page 1

HW 6

15 pts

1.

(a) Trapezoid Rule.

$$\int_a^b \ln x \, dx \approx \frac{(b-a)}{2}\left[\ln(a) + \ln(b)\right]$$

$$= \frac{(2-1)}{2}\left[\ln 2 + \ln 1\right]$$

5

$$\approx \boxed{0.3466}$$

Simpson's Rule

$$\int_a^b f(x)dx \approx \frac{(b-a)}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

$$\int_1^2 \ln x \, dx \approx \frac{(2-1)}{6}\left[\ln(1) + 4\ln\left(\frac{1+2}{3}\right) + \ln(2)\right]$$

5

$$= \frac{1}{6}\left[\ln(1) + 4\ln(1.5) + \ln(2)\right]$$

$$\approx \boxed{0.3858}$$

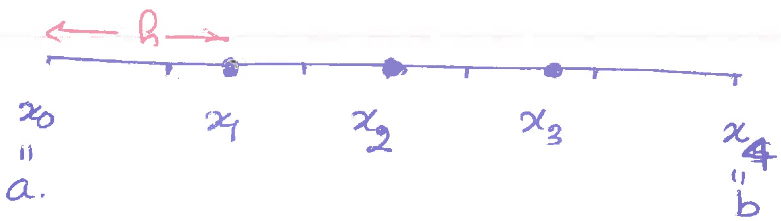
Problem 2.

15 pts

Approximate $\int_0^1 e^{x^2}\,dx$ using 4- subintervals.

$n=4.$ $\quad b=1 \quad a=0 \quad h=\frac{b-a}{n}=\frac{1-0}{4}=0.25.$

(a) Composite Trapezoidal Rule.



$$\int_a^b f(x)\,dx \approx \sum_{i=1}^{4} \frac{h}{2}\left[f(x_{i-1})+f(x_i)\right]$$

$$=\frac{h}{2}\left[f(x_0)+f(x_4)+2\left[f(x_1)+f(x_2)+f(x_3)\right]\right]$$

$$=\frac{0.25}{2}\left[f(0)+f(1)+2\left[f(0.25)+f(0.50)+f(0.75)\right]\right]$$

where $f(x)=e^{x^2}$.

$$\int_0^1 e^{x^2}\,dx \approx \boxed{1.4907}$$

2.

(b) Composite Midpoint Rule

$$\int_a^b f(x)\,dx \approx h \sum_{i=1}^{n} f\left(\frac{x_{i-1}+x_i}{2}\right)$$

$$\int_0^1 e^{x^2}\,dx \approx 0.25\left[f\left(\frac{x_0+x_1}{2}\right)+f\left(\frac{x_1+x_2}{2}\right)+f\left(\frac{x_2+x_3}{2}\right)+f\left(\frac{x_3+x_4}{2}\right)\right],$$

where $x_0=0$, $x_1=0.25$, $x_2=0.50$, $x_3=0.75$, and $x_4=1.0$

$$= 0.25\left[f(0.125)+f(0.3750)+f(0.6250)+f(0.8750)\right]$$

$$\int_0^1 e^{x^2}\,dx \approx \boxed{1.4487}$$

2 (c) Composite Simpson's Rule

Recall

$f(x) = e^{x^2}$

0	0.25	0.50	0.75	1
$x_0 = a$	x_1	x_2	x_3	$x_4 = b$
1	4	1		
		1	4	1.

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[f(x_0) + f(x_4) + 2f(x_2) + 4\left(f(x_1) + f(x_3) \right) \right]$$

$$= \frac{0.25}{3}\left[f(0) + f(1) + 2 f(0.5) + 4\left(f(0.25) + f(0.75) \right) \right]$$

$$= \frac{0.25}{3}\left[e^0 + e^1 + 2 e^{(0.5)^2} + 4\left(e^{(0.25)^2} + e^{(0.75)^2} \right) \right]$$

$$\int_0^1 e^{x^2}\,dx \approx \boxed{1.4637.}$$

Problem 3

Approximate $\int_1^4 \sin x \, dx$ using 6-subintervals

$n = 6$. $\quad h = \dfrac{b-a}{n} = \dfrac{4-1}{6} = 0.5.$

1.0	1.5	2.0	2.5	3.0	3.5	4.0
x_0	x_1	x_2	x_3	x_4	x_5	x_6

h.

i) Composite ~~Midpoint~~ Trapezoidal Rule

$$\int_a^b f(x)\,dx \approx \sum_{i=1}^{6} \frac{h}{2}\left[f(x_{i-1}) + f(x_i)\right]$$

$$= \frac{h}{2}\left[f(x_0) + f(x_6) + 2\left(f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)\right)\right],$$

where $f(x) = \sin x$.

$$\int_1^4 \sin x\,dx \approx \frac{0.5}{2}\Big[\sin(1) + \sin(4) + 2\big(\sin(1.5) + \sin(2.0) + \sin(2.5) + \sin(3.0) + \sin(3.5)\big)\Big]$$

$$\int_1^4 \sin x\,dx \approx \boxed{1.1690.}$$

3. (b). Composite Midpoint Rule.

$$\int_a^b f(x)\,dx \approx h \sum_{i=1}^{n} f\left(\frac{x_{i-1}+x_i}{2}\right)$$

$$= 0.5\left[f\left(\frac{x_0+x_1}{2}\right) + f\left(\frac{x_1+x_2}{2}\right) + f\left(\frac{x_2+x_3}{2}\right) + f\left(\frac{x_3+x_4}{2}\right) + \right.$$

$$\left. + f\left(\frac{x_4+x_5}{2}\right) + f\left(\frac{x_5+x_6}{2}\right)\right]$$

$$\int_1^4 \sin x\,dx = 0.5\left[\sin(1.25) + \sin(1.75) + \sin(2.25) + \sin(2.75) + \right.$$

$$\left. \sin(3.25) + \sin(3.75)\right]$$

$$\approx \boxed{1.2065}$$

3 (c) Composite Simpson's Rule.

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[f(x_0) + f(x_6) + 4\left(f(x_1)+f(x_3)+f(x_5)\right) + 2\left(f(x_2)+f(x_4)\right)\right]$$

$$\int_1^4 \sin x\,dx \approx \frac{0.5}{3}\left[\sin(1) + \sin(4) + 4\left(\sin(1.5)+\sin(2.5)+\sin(3.5)\right) + 2\left(\sin(2)+\sin(3)\right)\right]$$

$$\int_1^4 \sin x\,dx \quad \boxed{\approx 1.1944}$$

15 Pts

Problem 4

Find the step size h and the number of subintervals n required to approximate $\int_1^3 x^2 \ln x\, dx$ Correct within 10^{-4}.

(a) Composite Trapezoidal Rule.

$$E_{T_n} \leq +\frac{(b-a)h^2}{12} \max_{x\in[a,b]} |f''(x)|$$

$$f(x) = x^2 \ln x$$

$$f'(x) = 2x.\ln x + x^2 \times \frac{1}{x} = 2x \ln x + x$$

$$f''(x) = 2.\ln x + 2x \times \frac{1}{x} + 1$$

$$f''(x) = 2\ln x + 3$$

$$b = 3 \quad a = 1.$$

$$E_{T_n} \leq \frac{(3-1)}{12} h^2 \max_{x\in[1,3]} |2\ln x + 3| < 10^{-4}.$$

$$\frac{1}{6} h^2.(2\ln 3 + 3) < 10^{-4}.$$

$$h \leq \sqrt{\frac{10^{-4} \times 6}{(2\ln 3 + 3)}} \qquad \boxed{h \leq 0.0107}$$

let $\boxed{h = 0.01.}$ $\boxed{n = \frac{b-a}{h} = 200}$

(b) Composite Midpoint Rule

$$E_{M_n} = \frac{(b-a)}{24} h^2 f''(c)$$

$$E_{M_n} \leq \frac{(b-a)}{24} h^2 \left| f''(x) \right|_{\substack{max \\ x \in [a,b]}}$$

$$\frac{(3-1)}{24} h^2 \left| 2\ln x + 3 \right|_{max} \leq 10^{-4}$$

$$h^2 \leq \frac{12 \times 10^{-4}}{(2\ln x + 3)}$$

$$h \leq \sqrt{\frac{12 \times 10^{-4}}{2\ln x + 3}}$$

$$\boxed{h \leq 0.0152}$$

let $\boxed{h = 0.01, \text{ then } n = 200}$

(c) Composite Simpson's Rule

$$E_{S_n} = \frac{-(b-a)}{180} h^4 f^{(4)}(c)$$

$$E_{S_n} \le \frac{(b-a)}{180} h^4 \left| f^{(4)}(x) \right|_{max}$$

Recall:

$$f''(x) = 2\ln x + 3$$

$$f'''(x) = 2 \cdot \frac{1}{x}$$

$$f^{(4)}(x) = \frac{-2}{x^2}$$

$$\frac{(3-1)}{180} h^4 \left| \frac{-2}{x^2} \right|_{\substack{max \\ x \in [1,3]}} \le 10^{-4}$$

$$\frac{\cancel{2}}{\cancel{180}} \times h^4 \, \cancel{2} \le 10^{-4}$$

$\cancel{90}$

45

$$h^4 \le 45 \times 10^{-4}$$

$$h \le \sqrt[4]{45 \times 10^{-4}}$$

$$h \le 0.2590$$

Let $\boxed{h = 0.25}$

then

$$n = \frac{b-a}{h} = \frac{3-1}{1/4} = \boxed{8.}$$