2.4 Order of Convergence

2.6.19

- How fast does an algorithm converge?
  → one way to measure the speed of convergence is to use the ratio of the errors between successive iterations.

$$\frac{e_{n+1}}{e_n} \approx \frac{|x_{n+1} - x_n|}{|x_n - x_{n-1}|}$$

- To measure the speed of convergence we use a concept called the "order of convergence"

Order of convergence: Suppose the sequence $\{x_n\}_{n=0}^{\infty}$ converges to the value p with $x_n \neq p$ for all n. If there exists positive constants α and λ such that

$$\frac{|x_{n+1} - p|}{|x_n - p|^{\alpha}} = \lambda$$

* since we do not know p, we use $\frac{|x_{n+1} - x_n|}{|x_n - x_{n-1}|^{\alpha}} = \lambda$

then $\{x_n\}$ is said to converge to p of order α with asymptotic error constant λ.

- If $\alpha = 1$ and $\lambda < 1$ the convergence is linear
- If $\alpha = 2$ the convergence is quadratic
- Larger α means "faster convergence"

Theorem

Assume that $g(x)$ is continuously differentiable $g(p)=p$ and $S=|g'(x)|<1$. Then the FPI converges linearly with rate S to the fixed point p for initial guess sufficiently close to p.

*Note

- the order of convergence of the bisection method is linear
- Newton's method is quadratically convergent.

homework & quiz due Friday 02/08