ex 1 use RK4 method with $h=0.5$ to approximate the solution of the IVP:

$$\frac{dy}{dt} = t^2 - y \qquad y(0) = 1 \qquad \begin{matrix} t_0 = 0 \\ y_0 = 1 \\ h = 0.5 \end{matrix} \qquad 0 \le t \le 2$$

$f(t,y) = t^2 - y$

$k_1 = f(t_i, y_i) = t_i^2 - y_i \rightarrow t_0^2 - y_0 = \boxed{-1}$

$k_2 = f\left(t_i + \frac{h}{2}, y_i + \frac{h}{2}k_1\right) = \left(t_i + \frac{h}{2}\right)^2 - \left(y_i + \frac{h}{2}k_1\right) \rightarrow \left(t_0 + \frac{0.5}{2}\right)^2 - \left(y_0 + \frac{0.5}{2}(-1)\right) = \boxed{0.6875}$

$k_3 = f\left(t_i + \frac{h}{2}, y_i + \frac{h}{2}k_2\right) = \left(t_i + \frac{h}{2}\right)^2 - \left(y_i + \frac{h}{2}k_2\right)$

$\left(t_0 + \frac{0.5}{2}\right)^2 - \left(y_0 + \frac{0.5}{2}(0.6875)\right) = \boxed{-0.7656}$ (k_3)

$k_4 = f(t_i + h, y_i + hk_3) = (t_i + h)^2 - (y_i + hk_3)$

$(t_0 + 0.5)^2 - (y_0 + 0.5(-0.7656))$

$(0 + 0.5)^2 - (1 + 0.5(-0.7656))$

$0.25 - 0.6172 = \boxed{-0.3672}$ (k_4)

$y_1 = y_0 + \frac{0.5}{6}\left[-1 + 2(-0.6875) + 2(-0.7656) + (-0.3672)\right]$

$\boxed{y_1 = 0.6439}$

6.1 Introduction to Linear Algebra

Introduction to Linear Algebra

Matrix

An $m \times n$ matrix A is an $m \times n$ array of scalars of the form:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & & a_{2n} \\ \vdots & \vdots & & & \\ a_{m1} & a_{m2} & \cdots & & a_{mn} \end{bmatrix}$$

- The size (order) of a A is $m \times n$

The (i,j) entry of A is a_{ij}

- The identity matrix of order n, denoted I_n, is the $n \times n$ diagonal matrix, whose elements are 1.

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3 \times 3}$$

- An $n \times 1$ matrix is called a <u>column vector</u>:

ex $\vec{x} = \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix}_{3 \times 1}$

- Transpose

the transpose of an $m \times n$ matrix A, denoted A^T, is an $n \times m$ matrix whose columns are corresponding rows of A

$$(A^T)_{ij} = a_{ji}$$

<u>ex</u> $A = \begin{bmatrix} 1 & -1 & 0 \\ 3 & -2 & 5 \end{bmatrix}_{2 \times 3}$ $A^T = \begin{bmatrix} 1 & 3 \\ -1 & -2 \\ 0 & 5 \end{bmatrix}_{3 \times 2}$