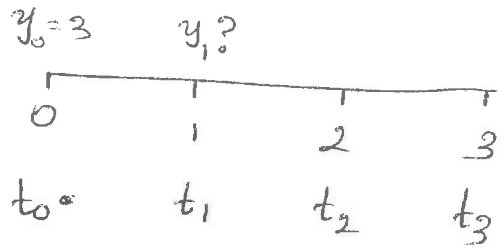
Problem 1

(a) RK4 $h=1$

$$\frac{dy}{dt} = (1-y)\cos t \qquad 0 \le t \le 3 \qquad y(0)=3$$

$$f(t,y) = (1-y)\cos t$$



$$K_1 = f(t_i, y_i)$$

$$= f(t_0, y_0) = f(0,3) = (1-3)\cos(0) = -2$$

$$K_2 = f\left(t_0 + \frac{h}{2}, y_0 + \frac{h}{2}K_1\right) = f(0.5, 2) = (1-2)\cos(0.5)$$

$$= -0.8776$$

$$K_3 = f\left(t_0 + \frac{h}{2}, y_0 + \frac{h}{2}K_2\right) = f(0.5, 2.5612)$$

$$= (1-2.5612)\cos(0.5)$$

$$= -1.3701$$

$$K_4 = f(\text{both}, y_0 + hK_3) = f(1, 1.6299)$$

$$= (1-1.6299)\cos(1)$$

$$= -0.3403$$

$$y_1 = y_0 + \frac{h}{6}\left[K_1 + 2K_2 + 2K_3 + K_4\right] = \boxed{1.8607}$$

1. (b) $y = 1 + 2e^{-\sin t}$.

True value at $t = 1$

$$y\Big|_{t=1} = 1 + 2e^{-\sin(1)} = \boxed{1.8622}$$

Absolute error

$$|y(1) - y_1| = |1.8622 - 1.8607|$$

$$= \boxed{0.0015}$$

$$A\vec{x} = \vec{b}$$