\Upsilon(4S) \rightarrow e^- \bar{\nu}_e \mu^+ \nu_\mu D^{*+} D^{*-}$ )         | 268    | 110   | 358    |
| 4     | $\Upsilon(4S) \rightarrow B^0 \bar{B}^0, B^0 \rightarrow D^{*-} D_s^{*+}, \bar{B}^0 \rightarrow \mu^- \bar{\nu}_\mu D^{*+}$<br>( $\Upsilon(4S) \rightarrow \mu^- \bar{\nu}_\mu D^{*+} D^{*-} D_s^{*+}$ )           | 2063   | 72    | 430    |
| 5     | $\Upsilon(4S) \rightarrow B^0 \bar{B}^0, B^0 \rightarrow e^+ \nu_e D^{*-}, \bar{B}^0 \rightarrow e^- \bar{\nu}_e D^{*+}$<br>( $\Upsilon(4S) \rightarrow e^+ e^- \nu_e \bar{\nu}_e D^{*+} D^{*-}$ )                 | 95     | 71    | 501    |
| rest  | $\Upsilon(4S) \rightarrow \text{others (81609 in total)}$<br>( $\Upsilon(4S) \rightarrow \text{corresponding to others}$ )   | —      | 99499 | 100000 |

877 Sometimes, we do not care about the decay of some particles. One can make the program  
878 ignore their decay branches with the following item. With the setting in the example, the decay  
879 of  $B^0$  and  $\bar{B}^0$  will be ignored by the program.

```
880 % Ignore the decay of the following particles
881 {
882     B0
883     anti-B0
884 }
885
```

886  
887 At some other times, we have interest in the decay of some particles but not in the decay of their  
888 daughters. To handle this case, the following item is developed to make the program ignore the  
889 decay of their daughters. In the following example, the decay of the daughters of  $B^0$  and  $\bar{B}^0$  will  
890 be ignored by the program.

```
891 % Ignore the decay of the daughters of the following particles
892 {
893     B0
894     anti-B0
895 }
896
```

897  
898 The two settings above have the same effects as those in the previous paragraph which set the  
899 maximum hierarchy at one and two, and hence the corresponding results are identical to those  
900 shown in Tables 21 and 22.

901 As mentioned in Section 2.4, the decay  $\pi^0 \rightarrow \gamma\gamma$  is ignored by default. On the occasions  
902 when we need to identify the signals involving the decay, we can make the program retain the

903 decay with the item below set to “Y”.

```
904
905 % Retain the decay of pi0 to gamma gamma (Two options: Y and N. Default: N)
906 {
907     Y
908 }
```

909 Besides, if needed, one can make the program ignore other final decay branches, such as  $\eta \rightarrow \gamma\gamma$   
910 and  $K_S^0 \rightarrow \pi^+\pi^-$ , with the following item.

```
911
912 % Ignore the following final decay branches
913 {
914     eta --> gamma gamma
915     K.S0 --> pi+ pi-
916 }
917
```

### 918 5.1.3. Initial and final state radiation photons

919 Initial state radiation (ISR) and final state radiation (FSR) are inevitable physical effects in  
920  $e^+e^-$  colliding experiments. Therefore, ISR and FSR photons are often involved in inclusive  
921 MC samples. The program processes them together with other particles in the default case. To  
922 distinguish them from other photons, the program tries to label them in the output plain text, tex  
923 source, and pdf files. Sometimes, these photons are marked out beforehand with special PDG  
924 codes according to particle status information from generators. One can inform the program of  
925 these PDG codes by the following two items.

```
926
927 % PDG code of ISR photons (Default: 222222222)
928 {
929     222222222
930 }
931
932 % PDG code of FSR photons (Default: -22)
933 {
934     -22
935 }
936
```

937  
938 In this case, the program is able to label the ISR and FSR photons as  $\gamma^i$  (gammai) and  $\gamma^f$  (gam-  
939 maf) in the output pdf (plain text) files, respectively.

940 On other occasions, ISR and FSR photons are not marked out in advance due to some reasons.  
941 In such cases, the program has to identify them by itself according to the following rules: photons  
942 who have no mothers recorded in the arrays of the PDG codes and mother indices are considered  
943 as generalized ISR photons, while other photons who have at least one  $e^\pm$ ,  $\mu^\pm$ ,  $\pi^\pm$ ,  $K^\pm$ ,  $p$ , or  $\bar{p}$   
944 sister are taken as generalized FSR photons. Here, the modifier “generalized” is used because  
945 the rules can not determine the types of the photons in absolute accuracy. For example, photons  
946 from radiative decays might be mistaken as FSR photons. Despite this, generalized ISR and FSR  
947 photons are good concepts, particularly in cases where the sources of the photons are not required  
948 to be distinguished clearly. The program will label the generalized ISR and FSR photons as  $\gamma^I$   
949 (gammaI) and  $\gamma^F$  (gammaF) in the output pdf (plain text) files, respectively.

