3 Point Forward Difference Formula

$$f'(x_0) \approx \frac{-3f(x_0) + 4f(\overset{x_0+h}{x_1}) - f(\overset{x_0+2h}{x_2})}{2h}$$

3 Point Backwards Difference Formula

$$f'(x_0) \approx \frac{3f(x_0) - 4(x_0 - h) + f(x_0 - 2h)}{2h}$$

example) From the following table approximate $f'(1)$ by the three point FDF & BDF.

x	0.6	0.8	1.0	1.2	1.4
$f(x)$	0.65	1.42	2.71	4.78	7.94

* $f(x) = x^2 e^x$

$f'(x) = 2xe^x + x^2 e^x$

$f'(1) = 2e^1 + e^1 = 8.1548$

FDF: $$f'(1) \approx \frac{-3f(1) + 4f(1.2) - f(1.4)}{2(0.2)}$$

$$\approx \frac{-3[2.71] + 4[4.78] - 1[7.94]}{0.4} = 7.695$$

BDF: $$f'(1) \approx \frac{3f(1) - 4f(0.8) + f(0.6)}{0.4} = 7.75$$

example) Given the table with $(x_n, f(x_n))$ use the appropriate formula to approximate $f'(x_n)$ $n = 0, 1, 2, 3$

x	1.1	1.2	1.3	1.4
y	3.5	3.7	2.9	2.6

Review:

Forward Difference Method:

$$f'(x_0) = \frac{f(x_0+h) - f(x_0)}{h}$$

Max error: $\dfrac{|f''(\xi)|h}{2}$ max (x_0, x_0+h)

Backward Difference Method:

$$f'(x_0) = \frac{f(x_0) - f(x_0-h)}{h}$$

Max error: $\dfrac{|f''(\xi)|h}{2}$ max (x_0-h, x_0)

Center Difference Method:

$$f'(x_0) = \frac{f(x_0+h) - f(x_0-h)}{2h}$$

Max error: $\dfrac{|f'''(\xi)|h^2}{6}$ max (x_0-h, x_0+h)

$n=0$ FDF

$$f'(x_0) = f'(1.1) \approx \frac{f(1.2) - f(1.1)}{0.1} = 2$$

$n=1$ CDF

$$f'(x_1) = f'(1.2) \approx \frac{f(1.3) - f(1.1)}{2(0.1)} = -3$$

$n=2$ CDF

$$f'(x_2) = f'(1.3) \approx \frac{f(1.4) - f(1.2)}{2(0.1)}$$

$n=3$ BDF

$$f'(x_3) = f'(1.4) \approx \frac{f(1.4) - f(1.3)}{0.1}$$