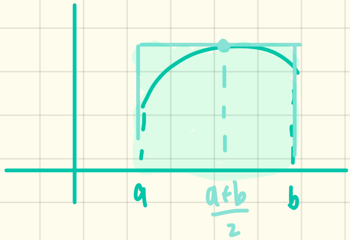
4.3 Elements of Numerical Integration

03.04.19

* Sometimes its hard to calculate a definite integral analytically.
* The basic method involved in approximating $\int_a^b f(x)\,dx$ is called numerical quadrature.

Mid Point Rule:

$$\int_a^b f(x)\,dx \approx f\left(\frac{a+b}{2}\right)(b-a)$$



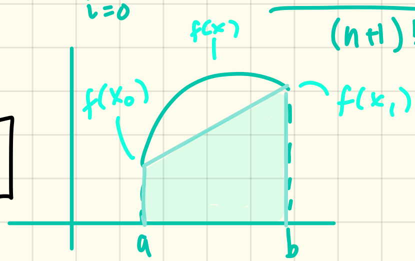
Other Quadrature Rules:

*Review: the Lagrange interpolating polynomial

$$f(x) = \sum_{i=0}^{n} f(x_i)L_i(x) + \prod_{i=0}^{n}(x-x_i)\frac{f^{(n+1)}(\xi(x))}{(n+1)!}$$

Trapezoidal Rule:

$$\int_a^b f(x)\,dx \approx (b-a)\left[\frac{f(b)+f(a)}{2}\right]$$



$$L_0(x) = \frac{(x-x_1)}{(x_0-x_1)} \qquad L_1(x) = \frac{(x-x_0)}{(x_1-x_0)}$$

$$f(x) = f(x_0)\frac{(x-x_1)}{(x_0-x_1)} + f(x_1)\frac{(x-x_0)}{(x_1-x_0)} + \frac{f''(\xi(x))(x-x_0)(x-x_1)}{2!}$$

integrate

$$\int_{x_0}^{x_1} f(x)\,dx = \int_{x_0}^{x_1} f(x_0)\frac{(x-x_1)}{(x_0-x_1)}\,dx + \int_{x_0}^{x_1} f(x_1)\frac{(x-x_0)}{(x_1-x_0)}\,dx$$

$$+ \int_{x_0}^{x_1} f''(\xi(x))\frac{(x-x_0)(x-x_1)}{2!}\,dx$$

$$\ldots \Rightarrow \int_{x_0}^{x_1} f(x)\,dx = \frac{(x_1-x_0)}{2}\left[f(x_0)+f(x_1)\right]$$