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A possible explanation for the discrepancy in Galaxy Rotation Curves

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ABSTRACT
In this work we propose a possible explanation for the discrepancy between observed galaxy rotation curves and theoretical predictions. By careful examination of the facts, we have found a simple solution for the discrepancy which does not require the dark matter hypothesis or changes in the current fundamental laws of nature. Applying the solution on a sample of more than 30 galaxies provides very promising results. We hope that this work will lead to reexamination of other observational problems that are currently explained by using dark matter.

Key words: Reference Frames – Celestial Mechanics – Cosmology: dark matter

1 INTRODUCTION

A Galaxy Rotation Curve (RC) describes the rotational velocities of stars in a galaxy as a function of their radial distance from the galactic center. One of the pioneers in this field was the American astronomer Horace Babcock who measured Andromeda's RC in 1939 and unexpectedly found that the orbital velocities of its stars increase with distance (Babcock 1939). This result was surprising due to the fact that Andromeda's mass distribution pointed out that the rotational velocities should actually decrease with distance, as in our own solar system for example. An illustration of this discrepancy is shown in figure 1.

During the seventies a meaningful progress was made when Vera Rubin constructed RCs for a wide range of galaxies (Rubin, Ford & Thonnard 1980). Looking at the measured data she found that none of the rotation curves were declining with the radial distance. Some RCs showed a monotonically increasing trend over the entire range of distances while other showed an asymptotically flat behavior. Her published work paved the way to the following conclusion: RCs cannot be explained by the visible matter only. A large amount of mass (5-10 times the mass of the visible matter) should be found mainly in the outer parts of a galaxy in order to explain the measured RCs. This kind of mass was eventually named "Dark Matter". The concept of dark matter became very popular over the years. Many studies were conducted in order to reveal the essence of this matter. Today's common view assumes that dark matter is composed of special particles that cannot interact with their surroundings through electromagnetic forces.

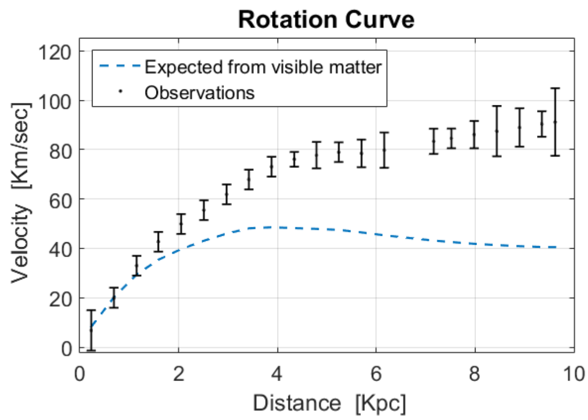

Figure 1. A galaxy measured RC (black error bars) and predicted RC (blue dashed line) are plotted together

Some researchers didn't feel comfortable with the mysterious invisible dark matter and proposed alternative theories. A well known theory of this kind has been proposed by Mordehai Milgrom (Milgrom 1983). His theory modifies Newton's laws of motion in such a way that there is no need for extra matter and the predicted curves are always asymptotically flat. A large amount of RCs were fitted using his theory. The Modified Newtonian Dynamics

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(MOND) can be surely regarded as a breakthrough but it still faces theoretical challenges.

The aim of this current work is to propose a solution to the RCs problem without the need for extra matter or the need of changing Einstein's General Relativity and Newton's laws. In section 2 we introduce the proposed solution. In sections 3, 4 and 5 we use this solution in order to construct RCs. In section 3 we construct RCs in a schematic approach, in section 4 we construct RCs for several different galaxies in an analytical approach and in section 5 we construct RCs for a large amount of galaxies in a numerical approach. In section 6 we discuss some future directions.

2 THE PROPOSED SOLUTION TO THE ROTATION CURVES PROBLEM

It is well known in physics that the fundamental quantities like "position" and "time" (and other quantities that are derived from them as well, like "velocity") are relative and have a meaning only relative to a well defined coordinate system. Newton's laws of motion, which are used successfully to explain a huge amount of physical phenomena, cannot be treated as "absolute" either. These laws are acceptable (as an approximation to more generalized physical theories) only relative to coordinate systems which we call "Inertial" (Einstein 1920). So if we wish to describe a motion of a star in a galaxy, for example, and we believe that Newton's laws are sufficient in order to explain its motion, we still have to make sure that its motion is given relative to an Inertial frame of reference.

