# 2-8-19

Friday, February 8, 2019 11:29 AM

$$\lim_{n\to\infty} \frac{|x_{n+1} - p|}{|x_n - p|^{\alpha}} = \lambda$$

For the linear convergence, $\alpha = 1$ and $0 < \lambda < 1$

$\{x_n\} \to p$

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**Example!** Show that the sequence $[1/2^n]$ converges to $0$ linearly

$x_n = \frac{1}{2^n}$

$x_{n+1} = \frac{1}{2^{n+1}}$

$$\lim_{n\to\infty} \frac{|x_{n+1} - p|}{|x_n - p|} = \lim_{n\to\infty} \frac{\left|\frac{1}{2^{n+1}} - 0\right|}{\left|\frac{1}{2^n} - 0\right|} = \lim_{n\to\infty}\left[\frac{1/2^{n+1}}{1/2^n}\right] = \lim_{n\to\infty}\frac{2^n}{2^{n+1}} = \boxed{\frac{1}{2}} = \lambda$$

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## Chapter 3: Interpolation and Polynomial Approximation

**Problem:** Given $(n+1)$ data points

say $(x_0, y_0)$
$(x_1, y_1)$
$\vdots$
$(x_n, y_n)$

**Goal:** Find a polynomial of degree $n$, $P_n(x)$, such that

$$P_n(x_i) = y_i \qquad i = 0, 1, 2, 3, \ldots, n$$

$$P(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$$

$(n+1)$ unknown

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**Example:**

| | $x_i$ | $y_i$ | |
|---|---|---|---|
| $x_0$ | 0 | 1 | $y_0$ |
| $x_1$ | 1 | 0 | $y_1$ |
| $x_2$ | 2/3 | 1/2 | $y_2$ |

$P_2(x) = a_0 + a_1 x + a_2 x^2$

$P_2(x_0) = a_0 + a_1(0) + a_2(0) = 1 \longrightarrow a_0 + 0a_1 + 0a_2 = 1$

$P_2(x_1) = a_0 + a_1(1) + a_2(1)^2 = 0 \longrightarrow a_0 + a_1 + a_2 = 0$

$P_2(x_2) = a_0 + a_1(2/3) + a_2(2/3)^2 = 1/2 \longrightarrow a_0 + a_1(2/3) + a_2(4/9) = 1/2$

$$\begin{array}{c} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2/3 & 4/9 \end{bmatrix} \\ \begin{matrix} a_0 & \;a_1 & \;a_2 \end{matrix} \end{array} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1/2 \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = X^{-1}\vec{b} = \begin{bmatrix} 1 \\ -1/4 \\ -3/4 \end{bmatrix}$$

$$\boxed{P_2(x) = 1 - \frac{1}{4}x - \frac{3}{4}x^2}$$