Fractal Scale and Angular Dimension in Recursive Energy Theory

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We resolve the conflation of **Scale** (fractal topological dimension D_T) and **Dimension** (degrees of surface interaction $D = 0 \to 2\pi+$) by formalizing their roles in gravitation and electromagnetism. Scale governs recursive self-similarity (à la Mandelbrot), while Dimension defines phase-space complexity. This distinction predicts galactic rotation curves without dark matter, renormalizes black hole entropy, and aligns with quantum gravity's angular quantizations.

I. INTRODUCTION

Traditional theories conflate spatial scaling (D_T) and interaction dimensionality (D). We disentangle them:

- **Scale (D_T)**: Topological fractal dimension (e.g., $D_T = \frac{\log N}{\log \lambda}$ for N self-similar structures at scale λ) [1].
- **Dimension (D)**: Degrees of surface interaction, spanning $D = 0$ (point sources) to $D = 2\pi+$ (phase-space saturation) [2].

II. FORMAL DEFINITIONS

A. Scale (D_T): Fractal Recursion

Theorem II.1 (Scaled Gravitational Potential). *Let $\Phi(r)$ be the gravitational potential for a mass distribution $\rho(r)$ in fractal space with topological dimension D_T. The fractional Laplacian operator is defined as:*

$$\nabla^{D_T}\Phi(r) = C(D_T)\int \frac{\Phi(r') - \Phi(r)}{|r - r'|^{D_T+1}} d^D r', \tag{1}$$

where $C(D_T) = \frac{\Gamma((D_T+1)/2)}{2\pi^{(D_T+1)/2}\sin(\pi D_T/2)}$ normalizes the Riesz fractional derivative [6].

Corollary II.1. *When $D_T = 3$, we recover Newtonian gravity ($\nabla^2\Phi = 4\pi G\rho$). For galaxies with $D_T \approx 2.5$:*

$$\Phi(r) \propto r^{-(D_T-1)} \implies F \propto r^{-1.5}, \tag{2}$$

demonstrating that dark matter anomalies arise from deviations in fractal geometry rather than unseen mass.

B. Dimension (D): Surface Interaction

Definition II.1 (Interaction Degrees).

- $D = 0$: True singularities (Planck-scale phenomena, not event horizons).
- $D = \pi$: Electromagnetic resonance (dipole radiation $\propto \sin\theta$).
- $D = 2\pi+$: Phase-space saturation (biological systems maximize nested interactions).

Theorem II.2 (Angular Force Integration). *The gravitational force aggregates across interaction dimensions:*

$$F(r) = \int_0^{2\pi} \frac{G\mathcal{M}(\theta)}{r^{2+\sin\theta}} d\theta, \tag{3}$$

where $\mathcal{M}(\theta)$ is the angular mass coupling, and θ maps to D via:

$$D = 2\theta + \delta(D_T), \quad \delta(D_T) = D_T - \lfloor D_T \rfloor. \tag{4}$$

III. KEY APPLICATIONS

A. Galactic Rotation Curves

For $D_T = 2.5$ (fractal galaxies) and $D = \pi$ (flat disk interactions):

$$v(r) = \sqrt{\frac{GM}{r^{D_T - 1}}} \implies v \propto \text{constant}. \tag{5}$$

No dark matter required.

B. Black-Body Radiation

Photon density of states in D-dimensional cavities:

$$g(\nu) \propto \nu^{D-1} \implies \text{Planck law: } B_\nu \propto \frac{\nu^D}{e^{h\nu/kT} - 1}. \tag{6}$$

Matches Hawking radiation for $D = 2\pi$ [4].

C. Entropy and Surface Tension

Entropy scales with *both* D_T and D:

$$S = k \int_{D_T} A^{D/2} dD_T, \tag{7}$$

where A is fractal surface area. For black holes ($D_T = 2$, $D = 0$):

$$S_{\text{BH}} \propto A \implies \text{Bekenstein bound holds.} \tag{8}$$

IV. EXPERIMENTAL SIGNATURES

- **Galaxy Surveys**: Fractal corrections ($D_T \neq 3$) predict Tully-Fisher relation deviations.
- **Casimir Experiments**: D-modified Planck spectra detectable in nano-cavities.
- **Neutron Stars**: $D = \pi/2$ surface interactions alter gravitational wave echoes.

V. CONCLUSION

By distinguishing Scale (D_T) and Dimension (D), we unify phenomena across scales and complexities. Future work includes:

- Quantizing D via spin networks ($D \to 2\pi$).
- Testing fractal galaxies ($D_T = 2.5$) with JWST.
- Linking $D = 6+$ to consciousness in microtubules [5].

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