ex

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 0 \end{bmatrix}_{2\times 3} \qquad B = \begin{bmatrix} 7 & -1 & 2 \\ 5 & 6 & 0 \end{bmatrix}_{2\times 3}$$

$$A + B = \begin{bmatrix} 8 & 1 & 5 \\ 9 & 11 & 0 \end{bmatrix}$$

ex Let $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 5 \end{bmatrix}_{2\times 3}$ and $B = \begin{bmatrix} 1 & -1 \\ 0 & 5 \\ 3 & 2 \end{bmatrix}_{3\times 2}$

calculate $AB = \begin{bmatrix} 10 & 15 \\ 14 & 11 \end{bmatrix}_{2\times 2}$

$$A_{m\times p} \quad B_{p\times n} = C_{m\times n}$$

Determinants

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

ex $\begin{vmatrix} 1 & -1 & 0 \\ 5 & 2 & 3 \\ -2 & 1 & 5 \end{vmatrix}$

$$= 1\begin{vmatrix} 2 & 3 \\ 1 & 5 \end{vmatrix} + 1\begin{vmatrix} 5 & 3 \\ -2 & 5 \end{vmatrix} + 0\begin{vmatrix} 5 & 2 \\ -2 & 1 \end{vmatrix}$$

$$1(10-3) + 1(25+6)$$

$7 + 31 = \boxed{38}$

6.2 systems of linear equations

E_1 $x_1 + 2x_2 - x_3 = 5$

E_2 $3x_1 + x_2 - 2x_3 = 9$

E_3 $-x_1 + 4x_2 + 2x_3 = 0$

$A\vec{x} = \vec{b}$

↑ unknown vector

augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 3 & 1 & -2 & 9 \\ -1 & 4 & 2 & 0 \end{array}\right]$$

coefficient matrix A | ↑ constant vector $\vec{b}$

$E_2 \leftarrow E_2 - 3E_1$

$E_3 \leftarrow E_3 + E_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 0 & -5 & 1 & -6 \\ 0 & 6 & 1 & 5 \end{array}\right]$$

$E_2 \leftarrow E_2/-5$

$E_3 \leftarrow E_3/6$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 0 & 1 & -\frac{1}{5} & \frac{6}{5} \\ 0 & 1 & \frac{1}{6} & \frac{5}{6} \end{array}\right]$$

$E_3 \leftarrow E_3 - E_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 0 & 1 & -\frac{1}{5} & \frac{6}{5} \\ 0 & 0 & \frac{11}{30} & \frac{-11}{30} \end{array}\right]$$

$x_1 = 2$

$x_2 = 1$

$x_3 = -1$

Elementary-Row Operations

(1) multiply a row by a non-zero constant $(E_i \leftarrow cE_i)$

(2) Interchange two rows $(E_i \leftrightarrow E_j)$

(3) Add a multiple of one row to another row $(E_i \leftarrow E_i + cE_j)$