WEDNESDAY JULY 31, 2019

CH. 6 LINEAR ALGEBRA

6.3 Iterative Methods for Solving sys$^{\underline{m}}$ of Linear eq$^{\underline{ns}}$

·6.3.1 Jacobi Method (JM) [Final Exam – 2 iterations]

This method is a form of a fixed-point iteration for a sys$^{\underline{m}}$ of eq$^{\underline{ns}}$

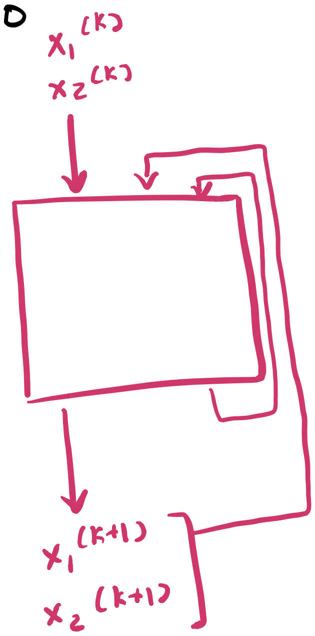
- Here, the 1st step is to solve the ith - equation for the ith variable
- Then, iterate as in Fixed-point iteration, starting with an initial guess

↑ given

ex. 1) Apply Jacobi m/d to solve the sys$^{\underline{m}}$:

$$3x_1 + x_2 = 5 \quad \text{— (1)}$$
$$x_1 + 2x_2 = 5 \quad \text{— (2)}$$

w/ $x_1^{(0)} = 0$, $x_2^{(0)} = 0$



STEP ONE:

$$x_1 = \frac{5}{3} - \frac{1}{3}x_2$$

$$x_2 = \frac{5}{2} - \frac{1}{2}x_1$$

STEP TWO:

$$x_1^{(k+1)} = \frac{5}{3} - \frac{1}{3}x_2^{(k)}$$

$$x_2^{(k+1)} = \frac{5}{2} - \frac{1}{2}x_1^{(k)}$$

k – iteration number

Iteration 1: k = 0

$$x_1^{(1)} = \frac{5}{3} - \frac{1}{3}x_2^{(0)} = \frac{5}{3} \qquad x_2^{(0)} = 0$$

$$x_2^{(1)} = \frac{5}{2} - \frac{1}{2}x_1^{(0)} = \frac{5}{2} \qquad x_1^{(0)} = 0$$

Iteration 2: k = 1

$$x_1^{(2)} = \frac{5}{3} - \frac{1}{3}x_2^{(1)} = \overset{\frac{10}{6}}{\frac{5}{3}} - \overset{\frac{5}{6}}{\frac{1}{3}\left(\frac{5}{2}\right)} = \frac{5}{6}$$

$$x_2^{(2)} = \frac{5}{2} - \frac{1}{2}x_1^{(1)} = \overset{\frac{15}{6}}{\frac{5}{2}} - \overset{\frac{5}{6}}{\frac{1}{2}\left(\frac{5}{3}\right)} = \frac{10}{6}$$

ex.2) Do 2-iteration of the Jacobi Method with $\vec{x}^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ to approximate the solu$^{\underline{n}}$ of the sys$^{\underline{m}}$

$$5x_1 - 2x_2 + 3x_3 = -1$$
$$-3x_1 + 9x_2 + x_3 = 2$$
$$2x_1 - x_2 - 7x_3 = 3$$

STEP 1:

$$x_1 = -\frac{1}{5} + \frac{2}{5}x_2 - \frac{3}{5}x_3$$

$$x_2 = \frac{2}{9} + \frac{3}{9}x_1 - \frac{1}{9}x_3$$

$$x_3 = -\frac{3}{7} - \frac{1}{7}x_2 + \frac{2}{7}x_1$$

STEP 2:

$$x_1^{(k+1)} = -\frac{1}{5} + \frac{2}{5}x_2^{(k)} - \frac{3}{5}x_3^{(k)}$$

$$x_2^{(k+1)} = \frac{2}{9} + \frac{3}{9}x_1^{(k)} - \frac{1}{9}x_3^{(k)}$$

$$x_3^{(k+1)} = -\frac{3}{7} - \frac{1}{7}x_2^{(k)} + \frac{2}{7}x_1^{(k)}$$

Iteration 1: $k=0$

$$x_1^{(1)} = -\frac{1}{5}$$

$$x_2^{(1)} = \frac{2}{9}$$

$$x_3^{(1)} = -\frac{3}{7}$$

Iteration 2: $k=1$

$$\left.\begin{aligned} x_1^{(2)} &= -\frac{1}{5} + \frac{2}{5}\left(\frac{2}{9}\right) - \frac{3}{5}\left(-\frac{3}{7}\right) \\ x_2^{(2)} &= \frac{2}{9} + \frac{3}{9}\left(-\frac{1}{5}\right) - \frac{1}{9}\left(-\frac{3}{7}\right) \\ x_3^{(2)} &= -\frac{3}{7} - \frac{1}{7}\left(\frac{2}{9}\right) + \frac{2}{7}\left(-\frac{1}{5}\right) \end{aligned}\right\}$$

Leave answer unsimplified for the final exam

$$\begin{bmatrix} x_1^{(k+1)} \\ x_2^{(k+1)} \\ x_3^{(k+1)} \end{bmatrix} = \begin{bmatrix} 0 & \frac{2}{5} & -\frac{3}{5} \\ \frac{3}{9} & 0 & -\frac{1}{9} \\ \frac{2}{7} & -\frac{1}{7} & 0 \end{bmatrix} \begin{bmatrix} x_1^{(k)} \\ x_2^{(k)} \\ x_3^{(k)} \end{bmatrix} + \begin{bmatrix} -\frac{1}{5} \\ \frac{2}{9} \\ -\frac{3}{7} \end{bmatrix}$$

constant vector

$$\vec{x}^{(k+1)} = M x^{(k)} + \vec{r}$$