950 Notably, we are not concerned about these ISR and FSR photons in many cases, particularly  
951 when we want to identify our signals from some samples. If they have already been marked out  
952 beforehand, one can make the program ignore them accurately by setting the following two items  
953 to “Ys”.

```
954
955 % Ignore ISR photons (Three options: Ys, Yg and N. Default: N)
```

```

956     {
957         Ys
958     }
959
960
961     % Ignore FSR photons (Three options: Ys, Yg and N. Default: N)
962     {
963         Ys
964     }
965

```

966 In cases that these photons are not marked in advance, the option “Yg” can be used to ignore the  
967 generalized ISR and FSR photons. In “Ys” and “Yg”, “s” and “g” are the initials of the words  
968 “strict” and “generalized”, respectively.

## 969 5.2. Settings on the functionalities of the program

### 970 5.2.1. Candidate based analysis

971 According to the number of signal candidates in an event that are selected and retained to  
972 extract physics results, data analysis in high energy experiments can be divided into the following  
973 two categories: event based analysis and candidate based analysis. While at most one candidate  
974 in an event is kept in event based analysis, one or more candidates in an event can be retained in  
975 candidate based analysis. Generally, the quantities related to a candidate are stored in an entry of  
976 the TTree objects in the root files. Thus, one or more entries relate to an event in candidate based  
977 analysis, while only one entry corresponds to an event in event based analysis. Normally, the  
978 indices of candidates within an event are stored in the corresponding entries in candidate based  
979 analysis.

980 By default, the program analyzes the input entries one by one. In this case, the events with  
981 multiple candidates will be processed repeatedly. Particularly, the number of physics processes at  
982 the sample level will be overcounted. One can make the program avoid the problem by inputting  
983 “Y” to the following item.

```

984
985     % Avoid over counting for candidate based analysis (Two options: Y and N. Default: N)
986     {
987         Y
988     }
989

```

990 Also, the indices of candidates within an event are required. We can tell the program the related  
991 TBranch name with the following item.

```

992
993     % TBranch name of the indices of candidates in an event (Default: _candidate_)
994     {
995         iCandidate
996     }
997

```

998 With the settings, the program will process the first entry of each event in a normal way, including  
999 obtaining and storing the topology tags; it will not analyze the other entries of the same event,  
1000 but only store the same topology tags to them.

### 1001 5.2.2. Charge conjugation

1002 Charge conjugation is an important concept in high energy physics. By default, charge con-  
1003 jugate objects (particles and decays) are processed separately in the program. However, we need  
1004 to handle them together in many physics studies because of the sameness between them. One  
1005 can have the program process them together with the item below set to “Y”.

Table 23: Topology tags related to charge conjugation involved in each kind of component analysis. For the latter seven kinds of component analysis, the topology tags in the (1) and (2) groups are only designed for the self-charge-conjugate and non-self-charge-conjugate particles and decay branches, respectively. The acronyms “cc” and  $\text{index}_{\text{cc}}$  are short for “charge conjugate” and “charge conjugate index”, respectively. For self-charge-conjugate objects (particles or decays), the charge conjugate indices have the value 0; for non-self-charge-conjugate objects, they have the value 1 or  $-1$ : while 1 tags the objects presented in the topology maps,  $-1$  indicates their charge conjugate objects.

