Problem $\boxed{8}$

$f(x) = x - \cos x$ on $[0, \frac{\pi}{3}]$ accuracy 10^{-8} $x_0 = \frac{\pi}{4}$

$x = g(x)$

$x = \cos x$

$g(x) = \cos x$

$\cos x \in [0, \frac{\pi}{3}]$ on $[0, \frac{\pi}{3}]$

$g'(x) = \sin x$

$\sin x$ is an increasing fun^n on $[0, \frac{\pi}{3}]$

$$\max_{x \in [0, \frac{\pi}{3}]} |g'(x)| = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} < 1$$

Error analysis

$$|P - x_n| \leq K^n \max\{x_0 - a, b - x_0\}$$

we need $|P - x_n| < 10^{-8}$.

If we find n such that

$$K^n \max\{x_0 - a, b - x_0\} < 10^{-8},$$

the $|P - x_n| < 10^{-8}$.

$$K = \max_{x \in [0, \frac{\pi}{3}]} |g'(x)| = \frac{\sqrt{3}}{2}$$

$$\left(\frac{\sqrt{3}}{2}\right)^n \max\left\{ \frac{\pi}{4} - 0, \frac{\pi}{3} - \frac{\pi}{4} \right\} < 10^{-8}$$

$$\left(\frac{\sqrt{3}}{2}\right)^n \frac{\pi}{4} < 10^{-8}$$

$$\left(\frac{\sqrt{3}}{2}\right)^n < \frac{4}{\pi} \times 10^{-8}$$

$$\log_{10}\left[\frac{\sqrt{3}}{2}\right]^n < \log_{10}\left[\frac{4}{\pi} \times 10^{-8}\right]$$

$$n \log_{10}\left[\frac{\sqrt{3}}{2}\right] < \log_{10}\left[\frac{4}{\pi} \times 10^{-8}\right]$$

$$n \overset{*}{>} \frac{\log_{10}\left[\frac{4}{\pi} \times 10^{-8}\right]}{\log_{10}\left[\frac{\sqrt{3}}{2}\right]}$$

Note*
$\log_{10}\left(\frac{\sqrt{3}}{2}\right)$ is a negative number.

$$n > 126.38$$

$$\boxed{n = 127}$$