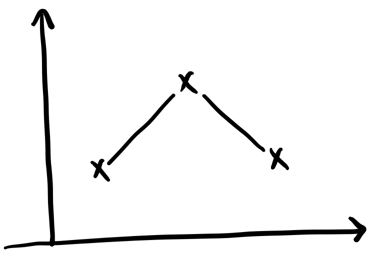
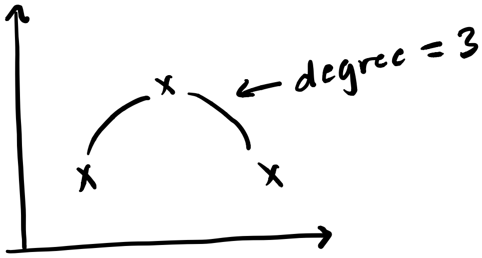
TUESDAY JULY 16, 2019

CH. 3 INTERPOLATION AND POLYNOMIAL APPROXIMATION

3.2 CUBIC SPLINES

The idea of the "splines" is to use several polynomials each a lower degree, to pass through the data points.



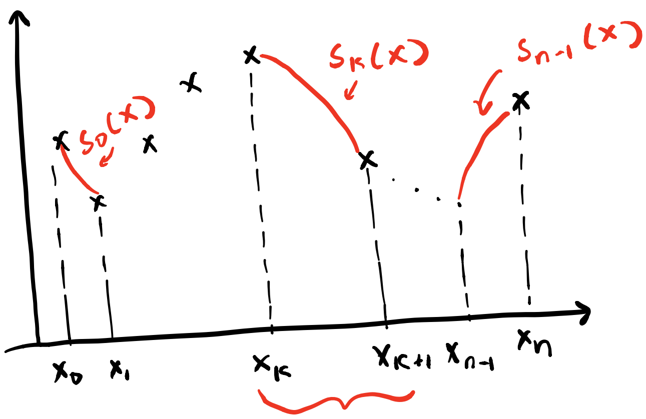


In cubic spline, 3rd degree polynomials are used to interpolate over each interval between data points.

- suppose there $(n+1)$ data points
  $(x_0, y_0), (x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$

  NOTE:
  - $(n+1)$ data points
  - Hence, n-number of intervals

  $\Rightarrow$ number of cubic splines



$$s_k(x) = a_k + b_k(x - x_k) + c_k(x - x_k)^2 + d_k(x - x_k)^3$$

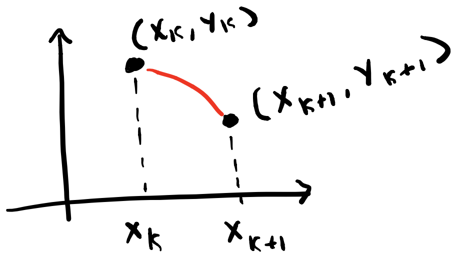
on the interval $[x_k, x_{k+1}]$

$k = 0, 1, 2, \dots, n-1$

$$= \begin{cases} S_0(x) = a_0 + b_0(x - x_0) + c_0(x - x_0)^2 + d_0(x - x_0)^3 \Leftarrow [x_0, x_1] \\ S_1(x) = a_1 + b_1(x - x_1) + c_1(x - x_1)^2 + d_1(x - x_1)^3 \Leftarrow [x_1, x_2] \\ \vdots \\ S_{n-1}(x) = a_{n-1} + b_{n-1}(x - x_{n-1}) + c_{n-1}(x - x_{n-1})^2 + d_{n-1}(x - x_{n-1})^3 \Leftarrow [x_{n-1}, x] \end{cases}$$

Property 1 (Left-endpoint)

$S_k(x_k) = y_k \leftarrow n$-eq^{ns}

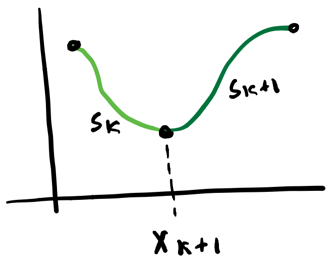


Property 2 (Right-endpoint)

$S_k(x_{k+1}) = y_{k+1} \leftarrow n$-eq^{ns}

Property 3

$S_k'(x_{k+1}) = S'_{k+1}(x_{k+1}) \leftarrow (n-1)$-eq^{ns}



Property 4

$S''_k(x_{k+1}) = S''_{k+1}(x_{k+1}) \leftarrow (n-1)$-eq^{ns}

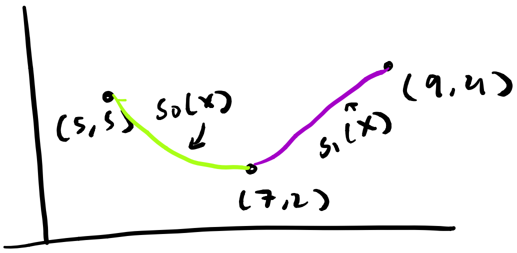
Property 5 Natural spline (free): $S_0''(x_0) = 0$, $S''_{n-1}(x_n) = 0$

OR

Clamped Cubic spline: $S_0'(x_0) = \alpha_1$, $S'_{n-1}(x_n) = \alpha_2$ — α_1 & α_2 are user-specified values ex. Construct a piecewise cubic spline interpolant for the curve passing through (5,5) with natural boundary conditions

(7,2)

(9,4)



$$S_0(x) = a_0 + b_0(x-5) + c_0(x-5)^2 + d_0(x-5)^3$$

$$S_1(x) = a_1 + b_1(x-7) + c_1(x-7)^2 + d_1(x-7)^3$$

$S_0(5) = a_0 = 5$ — 1

$S_0(7) = a_0 + b_0(7-5) + c_0(7-5)^2 + d_0(7-5)^3 = 2$

$= a_0 + 2b_0 + 4c_0 + 8d_0 = 2$ — 2

$S_1(7) = a_1 = 2$ — 3

$S_1(9) = a_1 + b_1(9-7) + c_1(9-7)^2 + d_1(9-7)^3 = 4$

$= a_1 + 2b_1 + 4c_1 + 8d_1 = 4$ — 4

$\underline{S_0'(7) = S_1'(7)}$

$S_0'(x) = b_0 + 2c_0(x-5) + 3d_0(x-5)^2$

$S_1'(x) = b_1 + 2c_1(x-7) + 3d_1(x-7)^2$

$\left.\begin{array}{l} S_0'(7) = b_0 + 4c_0 + 12d_0 \\ S_1'(7) = b_1 \end{array}\right\} b_0 + 4c_0 + 12d_0 = b_1$ — 5

$\underline{S_0''(7) = S_1''(7)}$

$S_0''(x) = 2c_0 + 6d_0(x-5)$

$S_1''(x) = 2c_1 + 6d_1(x-5)$

@ $x=7 \Rightarrow 2c_0 + 12d_0 = 2c_1$ — 6 with natural boundary conditions:

$S_0''(5) = 0$ $\qquad$ $S_1''(9) = 0$

$2c_0 = 0$ — (7) $\qquad$ $2c_1 + 12d_1 = 0$ — (8)