과제1

- 컴퓨터공학과 12171676 이종법
- 언어: python / 문제풀이는 손으로 풀고 pdf 합치기했습니다.

In [1]:

```
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
import os
from glob import glob
import sys
```

In [2]:

```
#시각화 패키지들
import seaborn as sns
import matplotlib.pyplot as plt
%matplotlib inline

plt.style.use("ggplot")
```

문제 1.

- (1) scatter plot 및 해석
- (2) x_bar, 공분산 행렬, 상관계수 행렬

In [3]:

```
# 파일 불러오기
X = pd.DataFrame( np.loadtxt('../data/P1-4.DAT', unpack = True).T,columns=['x1','x2','x3']
X
```

Out[3]:

	x1	x2	x3
0	108.28	17.05	1484.10
1	152.36	16.59	750.33
2	95.04	10.91	766.42
3	65.45	14.14	1110.46
4	62.97	9.52	1031.29
5	263.99	25.33	195.26
6	265.19	18.54	193.83
7	285.06	15.73	191.11
8	92.01	8.10	1175.16
9	165.68	11.13	211.15

In [4]:

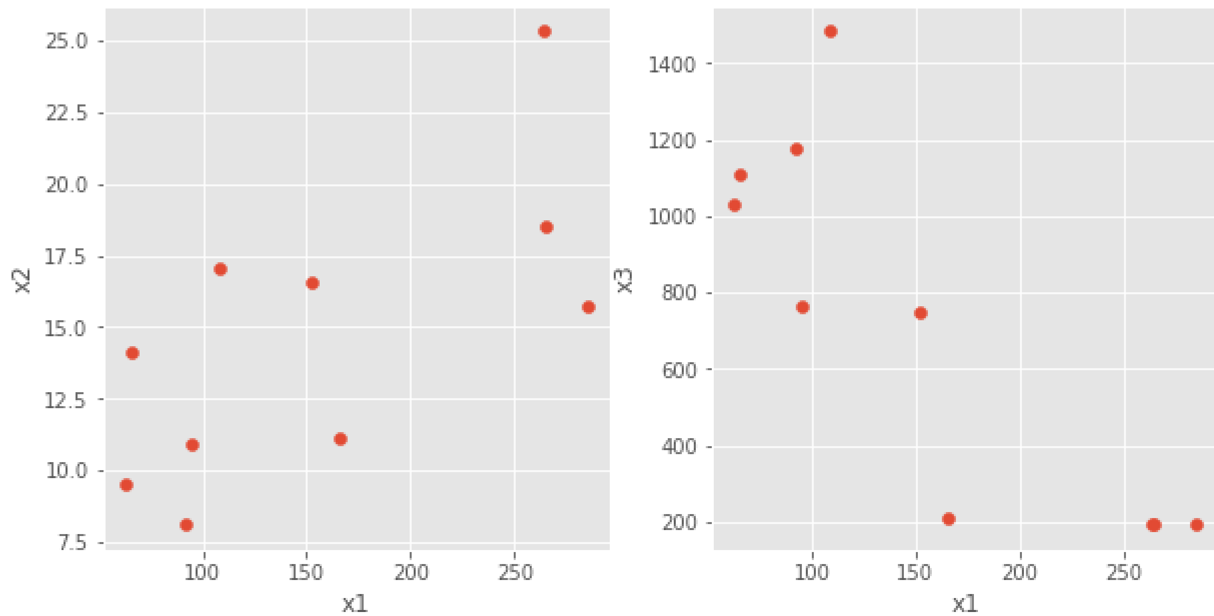
```
plt.figure(figsize=(10,5))


plt.subplot(1,2,1)
```

```
plt.scatter(X['x1'],X['x2'])
plt.xlabel("x1")
plt.ylabel("x2")


plt.subplot(1,2,2)

plt.scatter(X['x1'],X['x3'])
plt.xlabel("x1")
plt.ylabel("x3")
plt.show()
```



x1 과 x2는 양의 상관관계를 가지고, x1과 x3는 음의 상관관계를 가진다.