Let us focus on the RCs problem. As was explained earlier, a RC is a plot of the measured rotational velocities of bodies in their orbits around a galactic center as a function of their distance from this center. Using Newton's laws of motion (together with the assumed mass distribution) in order to predict these measured velocities leads to a discrepancy. In order to solve the discrepancy two main theories were proposed:

1. Newton's laws don't apply in these cases. A modification of the laws (and of general relativity) is needed.

2. Newton's laws do apply in these cases but an extra amount of matter should be taken into account.

We propose a third possibility:

3. Newton's laws do apply in these cases and no extra matter should be taken into account. However, the measured velocities (the black error bars in Figure 1) are not given relative to the galaxy's local inertial frame. Therefore the discrepancy arises.

Our proposition is based on the following argument: when extracting the measured velocities of a given galaxy, astronomers rely on a quantity called "the line-of-sight velocity". Therefore, the extracted velocities are correct only relative to systems of coordinates where the line-of-sight is fixed. However, no one guarantees that this direction is fixed relative to the inertial frame, where Newton's Laws are valid.

It is true that relative to an inertial frame "far bodies" are at rest, therefore the lines of sight to these bodies are fixed. But this is only a general statement. It's clear that a specific galaxy will not be exactly at rest relative to the Inertial frame. As is shown in the next sections, extremely slow movement of the line of sight relative to the Inertial frame ($\sim 10^{-16}$ rad/sec) can lead to significant changes in the measured velocities.

In the appendix we discuss how these measured velocities are extracted, and why the frames of reference (relative to which these velocities are given) cannot be regarded as Inertial. Next, let us develop a method dealing with the discrepancy. One important remark regarding the method is that instead of "fixing" the measured velocities (the black error bars), we change the predicted curve to fit these non-inertial velocities.

Let us assume that there is an inertial frame K, i.e. a frame of reference relative to which Newton's laws of motion apply (in good approximation) for the purpose of describing the motions of bodies in a specific galaxy. Without limiting the discussion, one can always arrange this frame in such a way that its $x - y$ plane will coincide with the galactic plane and its origin will coincide with the galactic center. Now let us define a system of coordinates called K'. K' is defined to be the system relative to which the measured rotational velocities are given. It's important to emphasize that each galaxy is characterized by a unique K', and in general these K' frames are not fixed one with respect to the others. In the appendix, one can see that for a given galaxy, system K' is also a system of coordinates which its $x' - y'$ plane coincides with the galactic plane and its origin coincides with the galactic center. Therefore, the only possible relative motion between K and K' is an angular velocity of one system around the $z - axis$ of the other (see Figure 2). In general, this angular velocity can be any function of time. In this work, however, we have chosen to restrict ourselves only for constant angular velocities, i.e. we assume that the desired inertial system is revolving at a constant angular velocity relative to system K' . Without any further information, this constant angular velocity is unknown. Therefore, at this work, it will be regarded as a free parameter used to find the assumed inertial system K.

Now, suppose that relative to system K, a body performs a uniform circular motion in the $x - y$ plane around the origin. This system revolves at a constant angular velocity around the z'-axis of system K'. Hence, as long as general relativity effects are negligible, the body will also perform a uniform circular motion around the origin of K' , and the following relation will certainly hold:

$$v_{K'} = v_K + \omega r \tag{1}$$

where $v_{K'}$ is the rotational velocity of the body relative to K', v_K is the rotational velocity of the body relative to K, ω is the constant angular velocity in which system K revolves relative to system K' and r is the radius of the circular motion. Of course, one could also derive this relation

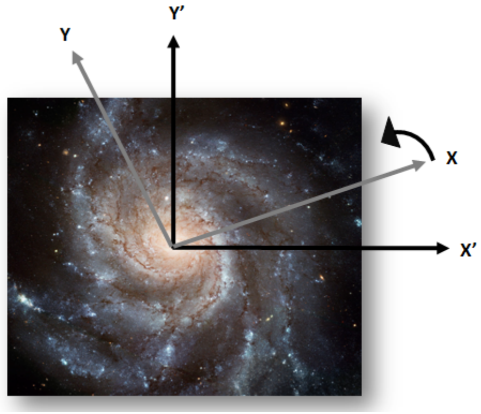

Figure 2. The inertial system of coordinates (K) revolves relative to the non-inertial one (K'). The measured velocities presented in RCs are valid only with respect to K', since the line of sight to the observer is fixed in that frame.

by taking into account the addition of fictitious forces in system K', forces whose existence in non inertial systems is valid in the frame of classical mechanics. Moreover, it is important to notice that v, v', ω are defined to be positive for a counterclockwise motion.

In our case, K is the assumed inertial system in which Newton's laws are valid, and the assumption is that a specific body, under the influence of all other bodies in the galaxy, performs a uniform circular motion relative to it. Therefore, it will perform a uniform circular motion relative to K' too, just with the addition of ωr to its velocity. All the cases examined and shown in the next sections are based on this simple correction.

