Exam 2 information
The exam will be in class, Friday, March 29.

The exam 2 covers chapters 4 − 5.2.

- **Numerical Differentiation**
- **Elements of Numerical Integration (Non-composite numerical integration)**
- **Composite Numerical Integration**
- **Elementary theory of Initial-Value Problems**
- **Euler's method**

This list of problems is not guaranteed to be a complete review. For a complete review make sure that you know how to do the problems discussed in the class and the problems in Homework sets 5, 6, and quiz 4.

1. Use non-composite trapezoid and Simpson rules to approximate $\int_0^1 x^2\,dx$ and find an upper bound for the error in the your approximation.

2. State Euler's method for computing an approximate solution to the IVP:

$$\frac{dy}{dt} = f(t, y) \quad \text{for} \;\; a \le t \le b \text{ with } y(a) = \alpha$$

and show that it can be derived from a truncated Taylor series expansion.

3. Prove that the two-point forward difference formula is an order 1 approximation ($\mathcal{O}(h)$).

4. State the existence and uniqueness theorem for IVPs.

5. Use Euler's method to approximate the solution for the initial value problem:

$$\frac{dy}{dt} = 1 + (t - y)^2, \;\; y(2) = 1$$

for $2 \le t \le 3.5$ and $h = 0.5$. Show your work. Be sure to label your approximations for $y(2.5)$, $y(3)$, and $y(3.5)$.

6. Determine the step size h required in order for the Composite Simpson's Rule to approximate the integral

$$\int_0^8 x \ln(x)\,dx$$

with an error of at most 10^{-4}.

7. (a) Show that the function $f(t, y) = \cos(t^2 y)$ satisfies a Lipschitz condition on the domain $D = \{(t, y) \;\text{ such that }\; 0 \le t \le 2, -\infty < y < \infty\}$ and find the Lipschitz constant L.

(b) Prove that the initial value problem:

$$\frac{dy}{dt} = \cos(t^2 y), \;\; y(0) = 1$$

has a unique solution for t in $[0, 2]$.

8. Consider the data in the following table.

x	1.0	1.1	1.2	1.3	1.4
$f(x)$	6	10	12	9	4

(a) Use the Two-point FDF, BDF, and CDF formulas to approximate $f'(1.2)$.
(b) Use the Three-point backward-difference formula to approximate $f'(1.3)$.

I am going to provide the following formula sheet during the 2nd exam. I hope this will help you to **reduce** the number of formulas that you need to memorize for the exam.

Error bounds for two-point FDF and CDF

$$E_{\text{FDF}} \leq \frac{h}{2} \max_{x \in (x_0,\, x_0+h)} |f''(x)| \qquad E_{\text{CDF}} \leq \frac{h^2}{6} \max_{x \in (x_0-h,\, x_0+h)} |f''(x)|$$

Three-point FDF:

$$f'(x_0) = \frac{-3f(x_0) + 4f(x_0+h) - f(x_0+2h)}{2h}$$

Error formulas for Non-Composite Numerical Integration

$$E_{\text{T}} = \frac{-(b-a)^3}{12} f''(c) \qquad E_{\text{S}} = \frac{-(b-a)^5}{90 \cdot 2^5} f^{(4)}(c)$$

Composite Simpson's Rule

$$\frac{h}{3}\left[f(x_0) + f(x_n) + 4\sum_{i=1}^{n/2} f(x_{2i-1}) + 2\sum_{i=1}^{(n/2)-1} f(x_{2i}) \right]$$

Error formulas for Composite Numerical Integration

$$E_{\text{CMR}} = \frac{(b-a)\, h^2}{24} f''(c) \qquad E_{\text{CTR}} = \frac{-(b-a)\, h^2}{12} f''(c) \qquad E_{\text{CSR}} = \frac{-(b-a)\, h^4}{180} f^{(4)}(c)$$

Error bound for Euler's method (only for Sec: 001)

$$|y(t_i) - y_i| \leq \frac{h\, M}{2\, L}\left(e^{L(t_i-a)} - 1\right)$$