Derivative from Lagrange Polynomial

- If $f(x)$ is not explicitly given, and we know $(x_i, f(x_i))$ for $i=0,\dots n$ then f can be approximated using the Lagrange polynomial.

* Review:

$$f(x) = \sum_{i=0}^{n} f(x_i) L_i(x) + \frac{f^{(n+1)}(\xi)}{(n+1)!} \prod_{i=0}^{n} (x-x_i)$$

$L_i(x) =$

$\underline{\hspace{4cm}}$

Differentiating both sides wrt the x-variable and evaluating at $x=x_j$ we get

$$f'(x_j) = \sum_{i=0}^{n} f(x_i) L_i'(x_j) + \frac{f^{(n+1)}(\xi(x))}{(n+1)!} \prod_{\substack{i=0 \\ i\neq j}}^{n} (x_j - x_i)$$

(n+1) point formula to approximate $f'(x_j)$

Three point Formula

$$L_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} \qquad L_0'(x) = \frac{(x-x_2)+(x-x_1)}{(x_0-x_1)(x_0-x_2)}$$

$$L_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} \qquad L_1'(x) = \frac{(x-x_2)+(x-x_0)}{(x_1-x_2)(x_1-x_0)}$$

$$L_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} \qquad L_2'(x) = \frac{(x-x_0)+(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

$$f'(x_j) = f(x_0)\left[\frac{(x_j-x_2)+(x_j-x_1)}{(x_0-x_1)(x_0-x_2)}\right] + f(x_1)\left[\frac{(x_j-x_2)+(x_j-x_0)}{(x_1-x_0)(x_1-x_2)}\right]$$

$$+ f(x_2)\left[\frac{(x_j-x_1)+(x_j-x_0)}{(x_2-x_1)(x_2-x_0)}\right] + \frac{f^3(\xi)}{3} \prod_{\substack{i=0 \\ i\neq j}}^{2} (x_j - x_i)$$

- If the nodes are equally spaced and $x_1 = x_0 + h$ and $x_2 = x_0 + 2h$ then,

$$f'(x_0) = f(x_0)\left[\frac{-2h-h}{(-h)-2h}\right] + f(x_0+h)\left[\frac{-h}{(2h)(h)}\right]$$

$$+ f(x_0+2h)\left[\frac{-h}{(2h)h}\right] + \frac{f^3(z)}{6}(-h)(2h)$$

$$f'(x_0) = \frac{-3f(x_0) + 4(x_0+h) - f(x_0+2h)}{2h} + \frac{f^3(z)h^2}{3}$$