In [5]:

```
print("x_bar : ",np.array(X).mean(axis=0) )
```

```
x_bar :  [155.603  14.704 710.911]
```

In [6]:

```
print("S_n(공분산 행렬) : ")
print(X.cov())
```

```
S_n(공분산 행렬) :
              x1           x2             x3
x1   7476.453246   303.618620  -35575.959570
x2    303.618620    26.190316   -1053.827393
x3 -35575.959570 -1053.827393  237054.269832
```

In [7]:

```
print("Sample Correlation R : ")
print(X.corr())
```

```
Sample Correlation R :
          x1        x2        x3
x1  1.000000  0.686136 -0.845055
x2  0.686136  1.000000 -0.422937
x3 -0.845055 -0.422937  1.000000
```

문제 2

- (1) Pairwise Scatter Plot
- (2) x_bar, 공분산 행렬, 상관계수 행렬

In [8]:

```python
# 파일 불러오기
X = pd.DataFrame( np.loadtxt('../data/T1-8.DAT', unpack = True).T,columns=['D.radius','Rad
X
```

Out[8]:

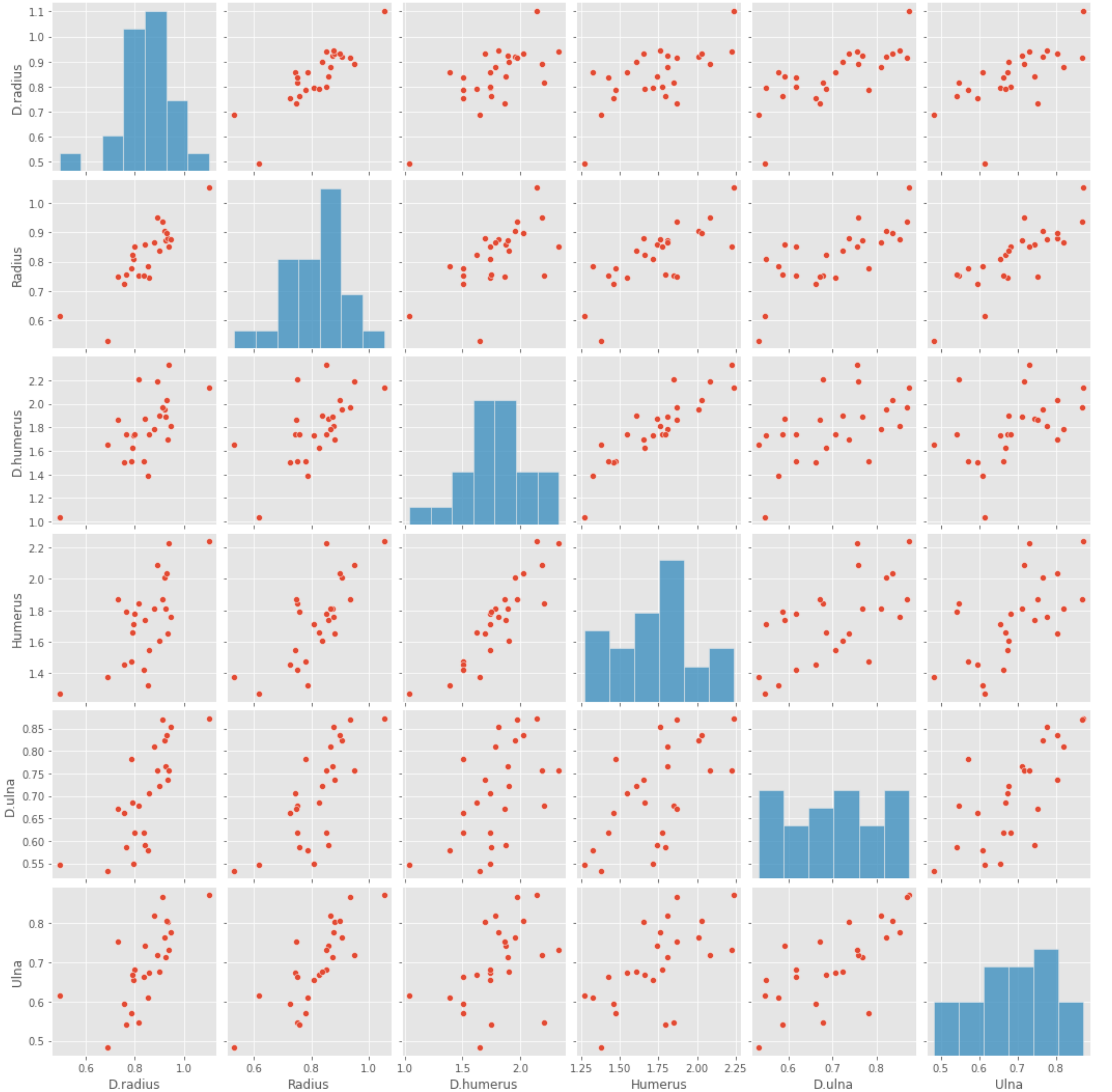
	D.radius	Radius	D.humerus	Humerus	D.ulna	Ulna
0	1.103	1.052	2.139	2.238	0.873	0.872
1	0.842	0.859	1.873	1.741	0.590	0.744
2	0.925	0.873	1.887	1.809	0.767	0.713
3	0.857	0.744	1.739	1.547	0.706	0.674
4	0.795	0.809	1.734	1.715	0.549	0.654
5	0.787	0.779	1.509	1.474	0.782	0.571
6	0.933	0.880	1.695	1.656	0.737	0.803
7	0.799	0.851	1.740	1.777	0.618	0.682
8	0.945	0.876	1.811	1.759	0.853	0.777
9	0.921	0.906	1.954	2.009	0.823	0.765
10	0.792	0.825	1.624	1.657	0.686	0.668
11	0.815	0.751	2.204	1.846	0.678	0.546
12	0.755	0.724	1.508	1.458	0.662	0.595
13	0.880	0.866	1.786	1.811	0.810	0.819
14	0.900	0.838	1.902	1.606	0.723	0.677
15	0.764	0.757	1.743	1.794	0.586	0.541
16	0.733	0.748	1.863	1.869	0.672	0.752
17	0.932	0.898	2.028	2.032	0.836	0.805
18	0.856	0.786	1.390	1.324	0.578	0.610
19	0.890	0.950	2.187	2.087	0.758	0.718
20	0.688	0.532	1.650	1.378	0.533	0.482
21	0.940	0.850	2.334	2.225	0.757	0.731
22	0.493	0.616	1.037	1.268	0.546	0.615
23	0.835	0.752	1.509	1.422	0.618	0.664
24	0.915	0.936	1.971	1.869	0.869	0.868