| Component type                      | Topology tag             | Interpretation  |
|-------------------------------------|--------------------------|---|
| Decay trees                         | iCcDcyTr                 | $\text{index}_{\text{cc}}$ of decay tree  |
| Decay initial-final states          | iCcDcyIFSts              | $\text{index}_{\text{cc}}$ of decay initial-final states  |
| Decay branches of particles         | iCcPDcyBr.i              | $\text{index}_{\text{cc}}$ of particle <sub>i</sub>   |
|                                     | (1) iCcDcyBrP.i.j        | $\text{index}_{\text{cc}}$ of decay branch of the j <sup>th</sup> particle <sub>i</sub>               |
|                                     | (2) nCcPDcyBr.i          | number of cc particle <sub>i</sub> s (decay branches)   |
|                                     | (2) iDcyBrCcP.i.j        | index of decay branch of the j <sup>th</sup> cc particle <sub>i</sub>                                 |
| Production branches of particles    | (2) nAllPDcyBr.i         | number of all particle <sub>i</sub> s (decay branches)  |
|                                     | iCcPProdBr.i             | $\text{index}_{\text{cc}}$ of particle <sub>i</sub>   |
|                                     | (1) iCcProdBrP.i.j       | $\text{index}_{\text{cc}}$ of production branch of the j <sup>th</sup> particle <sub>i</sub>          |
|                                     | (2) nCcPProdBr.i         | number of cc particle <sub>i</sub> s (production branches)  |
| Mothers of particles                | (2) iProdBrCcP.i.j       | index of production branch of the j <sup>th</sup> cc particle <sub>i</sub>                            |
|                                     | (2) nAllPProdBr.i        | number of all particle <sub>i</sub> s (production branches)   |
|                                     | iCcPMoth.i               | $\text{index}_{\text{cc}}$ of particle <sub>i</sub>   |
|                                     | (1) iCcMothP.i.j         | $\text{index}_{\text{cc}}$ of mother of the j <sup>th</sup> particle <sub>i</sub>                     |
| Cascade decay branches of particles | (2) nCcPMoth.i           | number of cc particle <sub>i</sub> s (mothers)  |
|                                     | (2) PDGMothCcP.i.j       | PDG code of mother of the j <sup>th</sup> cc particle <sub>i</sub>                                    |
|                                     | (2) nAllPMoth.i          | number of all particle <sub>i</sub> s (mothers)   |
|                                     | iCcPCascDcyBr.i          | $\text{index}_{\text{cc}}$ of particle <sub>i</sub>   |
| Decay final states of particles     | (1) iCcCascDcyBrP.i.j    | $\text{index}_{\text{cc}}$ of cascade decay branch of the j <sup>th</sup> particle <sub>i</sub>       |
|                                     | (2) nCcPCascDcyBr.i      | number of cc particle <sub>i</sub> s (cascade decay branches)   |
|                                     | (2) iCascDcyBrCcP.i.j    | index of cascade decay branch of the j <sup>th</sup> cc particle <sub>i</sub>                         |
|                                     | (2) nAllPCascDcyBr.i     | number of all particle <sub>i</sub> s (cascade decay branches)  |
| Inclusive decay branches            | iCcPDcyFSt.i             | $\text{index}_{\text{cc}}$ of particle <sub>i</sub>   |
|                                     | (1) iCcDcyFStP.i.j       | $\text{index}_{\text{cc}}$ of decay final state of the j <sup>th</sup> particle <sub>i</sub>          |
|                                     | (2) nCcPDcyFSt.i         | number of cc particle <sub>i</sub> s (decay final states)   |
|                                     | (2) iDcyFStCcP.i.j       | index of decay final state of the j <sup>th</sup> cc particle <sub>i</sub>                            |
| IRA decay branches                  | (2) nAllPDcyFSt.i        | number of all particle <sub>i</sub> s (decay final states)  |
|                                     | iCcIncDcyBr.i            | $\text{index}_{\text{cc}}$ of inclusive decay branch <sub>i</sub>                                     |
|                                     | (1) iCcDcyBrIncDcyBr.i.j | $\text{index}_{\text{cc}}$ of decay branch of the j <sup>th</sup> inclusive decay branch <sub>i</sub> |
|                                     | (2) nCcIncDcyBr.i        | number of cc inclusive decay branch <sub>i</sub> s  |
| IRA decay branches                  | (2) iDcyBrCcIncDcyBr.i.j | index of decay branch of the j <sup>th</sup> cc inclusive decay branch <sub>i</sub>                   |
|                                     | (2) nAllIncDcyBr.i       | number of all inclusive decay branch <sub>i</sub> s   |
|                                     | iCcIRADcyBr.i            | $\text{index}_{\text{cc}}$ of IRA decay branch <sub>i</sub>   |
|                                     | (1) iCcDcyBrIRADcyBr.i.j | $\text{index}_{\text{cc}}$ of decay branch of the j <sup>th</sup> IRA decay branch <sub>i</sub>       |
|                                     | (2) nCcIRADcyBr.i        | number of cc IRA decay branch <sub>i</sub> s  |
|                                     | (2) iDcyBrCcIRADcyBr.i.j | index of decay branch of the j <sup>th</sup> cc IRA decay branch <sub>i</sub>                         |
|                                     | (2) nAllIRADcyBr.i       | number of all IRA decay branch <sub>i</sub> s   |

```

1006
1007
1008 % Process charge conjugate objects together (Two options: Y and N. Default: N)
1009 {
1010     Y
1011 }
1012

```

1013 Performing topology analysis with this setting inserts new topology tags in the output root files  
1014 and adds new counters to topology maps in the output plain text, tex source, and pdf files. Tables  
1015 23 and 24 list and interpret all of the topology tags related to charge conjugation involved in the  
1016 component analysis and signal identification functionalities, respectively.

