Calculus Review

1.14.19

continuity of a function

A function (fun$^{\underline{n}}$) $f(x)$ is continuous at $x=a$ if:

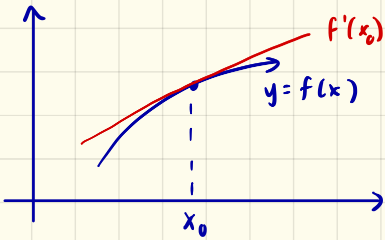
(a) $f(a)$ exists

(b) $\lim\limits_{x \to a} f(x)$ exists

(c) $\lim\limits_{x \to a} f(x) = f(a)$

differentiability of a function

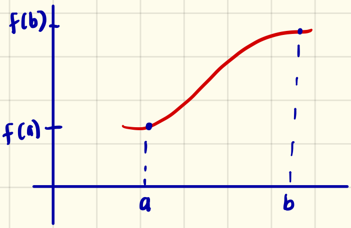
Let $f(x)$ be a fun$^{\underline{n}}$ defined in an open interval containing x_0. Then, the fun$^{\underline{c}}$ $f(x)$ is differentiable



example) If $f(x) = e^x \cos(2x)$

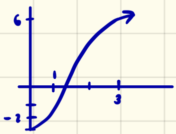
$f'(x) = -2e^x \sin(2x) + \cos(2x)\,e^x$ by the product rule

intermediate value theorem



Let $f(x)$ be a continuous function on the interval $[a,b]$. If y^* is a number between $f(a)$ and $f(b)$, then there exists c in (a,b) for which $f(c) = y^*$

example) Show that $f(x) = x^3 - 3$ on the interval $[1,3]$ must take on the values -2 to 6.



$f(1) = -2$

$f(3) = 6$

so by the IVT, any c in $[1,3]$ must produce $f(c)$ within $[-2,6]$.

Mean value theorem

Let $f(x)$ be a continuously differentiable function fun^{n} on the interval $[a,b]$ there exists a number c between a and b that:

$$\boxed{\frac{f(b)-f(a)}{b-a} = f'(c)}$$

example) Find a value c satisfying the MVT for $f(x)=x^2-3$ on the interval $[1,3]$

$a=1$ $\quad f(a)=-2$

$b=3$ $\quad f(b)=6$

$$\frac{6-(-2)}{3-1} = \frac{8}{2} = 4 = f'(c)$$

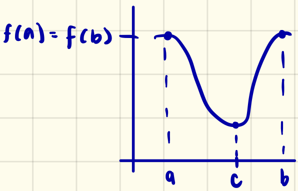
$f'(x)=2x$

$4=2c$

$c=2$

Rolle's Theorem

Let $f(x)$ be a continuously differentiable function fun^{n} on the interval $[a,b]$ and assume that $f(a)=f(b)$. There is a number c between (bet^{n}) a and b such that $f'(c)=0$



$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$$f'(c)=0$$

example) Show that $x^5-2x^3+3x^2-1=0$ has a root in the interval $[0,1]$

$f(0)=-1$

$f(1)=1$

By the IVT, there should be some c in $[0,1]$ that is equal to 0.