MONDAY JULY 8, 2019

REVIEW OF CALCULUS

- **CONTINUITY OF A FUNCTION —**

  A function of $f(x)$ is said to be continuous at $x = a$, if the following conditions are satisfied:

  (1) $f(a)$ exists

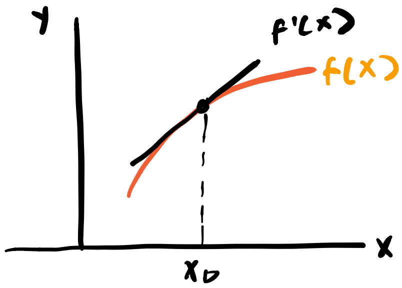
  (2) $\lim_{x \to a} f(x)$ exists

  (3) $\lim_{x \to a} f(x) = f(a)$

  ***NOTE**: The graph of a continuous function has no gaps

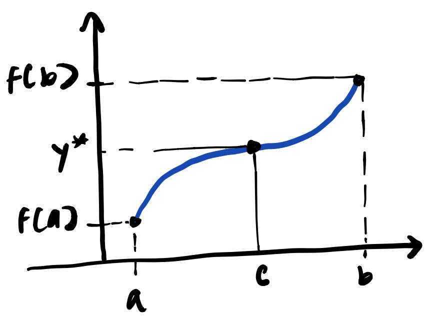
- **DIFFERENTIABILITY —**

  Let $f(x)$ be a fun$^{\underline{n}}$ defined on an open interval containing x_0. Then fun$^{\underline{n}}$ $f(x)$ is differentiable at x_0 if $f'(x) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$ exists.



- **INTERMEDIATE VALUE THEOREM (IVT) —**

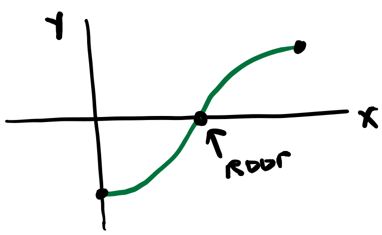
  Let $f(x)$ be a continuous fun$^{\underline{n}}$ on the interval $[a,b]$ if y^* is a number between $f(a)$ and $f(b)$, then there exists a number "c" in (a,b) such that $y^* = f(c)$



ex) show that $x^5 - 2x^3 + 3x^2 - 1$ has a root in the interval $[0,1]$

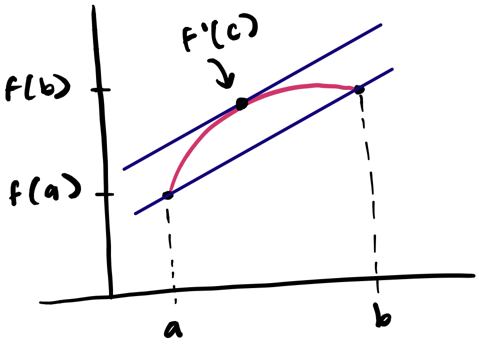
$f(0) = -1$; [$f(0) < 0$ & $f(1) > 0$ so there must be a root in the interval $[0,1]$]

$f(1) = 1$



• MEAN VALUE THEOREM (MVT) —

Let $f(x)$ be a continuously differentiable funⁿ on the interval $[a,b]$. Then, there exists a number "c" between 'a' and 'b' such that $f'(c) = \dfrac{f(b)-f(a)}{b-a}$



ex) Find "c" satisfying the MVT for $f(x) = x^2 - 3$ on the interval $[1,3]$

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

① calculate slope

$$\frac{f(3)-f(1)}{3-1} = \frac{6-(-2)}{3-1} = 4$$

$f(3) = 6$
$f(1) = -2$

② find c-value

$$f'(x) = 2x$$

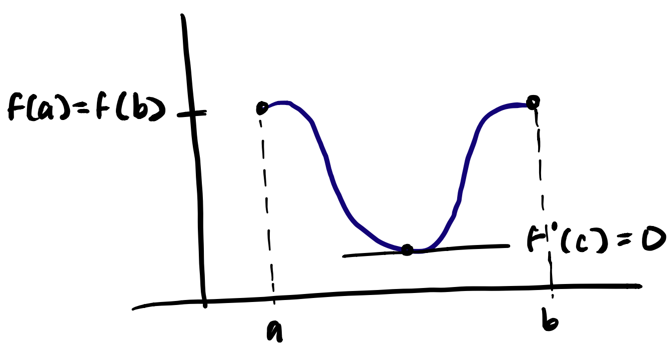
$$f'(c) \Rightarrow 2x = 4$$

$$x = \frac{4}{2} = 2$$

$$\boxed{c = 2}$$

• ROLLE'S THEOREM —

Let $f(x)$ be a continuously differentiable funⁿ on the interval $[a,b]$, and assume that $f(a) = f(b)$. Then, there exists a number "c" betⁿ 'a' and 'b' such that $f'(c) = 0$.



• TAYLOR'S TH$^{\underline{m}}$ w/ REMAINDER —

Let x and x_0 be real numbers, and let $f(x)$ be $(k+1)$-times continuously differentiable on the interval x and x_0. Then, there is a number $\xi(x)$ bet$^{\underline{n}}$ x and x_0 such that

$$f(x) = \underbrace{\frac{f'(x_0)(x-x_0)}{1!} + \frac{f''(x_0)(x-x_0)^2}{2!} + \dots + \frac{f^{(k)}(x_0)(x-x_0)^k}{k!}}_{\text{degree "k" Taylor Polynomial } [P_k(x)]} + \underbrace{\frac{f^{(k+1)}(\xi(x))(x-x_0)^{k+1}}{(k+1)!}}_{\text{Taylor Remainder } [R_k(x)]}$$

ex) use the Taylor's Th$^{\underline{m}}$ on $f(x) = \cos x$ about $x_0 = 0$ to write $f(x)$ as $f(x) = P_4(x) + R_4(x)$, where $P_4(x)$ is the Taylor Polynomial degree 4.

$$\begin{aligned}
f(x) &= \cos x & f(0) &= 1 \\
f'(x) &= -\sin x & f'(0) &= 0 \\
f''(x) &= -\cos x & f''(0) &= -1 \\
f'''(x) &= \sin x & f'''(0) &= 0 \\
f^4(x) &= \cos x & f^4(0) &= 1
\end{aligned}$$

$$P_4(x) = f(0) + \frac{f'(0)(x-0)}{1!} + \frac{f''(0)(x-0)^2}{2!} + \frac{f'''(0)(x-0)^3}{3!} + \frac{f^4(0)(x-0)^4}{4!}$$

$$= 1 + \frac{0 \cdot x}{1!} - \frac{x^2}{2!} + \frac{0 \cdot x^3}{3!} + \frac{1 \cdot x^4}{4!}$$

$$\boxed{P_4(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}}$$

$$R_4(x) = \frac{f^5(\xi(x))(x-x_0)^5}{5!}$$

$$\boxed{R_4(x) = \frac{-\sin(\xi(x))(x-0)^5}{5!}}$$

$$\left[\cos x = P_4(x) + R_4(x)\right]$$