Table 24: Topology tags related to charge conjugation involved in each kind of signal identification. For the latter six kinds of signal identification, the topology tags in the (\*) groups are only designed for the non-self-charge-conjugate particles and decay branches. The acronyms “cc” and  $\text{index}_{\text{cc}}$  are short for “charge conjugate” and “charge conjugate index”, respectively. For self-charge-conjugate objects (particles or decays), the charge conjugate indices have the value 0; for non-self-charge-conjugate objects, they have the value 1 or  $-1$ : while 1 tags the objects presented in the topology maps,  $-1$  indicates their charge conjugate objects.

| Signal type                      | Topology tag              | Interpretation   |
|----------------------------------|---------------------------|--|
| Decay trees                      | iCcSigDcyTr               | $\text{index}_{\text{cc}}$ of signal decay tree                                  |
| Decay initial-final states       | iCcSigDcyIFSts            | $\text{index}_{\text{cc}}$ of signal decay initial-final states                  |
| Particles                        | iCcSigP.i                 | $\text{index}_{\text{cc}}$ of signal particle <sub>i</sub>                       |
|                                  | (*) nCcSigP.i             | number of cc signal particle <sub>i</sub> s                                      |
|                                  | (*) nAllSigP.i            | number of all signal particle <sub>i</sub> s                                     |
| Decay branches                   | iCcSigDcyBr.i             | $\text{index}_{\text{cc}}$ of signal decay branch <sub>i</sub>                   |
|                                  | (*) nCcSigDcyBr.i         | number of cc signal decay branch <sub>i</sub> es                                 |
|                                  | (*) nAllSigDcyBr.i        | number of all signal decay branch <sub>i</sub> es                                |
| Cascade decay branches           | iCcSigCascDcyBr.i         | $\text{index}_{\text{cc}}$ of signal cascade decay branch <sub>i</sub>           |
|                                  | (*) nCcSigCascDcyBr.i     | number of cc signal cascade decay branch <sub>i</sub> es                         |
|                                  | (*) nAllSigCascDcyBr.i    | number of all signal cascade decay branch <sub>i</sub> es                        |
| Inclusive decay branches         | iCcSigIncDcyBr.i          | $\text{index}_{\text{cc}}$ of signal inclusive decay branch <sub>i</sub>         |
|                                  | (*) nCcSigIncDcyBr.i      | number of cc signal inclusive decay branch <sub>i</sub> es                       |
|                                  | (*) nAllSigIncDcyBr.i     | number of all signal inclusive decay branch <sub>i</sub> es                      |
| Inclusive cascade decay branches | iCcSigIncCascDcyBr.i      | $\text{index}_{\text{cc}}$ of signal inclusive cascade decay branch <sub>i</sub> |
|                                  | (*) nCcSigIncCascDcyBr.i  | number of cc signal inclusive cascade decay branch <sub>i</sub> es               |
|                                  | (*) nAllSigIncCascDcyBr.i | number of all signal inclusive cascade decay branch <sub>i</sub> es              |
| IRA decay branches               | iCcSigIRADcyBr.i          | $\text{index}_{\text{cc}}$ of signal IRA decay branch <sub>i</sub>               |
|                                  | (*) nCcSigIRADcyBr.i      | number of cc signal IRA decay branch <sub>i</sub> es                             |
|                                  | (*) nAllSigIRADcyBr.i     | number of all signal IRA decay branch <sub>i</sub> es                            |

1017 As an example, we perform the component analysis over decay trees with the charge con-  
1018 jugate item. Table 25 shows the obtained topology map. Besides the columns in Table 2, two  
1019 additional columns with the headers “nCcEtr” and “nAllEtr” are inserted in the table. Here, “nC-  
1020 cEtr” represents the number of entries involving the charge conjugate decay trees, and “nAllEtr”  
1021 is the sum of “nEtr” and “nCcEtr”. In addition to “iDcyTr”, “iCcDcyTr” is also inserted in the  
1022 output root files as a topology tag. It is short for charge conjugate index of decay tree. For self-  
1023 charge-conjugate decay trees, it has the value 0; for non-self-charge-conjugate decay trees, it has  
1024 the value 1 or  $-1$ : while 1 tags the decay trees listed in the topology maps,  $-1$  indicates their  
1025 charge conjugate decay trees. Whereas the equal values of “iDcyTr” for each decay tree and its  
1026 charge conjugate decay tree indicate their sameness, the opposite values of “iCcDcyTr” for them  
1027 reflect their difference.