3 A SCHEMATIC DEMONSTRATION OF THE PROPOSED SOLUTION

When trying to predict an RC, one should first model the galaxy's mass distribution. This is often done by dividing the galaxy into its different components (i.e. Bulge, Stellar disk, Gaseous disk and Dark halo) and calculating the contribution of each component to the velocity. The velocity is then given by:

$$V(r) = \sqrt{v_1^2(r) + v_2^2(r) + v_3^2(r) + v_{dark}^2(r)} \tag{2}$$

Where $V(r)$ is the predicted RC, $v_1(r), v_2(r), v_3(r)$ are the contributions of the visible components and $v_{dark}(r)$ is the contribution of the dark halo.

Our solution however, claims that there is no 'dark halo' contribution. We rather claim that the predicted RC should be given by:

$$V(r) = \sqrt{v_1^2(r) + v_2^2(r) + v_3^2(r)} + \omega r = V_{inertial}(r) + \omega r \tag{3}$$

Where ω is the constant angular velocity in which the inertial system revolves relative to the system in which the velocities are given, as was explained in the previous section.

In order to demonstrate schematically the shapes of RCs that can be obtained by using our method, we have drawn four RCs. We have chosen a typical $V_{inertial}(r)$ to be the same in all four cases and changed only the value of ω, the slope of the linear correction term. The results are shown in Figure 3. One can notice that the schematic RCs obtained can represent in general the real shapes of actual measured RCs.

4 APPLYING THE PROPOSED SOLUTION USING ANALYTICAL MASS MODEL

As mentioned in the previous section, when one wishes to predict an RC of a specific galaxy, he should first model the galaxy's mass distribution. For this purpose we have chosen the most common analytical mass model. The model includes three different components (or parts of them): a spherical bulge, a stellar disk and a gaseous disk. Let us introduce shortly the model's ingredients.

- The spherical bulge represents the central group of stars found in most spiral galaxies. We used a Hernquist density profile of the form:

$$\rho(r) = \frac{M_b}{2\pi r_b^3} \frac{1}{\frac{r}{r_b}(1 + \frac{r}{r_b})^3} \tag{4}$$

where $\rho(r)$ is the volume mass density, r is the radial distance from the galactic center and r_b is the sphere characteristic scale length. Assuming circular motion results in the following velocity (Hernquist 1989):

$$v_b^2 = \frac{GM_b r}{(r + r_b)^2} \tag{5}$$

- The stellar disk represents the flat part of the galaxy which continues beyond the central bulge and is mainly composed of galaxy's stars. It is modeled as a two dimensional disk with a surface mass density of the form:

$$\Sigma(r) = \Sigma_0 e^{-\frac{r}{r_d}} \tag{6}$$

where $\Sigma(r)$ is the surface mass density, Σ_0 is the central mass density, r is the distance from the disk center and r_d is a characteristic scale length. The disk mass is given by:

$$M_d = 2\pi r_d^2 \Sigma_0 \tag{7}$$

Assuming circular motion results in the following velocity (Freeman 1970):

$$v_d^2 = 4\pi G \Sigma_0 r_d y^2 [I_0(y)K_0(y) - I_1(y)K_1(y)] \tag{8}$$

where $y \equiv \frac{r}{2r_d}$ and I, K are the Modified Bessel Functions of first and second kind.

- The gaseous disk, like the stellar one, represents the flat part of the galaxy around the central bulge that is mainly composed of the galaxy gas and dust. In many cases a Gaussian density distribution can be assumed and one gets:

$$\sigma(r) = \sigma_0 e^{-\frac{r^2}{2r_g^2}} \tag{9}$$

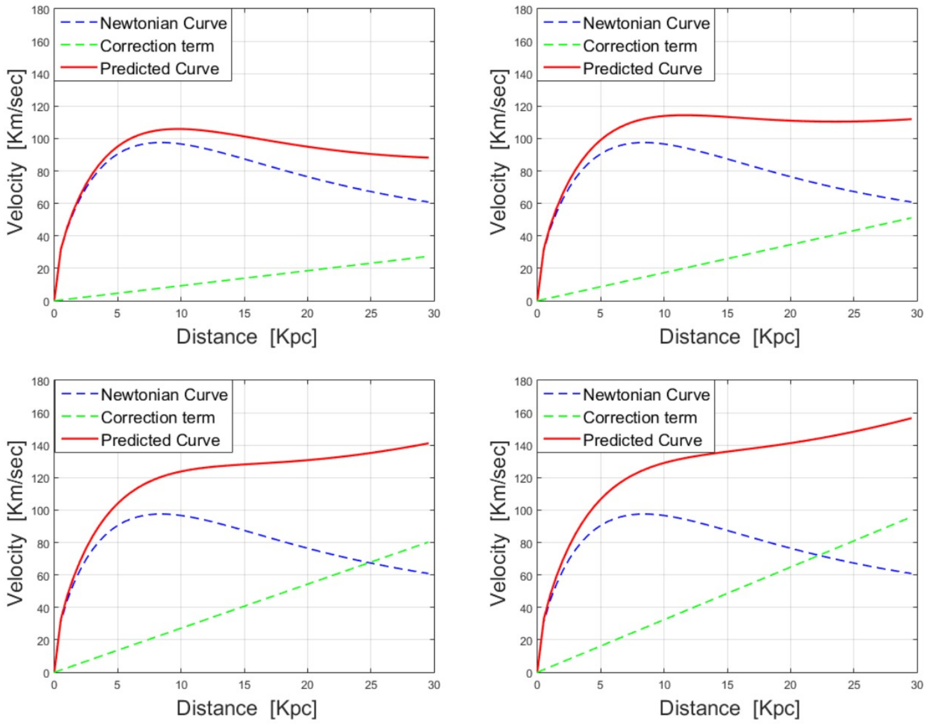

Figure 3. Four schematic RCs. The blue line represents the Newtonian rotation curve of a typical galaxy, the green line represents the linear correction term and the red line represents the predicted curve. One can see four trends: a slowly declining RC, A flat RC, A slowly rising RC and a rising RC. All four trends were actually observed in real measured RCs.

where $\sigma(r)$ is the surface mass density, σ_0 is the central mass density, r is the distance from the disk center and r_g is a characteristic scale length. The disk mass is given by:

$$M_g = 2\pi r_g^2 \sigma_0 \tag{10}$$

Assuming circular motion results in the following velocity (Toomre 1963):

$$v_g^2 = \frac{\sqrt{\pi}}{2\sqrt{2}} G M_g \frac{r^2}{r_g^3} F(1.5, 2, -\frac{r^2}{2r_g^2}) \tag{11}$$

where F is the Generalized Hyperbolic Function.

It is important to notice that the derivation of the velocities v_b, v_d, v_g from their corresponding mass distributions assumes the validity of Newton's laws and therefore these velocities can be regarded as correct only relative to inertial reference frames. Especially, relative to the assumed inertial system K, whose origin coincides with the galactic center, the velocity of a star which lies on the galactic plane and is located at a distance r from the origin is given by:

$$V_{inertial}(r) = V_K(r) = \sqrt{v_b^2(r) + v_d^2(r) + v_g^2(r)} \tag{12}$$

If we wish to describe the star's motion relative to system K' (the frame of reference in which measurements are given) which is assumed to be rotating with a uniform angular velocity relative to K, we should add the correction term ωr to the velocity $V_K(r)$, as was shown in Eq 3 and explained in section 2.

Galaxy	NGC 3109	NGC 55	M33	M31
r_b $[Kpc]$	-	-	-	0.61
r_d $[Kpc]$	1.8	1.8	1.27	4.4
r_g $[Kpc]$	5.5	3.5	4.5	-
$M_b[10^9 M_\odot]$	-	-	-	33
$M_d[10^9 M_\odot]$	1.45	2.15	3.77	90
$M_g[10^9 M_\odot]$	0.7	1.4	2.7	-
ω $[10^{-16} rad/sec]$	1.35	1.4	1.77	0.9

Table 1. The model parameters for the different galaxies. NGC 3109 (Carignan 1985) (Carignan et al. 2013) NGC 55 (Puche, Carignan & Wainscoat 1991), M33 (Corbelli & Salucci 2000) and M31 (Geehan et al. 2006) masses and scale lengths are presented.

This method was used for analyzing 4 different galaxies. For each galaxy $V_K(r)$ was calculated by Eq 12 and according to the characteristic masses and scale lengths of the specific galaxy. Then the value of ω was fixed to give the best fit to the measured RC. The values of the galaxies characteristic masses and scale lengths are summarized in Table 1. The measured RCs together with those predicted by our model are introduced in figure 4. Below the figure we indicate the references to the papers from which the measured RCs data was taken.

