Numerical Analysis MAT 362: Homework 2

Due on Monday, February 4 in class

Instructions

- Answer all the problems.
- **Show all the steps that you go between the question and the answer. Show how you derived the answer. For your work to be complete, you need to explain your reasoning and make your computations clear.**
- **You will be graded on the readability of your work.**
- **The correct answer with no or incorrect work will earn you NO marks**
- **Show ALL your work**
- Use only four decimal places for all numbers.

Problem 1

Use the fixed-point iteration theorem to show that $g(x) = 2^{-x}$ has a unique fixed point on $\left[\frac{1}{3}, 1\right]$. Hint: $\dfrac{d}{dx}a^{-x} = -a^{-x}\ln a$.

Problem 2

Consider three functions:

$$g_1(x) = \frac{x^2 - 3}{2}, \quad g_2(x) = \sqrt{2x+3}, \quad \text{and} \quad g_3(x) = \frac{3}{x-2}.$$

(a) Show that fixed points of each $g_i(x)$ are roots of $f(x) = x^2 - 2x - 3$.

(b) Which $g_i(x)$ has a unique fixed point in $[1, 4]$ guaranteed by the Fixed Point Theorem?

Problem 3

Use the fixed point iteration to find the solution of $e^x = 3x$ on $[1, 2]$ with $x_0 = 1.5$ correct to roughly within 10^{-3}. Hint: Use the correct $g(x)$ for $f(x) = e^x - 3x$.

Problem 4

Find n for which the nth iteration by the Fixed-point method guarantees to approximate the root of $f(x) = x - \cos x$ on $[0, \frac{\pi}{3}]$ with accuracy within 10^{-8} using $x_0 = \frac{\pi}{4}$.