Table 25: Decay trees and their respective initial-final states (with the charge conjugation setting).

| rowNo | decay tree<br>(decay initial-final states)   | iDcyTr | nEtr | nCcEtr | nAllEtr | nCEtr  |
|-------|--|--------|------|--------|---------|--------|
| 1     | $\Upsilon(4S) \rightarrow B^0 \bar{B}^0, B^0 \rightarrow e^+ \nu_e D^{*-} \gamma^F, \bar{B}^0 \rightarrow \mu^- \bar{\nu}_\mu D^{*+}, D^{*-} \rightarrow \pi^- \bar{D}^0,$<br>$D^{*+} \rightarrow \pi^+ D^0, \bar{D}^0 \rightarrow \pi^0 \pi^- K^+, D^0 \rightarrow \pi^0 \pi^+ K^-$<br>$(\Upsilon(4S) \rightarrow e^+ \nu_e \mu^- \bar{\nu}_\mu \pi^0 \pi^0 \pi^+ \pi^- \pi^- K^+ K^- \gamma^F)$                  | 20870  | 3    | 0      | 3       | 3      |
| 2     | $\Upsilon(4S) \rightarrow B^0 \bar{B}^0, B^0 \rightarrow \mu^+ \nu_\mu D^-, \bar{B}^0 \rightarrow e^- \bar{\nu}_e D^{*+}, D^- \rightarrow e^- \bar{\nu}_e \pi^- K^+,$<br>$D^{*+} \rightarrow \pi^+ D^0, D^0 \rightarrow \pi^0 \pi^+ K^-$<br>$(\Upsilon(4S) \rightarrow e^- e^+ \bar{\nu}_e \bar{\nu}_e \mu^+ \nu_\mu \pi^0 \pi^+ \pi^- \pi^- K^+ K^-)$   | 3722   | 1    | 1      | 2       | 5      |
| 3     | $\Upsilon(4S) \rightarrow B^0 \bar{B}^0, B^0 \rightarrow \pi^0 \pi^+ \pi^- \rho^- D^-, \bar{B}^0 \rightarrow \mu^- \bar{\nu}_\mu D^{*+}, \rho^- \rightarrow \pi^0 \pi^-,$<br>$D^- \rightarrow \pi^- \pi^- K^+, D^{*+} \rightarrow \pi^+ D^0, D^0 \rightarrow K_L^0 \pi^+ \pi^-$<br>$(\Upsilon(4S) \rightarrow e^+ \nu_e \pi^0 \pi^0 \pi^+ \pi^+ \pi^- \pi^- \pi^- \pi^- K^+ K^- \gamma^F)$                         | 5295   | 2    | 0      | 2       | 7      |
| 4     | $\Upsilon(4S) \rightarrow B^0 \bar{B}^0, B^0 \rightarrow e^+ \nu_e D^{*-} \gamma^F, \bar{B}^0 \rightarrow \pi^0 \pi^+ \pi^- \pi^- D^{*+},$<br>$D^{*-} \rightarrow \pi^0 D^0, D^{*+} \rightarrow \pi^+ D^0, D^- \rightarrow \pi^- \pi^- K^+, D^0 \rightarrow \pi^0 \pi^+ K^-$<br>$(\Upsilon(4S) \rightarrow e^+ \nu_e \pi^0 \pi^0 \pi^+ \pi^+ \pi^- \pi^- \pi^- \pi^- K^+ K^- \gamma^F)$                            | 10206  | 1    | 1      | 2       | 9      |
| 5     | $\Upsilon(4S) \rightarrow B^0 \bar{B}^0, B^0 \rightarrow \mu^+ \nu_\mu D^{*-}, \bar{B}^0 \rightarrow \mu^+ \nu_\mu D^{*-}, D^{*-} \rightarrow \pi^0 D^-,$<br>$D^{*-} \rightarrow \pi^- \bar{D}^0, D^- \rightarrow \pi^0 \pi^- K_S^0, \bar{D}^0 \rightarrow \pi^0 \pi^- K^+, K_S^0 \rightarrow \pi^+ \pi^-$<br>$(\Upsilon(4S) \rightarrow e^+ \mu^+ \nu_\mu \nu_\mu \pi^0 \pi^0 \pi^+ \pi^- \pi^- \pi^- \pi^- K^+)$ | 11916  | 1    | 1      | 2       | 11     |
| rest  | $\Upsilon(4S) \rightarrow \text{others (99969 in total)}$<br>$(\Upsilon(4S) \rightarrow \text{corresponding to others})$   | —      | —    | —      | 99989   | 100000 |

