[redacted]

Problem 12

(a) Please see the notes

(b)

$$f(x) = \underbrace{f(x_0) + f'(x_0)\frac{(x-x_0)}{1!_0} + f''(x_0)\frac{(x-x_0)^2}{2!_0} + f'''(x_0)\frac{(x-x_0)^3}{3!_0}}_{\text{degree 3 Taylor Polynomial}} + \underbrace{f^{(iv)}(c)\frac{(x-x_0)^4}{4!_0}}_{\text{Taylor Remainder}}$$

$f(x) = \ln x \qquad x_0 = 1$

$f'(x) = \frac{1}{x}$

$f''(x) = \frac{-1}{x^2}$

$f'''(x) = \frac{2}{x^3}$

$f^{(iv)}(x) = \frac{-6}{x^4}$

$$P_3(x) = \ln(1) + \left(\frac{1}{1}\right)\frac{(x-1)}{1!_0} + \left(\frac{-1}{1}\right)\frac{(x-1)^2}{2!_0} + \left(\frac{2}{1}\right)\frac{(x-1)^3}{3!_0}$$

$$P_3(x) = (x-1) - \frac{(x-1)^2}{2} + 2\frac{(x-1)^3}{3!_0}$$

(c) $\ln(1.1) \approx (1.1-1) - \frac{(1.1-1)^2}{2} + 2\frac{(1.1-1)^3}{3!_0}$

$\approx 0.1 - \frac{0.01}{2} + \frac{2}{6}(0.001)$

≈ 0.0953

(d) Taylor Remainder $= f^{(iv)}(c)\frac{(x-x_0)^4}{4!_0}$

$= \frac{-6}{c^4}\frac{(x-1)^4}{4!_0}$,

where $c \in (1, x)$