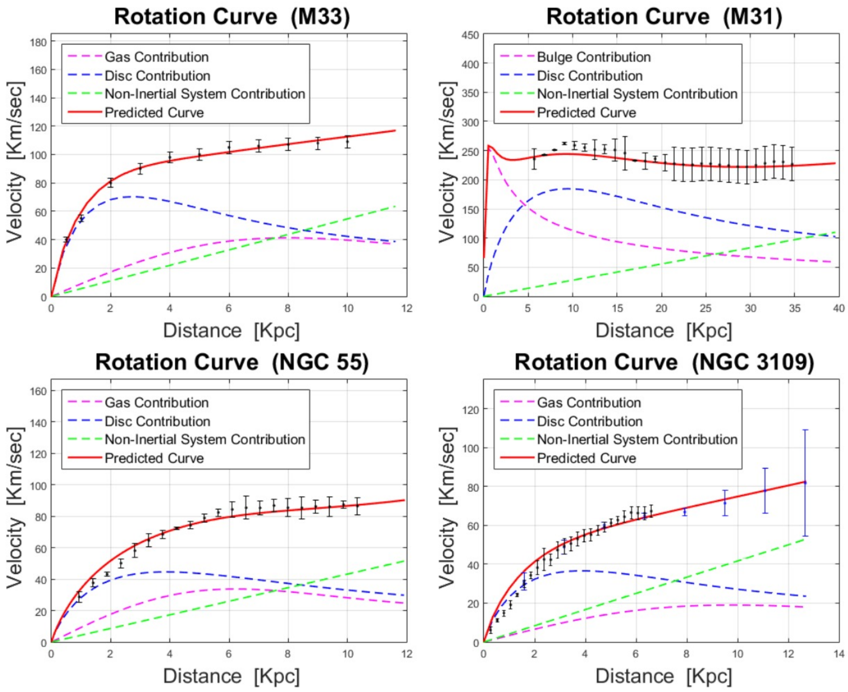

Figure 4. RCs of four different galaxies are presented. Top left panel: Rotation Curve of M33 (Corbelli & Salucci 2000), Top right panel: Rotation Curve of M31 (Carignan et al. 2006), Bottom left panel: Rotation Curve of NGC 55 (Puche, Carignan & Wainscoat 1991), Bottom right panel: Rotation Curve of NGC 3109 (Jobin & Carignan 1990) (Carignan et al. 2013). In each figure, one can see the measured values (black error bars) together with the predicted curve (red line). The blue dashed curve represents the stellar disk contribution v_d, the purple dashed curve represents the bulge contribution v_b or the gas contribution v_g and the green dashed curve represents the ωr correction.

5 APPLYING THE PROPOSED SOLUTION USING NUMERICAL MASS MODEL

The next stage is trying to construct RCs for a large number of galaxies using modern tools. In contrast to the last section, the Newtonian contribution to the velocity, $V_K(r)$, is now being calculated numerically by solving the Poisson equation. A detailed explanation of this process can be found in Sanders & McGaugh Annual Review (Sanders & McGaugh 2002). Unless a bulge is present, the only parameter needed when using this method is the M/L (Mass to Light ratio), which is assumed to be constant for a given galaxy. The Newtonian velocity is then given by:

$$V_{inertial}(r) = V_K(r) = \sqrt{(M/L) \cdot v_{disk}^2(r) + v_g^2(r)} \qquad (13)$$

Where M/L is the free parameter, $v_{disk}(r)$ is the stellar disk contribution (assuming $M/L = 1$) and $v_g(r)$ is the gaseous disk contribution.

In this work, the galaxies data (i.e. the observed RCs and the model contributions $v_{disk}(r)$ and $v_g(r)$ for each individual galaxy) was kindly given to us by Prof. Stacy McGaugh. The data can be found in McGaugh's Data Pages (McGaugh 2017). This is the same sample used by McGaugh in his Baryonic Tully-Fisher work (McGaugh 2005). For the sake of simplicity, in this work we included only galaxies without a bulge contribution.

As shown in previous sections, an ωr term should be added to the Newtonian term in order to predict the observed RC. Therefore a model with 2-parameters (M/L and ω) was used in order to predict each individual RC. Least-Squares method was used in order to find the best M/L and ω for each galaxy. A fair selection of our fits to RCs is given in Figures 5 and 6. Best fit M/L's and ω's are given in Table 2. Connections between M/L , ω and other galaxy parameters will be discussed in future works.

6 DISCUSSION

In the scope of this work, we assumed that for a given galaxy, Newton's laws and the detectable mass distribution are sufficient for predicting the orbital motions of stars. However, we have proposed that the measured velocities are not given relative to the galaxy's local inertial frame, therefore generating a discrepancy. A method which utilizes the only degree of freedom exist in this problem was developed in order to predict these non-inertial velocities. Applying the method analytically and numerically, we've got promising agreements between the observed and predicted RCs without changing the current fundamental laws of nature or assuming the existence of dark matter.