In [9]:

```python
sns.pairplot(X)
```

Out[9]:

```
<seaborn.axisgrid.PairGrid at 0x20a4684b788>
```



RADIUS, HUMERUS , uLna 간에 모두 양의 상관관계를 보인다

In [10]:
```
print("x_bar : ",np.array(X).mean(axis=0) )
```

```
x_bar :  [0.8438   0.81832 1.79268 1.73484 0.7044   0.69384]
```

In [11]:
```
print("S_n(공분산 행렬) : ")
print(X.cov())
```

```
S_n(공분산 행렬) :
           D.radius    Radius  D.humerus   Humerus    D.ulna      Ulna
D.radius   0.013002  0.010378   0.022350  0.020086  0.009121  0.007958
Radius     0.010378  0.011418   0.018535  0.021100  0.008530  0.008909
D.humerus  0.022350  0.018535   0.080357  0.066776  0.016837  0.012847
Humerus    0.020086  0.021100   0.066776  0.069484  0.017735  0.016794
D.ulna     0.009121  0.008530   0.016837  0.017735  0.011568  0.008071
Ulna       0.007958  0.008909   0.012847  0.016794  0.008071  0.010599
```

In [12]:

```
print("Sample Correlation R : ")
print(X.corr())
```

```
Sample Correlation R :
           D.radius    Radius  D.humerus   Humerus    D.ulna      Ulna
D.radius   1.000000  0.851807   0.691459  0.668258  0.743693  0.677894
Radius     0.851807  1.000000   0.611916  0.749093  0.742178  0.809798
D.humerus  0.691459  0.611916   1.000000  0.893646  0.552222  0.440205
Humerus    0.668258  0.749093   0.893646  1.000000  0.625550  0.618820
D.ulna     0.743693  0.742178   0.552222  0.625550  1.000000  0.728892
Ulna       0.677894  0.809798   0.440205  0.618820  0.728892  1.000000
```

문제 3.

