다변량 통계 과제 2

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- 언어 : python

In [225…
```
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
import os
from glob import glob
import sys

import scipy.stats as stats # qqplot에 사용
```

In [2]:
```
#시각화 패키지들
import seaborn as sns
import matplotlib.pyplot as plt
%matplotlib inline

plt.style.use("ggplot")
```

문제 1.

x1:석유, x2: 천연가스, x3:수력 발전, x4:원자력 발전 의 에너지 소비량의 표본 평균과 표본 공분산이다.

$$\bar{\boldsymbol{X}} = \begin{bmatrix} 0.766 \\ 0.508 \\ 0.438 \\ 0.161 \end{bmatrix}, \quad \boldsymbol{S} = \begin{bmatrix} 0.856 & 0.635 & 0.173 & 0.096 \\ 0.635 & 0.568 & 0.128 & 0.067 \\ 0.173 & 0.128 & 0.171 & 0.039 \\ 0.096 & 0.067 & 0.039 & 0.043 \end{bmatrix},$$

In [25]:
```
x_bar = np.array([[0.766],
                  [0.508],
                  [0.438],
                  [0.161]])
S = np.array([[0.856,0.635,0.173,0.096],
              [0.635,0.568,0.128,0.067],
              [0.173,0.128,0.171,0.039],
              [0.096,0.067,0.039,0.043]])
```

1 - a)

X1+X2+X3+X4의 표본평균과 표본분산

- E(X1+X2+X3+X4)=E(X1)+E(X2)+E(X3)+E(X4)

- VAR(X1+X2+X3+X4) = COV(X1+X2+X3+X4, X1+X2+X3+X4) = SUM(공분산 행렬)

In [53]:
```
x_all_mean = x_bar.T @ np.array([1,1,1,1])
print("표본평균 : ", x_all_mean)
```

```
표본평균 :  [1.873]
```

In [54]:
```
x_all_var=np.array([1,1,1,1]) @ S @ np.array([1,1,1,1])
print("표본분산 : ",x_all_var)
```

```
표본분산 :  3.914
```

1 - b)

X1-X2의 표본평균과 표본분산

- E(X1-X2) = E(X1)-E(X2)
- VAR(X1-X2) = VAR(X1)+VAR(X2)-2COV(X1,X2)

In [55]:
```
x_diff_mean = x_bar.T @ np.array([1,-1,0,0])
print("표본평균 : ", x_diff_mean)
```

```
표본평균 :  [0.258]
```

In [56]:
```
x_diff_var=np.array([1,-1,0,0]) @ S @ np.array([1,-1,0,0])
print("표본분산 : {:.4f}".format(x_diff_var))
```

```
표본분산 : 0.1540
```

1 - c)

(X1+X2+X3+X4)과 (X1-X2)의 표본공분산(행렬) 구하기

- COV(X1+X2+X3+X4,X1-X2) = {VAR(X1)+COV(X2,X1)+COV(X3,X1)+COV(X4,X1)} - {COV(X1,X2)+VAR(X2)+COV(X3,X2)+COV(X4,X2)}
- 공분산은 공분산행렬S에서 1열 - 2열 한것의 결과와 같다,

In [64]:
```
cov_of_pair = S @ np.array([1,-1,0,0]) @np.array([1,1,1,1])
cov_matirix_pair = np.array([[x_all_var,cov_of_pair],
                             [cov_of_pair,x_diff_var]])
print("공분산 행렬 : \n",cov_matirix_pair)
```

```
공분산 행렬 :
 [[3.914 0.362]
 [0.362 0.154]]
```

문제 2.