Of course, the method developed in this work is valid only for the cases of disc galaxies. Its not straight forward to implement this method to other observational problems (like those related with elliptical galaxies or galaxy clusters

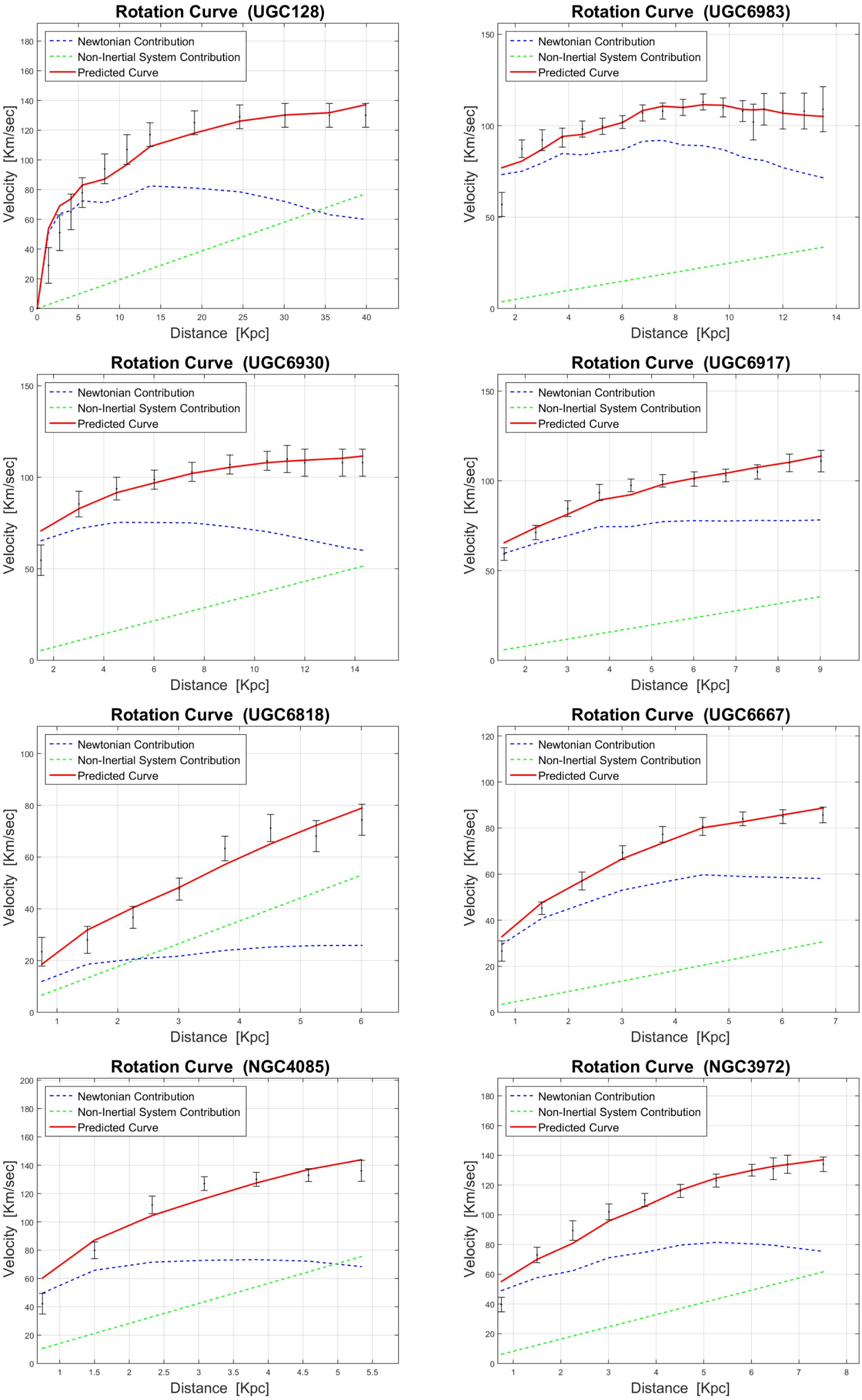

Figure 5. RCs for the different galaxies are presented. In each panel, one can see the measured values (black error bars) together with the predicted curve (red line). The blue dashed curve represents the Newtonian term $v_k(r)$ and the green dashed curve represents the correction term ωr.