- (a) scatter plot과 r
- (b) 세 개의 outlier 제거와, r
- (c) m^2 로 바뀔시 상관계수와 그 이유

In [4]:

```
# 파일 불러오기
X = pd.DataFrame( np.loadtxt('../data/T1-11.DAT', unpack = True).T,columns=['millions','ac
X
```

Out[4]:

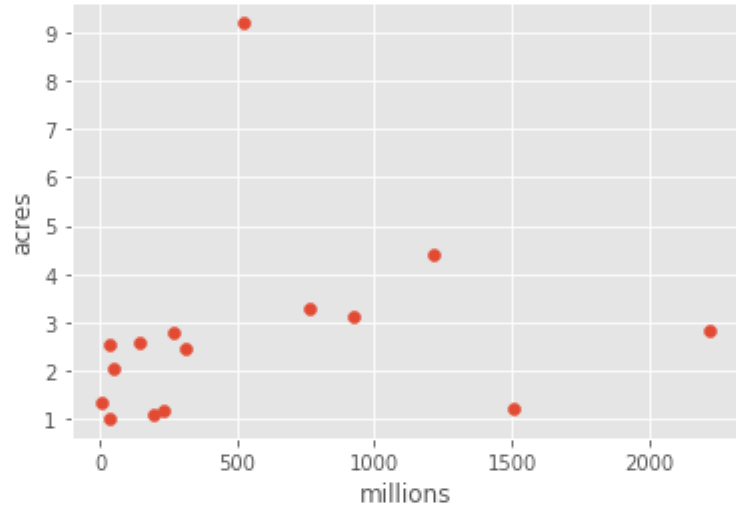
	millions	acres
0	47.4	2.05
1	35.8	1.02
2	32.9	2.53
3	1508.5	1.23
4	1217.4	4.40
5	310.0	2.46
6	521.8	9.19
7	5.6	1.34
8	922.7	3.14
9	235.6	1.17
10	265.8	2.80
11	199.0	1.09
12	2219.8	2.84
13	761.3	3.30
14	146.6	2.59

In [5]:

```
plt.scatter(X['millions'],X['acres'])
plt.xlabel("millions")
plt.ylabel("acres")
```

Out[5]:

```
Text(0, 0.5, 'acres')
```



In [6]:

```
r=X.corr()["acres"][0]
print("상관계수 : ",r)
#이상치 때문에 상관계수가 낮은것으로 생각된다.
#scatter plot에서는 outlier 1개, leverage 2개로 관측됨.
```

```
상관계수 :  0.1725274227916501
```

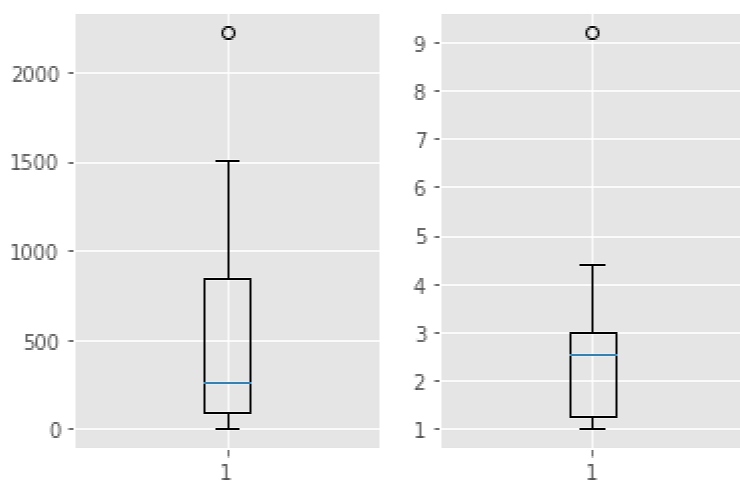
- outlier 제거와 r

In [7]:

```
plt.subplot(1,2,1)
plt.boxplot(X['millions'])
plt.subplot(1,2,2)
plt.boxplot(X['acres'])
```

