Intro to QM due: March 13, 2019

Problem Set 15

Problem 1: If 2 identical bosons are in orthonormal 1-particle states $|a\rangle$ and $|b\rangle$, then it was claimed in class that their normalized 2-particle state is $|ab\rangle_S = (|a\rangle|b\rangle + |b\rangle|a\rangle)/\sqrt{2}$. Now suppose that $|a\rangle$ and $|b\rangle$ are normalized but not orthogonal. What is their normalized 2-particle state?

For **problems 2 - 7** consider identical non-interacting fermions of mass m in a common 1-dimensional harmonic oscillator potential $V(x) = (m\omega^2/2)x^2$. Denote by $|n\rangle$ $(n = 0, 1, 2, \ldots)$ the usual normalized one-particle energy eigenstates.

Problem 2: Suppose that one fermion is in the state $|\psi_1\rangle$ and a second identical fermion is in the state $|\psi_2\rangle$ with

$$|\psi_1\rangle = \frac{1}{\sqrt{3}}\left(|1\rangle + i\sqrt{2}\,|2\rangle\right), \qquad\qquad |\psi_2\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |2\rangle\right).$$

What is the 2-particle state, $|\psi_{12}\rangle$?

Problem 3: The ground state (lowest energy eigenstate) of the two-fermion system is

$$|\text{g.s.}\rangle = \frac{1}{\sqrt{2}}\left(|0,1\rangle - |1,0\rangle\right).$$

What is the energy of this state?

Problem 4: Suppose now three identical fermions are in the oscillator potential. What is their ground state, and what is its energy?

Problem 5: What is the ground state energy of N identical fermions in the harmonic oscillator?

Problem 6: What is the degeneracy of the first (= ground state), second, and third energy levels when there are two identical fermions?

Problem 7: What is the degeneracy of the third energy level when there are N identical fermions?

Problem 8: Consider a composite object such as the hydrogen atom. Will it behave as a boson or a fermion? Argue in general that objects containing an even/odd number of fermions will behave as bosons/fermions.