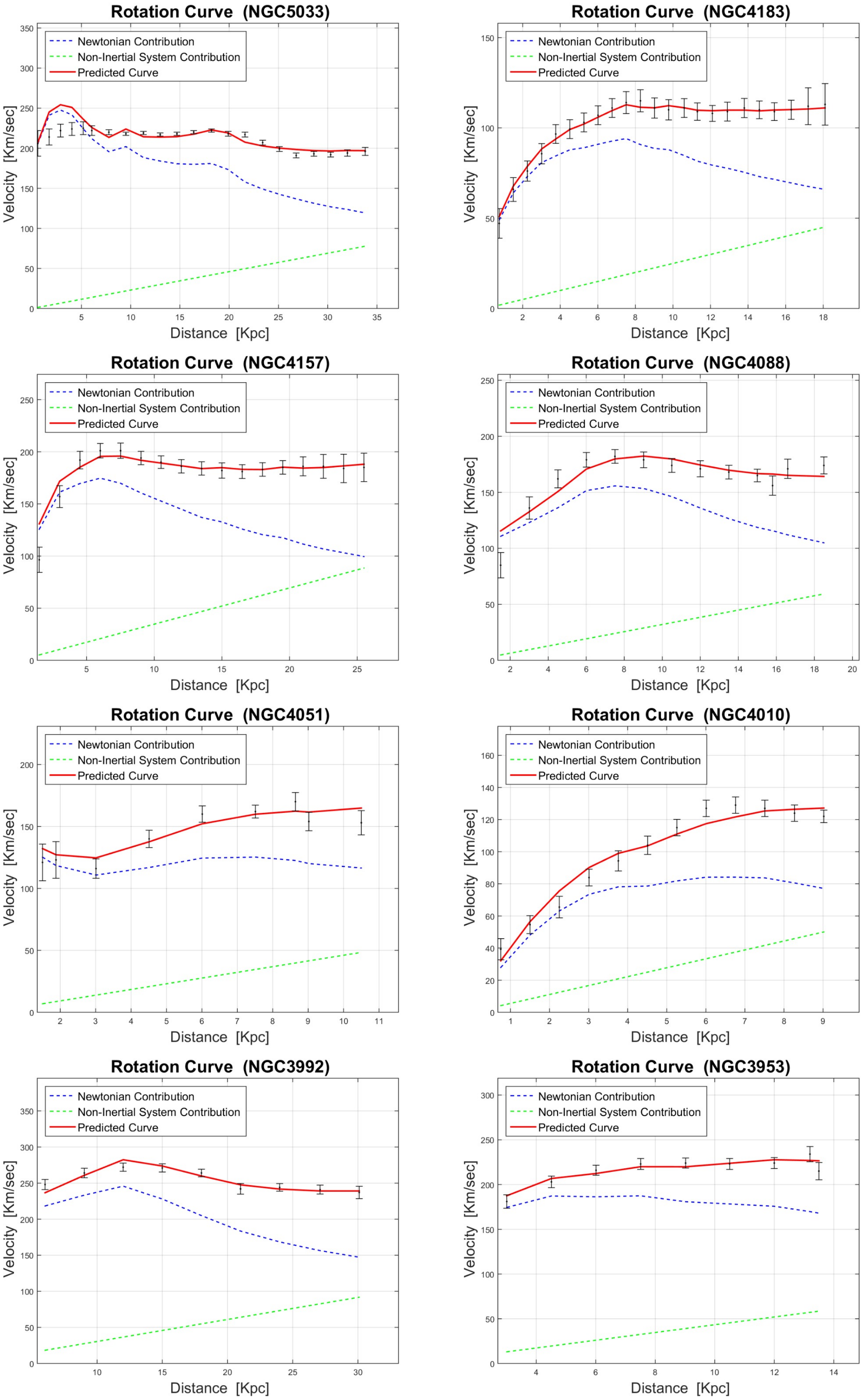

Figure 6. RCs for the different galaxies are presented. In each panel, one can see the measured values (black error bars) together with the predicted curve (red line). The blue dashed curve represents the Newtonian term $v_k(r)$ and the green dashed curve represents the correction term ωr.

Galaxy	ω $[10^{-16} rad/sec]$	M/L $[M_{sun}/L_{sun}]$	Galaxy	ω $[10^{-16} rad/sec]$	M/L $[M_{sun}/L_{sun}]$
M33	1.12	0.97	NGC 4085	4.58	0.50
NGC 1003	0.75	0.54	NGC 4088	1.04	1.16
NGC 1560	1.82	2.57	NGC 4100	1.07	2.36
NGC 247	1.51	1.76	NGC 4157	1.12	2.27
NGC 300	1.55	0.96	NGC 4183	0.81	1.45
NGC 3726	0.93	1.19	NGC 4217	1.73	1.72
NGC 3769	0.89	1.77	NGC 5033	0.74	5.06
NGC 3877	0.94	1.78	NGC 5585	1.50	0.87
NGC 3893	1.71	1.43	NGC 6946	0.72	0.79
NGC 3917	1.16	1.58	NGC 7793	1.49	1.40
NGC 3949	4.51	0.48	UGC 128	0.63	3.49
NGC 3953	1.40	2.25	UGC 6446	0.81	1.70
NGC 3972	2.66	1.00	UGC 6667	1.47	1.74
NGC 3992	0.99	4.26	UGC 6818	2.86	0.16
NGC 4010	1.80	1.37	UGC 6917	1.28	2.01
NGC 4013	1.13	2.90	UGC 6930	1.17	1.30
NGC 4051	1.49	1.11	UGC 6983	0.80	3.11

Table 2. For each galaxy, the best-fit M/L and ω are presented. The M/L ratios are given in the B-band.

for example) and these attempts will be the subject of future works.