아래의 평균과 공분산을 갖는 이변량 정규분포의 n=1000 랜덤 표본 추출

$$\boldsymbol{\mu} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \qquad \boldsymbol{\Sigma} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

2 - a).

n=1000, rep=1000 xbar 와 S를 구해서 히스토그램과 수치로 비교

In [113…
```
bivariate_norm_mean= np.array([0,0])
bivariate_norm_cov= np.array([[2,1],[1,2]])
bivariate_norm_cov
```

Out[113…
```
array([[2, 1],
       [1, 2]])
```

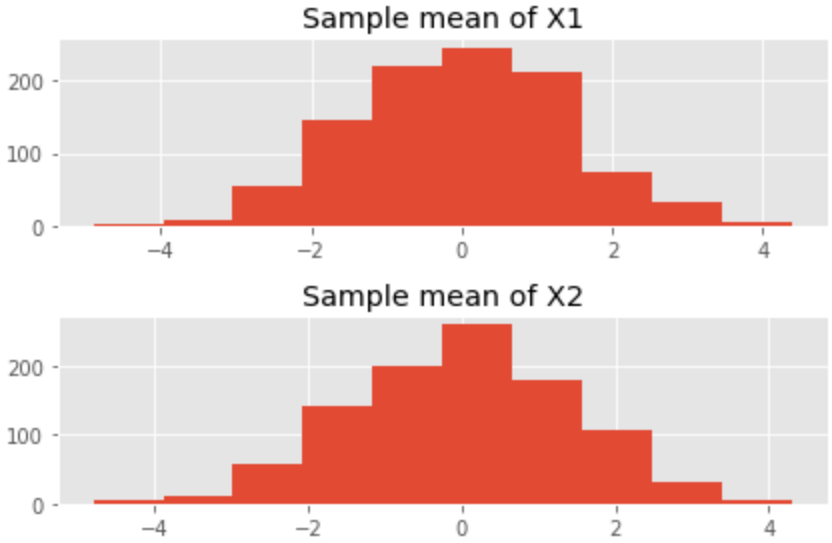
In [142…
```
np.stack([np.cov(x1,x2),np.cov(x1,x2),np.cov(x1,x2),np.cov(x1,x2)])[:,0,0]
```

Out[142…
```
array([2.36427718, 2.36427718, 2.36427718, 2.36427718])
```

In [144…
```
x1_means=[]
x2_means=[]

all_cov=[]

for nums in range(1000):
    x1,x2=np.random.multivariate_normal(bivariate_norm_mean,bivariate_norm_cov,1000).T
    x1_means.append(np.mean(x1))
    x2_means.append(np.mean(x2))

    all_cov.append(np.cov(x1,x2))
all_cov=np.stack(all_cov)
```

In [145…
```
print('x1의 평균 : ',np.mean(x1_means))
print('x2의 평균 : ',np.mean(x2_means))

plt.subplot(2,1,1)

plt.title("Sample mean of X1")
plt.hist(x1)

plt.subplot(2,1,2)
plt.title("Sample mean of X2")
plt.hist(x2)

plt.tight_layout()
```

