Review: 02.13.19

$$l_i(x) = \frac{(x-x_0)(x-x_1)\cdots(x-x_{i-1})(x-x_{i+1})(x-x_n)}{(x_i-x_0)(x_i-x_1)\cdots(x_i-x_n)}$$

$$P_n(x) = \sum_{i=0}^{n} l_i(x)\cdot y_i$$

example) Find the maximum error in approximating $f(x) = 4\ln x$ on $[1, 4]$ by the Lagrange polynomial $P_2(x)$ using points $x: \{1, 3, 4\}$

$$f(x) = P_n(x) + \frac{f^{(n+1)}(\xi(x))}{(n+1)!}(x-x_0)(x-x_1)\cdots(x-x_n)$$

$$f(x) - P_n(x) = \frac{f^3(\xi(x))(x-1)(x-3)(x-4)}{3!}$$

$$|f(x) - P_2(x)| = \frac{|f^3(\xi(x))||(x-1)(x-3)(x-4)|}{3!}$$

$$\max_{x\in[1,4]} f'''(\xi(x)) = 8$$

$$|f(x) - P_2(x)| \le \frac{8\,|(x-1)(x-3)(x-4)|}{3!}$$

$f(x) = 4\ln x$
$f'(x) = 4/x$
$f''(x) = -4/x^2$
$f'''(x) = 8/x^3$

Now find the maximum of $h(x) = (x-1)(x-3)(x-4)$

$h(x) = x^3 - 8x^2 + 19x - 12$

$h'(x) = 3x^2 - 16x + 19 = 0$

$$x = \frac{8 \pm \sqrt{7}}{3}$$

≈ 1.784

$$h\left(\frac{8+\sqrt{7}}{3}\right) = -0.631$$

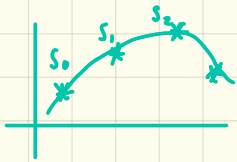
$$h\left(\frac{8-\sqrt{7}}{3}\right) = 2.113$$

$$|f(x) - P_2(x)| \le \frac{8\cdot(2.113)}{6} = 2.817$$

3.2 Cubic Splines

the idea of splines is to use several polynomials, each a lower degree to pass through the data points

the cubic spline is a 3rd degree polynomial used to interpolate over each interval between data points.



- one of the best techniques for polynomial interpolation