example) Show that $f(x) = x^3 + x - 1$ has a root on the interval $[0, 1]$

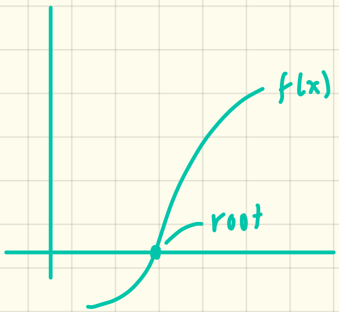
1.22.19
lecture

$f(0) = -1$ so by the IVT there must be some
$f(1) = 1$ value $x = c$ where $f(c) = 0$.

CHAPTER two:

Solution of equations of one variable

focus: Finding numerical solutions of equations in the general form: $f(x) = 0$



depending on the nature of the curve of $f(x)$, we may have a unique solution, multiple solutions or no solutions.

2.1 Bisection Method

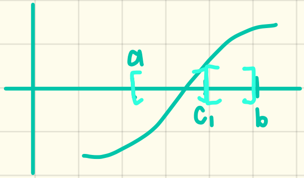
- Requires that an initial interval containing the root be identified (use IVT)

* $f(x)$ has a root at $x = r^*$ if $f(r^*) = 0$

Theorem

Let $f(x)$ be a continuous function on $[a, b]$ satisfying $f(a)f(b) < 0$, then $f(x)$ has a root between a & b.

Bisection Method Procedure



1. Locate the Midpoint of [a,b]
   * $c = \frac{a+b}{2}$
2. If $f(a)$ & $f(c_1)$ have opposite signs, or $f(a)\cdot f(c_1)<0$, then the root is contained within $[a, c_1]$.
   → Repeat @ step 1 with $[a, c_1]$.
3. If $f(c_1)$ & $f(b)$ have opposite signs, or $f(c_1)\cdot f(b)<0$, then the root is contained within $[c_1, b]$.
   → Repeat @ step 1 with $[c_1, b]$
4. Repeat until the length of the most recent interval evaluated has the desired accuracy.

Example) Show that $f(x) = x^3+x-1$ has a root in the interval [0,1] then use the bisection method to find c_5.

i	a_i	$f(a_i)$	c_i	$f(c_i)$	b_i	$f(b_i)$
1	0	−	0.5	−	1	+
2	0.5	−	0.75	+	1	+
3	0.5	−	0.625	−	0.75	+
4	0.625	−	.6875	+	0.75	+
5	0.625	−	.65625	−	0.6875	+