A subject worth mentioning is the term ωr itself. One may think that there are many 1-parameter functions which could be added to the Newtonian term in order to get a well fitted curve. If this is the case, then the fitted curves obtained above are not to be unexpected . Checking our method by adding to the Newtonian term other types of 1-parameter functions $(Ar^2, Ar^3, Aln(r))$ leads to a clear disagreement with the observed data independently of the parameters values. Thus, it would be fair to state that adding an ωr term (with ω as a free parameter) and obtaining a well fitted curve, is not an obvious result!

The main goal of this work was to demonstrate the validity of the proposed solution in the context of galaxy rotation curves. Here are some open questions and subjects for future investigation:

1. Is it possible to pre-determine the value of ω? Can one find the local inertial frame of a given galaxy without using the dynamics of its stars?

2. Is it possible to find an explanation for the mass discrepancy related with gravitational lensing in the light of the proposed solution?

3. Using our method, the rotation kinetic energy of a galaxy is much smaller when calculated relative to System K. Therefore it would be interesting to investigate it in the context of disk galaxies' stability, where this quantity plays a major role.

4. Is it possible to establish a Baryonic-Tully-Fisher relation in the context of our work? (i.e. to replace the well known V_{flat} used today with a typical velocity that relates to System K).

Finally, we would like to thank our dear friends Dan Michaels and Erez Raicher for some valuable advices. In addition, we would like to thank Prof. Stacy McGaugh for sharing with us his data and for some very helpful insights.

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APPENDIX

The goal of this appendix is to demonstrate how the rotational velocities (plotted in RCs) are calculated and to define the system of coordinates relative to which these velocities are given.

When observing a distant body with a spectrograph, using Doppler Effect and spectroscopic techniques, one can extract a quantity called the 'line-of-sight velocity'. This quantity is the component of the body's velocity along the line-of-sight.

The next step is deriving the rotational velocity. This is done by using the relation:

$$v = \frac{v_{obs} - v_{sys}}{sin(i)} \tag{1}$$

where v is the rotational velocity of the orbiting body, v_{obs} is the star's line-of-sight velocity, v_{sys} is the galactic line-of-sight velocity, and i is the inclination angle of the galactic plane ("Edge On" galaxies have $i = 90^o$). It is important to emphasize that v is the velocity that appears in RCs. Relation 1 is used only when the body is located on the galaxy's main axis. However, more complex relations or models would not change the basic idea. A schematic demonstration is given in Figure 1.

In order to derive this relation certain assumptions were made :

1. The body performs circular motion.
2. The velocity v is related to systems of coordinates where the plane of the circular orbit is fixed.
3. The velocity v is related to systems of coordinates in which the line-of-sight is fixed.

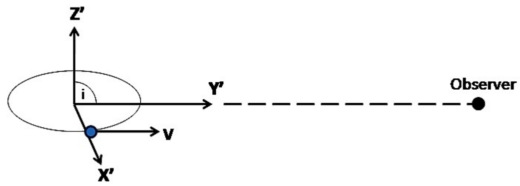

Figure 1. A body orbiting a galactic center. In this simple case $v_{sys} = 0$ and $sin(i) = 1$ so the rotational velocity is equal to the line-of-sight measured velocity. This rotational velocity is given relative to a system of coordinates in which the line of sight is fixed (as opposed to systems of coordinates where the line-of-sight might rotate around the $z - axis$)

The reader may take a minute to be fully convinced at these three points. Thus, without limiting the discussion and based on these points, one can define a system of coordinates relative to which the velocity v is given, as the system of coordinates whose fundamental plane coincides with the galaxy plane, its origin coincides with the galactic center and its primary axis is pointing towards the observer (or the projection of this direction on the fundamental plane, in the general case). This frame of reference is defined in the main text as K'.

The frame of reference K' cannot be regarded a priori as inertial for the purpose of describing the motions of bodies in a specific galaxy. It's true that the galaxy plane is static relative to this frame (as required by inertial frames, given a central force problem). But the galaxy's primary axis is defined in an arbitrary way. It's defined by the requirement that the line of sight to the observer will be static. In the case where $i = 90^o$ it can be defined as the line pointing towards the observer. Why should the primary axis of an inertial system (used to describe the motions of bodies of a specific galaxy) point towards an (irrelevant) observer?

The subject of defining inertial frames (in reality) is not obvious at all. Nowdays, inertial frames are commonly defined as the frames of reference relative to whom Newton's laws are valid in their simplest form (i.e. without the addition of fictitious forces). If this is the only requirement, then one has the freedom to search for inertial frames. And indeed, in this work the desired inertial frame was searched with the addition of the parameter ω for each galaxy. Saying that, the authors must address the work of Thirring from 1918 (Thirring 1918), based on Einstein's General Relativity. In this work a local inertial frame would be determined by the condition that the fixed stars (i.e. distant objects) are on average at rest. Anyway, not by the condition that a single body (i.e. the observer) is at rest.