As another example, we carry out the component analysis over the decay branches of  $D^{*+}$  and  $J/\psi$ . The resulting topology map of  $D^{*+}$  is displayed in Table 26. Compared with Table 4, two new columns are added to the table, and their headers “nCcCase” and “nAllCase” have similar meanings as “nCcEtr” and “nAllEtr” in Table 25. For a specified particle, what we want to further record with topology tags are as follows: (1) whether it is self-charge-conjugate; (2) whether its decay branches are self-charge-conjugate, if it is self-charge-conjugate; (3) the number and the indices of the decay branches of its charge-conjugate particle, if it is not self-charge-conjugate. Hence, in addition to “nPDCyBr.i” and “iDcyBrP.i.j”, the following topology tags are also inserted in the output root files: “iCcPDcyBr.i” for all specified particles; “iCcDcyBrP.i.j” for self-charge-conjugate particles only; and “nCcPDcyBr.i”, “iDcyBrCcP.i.j”, and “nAllPDcyBr.i” for non-self-charge-conjugate particles only. Here, “iCcPDcyBr.i” tags whether the  $i^{\text{th}}$  particle is self-charge-conjugate. For self-charge-conjugate particles, it has the value 0; for non-self-charge-conjugate particles, it has the value 1.

Table 26: Decay branches of  $D^{*+}$  (with the charge conjugation setting).

| rowNo | decay branch of $D^{*+}$                | iDcyBrP | nCase | nCcCase | nAllCase | nCCCase |
|-------|---|---------|-------|---------|----------|---------|
| 1     | $D^{*+} \rightarrow \pi^+ D^0$          | 0       | 31180 | 31291   | 62471    | 62471   |
| 2     | $D^{*+} \rightarrow \pi^0 D^+$          | 1       | 13978 | 14166   | 28144    | 90615   |
| 3     | $D^{*+} \rightarrow D^+ \gamma$         | 2       | 700   | 721     | 1421     | 92036   |
| 4     | $D^{*+} \rightarrow \pi^+ D^0 \gamma^F$ | 3       | 28    | 36      | 64       | 92100   |
| 5     | $D^{*+} \rightarrow \pi^0 D^+ \gamma$   | 4       | 0     | 1       | 1        | 92101   |

The topology tag “iCcDcyBrP.i.j” records the charge conjugation property of the decay branch of the  $j^{\text{th}}$  instance of the  $i^{\text{th}}$  particle. It is to “iDcyBrP.i.j” what “iCcDcyTr” is to “iDcyTr”. The topology tag “iDcyBrCcP.i.j” is designed for the charge conjugate particle of the  $i^{\text{th}}$  particle (for  $D^{*-}$  in this example). It has a similar meaning as “iDcyBrP.i.j”. Particularly, the values of “iDcyBrP.i.j” and “iDcyBrCcP.i.j” tagging charge conjugate decay branches are equal to each other. The topology tag “nCcPDcyBr.i” stands for the number of the charge conjugate

1047  $i^{\text{th}}$  particles (or their decay branches) found in each event, and “nAllPDcyBr\_i” is the sum of  
 1048 “nPDcyBr\_i” and “nCcpDcyBr\_i”.

### 1049 5.2.3. Settings only on signal identification

1050 Normally, the signals specified in the signal identification functionality items are both tagged  
 1051 and counted by executing the program one time. In the case of a huge sample that will take a long  
 1052 time, it is a good idea to first tag the signals with multiple jobs each running on one machine, and  
 1053 then count the tagged signals together. One can make the program carry out the idea by setting  
 1054 the following item to “T” and “C” in the first and second steps, respectively. Here, “T” and “C”  
 1055 stand for tagging and counting, respectively.

```
1056 % Analysis tasks for signal identifications (Three options: TC, T and C. Default: TC)
1057 {
1058   T
1059 }
1060
```

1061 By default, the signals set in the signal identification functionality items are listed in the out-  
 1062 put plain text, tex source, and pdf files in the sequence they are specified. In cases of plenty of  
 1063 signals, there is probably a need to sort them according to the number of cases found in the input  
 1064 samples. One can have the program do the sorting by inputting “Y” to the item below.

```
1065 % Sort the signals in the topology maps related to signal identifications (Two options: Y and N. Default: N)
1066 {
1067   Y
1068 }
1069
```

### 1071 5.3. Settings on the output of the program

1072 By default, decay objects (trees, initial-final states, and branches) are left-aligned in the out-  
 1073 put pdf files. If one likes it, he/she can request the program to center them by setting the following  
 1074 item to “Y”.

```
1075 % Center decay objects in output pdf files (Two options: Y and N. Default: N)
1076 {
1077   Y
1078 }
1079
```

1080 As mentioned in Section 2.4, after the execution of the program, one or more root files will  
 1081 be output to save topology tags. By default, the program switches to a new output file whenever  
 1082 the size of the TTree object in memory exceeds 3 GB. In addition to this, the program provides  
 1083 an item to control the switch of output files by setting the maximum number of entries to be  
 1084 saved in a single output file. The following example shows the item with the maximum number  
 1085 set to 1 million.

```
1086 % Maximum number of entries to be saved in a single output root file
1087 {
1088   1000000
1089 }
1090
```