Out[7]:

```
{'whiskers': [<matplotlib.lines.Line2D at 0x2f91f6f0448>,
  <matplotlib.lines.Line2D at 0x2f91f6f0c08>],
 'caps': [<matplotlib.lines.Line2D at 0x2f91f6f0388>,
  <matplotlib.lines.Line2D at 0x2f91f6b3fc8>],
 'boxes': [<matplotlib.lines.Line2D at 0x2f91f6e9ac8>],
 'medians': [<matplotlib.lines.Line2D at 0x2f91f69ed88>],
 'fliers': [<matplotlib.lines.Line2D at 0x2f91f6f05c8>],
 'means': []}
```



In [8]:

```
term1= X['millions']>1500
term2 =X['acres']>5

X[ term1 | term2]
#outlier 3개
```

Out[8]:

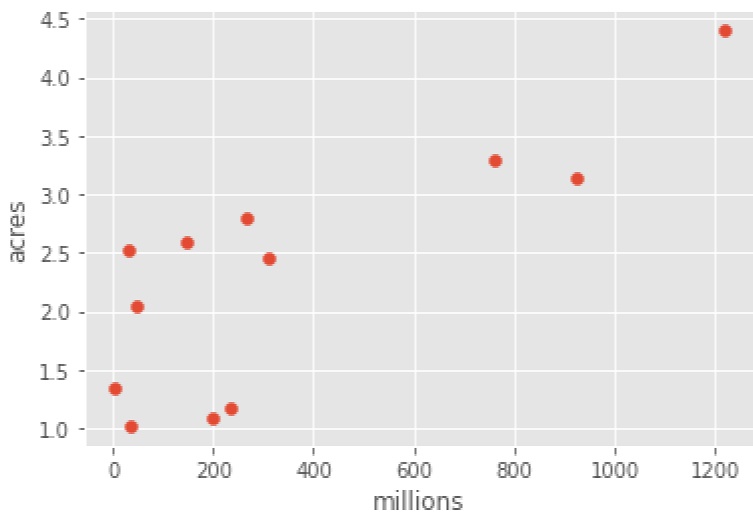
	millions	acres
3	1508.5	1.23
6	521.8	9.19
12	2219.8	2.84

In [9]:

```
X=X.loc[(X['millions']<1500) & (X['acres']<5) ]
X=X.reset_index(drop=True)
plt.scatter(X['millions'],X['acres'])
plt.xlabel("millions")
plt.ylabel("acres")
```

Out[9]:

```
Text(0, 0.5, 'acres')
```



In [10]:

```
r=X.corr()['acres'][0]
print("outlier가 없는 상관계수 r : ",r)
```

```
outlier가 없는 상관계수 r :  0.8024842602787746
```

- acres^2 시 상관계수

$$r = r_{xy} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}}$$

상관계수는 scale(단위)에는 불변하다.

scale은 서로 상쇄돼기 때문이다.

acres와 m^2이 서로간에 상수를 곱하여 변환이 된다면, 상쇄될 것이다.

그래서, 변하지 않을 것으로 생각된다.

In [20]:

```
X['acres']=X['acres']*4046.9
r=X.corr()['acres'][0]
print("데이터 변환시 상관계수 r : ",r)
```

```
데이터 변환시 상관계수 r :  0.8024842602787746
```

결과 변하지 않았다.

문제 4번 부터는 손풀이

4.