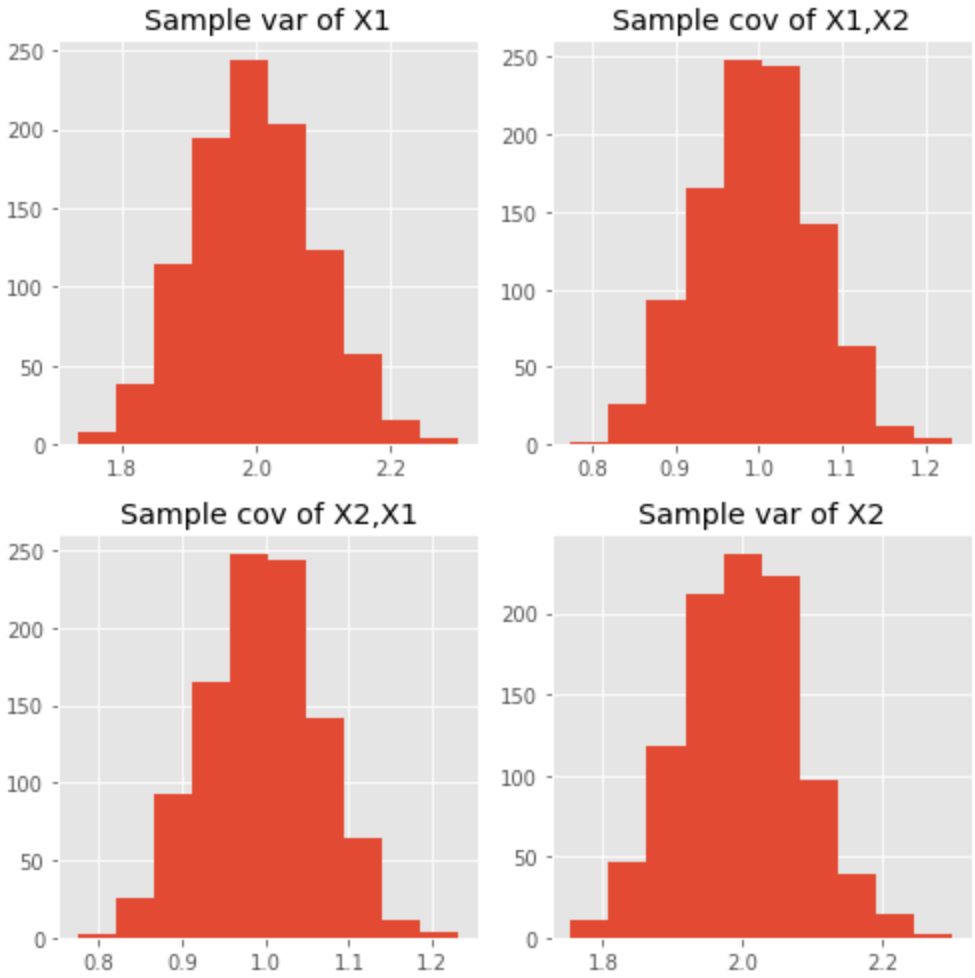
```
x1의 평균 :  0.002004897339473411
x2의 평균 :  0.0008866986650858212
```



In [154…

```
print("S11 : ",np.mean(all_cov[:,0,0]))
print("S12 : ",np.mean(all_cov[:,0,1]))
print("S21 : ",np.mean(all_cov[:,1,0]))
print("S22 : ",np.mean(all_cov[:,1,1]))

plt.figure(figsize=(7,7))
plt.subplot(2,2,1)

plt.title("Sample var of X1")
plt.hist(all_cov[:,0,0])

plt.subplot(2,2,2)
plt.title("Sample cov of X1,X2")
plt.hist(all_cov[:,0,1])

plt.subplot(2,2,3)
plt.title("Sample cov of X2,X1")
plt.hist(all_cov[:,1,0])

plt.subplot(2,2,4)
plt.title("Sample var of X2")
plt.hist(all_cov[:,1,1])

plt.tight_layout()
```

```
S11 :  1.996653232770947
S12 :  0.9970238842444674
S21 :  0.9970238842444674
S22 :  1.9986437551327636
```



2 - b)

n=1000 표본에서 1000개의 |S| 와 tr(S)의 히스토그램 그리고, 평균을 나타내기

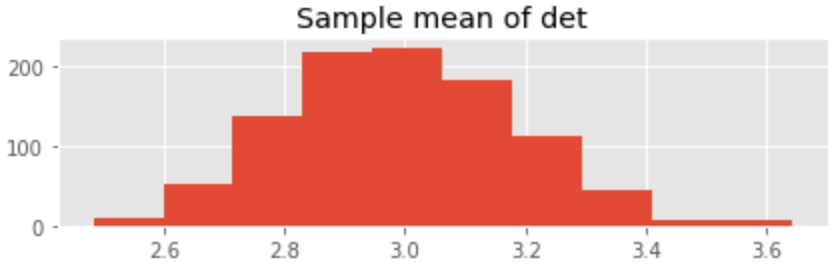
In [162…
```
all_cov.shape
```

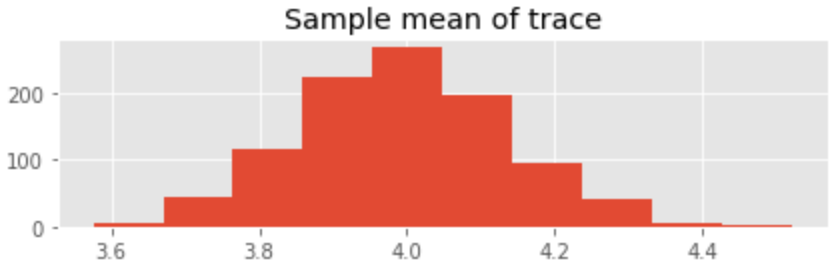
Out[162…
```
(1000, 2, 2)
```

In [174…
```
all_cov_det=np.linalg.det(all_cov)
all_cov_trace=np.trace(all_cov,axis1=1,axis2=2)
```

In [177…
```
print("Mean of sample det : ",np.mean(all_cov_det))
print("Mean of sample trace : ",np.mean(all_cov_trace))

plt.subplot(2,1,1)

plt.title("Sample mean of det")
plt.hist(all_cov_det)

plt.subplot(2,1,2)
plt.title("Sample mean of trace")
plt.hist(all_cov_trace)

plt.tight_layout()
```

```
Mean of sample det :  2.993990140552404
Mean of sample trace :  3.99529698790371
```





실제 determinant : 3, trace : 4로, 거의 일치한 수치를 얻을 수 있다.

2 - c)

1000개 표본, 1000 rep 에서 S의 leading eigen value 구하고 비교

In [182…

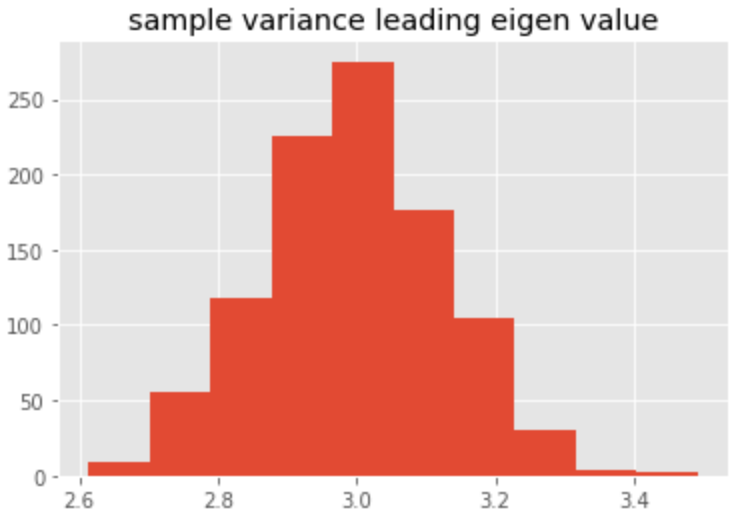
```
print("실제 cov의 eigen value : ",np.linalg.eigvals(bivariate_norm_cov))
```

```
실제 cov의 eigen value :  [3. 1.]
```

In [209…

```
all_cov_leading_eigval =np.sort(np.linalg.eigvals(all_cov),)[:,1]
print("Mean of sample variance leading eigen value",np.mean(all_cov_leading_eigval))

plt.title("sample variance leading eigen value")
plt.hist(all_cov_leading_eigval)

plt.show()
```

```
Mean of sample variance leading eigen value 2.9960892241327493
```



실제값 3과 매우 유사함을 알 수 있다.

문제 3.

- T4-6.DAT 130명 페루 청소년 심리검사 결과

- 첫 5개 변수 : independence, support, benevolence, conformity, leadership

In [224…
```
# 파일 불러오기
X = pd.DataFrame( np.loadtxt('../data/T4-6.DAT', unpack = True).T[:,:5],
                  columns=['independence','support','benevolence','conformity','leadership'
X
```

