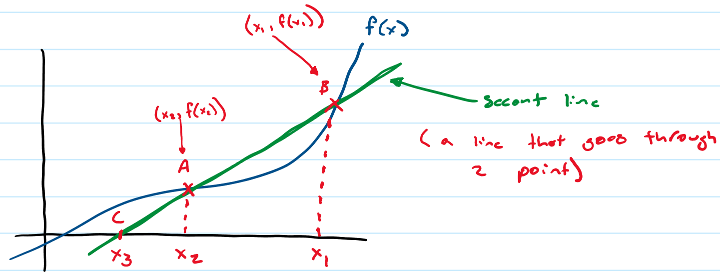
2-4-19

Monday, February 4, 2019 11:33 AM

2.3.2 Secant Method

Main Idea • avoid computing $f'(x)$
• need two initial points



Slope of Secant line:

$$m = \frac{f(x_1) - f(x_2)}{x_1 - x_2}$$

Slope of the line segment AC is: $\frac{f(x_2) - 0}{x_2 - x_3}$

Then $\frac{f(x_2)}{x_2 - x_3} = \frac{f(x_1) - f(x_2)}{x_1 - x_2}$

$$\frac{f(x_2)}{\left[\frac{f(x_1) - f(x_2)}{x_1 - x_2}\right]} = x_2 - x_3 \rightarrow x_3 = x_2 - f(x_2)\left[\frac{x_1 - x_2}{f(x_1) - f(x_2)}\right]$$

In General,

$$\boxed{x_{n+1} = x_n - f(x_n)\left[\frac{x_{n-1} - x_n}{f(x_{n-1}) - f(x_n)}\right]}$$

Example:

Apply the Secant method with starting guesses $x_0 = 0$, $x_1 = 1$ to find the root of $f(x) = x^3 + x - 1$. Find x_2 and x_3

$$x_{n+1} = x_n - f(x_n)\left[\frac{x_{n-1} - x_n}{f(x_{n-1}) - f(x_n)}\right]$$

$x_n = x_1$
$x_{n-1} = x_0$
$x_{n+1} = x_2$
$x_{n+2} = x_3$

$$x_2 = x_{1+1} = 1 - 1\left[\frac{0-1}{-1-1}\right] \rightarrow 1 - 1\left[\frac{-1}{-2}\right] \rightarrow 1 - \frac{1}{2} = \boxed{0.5 = x_2}$$

$$x_3 = x_{2+1} = 0.5 - (-.375)\left[\frac{1 - 0.5}{1 - (-.375)}\right] \rightarrow 0.5 + 0.375\left[\frac{0.5}{1.375}\right] \rightarrow 0.5 + 0.136 = \boxed{0.636 = x_3}$$

Fixed-Point Iteration Review: Quiz Friday (2-8)

$x = g(x)$ $I = [a, b]$

(1) $g(x) \in [a, b]$ ← To Find Point

(2) $\max_{x \in I} |g'(x)| < 1$ ← Just for Unique Solution

Review Taylor's Th^m with Remainder

Let x and x_0 be Real numbers, and let $f(x)$ be $(k+1)$ times continuously differentiable on the interval between x and x_0. Then, there is a number c between x and x_0 such that

$$f(x) = \left[f(x_0) + \frac{f'(x_0)(x - x_0)}{1!} + \frac{f''(x_0)(x - x_0)^2}{2!} + \dots\right.$$

$$f(x) = \left[\frac{f(x_0) + f'(x_0)(x-x_0)}{1!} + \frac{f''(x_0)(x-x_0)^2}{2!} + \cdots \right.$$

$$\left. + \frac{f^{(k)}(x_0)(x-x_0)^k}{k!} \right] + \underbrace{\frac{f^{(k+1)}(c)(x-x_0)^{k+1}}{(k+1)!}}_{\text{Taylor's Remainder}}$$

degree k Taylor polynomial for $f(x)$ centered at x_0

Example: Find the degree and Taylor polynomial, $p_4(x)$, for $f(x) = \sin x$ centered at the point $x_0 = 0$

$f(x) = \sin x$

$f'(x) = \cos x$ $\qquad = \sin(0) + \frac{(x)}{1!} + 0 + \frac{(-1)x^3}{3!} + 0$

$f''(x) = -\sin x$

$f'''(x) = -\cos x$

$f^4(x) = \sin x$ $\qquad \boxed{\sin x \approx \frac{x}{1!} - \frac{x^3}{6}}$