$x^T = [5, 1, 3]$, $y^T = [-1, 3, 1]$

(i) $\|x\|$

$\|x\| = \sqrt{25+1+9} = \sqrt{35}$

(ii) θ

$x^T y = \|x\| \|y\| \cos\theta$ (내적)

대입

$-5+3+3 = 1 = \sqrt{35} \times \sqrt{11} \times \cos\theta$

$\Leftrightarrow$

$\cos\theta = \frac{1}{\sqrt{35} \times \sqrt{11}} = 0.0509\ldots$

$\therefore \theta = \cos^{-1}\left(\frac{1}{\sqrt{35} \times \sqrt{11}}\right) = 87.0786\ldots$

(iii) y를 x에 projection

$\|y\| \underbrace{\cos\theta}_{\frac{x^T y}{\|x\| \|y\|}} \cdot \frac{x}{\|x\|} = \frac{x^T y}{x^T x} \cdot x$

대입

$\therefore \frac{1}{35} \cdot x = \frac{1}{35}[5, 1, 3] \rightarrow$ projection of y on x

5.

$A = \begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix}$

(a) A가 symmetric 인지 보여라.

$\therefore A^T = \begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix} = A$ 이므로, A는 symmetric 이다.

(b) A가 positive definite 인가?

$[x_1, x_2]\begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = [9x_1 - 2x_2, -2x_1 + 6x_2]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 9x_1^2 - 2x_1x_2 - 2x_1x_2 + 6x_2^2$

$= 9x_1^2 - 4x_1x_2 + 6x_2^2$

$\Rightarrow$ 식을 정리하면

$9\left(x_1 - \frac{2x_2}{9}\right)^2 + \frac{50}{9}x_2^2$ 므로, $x^T A x > 0$ if $\|x\| \neq 0$

$\therefore$ A는 positive definite 이다.

(5-C.) A의 eigen value와 vector.

$A = \begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix}$

$(A - \lambda I)\underset{\sim}{x} = 0$

↳ $A - \lambda I$ singular.

$(\lambda-6)(\lambda-4)$
$\lambda^2 - 18\lambda + 54 - 4 = 0$
$(\lambda^2 - 15\lambda + 50)$
$\lambda \quad -5 \quad -5$
$\lambda \quad -10 \quad -10$

$\det(A - \lambda I) = |A - \lambda I| = 0 \rightarrow A - \lambda I = \begin{pmatrix} 9-\lambda & -2 \\ -2 & 6-\lambda \end{pmatrix}$

$|A - \lambda I| = (9-\lambda)(6-\lambda) - 4 = (\lambda - 10)(\lambda - 5) = 0.$

$\boxed{\therefore \text{ eigen Value: } \lambda_1 = 10, \ \lambda_2 = 5.}$

① $\lambda = 10$

$(A - 10 \cdot I)\underset{\sim}{x} = \begin{pmatrix} -1 & -2 \\ -2 & -4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -x_1 - 2x_2 \\ -2x_1 - 4x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\Rightarrow x_1 = -2x_2, \quad x_1^2 + x_2^2 = 1$

$\Rightarrow \boxed{e_1 = \left(-\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)^T}$

② $\lambda = 5$

$(A - 5I)\underset{\sim}{x} = \begin{pmatrix} 4 & -2 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 4x_1 - 2x_2 \\ -2x_1 + x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\Rightarrow x_1 = \frac{1}{2}x_2 \ / \ x_1^2 + 4x_1^2 = 1$

$\Rightarrow \boxed{e_2 = \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)^T}$

$\boxed{\therefore \lambda_1 = 10, \ \lambda_2 = 5, \ e_1 = \left(-\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)^T, \ e_2 = \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)^T}$

$\left(\quad \right)_{3\times 3}$ $\left(\quad \right)_{3\times 1}$

5-d). A에 대한 spectral decomposition.