1091 Besides, one can have the program generate one output file by one input file with the following  
 1092 item set to “Y”.

```
1093 % One output root file by one input root file (Two options: Y and N. Default: N)
1094 {
1095   Y
1096 }
1097
```

1100 In default cases, flat TBranch objects are used to store topology tags in the output root files.



1102 This is necessary for the Belle II experiment, as array TBranch objects are not recommended to  
 1103 use in physics analyses in order to use other tools such as NumPy [11] and pandas [12]. Howev-  
 1104 er, since array TBranch objects are elegant and efficient in organizing and storing homogeneous  
 1105 data, sometimes it is better to use them than flat TBranch objects in other experiments, such as  
 1106 the BESIII experiment. One can make the program use array TBranch objects to store topology  
 1107 tags by inputting “Y” to the item below.

```
1108
1109     % Use array tbranches to store topology tags in output root files when possible (Two options: Y and N. Default: N)
1110     {
1111         Y
1112     }
1113
```

1114 By default, to facilitate the validation of topology analysis results, the input TBranch objects  
 1115 are copied to the output root files along with other TBranch objects for physics analyses. How-  
 1116 ever, they often occupy too much disk space and are useless for following physics analyses. In  
 1117 the case of being flat, a massive amount of these TBranch objects also looks awkward. Thus,  
 1118 after the validation with a small sample, it would be better to remove these TBranch objects from  
 1119 the output root files. One can request the program to perform this removal operation before it  
 1120 terminates by setting the following item to “Y”.

```
1121
1122     % Remove the input tbranches from output root files (Two options: Y and N. Default: N)
1123     {
1124         Y
1125     }
1126
```

1127 In all of the previous examples, the program is applied to the inclusive MC samples in  $e^+e^-$   
 1128 colliding experiments. Besides, the program can also be used in other types of high energy ex-  
 1129 periments, for example, the PANDA experiment [17], a  $p\bar{p}$  annihilation experiment under con-  
 1130 struction at Darmstadt, Germany. On these occasions, we have to specify the right initial state  
 1131 particles with the following item to obtain the proper topology maps.

```
1132
1133     % Initial state particles (Default: e- e+)
1134     {
1135         anti-p-    p+
1136     }
1137
```

1138 With the setting, the default initial state  $e^+e^-$  is replaced by  $p\bar{p}$ , as shown in Table 27, which  
 1139 displays the results of a component analysis over decay trees of a small  $p\bar{p}$  annihilation sample.

Table 27: Decay trees and their respective initial-final states ( $p\bar{p}$  annihilation).

| rowNo | decay tree  | decay final state                | iDcyTr | nEtr | nCEtr |
|-------|---|----------------------------------|--------|------|-------|
| 1     | $p\bar{p} \rightarrow p\bar{p}$                       | $p\bar{p}$                       | 1      | 232  | 232   |
| 2     | $p\bar{p} \rightarrow \pi^+\pi^-\pi\bar{p}$           | $\pi^+\pi^-\pi\bar{p}$           | 24     | 53   | 285   |
| 3     | $p\bar{p} \rightarrow \pi^0 p\bar{p}$                 | $\pi^0 p\bar{p}$                 | 5      | 35   | 320   |
| 4     | $p\bar{p} \rightarrow \pi^0\pi^+\pi^-\pi\bar{p}$      | $\pi^0\pi^+\pi^-\pi\bar{p}$      | 0      | 33   | 353   |
| 5     | $p\bar{p} \rightarrow \pi^0\pi^0\pi^+\pi^-\pi^-\pi^+$ | $\pi^0\pi^0\pi^+\pi^-\pi^-\pi^+$ | 39     | 31   | 384   |
| rest  | $p\bar{p} \rightarrow \text{others (184 in total)}$   | corresponding to others          | —      | 616  | 1000  |

## 1140 6. Auxiliary facilities

1141 This section introduces some auxiliary facilities for the use of the program, including a card  
 1142 file to preset frequently used items and two commands implemented in tex source files. Different

1143 from that presented in the previous four sections, the content presented in this section is not the  
 1144 essential part of the program. However, with these auxiliary facilities, we can make the program  
 1145 do our jobs better and quicker on some occasions.

