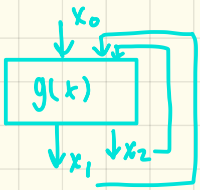
Fixed point Iteration

1.28.18
lecture

Algorithm

1. Start with initial guess x_0
2. For $n = 1, 2, 3 \ldots \quad x_n = g(x_{n-1})$



* $\{x_n\}_{n=1}^{\infty}$ may or may not converge. However, if it converges to a number P, and g(x) is continuous, then P is a fixed point.

Example) Using Fixed Point Iteration (FPI), determine the root of the function $f(x) = x - \frac{x}{2} - \frac{1}{x}$ with $x_0 = 1$

$f(x) = 0$

$x - \frac{x}{2} - \frac{1}{x} = 0$

$x = \underbrace{\frac{x}{2} + \frac{1}{x}}_{g(x)}$

$x_1 = g(x)$

$x_0 = 1$

$x_1 = g(x_{n-1}) = g(x_0) = \frac{1}{2} + 1 = \frac{3}{2}$

$\Downarrow$

$x_1 = 3/2$

$x_2 = g(x_{n-1}) = g(x_1) = \frac{\frac{3}{2}}{2} + \frac{1}{\frac{3}{2}} = \frac{17}{12}$

$\Downarrow$

$$\begin{bmatrix} x_3 = 1.4142 \\ \Downarrow \\ x_4 = 1.4142 \end{bmatrix}$$

example) Using FPI, determine the root of the function

$f(x) = x - \frac{\sin x}{2} - \frac{\cos x}{2}$ with $x_0 = 0$

$f(x) = 0$

$x = \underbrace{\frac{\sin x}{2} + \frac{\cos x}{2}}_{g(x)}$

$x_0 = 0$

$x_1 = g(x_0)$

$x_1 = \frac{\sin(0)}{2} + \frac{\cos(0)}{2} = \frac{1}{2}$

$\Downarrow$

$x_1 = \frac{1}{2}$

$x_2 = g(x_1) = \frac{\sin(\frac{1}{2}) + \cos(\frac{1}{2})}{2} = .6785$

$\Downarrow$

$x_3 = 0.7030$

$\Downarrow$

$x_4 = 0.7047$

$\Downarrow$

$x_5 = 0.7048$