Homework 1

Name: _____

Collaborators: _____

Reading

All readings are from the course textbook unless otherwise stated.

This week: Skim Ch2.1 to 2.3 for future reference, and read Ch3 for understanding.

Next week: Skim rest of Ch2, read Ch5 (as much as you can).

1. Fourier and Laplace Transforms (15 points)

(a) (10 points) For each of the following functions look up what their Fourier and Laplace transforms are. Note that neither transform is guaranteed to exist!

$f(t)$	$F(j\omega)$	$F(s)$
1		
e^{at}		
$cos(\omega_0 t)$		
$sin(\omega_0 t - \frac{\pi}{2})$		
$e^{at}cos(\omega_0 t)$		
$step(t) = \begin{cases} 0, t < 0 \\ 1, t \geq 0 \end{cases}$		
$pulse(t) = \begin{cases} 1, \lvert t \rvert \leq W \\ 0, \lvert t \rvert > W \end{cases}$		
$square(t) = \begin{cases} 1, cos(2\pi f_0 t) \geq 0 \\ 0, cos(2\pi f_0 t) < 0 \end{cases}$		
$\delta(t)$		
e^{t^2}		

List any sources you used.

(b) (5 points) Use a random number generator to pick one of the above functions, and prove one of the Laplace or Fourier transforms (or both if you are so inclined). You are encouraged to spend a little bit of time trying to do it independently, but if it's not obvious look it up (just cite your source!). Repeat this for more functions if you want the calculus practice, or are having trouble sleeping.

2. Modulation (11 points)

Do Exercise 2.10 from the textbook. Questions reproduced for convenience.

(a) (3 points) Sketch $X_1(f)$ and $X_2(f)$ Using the representative Frequency blobs in Figure 2.47:

(b) (3 points) Find the relationship betwen f_1, f_2, and f_M such that the signals from the two transmitters don't interfere with each other. Assume that $f_2 > f_1$.

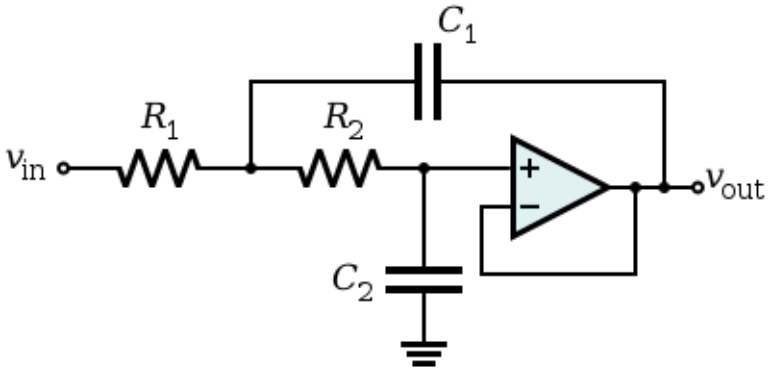
(c) (3 points) Assuming that linearity holds, please sketch $Y_1(f)$ and $Y_2(f)$ and describe why $\tilde{s_1}(t) = \frac{1}{2}s_1(t)$ and $\tilde{s_2}(t) = \frac{1}{2}s_2(t)$.

(d) (2 points) In the North American 2.4 GHz, 802.11 WiFi standard the range of frequencies used is 2.401GHz - 2.473GHz. The bandwidth of the WiFi signal is 22MHz (i.e $f_M = 11MHz$). How many independent WiFi systems can operate in the same physical location without overlapping? For further related information, you can read about 802.11 frequency bands at 2.4 GHz Wi-Fi Bands.

3. Analog Circuit Op-Amp Practice with the Sallen-Key Topology (10 points)

If you don't do this question, please cut these pages from the PDF. Images sourced from Wikipedia.

(a) (10 points) Find the transfer function $H(s) = \frac{V_O}{V_N}$ for this circuit, and describe what you think it does (sketching $|H(s)|$ might help).



(b) (0 points) (+5j) Optional: Examine the poles of your transfer function. What constraints (if any) should you impose on the component values to ensure the circuit is stable?

(c) (0 points) (+10j) Extra Optional: Find the transfer function $H(s) = \frac{V_O}{V_N}$ for the more generic Sallen-Key Circuit: