Review:

Forward Difference Method:

$$f'(x_0) = \frac{f(x_0+h) - f(x_0)}{h}$$

Max error:

$$\frac{|f''(\xi)|h}{2} \quad \max_{(x_0,\, x_0+h)}$$

Backward Difference Method:

$$f'(x_0) = \frac{f(x_0) - f(x_0-h)}{h}$$

Max error:

$$\frac{|f''(\xi)|h}{2} \quad \max_{(x_0-h,\, x_0)}$$

Center Difference Method:

$$f'(x_0) = \frac{f(x_0+h) - f(x_0-h)}{2h}$$

Max error:

$$\frac{|f'''(\xi)|h^2}{6} \quad \max_{(x_0-h,\, x_0+h)}$$

Example) Consider $f(x) = x^2 e^x$. Find the maximum error approximating $f'(1)$ by th FDF, BDF, & CDF with $h = 0.2$

$$f'(x) = 2xe^x + x^2e^x = (2x+x^2)e^x$$

$$f''(x) = (2+4x+x^2)e^x$$

$$f'''(x) = (6+6x+x^2)e^x$$

FDF: $\text{max error} = \max_{(1,\,1.2)} \frac{|f''(\xi)|h}{2} \leq (2+4(1.2)+(1.2)^2)e^{1.2}\frac{(0.2)}{2}$

≤ 2.736

BDF: $\text{max error} = \max_{(0.8,\,1.0)} \frac{|f''(\xi)|h}{2} \leq (2+4+1)e^1\frac{(0.2)}{2}$

≤ 1.903

CDF: $\text{max error} = \max_{(0.8,\,1.2)} \frac{|f'''(\xi)|h^2}{6} \leq \frac{(6+6(1.2)+(1.2)^2 e^{1.2}(0.2)}{6}$

≤ 0.324