### 1146 6.1. *The underlying card file*

1147 A card file, namely “underlying.topoana.card” under the directory “share”, to preset fre-  
 1148 quently used items is developed to assist the card file specified by the first argument of the  
 1149 command “topoana.exe”. Here, we refer to the former and latter card files as underlying and  
 1150 primary, respectively. In general, the primary card file is sufficient to set items for the execution  
 1151 of the program. However, considering some items are frequently used with constant inputs by a  
 1152 user or a group of users, it is better to move the items from the primary card file to the underlying  
 1153 card file, in order to make the primary card file more concise and make us more focused on the  
 1154 items specially set for the dedicated topology analysis.

1155 One can decide whether to set an item in the underlying card file according to his/her own  
 1156 needs. Here, we introduce some frequently used items that are suitable to be put in the underlying  
 1157 card file as follows. As mentioned in Section 2.3, the items related to the storage type and  
 1158 TBranches names of the input data are usually fixed for a user or a group of users. Thus, it is  
 1159 quite appropriate to move them to the underlying card file. We have to process charge conjugation  
 1160 particles and decays together in many physics studies. In such studies, it is also a good practice  
 1161 to put the item on charge conjugation in the underlying card file.

1162 The program first reads the items in the underlying card file and then reads those in the  
 1163 primary card file. The items set in the underlying card file can be reset in the primary card file.  
 1164 In such a case, the inputs in the underlying card file will be replaced by their counterparts in the  
 1165 primary card file.

### 1166 6.2. *Commands implemented in tex source files*

1167 The output pdf files can be checked after the execution of the program. If their styles are not  
 1168 to our taste, we can edit the corresponding tex source files to get the desired styles, according  
 1169 to the regular LaTeX rules. Besides the rules, two commands are implemented in the tex source  
 1170 files to help us edit the files quickly and easily for two common desired styles.

1171 By default, topology tags are listed along with topology maps in the output plain text, tex  
 1172 source, and pdf files. However, only the topology maps are needed on some occasions, especial-  
 1173 ly in presentations. In such cases, one can suppress the topology tags in the output tex source  
 1174 and pdf files by simply changing the definition of the cmtTopoTags command from the nominal  
 1175 one

1176  
 1177 
$$\backslash\text{newcommand}\{\backslash\text{topoTags}\}[1]\{\#1\}$$

1178  
 1179 to the alternative one

1180  
 1181 
$$\backslash\text{newcommand}\{\backslash\text{topoTags}\}[1]\{\}$$

1182  
 1183 in the preamble of the text source files. Here, “#1” is the formal parameter of the string for  
 1184 the topology tags. With the nominal definition, “\topoTags{#1}” returns the string exactly, while  
 1185 with the alternative definition it only returns an empty string. That is why the definition below is  
 1186 able to suppress the topology tags.

1187 After the revision of the tex source files, one can re-compile them with the `pdflatex` command.  
1188 Usually, the `pdflatex` command has to be executed two or three times for a fully compiled pdf  
1189 file, and many undesired files in other formats are generated during the compilation. To execute  
1190 the `pdflatex` command and remove the undesired files at one stroke, we develop a bash script,  
1191 namely “`getPdfFromTex.sh`” under the directory “utilities”. The script should be executed with  
1192 the following command line: `getPdfFromTexFl.sh texFileName`. Compiling the tex source  
1193 files with the script is recommended.

## 1194 7. Summary

1195 We develop a program, namely `TopoAna`, with C++, ROOT, and LaTeX for the topology  
1196 analysis of inclusive MC samples in high energy physics experiments. This user guide provides  
1197 a detailed description of the program, including a basic introduction to it, two categories of its  
1198 functionalities — component analysis and signal identification, and some common settings and  
1199 auxiliary facilities for its execution. The program has rich functionalities and aims to solve all  
1200 kinds of topology analysis tasks. Meanwhile, it is easy to use and has a high processing rate.  
1201 These features make the program a powerful tool to analyze the backgrounds involved in our  
1202 research works and to identify the physics processes of interests from the inclusive MC samples.

1203 Since it does not rely on any specific software frameworks, the program applies to many high  
1204 energy physics experiments. Up to now, it has been put into use in three experiments at  $e^+e^-$   
1205 colliders: the BESIII, Belle, and Belle II experiments. Besides these experiments, it can also be  
1206 used in other types of experiments, such as the PANDA experiment, a  $p\bar{p}$  annihilation experi-  
1207 ment. Also, the program is applicable to the future  $e^+e^-$  colliding experiments under research  
1208 and development, such as the circular electron-positron collider (CEPC) [18, 19] experiment in  
1209 China, the super Charm- $\tau$  factory (SCTF) experiment [20] in Russia, and the super  $\tau$ -Charm fac-  
1210 tory (STCF) experiment [21] in China. These experiments offer wide space for the application of  
1211 the program. With more user needs coming out in the future, we will further extend and perfect  
1212 it to make it more powerful and well-rounded.

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