(b)

(i) $E_T = -\dfrac{(b-a)^3}{12} f''(c)$

$|E_T| = \left|\dfrac{(b-a)^3}{12}\right| |f''(c)|$

$\leq \dfrac{(b-a)^3}{12} |f''(x)|_{max}$

$f(x) = \ln x$

$f'(x) = \dfrac{1}{x}$

$f''(x) = \dfrac{-1}{x^2}$

$x \in [1,2]$

$|f''(x)|_{max} = \left|\dfrac{1}{x^2}\right|_{\substack{max \\ x\in[1,2]}} = 1$

5

$|E_T| \leq \dfrac{(2-1)^3}{12}(1)$

$\boxed{|E_T| \leq 0.0833}$

$E_S = -\dfrac{(b-a)^5}{90} \dfrac{1}{2^5} f^{(4)}(c)$

$|E_S| = \dfrac{|(b-a)^5|}{90} \dfrac{1}{2^5} |f^{(4)}(c)|$

$|E_S| \leq \dfrac{(b-a)^5}{90} \times \dfrac{1}{2^5} |f^{(4)}(x)|_{\substack{max \\ x\in[a,b]}}$

$f'''(x) = \dfrac{2}{x^3}$

$f^{(4)}(x) = \dfrac{-6}{x^4}$

$|E_S| \leq \dfrac{(2-1)^5}{90} \times \dfrac{1}{2^5} |f^{(4)}(x)|_{\substack{max \\ x\in[1,2]}}$

$|E_S| \leq \dfrac{1}{90} \times \dfrac{1}{2^5}(6)$

$\boxed{|E_S| = 0.0021}$

5