Numerical Analysis MAT 362: Homework 7

Due on Wednesday, April 10 in class

Please read the Instructions

- **Show all the steps that you go between the question and the answer. Show how you derived the answer. For your work to be complete, you need to explain your reasoning and make your computations clear.**
- **You will be graded on the readability of your work.**
- **The correct answer with no or incorrect work will earn you NO marks**
- **Show ALL your work**
- Use only four decimal places for all numbers.
- **If possible, use** 8.5" × 11" **white paper (not torn from spiral binders) and staple sheets together**.
- Print your name legibly in the upper corner of the page.
- Write your solutions as though you're trying to convince someone that you know what you're talking about.
- **Failure to follow these instructions will result in loss points (up to the full amount of the homework total)**

Problem 1

Use Euler's method with step size $h = 0.5$ to approximate the solution of the IVP:

$$\frac{dy}{dt} = \frac{e^{y^2}}{t}, \quad 1 \leq t \leq 2, \quad y(1) = 0.$$

Problem 2

Consider the IVP:

$$\frac{dy}{dt} = ty + t^3, \quad 0 \leq t \leq 1, \quad y(0) = 1.$$

(a) Use Euler's method with step size $h = 0.25$ to approximate $y(0.5)$.
(b) Find the exact solution of the IVP.
(c) Find the maximum error in approximating $y(0.5)$ by y_2.
(d) Calculate the actual absolute error in approximating $y(0.5)$ by y_2.

Problem 3

Derive the $n^{th}-$order Taylor's method for the IVP:

$$\frac{dy}{dt} = f(t, y), \quad a \le t \le b, \quad y(a) = \alpha.$$

Problem 4

Use Taylor's method of order 2 with step size $h = 0.5$ to approximate the solution of the following IVP:

$$\frac{dy}{dt} = t - y^2, \quad 1 \le t \le 3, \quad y(1) = 0.$$

Problem 5

Use RK2 with step size $h = 0.5$ to approximate the solution of the following IVP:

$$\frac{dy}{dt} = \frac{e^y}{2t}, \quad 1 \le t \le 3, \quad y(1) = 0.$$

Problem 6

Consider the IVP:

$$\frac{dy}{dt} = (1 - y)\cos t, \quad 0 \le t \le 3, \quad y(0) = 3.$$

(a) Use RK4 with step size $h = 1$ to set up an iteration formula for y_{i+1} to approximate the solution of the above IVP. (Hint. write k_1, k_2, k_3, k_4 and y_{i+1} in terms of them).
(b) Approximate the solution of the IVP.
(c) Find the absolute error in approximating $y(1)$ by y_1 using the actual solution $y = 1 + 2^{-\sin t}$.