Problems 9,10, and 11

Prob 9

Apply two steps of Newton's method with initial guess $x_0 = 0$.

Newton's method:

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

(a) $x^3 + x - 2 = 0$

What is $f'(x_0)$?

$$f'(x_0) = 0 + 1 = 1$$

What is x_1?

$$x_1 = 0 - \frac{-2}{1} = 2$$

What is x_2?

$$x_2 = 2 - \frac{8}{13} = \frac{18}{13}$$

After two steps using Newton's Method, we find,

$$x_2 = \frac{18}{13} \approx 1.3846$$

(b) $x^4 - x^2 + x - 1 = 0$

What is $f'(x_0)$?

$$f'(x_0) = 0 - 0 + 1 = 1$$

What is x_1?

$$x_1 = 0 - \frac{-1}{1} = 1$$

What is x_2?

$$x_2 = 1 - \frac{0}{3} = 1$$

After two steps using Newton's Method, we find,

$$x_2 = 1$$

Problem 10

Apply two steps of ~~N~~ewton's method with initial guess $x_0 = 1$.

(a) $x^3 + x^2 - 1 = 0$

What is $f'(x_0)$?

$$f'(x_0) = 1 + 1 - 1 = 1$$

What is x_1?

$$x_1 = 1 - \frac{1}{5} = \frac{4}{5} = 0.8$$

What is x_2?

$$x_2 = 0.8 - \frac{(0.8)^3 + (0.8)^2 - 1}{3(0.8)^2 + 2(0.8)} = 0.7568$$

After two steps using Newton's Method, we find,

$$x_2 \approx 0.7568$$

(b) $5x - 10 = 0$

What is $f'(x_0)$?

$$f'(x_0) = 5$$

What is x_1?

$$x_1 = 1 - \frac{-5}{5} = 2$$

What is x_2?

$$x_2 = 2 - \frac{0}{5} = 2$$

After two steps using Newton's Method, we find,

$$x_2 = 2$$

Problem 11

Apply two steps of the Secant Method with initial guesses $x_0 = 1$ and $x_1 = 2$.

$$x_{x+1} = x_n - f(x_n)[\frac{x_{n+1} - x_n}{f(x_n - 1) - f(x_n)}]$$

(a) $e^x + x - 7 = 0$

For x_2:

$$x_2 = x_1 - f(x_1)[\frac{x_0 - x_1}{f(x_0) - f(x_1)}]$$
$$x_2 = 2 - (e^2 + 2 - 7)[\frac{1 - 2}{(e^1 + 1 - 7) - (e^2 + 2 - 7)}] = 1.5787$$

For x_3 :

$$x_3 = x_2 - f(x_2)[\frac{x_1 - x_2}{f(x_1) - f(x_2)}]$$
$$x_3 = 1.5787 - (e^{1.5787} + 1.5787 - 7)[\frac{2 - 1.5787}{(e^2 + 2 - 7) - (e^{1.5787} + 1.5787 - 7)}] = 1.6602$$

After two steps of the Secant Method, we find,

$$x_3 \approx 1.6602$$

(b) $x^3 - 2x - 2 = 0$

For x_2:

$$x_2 = x_1 - f(x_1)[\frac{x_0 - x_1}{f(x_0) - f(x_1)}]$$
$$x_2 = 0.0870$$

For x_3 :

$$x_3 = x_2 - f(x_2)[\frac{x_1 - x_2}{f(x_1) - f(x_2)}]$$
$$x_3 = 0.1797$$

After two steps of the Secant Method, we find,

$$x_3 \approx 0.1797$$