Out[224…

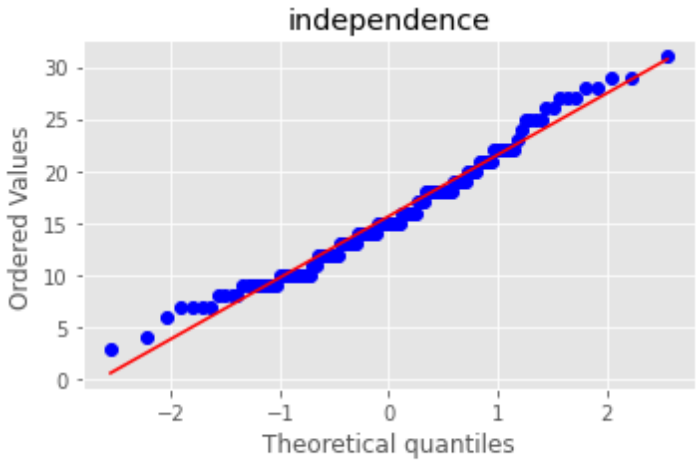
	independence	support	benevolence	conformity	leadership
0	27.0	13.0	14.0	20.0	11.0
1	12.0	13.0	24.0	25.0	6.0
2	14.0	20.0	15.0	16.0	7.0
3	18.0	20.0	17.0	12.0	6.0
4	9.0	22.0	22.0	21.0	6.0
…	…	…	…	…	…
125	10.0	11.0	26.0	17.0	10.0
126	14.0	12.0	14.0	11.0	29.0
127	19.0	11.0	23.0	18.0	13.0
128	27.0	19.0	22.0	7.0	9.0
129	10.0	17.0	22.0	22.0	8.0

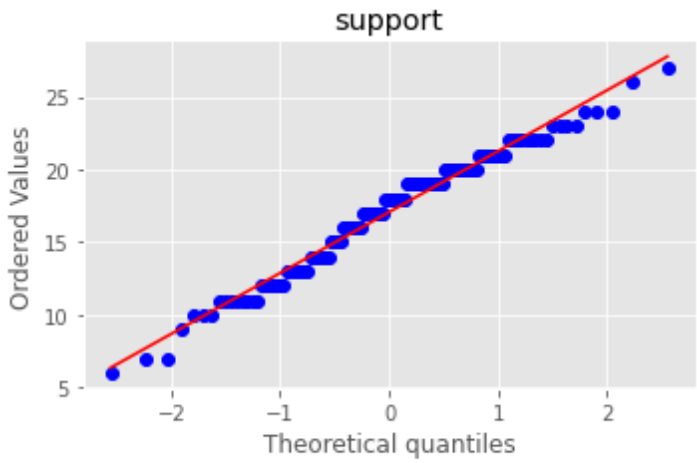
130 rows × 5 columns

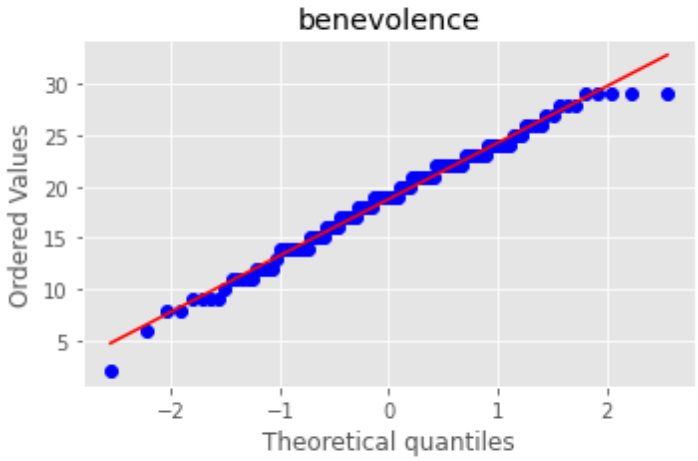
3 - a).

