HW4

Problem 13

(a) $P_2(x) = \frac{(x-2)(x-3)}{(0-2)(0-3)}[1] + \frac{(x-0)(x-3)}{(2-0)(2-3)}[3] + \frac{(x-0)(x-2)}{(3-0)(3-2)}[0]$

(b) is simalar to part (a)

(c) $P_3(x) = \frac{(x-2)(x-3)(x-5)}{(-1-2)(-1-3)(-1-5)}[0] + \frac{(x+1)(x-3)(x-5)}{(2+1)(2-3)(2-5)}[1] +$

$\frac{(x+1)(x-2)(x-5)}{(3+1)(3-2)(3-5)}[1] + \frac{(x+1)(x-2)(x-3)}{(5+1)(5-2)(5-3)}[2]$

Problem 3.

Problem 14

(a)

$$P_3(x) = \frac{(x-1850)(x-1900)(x-2000)}{(1800-1850)(1800-1900)(1800-200)}[280] +$$

$$\frac{(x-1800)(x-1900)(x-2000)}{(50)(-50)(-150)}[283] +$$

$$\frac{(x-1800)(x-1850)(x-2000)}{100(50)(-100)}[291] +$$

$$\frac{(x-1800)(x-1850)(x-1900)}{(200)(150)(100)}[370]$$

(b) $P_3(1950)$

$P_3(2050)$

Problem 4 Problem 15

$$|f(x) - P_2(x)| = \left| \frac{f^{(3)}(\xi)}{3!} (x+1)\, x\, (x-1) \right| \quad \text{on } [-1, 1]$$

$f(x) = e^x \qquad f^{(3)}(x) = e^x.$

$$|f(x) - P_2(x)| \leq \frac{|f^{(3)}(\xi)|}{6} \, |x^3 - x| \quad \text{on } [-1, 1]$$

$$\max |f^{(3)}(\xi)| = e^1$$

Let $h(x) = x^3 - x$. Now, let's find the maxi^m for $h(x)$

$h'(x) = 3x^2 - 1 = 0$

$x = \pm \frac{1}{\sqrt{3}}$

$x = \frac{1}{\sqrt{3}} \Rightarrow \frac{1}{(\sqrt{3})^3} - \frac{1}{\sqrt{3}}$

$x = \frac{-1}{\sqrt{3}} \Rightarrow \frac{-1}{\sqrt{3}^3} + \frac{1}{\sqrt{3}}$

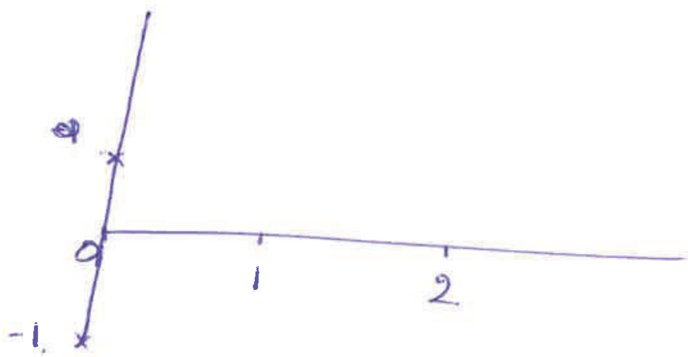
$\therefore \max_{x \in [-1,1)} |h(x)| = \left| \frac{3-1}{3\sqrt{3}} \right|$

$= \frac{2}{3\sqrt{3}}$

$\therefore |f(x) - P_2(x)| \leq \frac{e}{6} \frac{2}{3\sqrt{3}}.$

Problem 5. Problem 16

(a). $S(x) = \begin{cases} S_0(x) = x^3 + x - 1 \\ S_1(x) = 1 + 3(x-1) + 3(x-1)^2 - (x-1)^3 \end{cases}$



$S_0(1) = 1. \quad S_1(1) = 1.$

$\Rightarrow \boxed{S_0(1) = S_1(1)}$ ok.

$S_0'(1) \overset{?}{=} S_1'(1)$

$S_0'(x) = 3x^2 + 1 \Rightarrow S_0'(1) = 4$

$S_1'(x) = 3 + 6(x-1) - 3(x-1)^2 \Rightarrow S_1'(1) = 3$

$\Rightarrow S_0'(1) \neq S_1'(1)$

Not a cubic spline.

