Numerical Analysis MAT 362: Homework 4

Due on Wednesday, February 20 in class

Please read the Instructions

- **Show all the steps that you go between the question and the answer. Show how you derived the answer. For your work to be complete, you need to explain your reasoning and make your computations clear.**
- **You will be graded on the readability of your work.**
- **The correct answer with no or incorrect work will earn you NO marks**
- **Show ALL your work**
- Use only four decimal places for all numbers.
- **If possbile, use** 8.5" × 11" **white paper (not torn from spiral binders) and staple sheets together**.
- Print your name legibly in the upper corner of the page.
- Write your solutions as though you're trying to convince someone that you know what you're talking about.
- **Failure to follow these instructions will result in loss points (up to the full amount of the homework total)**

Problem 1

(a) State the Taylor's theorem with remainder.

(b) Construct the $3^{rd}-$order Taylor polynomial about $x_0 = 1$ approximating $\ln(x)$.

(c) Use this polynomial to approximate $\ln(1.1)$.

(d) Give an expression for the Taylor remainder.

Problem 2

Use Lagrange interpolation to find a polynomial that passes through the points:

(a) $(0, 1)$, $(2, 3)$, $(3, 0)$

(b) $(0, -2)$, $(2, 1)$, $(4, 4)$

(c) $(-1, 0)$, $(2, 1)$, $(3, 1)$, $(5, 2)$

Problem 3

The estimated mean atmospheric concentration of carbon dioxide in earth's atmosphere is given in the table that follows, in parts per million (ppm) by volume.

Year	1800	1850	1900	2000
CO_2 (ppm)	280	283	291	370

(a) Find the degree 3 Lagrange interpolating polynomial of the data set.

(b) Then, use it to estimate the CO_2 concentration in 1950 and 2050. (FYI, the actual concentration in 1950 was 310 ppm.)

Problem 4

Find the **maximum error** in approximating $f(x) = e^x$ on $[-1, 1]$ by the Lagrange polynomial $P_2(x)$ using points $x = \{-1, 0, 1\}$.

Problem 5

Decide whether the equations form a cubic spline.

(a) $S(x) = \begin{cases} S_0(x) = x^3 + x - 1 & \text{on } [0,1] \\ S_1(x) = 1 + 3(x-1) + 3(x-1)^2 - (x-1)^3 & \text{on } [1,2] \end{cases}$

(b) $S(x) = \begin{cases} S_0(x) = 2x^3 + x^2 + 4x + 5 & \text{on } [0,1] \\ S_1(x) = 12 + 12(x-1) + 7(x-1)^2 + (x-1)^3 & \text{on } [1,2] \end{cases}$

Problem 6

Find the **natural** cubic spline through $(0, 3)$, $(1, 2)$, and $(2, 1)$.

Optional problem

(a) Construct the Newton's method to solve $x^2 - a = 0$, where $a > 0$ for computing the root $\alpha = \sqrt{a}$. Show that the iteration scheme can be written in the form:

$$x_{k+1} = \frac{1}{2}\left(x_k + \frac{a}{x_k}\right).$$

(b) Let $e_k = x_{k+1} - x_k$. Show that $x_k = \dfrac{a}{e_k + \sqrt{e_k^2 + a}}$.