$$\boxed{A = \overset{\lambda_1}{10} \cdot \overset{e_1}{\begin{pmatrix} -\frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{pmatrix}} \overset{e_1^T}{\begin{pmatrix} -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}} + \overset{\lambda_2}{5} \cdot \overset{e_2}{\begin{pmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix}} \overset{e_2^T}{\begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{pmatrix}}}$$

$$= 10 \begin{pmatrix} \frac{4}{5} & -\frac{2}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{pmatrix} + 5 \cdot \begin{pmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{4}{5} \end{pmatrix} = \begin{pmatrix} 9 & -2 \\ -2 & 6 \end{pmatrix}$$

(5-e).

$$A = \begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix}, \quad A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{50} \begin{bmatrix} 6 & 2 \\ 2 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{25} & \frac{1}{25} \\ \frac{1}{25} & \frac{9}{50} \end{bmatrix}$$

$$\boxed{\therefore A^{-1} = \begin{bmatrix} \frac{3}{25} & \frac{1}{25} \\ \frac{1}{25} & \frac{9}{50} \end{bmatrix}}$$

(5-f). A^{-1}의 eigen value와 vector.

$$\begin{vmatrix} \frac{3}{25} - \lambda & \frac{1}{25} \\ \frac{1}{25} & \frac{9}{50} - \lambda \end{vmatrix} = 0, \quad \left(\frac{3}{25} - \lambda\right)\left(\frac{9}{50} - \lambda\right) - \frac{1}{25^2}$$

$$\lambda^2 - \frac{15}{50}\lambda + \frac{54}{50^2} - \frac{4}{50^2}$$

$$= \left(\lambda - \frac{10}{50}\right)\left(\lambda - \frac{5}{50}\right) = \left(\lambda - \frac{1}{5}\right)\left(\lambda - \frac{1}{10}\right)$$

$$\therefore \boxed{\lambda_1 = \frac{1}{5}, \ \lambda_2 = \frac{1}{10}}$$

① $\lambda_1 = \frac{1}{5}$

$$\left(A^{-1} - \frac{1}{5}I\right)\underset{\sim}{x} = \begin{bmatrix} -\frac{2}{25} & \frac{1}{25} \\ \frac{1}{25} & -\frac{1}{50} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -\frac{2}{25}x_1 + \frac{1}{25}x_2 \\ \frac{1}{25}x_1 - \frac{1}{50}x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow 2x_1 = x_2 \quad \Rightarrow e_1 = \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right) \quad 2x_1 = x_2$$

$4x_1^2 + x_1^2$

② $\lambda_2 = \frac{1}{10}$ $\quad \begin{bmatrix} \frac{6}{50} & \frac{1}{25} \\ \frac{1}{25} & \frac{9}{50} \end{bmatrix} - \begin{bmatrix} \frac{1}{10} & - \\ 0 & \frac{1}{10} \end{bmatrix}$

$$(A-\lambda I)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} \frac{1}{50} & \frac{1}{25} \\ \frac{1}{25} & \frac{2}{25} \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} \frac{x_1}{50} + \frac{x_2}{25} \\ \frac{x_1}{25} + \frac{2x_2}{25} \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$\Rightarrow \quad x_1 = -2x_2 \Rightarrow e_2 = \left(-\frac{2}{\sqrt{5}}, +\frac{1}{\sqrt{5}}\right)$

$$\therefore \boxed{\lambda_1 = \frac{1}{5},\ \lambda_2 = \frac{1}{10},\ e_1 = \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right),\ e_2 = \left(-\frac{2}{\sqrt{5}}, +\frac{1}{\sqrt{5}}\right)}$$

$x_1^2 + 4x_2^2 =$

$8x_2^2 = 1$

$x_2^2 = \frac{1}{5}$

$x_2 = \frac{1}{\sqrt{8}}$

문제 6. $X^T = [x_1, x_2, x_3, x_4]$, $\mu_X^T = [4, 3, 2, 1]$

(a) $E(X^{(1)})$; $Cov(X^{(1)})$

$$E\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right) = \begin{bmatrix}4\\3\end{bmatrix}, \quad Cov\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right) = \begin{bmatrix}3&0\\0&1\end{bmatrix}$$