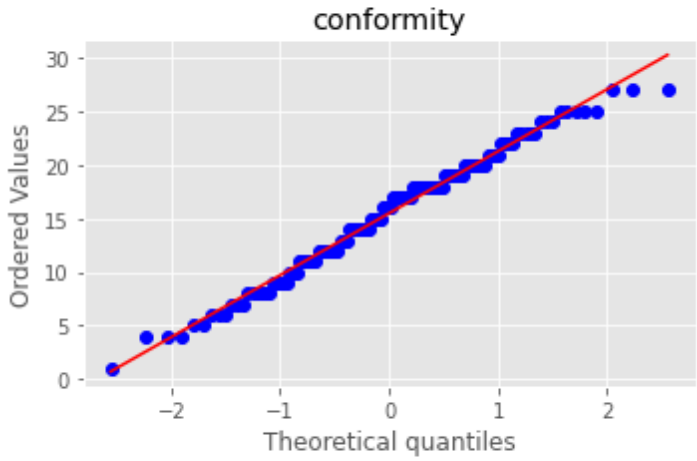
- Q-Q PLOT marginal normality

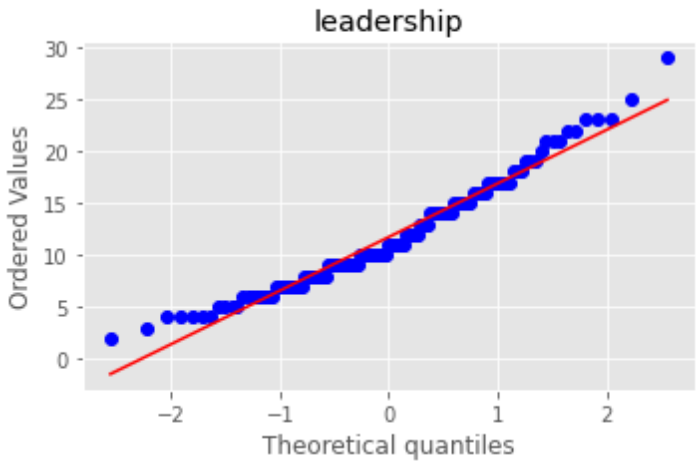
In [235…
```
plt.figure(figsize=(10,10))

plt.subplot(3,2,1)
stats.probplot(X['independence'],plot=plt)
plt.title('independence')

plt.subplot(3,2,2)
stats.probplot(X['support'],plot=plt)
plt.title('support')

plt.subplot(3,2,3)
stats.probplot(X['benevolence'],plot=plt)
plt.title('benevolence')

plt.subplot(3,2,4)
stats.probplot(X['conformity'],plot=plt)
plt.title('conformity')

plt.subplot(3,2,5)
stats.probplot(X['leadership'],plot=plt)
plt.title('leadership')

plt.tight_layout()
```











Indepence와 Leadership은 곡선형태로 관측된다.

Indepence와 Leadership은 정규성을 띄지않는다고 생각된다.

3 - b).

- Chi-square PLOT multivariate noramlity

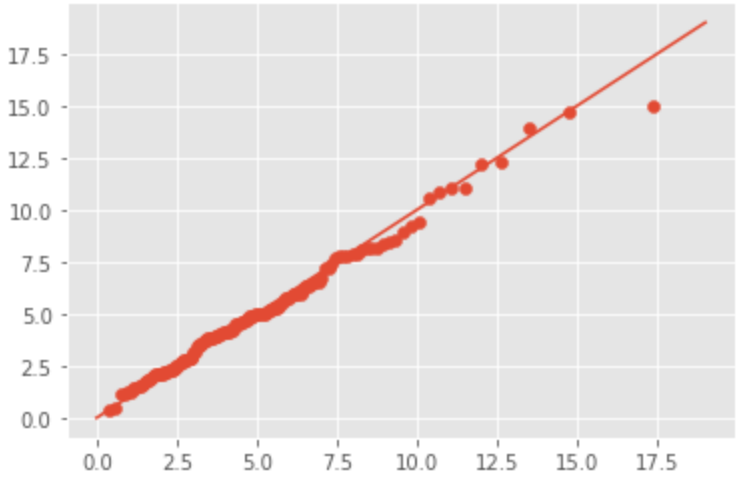
In [294…

```
deviation_matrix=X-X.mean()
cov_matrix = deviation_matrix.cov().to_numpy()
invcov_matrix=np.linalg.inv(cov_matrix)

chisq_list =[]

for i in range(130):
    row_one=deviation_matrix.to_numpy()[i,:]
    chisq_list.append(row_one.T @ invcov_matrix @ row_one)
```

```
plt.scatter(scipy.stats.chi2.ppf((np.arange(1,131)-0.5)/130,df=5),
    np.sort(chisq_list))
plt.plot(np.arange(0,20))
```

Out[294...

```
[<matplotlib.lines.Line2D at 0x1911ea61488>]
```



multivariate Chi-square plot을 그려본 결과.

마지막 한 값이 아웃라이어가 존재한다.

그것을 제외하면, 정규성을 띄는것으로 관측된다.