(b)
$$S(x) = \begin{cases} S_0(x) = 2x^3 + x^2 + 4x + 5 & \text{on } [0,1] \\ S_1(x) = 12 + 12(x-1) + 7(x-1)^2 + (x-1)^3 & \text{on } [1,2] \end{cases}$$

* Continuity of $S_0(x)$ and $S_1(x)$ at $x = 1$.

$S_0(1) = 2 + 1 + 4 + 5 = 12$

$S_1(1) = 12$

$$S_0(1) = S_1(1)$$

* Continuity of the 1st derivatives at $x = 1$.

$S_0'(x) = 6x^2 + 2x + 4$

$S_1'(x) = 12 + 14(x-1) + 3(x-1)^2.$

$$S_0'(1) = 12 = S_1'(1)$$

* Continuity of the 2nd derivative at $x = 1$.

$S_0''(x) = 12x + 2$

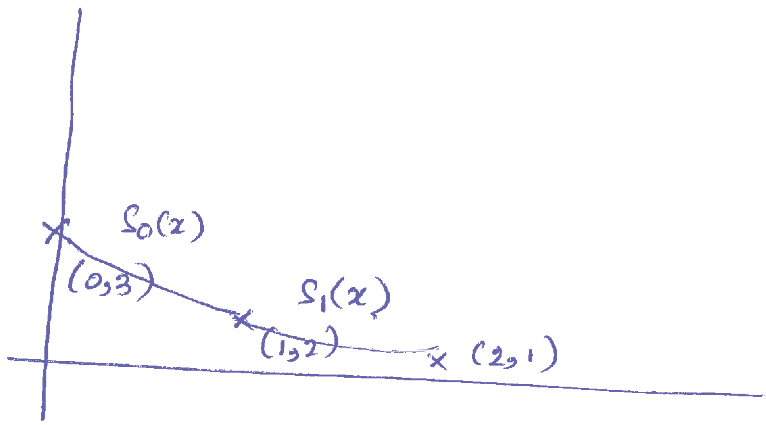
$S_1''(x) = 14 + 6(x-1)$

$$S_0''(1) = S_1''(1) = 14.$$

$\Rightarrow$ Cubic spline!

Problem 6 Problem 17

Natural cubic spline through $(0,3)$, $(1,2)$, and $(2,1)$



$$S_0(x) = a_0 + b_0(x+0) + c_0(x-0)^2 + d_0(x-0)^3 = a_0 + b_0 x + c_0 x^2 + d_0 x^3$$

$$S_1(x) = a_1 + b_1(x-1) + c_1(x-1)^2 + d_1(x-1)^3$$

$S_0(0) = 3$

$\boxed{a_0 = 3}$ — ①

$S_1(1) = 2$

$\boxed{a_1 = 2}$ — ③

$S_0(1) = 2$

$\boxed{a_0 + b_0 + c_0 + d_0 = 2}$ — ②

$S_1(2) = 1$

$\boxed{a_1 + b_1 + c_1 + d_1 = 1}$ — ④

$$S_0'(x) = b_0 + 2c_0 x + 3d_0 x^2$$

$$S_1'(x) = b_1 + 2c_1(x-1) + 3d_1(x-1)^2$$

Continuity of the 1^{st} derivative

$$S_0'(1) = S_1'(1)$$

$$\boxed{b_0 + 2c_0 + 3d_0 = b_1} \quad — (5)$$

$$S_0''(x) = 2c_0 + 6d_0 x$$

$$S_1''(x) = 2c_1 + 6d_1(x-1)$$

Continuity of the 2^{nd} derivative

$$S_0''(1) = S_1''(1)$$

$$\boxed{2c_0 + 6d_0 = 2c_1} \quad — (6)$$

Naturall BC^s

$$S_0''(0) = 0$$

$$\boxed{2c_0 = 0.} \quad — (7)$$

$$S_1''(2) = 0$$

$$\boxed{2c_1 + 6d_1 = 0} \quad — (8)$$