Numerical Analysis MAT 362: Homework 8

Due on Wednesday, April 22 in class

Please read the Instructions

- **Show all the steps that you go between the question and the answer. Show how you derived the answer. For your work to be complete, you need to explain your reasoning and make your computations clear.**
- **You will be graded on the readability of your work.**
- **The correct answer with no or incorrect work will earn you NO marks**
- **Show ALL your work**
- Use only four decimal places for all numbers.
- **If possible, use 8.5" × 11" white paper (not torn from spiral binders) and staple sheets together.**
- Print your name legibly in the upper corner of the page.
- Write your solutions as though you're trying to convince someone that you know what you're talking about.
- **Failure to follow these instructions will result in loss points (up to the full amount of the homework total)**

Problem 1

(a) Use RK4 with step size $h = 1$ to approximate the solution of the following IVP:

$$\frac{dy}{dt} = (1 - y)\cos t, \quad 0 \le t \le 3, \quad y(0) = 3.$$

at $y(1)$ (ONLY 1 step). (b) Find the absolute error in approximating $y(1)$ by y_1 using the actual solution
$y = 1 + 2e^{-\sin t}$.

Problem 2

(Introduction to Linear Algebra)
2.1 If A and B are 5×7 matrices, and C is a 2×5 matrix, which of the following are defined?
(a) AB (b) A^T (c) $A + B$ (d) $B^T C^T$ (e) CB (f) $B + C$
2.2 Evaluate the matrix product BA^T where, $A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$.

Problem 3

Use Gaussian Elimination to solve the system:

$$\begin{array}{rcrcrcl} 2x_1 & - & 6x_2 & + & 2x_3 & = & 4 \\ x_1 & & & + & 2x_3 & = & 5 \\ 3x_1 & - & 5x_2 & + & 3x_3 & = & 6 \end{array}$$

Problem 4

Do 2-iterations of the **Jacobi and Gauss-Seidel methods** with $\vec{x}^{(0)} = [0, 0, 0]^T$ to approximate the solution of the following system:

$$\begin{array}{rcrcrcl} 3x_1 & + & x_2 & + & x_3 & = & 6 \\ x_1 & + & 3x_2 & + & x_3 & = & 3 \\ x_1 & + & x_2 & + & 3x_3 & = & 5 \end{array}$$

Problem 5

For the following coefficient matrices A, determine if the Jacobi and Gauss-Seidel iterations converge.

(a) $A = \begin{bmatrix} 2 & 1 & 0 \\ -3 & -5 & 1 \\ 0 & -2 & -3 \end{bmatrix}$ (b) $A = \begin{bmatrix} 2 & 1 & 0 \\ -3 & -4 & 1 \\ 0 & -2 & 3 \end{bmatrix}$

Problem 6

(a) Do three iterations by the power method with $\vec{x}^{(0)} = [-1, 1]^T$ to approximate the largest eigenvalue of $A = \begin{bmatrix} 5 & -4 \\ -1 & 2 \end{bmatrix}$ with its eigenvector.
(b) Calculate the *actual* dominant eigenvalue and the corresponding eigenvector.