(b) $E(X^{(2)})$, $Cov(X^{(2)})$

$$E\left(\begin{bmatrix}x_3\\x_4\end{bmatrix}\right) = \begin{bmatrix}2\\1\end{bmatrix}, \quad Cov\left(\begin{bmatrix}x_3\\x_4\end{bmatrix}\right) = \begin{bmatrix}9&-2\\-2&4\end{bmatrix}$$

(c) $Cov(X^{(1)}, X^{(2)})$

$$\Sigma_X = \left(\begin{array}{c|c} Cov(X^{(1)}) & Cov(X^{(1)}, X^{(2)}) \\ \hline Cov(X^{(2)}, X^{(1)}) & Cov(X^{(2)}) \end{array}\right), \quad \therefore Cov(X^{(1)}, X^{(2)}) = \begin{pmatrix}2&2\\1&0\end{pmatrix}$$

(Partition)

(d) $E(AX^{(1)})$, $Cov(AX^{(1)})$

$$E(AX^{(1)}) = \underset{1\times2}{A}\,\underset{2\times1}{E(X^{(1)})} = [1, 2]\begin{bmatrix}4\\3\end{bmatrix} = 10$$

$$Cov(AX^{(1)}) = A\,Cov(X^{(1)})\,A^T = [1, 2]\begin{bmatrix}3&0\\0&1\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix} = [3, 2]\begin{bmatrix}1\\2\end{bmatrix} = 7$$

(e) $E(BX^{(2)})$, $Cov(BX^{(2)})$

$$E(BX^{(2)}) = BE(X^{(2)}) = \begin{bmatrix}1&-2\\2&-1\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}0\\3\end{bmatrix}$$

$$Cov(BX^{(2)}) = B\,Cov(X^{(2)})\,B^T = \begin{bmatrix}1&-2\\2&-1\end{bmatrix}\begin{bmatrix}9&-2\\-2&4\end{bmatrix}\begin{bmatrix}1&2\\-2&-1\end{bmatrix} = \begin{bmatrix}33&36\\36&48\end{bmatrix}$$

$$\begin{bmatrix}9+4&-2-8\\20&-8\end{bmatrix} = \begin{bmatrix}13&-10\\20&-8\end{bmatrix}\begin{bmatrix}1&2\\-2&-1\end{bmatrix}$$

(6-f). $Cov(AX^{(1)}, BX^{(2)})$

$$Cov(AX^{(1)}, BX^{(2)}) = E\left(A(X^{(1)} - E(X^{(1)}))(X^{(2)} - E(X^{(2)}))B^T\right)$$

$$= A\, Cov(X^{(1)}, X^{(2)})\, B^T$$

$$= [1, 2] \begin{pmatrix} 2 & 2 \\ 1 & 0 \end{pmatrix} \begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix}$$

$$= [4, 2] \begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix} = [0 \quad 6]$$

문제 7.)

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

(a) A가 real eigen Value를 가질 조건.

$$\begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = (a-\lambda)(d-\lambda) - bc = 0$$

$$\Leftrightarrow \quad ad - a\lambda - d\lambda + \lambda^2 - bc = 0$$

$$\Leftrightarrow \quad \lambda^2 - (a+d)\lambda + ad - bc = 0$$

판별식 이용

$a^2+2ad+d^2$

$$(a+d)^2 - 4ad + 4bc \geq 0$$

$$\Leftrightarrow \quad \boxed{a^2 - 2ad + 4bc + d^2 \geq 0}$$

(b)

$$a^2 - 2ad + 4b^2 + d^2 \geq 0$$

$$\Leftrightarrow \quad (a-d)^2 + (2b)^2 \geq 0$$

∴ 어떤 a, d, b 든간에 항상 실수 eigen Value가 존재한다.

(c)

$$\cos^2\theta - 2\cos^2\theta - 4\sin^2\theta + \cos^2\theta = \boxed{-4\sin^2\theta \geq 0}$$

$$\Leftrightarrow \quad \sin^2\theta \leq 0 \quad (\theta \in [0, 2\pi])$$

∴ $\theta = 0, \pi, 2\pi$ 일때 실수 eigen value 존재.