$$H_1(z) = \frac{1}{z^2 - 2rz\cos w + r^2}$$ This is how we got for basic second-order filter

$$H_2(z) = \frac{r\sin w_0 z}{z^2 - 2rz\cos w + r^2}$$ with $h_2(n) = r^n \sin w_0 n \, u(n)$, This is we got from Z-trans table

profile oscillation

$$H_2(z) = r\sin(w_0) H_1(z+1)$$

$$h_1 = \frac{r^n}{\sin(w_0)} \sin(w_0 n)\, u(n)$$

Now given two second-order filter H_A and H_B with, $h_A = \frac{r_A^n}{\sin(w_0)} \sin(w_0 n)\, u(n)$

$h_B = \frac{r_B^n}{\sin(w_0)} \sin(w_0 n)\, u(n)$

Leave the oscillation part (for we only care about profile which could indicate the peak and decay.

let $k_A = r_A^n u(n)$ $k_B = r_B^n u(n)$

$$\text{profile}(n) = k_A * k_B \quad n \geq 0 \quad = \int_{-\infty}^{+\infty} r_A^{n-k} u(n-k) \cdot r_B^k u(k)\, dk \quad \left.\begin{array}{l} n-k>0 \\ k>0 \end{array}\right\} \Rightarrow = \int_0^{n} r_A^{n-k} r_B^k \, dk = \sum_{k=1}^{n-1} r_A^{n-k} r_B^k$$

Find

$$\frac{d\,\text{profile}(n)}{dn} = 0 \Leftrightarrow$$

$$\text{profile}(n) = r_A^{n-1} r_B + r_A^{n-2} r_B^2 + \cdots + r_A^1 r_B^{n-1} \quad \cdots (n-1) \text{ elements}$$

$$\cancel{\frac{d\,\text{profile}(n)}{d(n)} = (n-1) r_A^{n-2} r_B + (n-2) r_A^{n-3} r_B^2 + \cdots + (n-1) r_A r_B^{n-2}}$$

$$\cancel{= r_A^{n-1} r_B \ln r_A}$$

$\text{profile}(n-1) = r_A^{n-2} r_B + r_A^{n-3} r_B^2 + \cdots + r_A^1 r_B^{n-2}$ $(n-2)$ elements

$\boxed{\text{profile}(n) - \text{profile}(n-1) < 0}$ Then $n-1$ is the peak we need

$= r_A^{n-1} r_B + r_A^{n-2}(r_B^2 - r_B) + \cdots + r_A^1 (r_B^{n-1} - r_B^{n-2})$

$= r_A^{n-1} r_B + (r_B - 1)(r_A^{n-2} r_B + r_A^{n-3} r_B^2 + \cdots + r_A^1 r_B^{n-2})$

$= r_A^{n-1} r_B + (r_B - 1)\,\text{profile}(n-1)$

In this case Find $n-1$ s.t. $\text{profile}(n) - \text{profile}(n-1) < 0$ is not easy, we alternative find

n. s.t. $\boxed{\text{profile}(n) - \text{profile}(n-1) \to 0 \quad \text{or} \quad \approx 0.}$

That is $\text{profile}(n-1) = -\dfrac{r_A^{n-1} r_B}{r_B - 1} = \dfrac{r_A^{n-1} r_B}{1 - r_B}$ and $n-1$ could be the peak time,

That is Find n, $\sum_{k=1}^{n-1} r_A^{n-k} r_B^k = \dfrac{r_A^{n} r_B}{1 - r_B}$ where n satisified this equation.

However in this question, we specify n, to find suitable r_A and r_B to satisify this equation

$$\text{So} \quad \sum_{k=1}^{n_0-1} r_A^{n-k} r_B^k = \frac{r_A^{n_0} r_B}{1 - r_B} \quad \cdots \quad (1)$$

The next equation comes while setting up decay time

When consider decaying

$$\text{profile}(n_0) = \sum_{k=1}^{n_0-1} r_A^{n_0-k} r_B^{k} = \frac{1}{100} \cdot \frac{r_A^{n_0-1}\, r_B}{1-r_B} \quad \cdots (2)$$

where the right side of equation is peak we got last time.

Solve this equation find r_A and r_B. However don't know How to solve...