Intro to QM due: April 10, 2019

Problem Set 16

All problems in this problem set will be worth 2 points instead of the usual 1 point.

Problem 1: Use the Born approximation to compute the differential cross section for a particle of mass μ to scatter off the potential $V = C/r^2$ where C is a constant.

Problem 2: Use the Born approximation to compute the differential cross section for a particle of mass μ to scatter off the potential $V = V_0 e^{-r/a}$ where V_0 and $a > 0$ are constants.

Problem 3: Use the Born approximation to compute the total cross section for a particle of mass μ to scatter off the spherical potential well

$$V(r) = \begin{cases} -V_0 & r < a \\ 0 & r > a \end{cases}$$

where V_0 and a are positive constants. Show that in the $k \to 0$ limit your result agrees with that obtained in the text from the S-wave phase shift.

Problem 4: A particle is scattered by a spherically symmetric potential at sufficiently low energy that only the $\ell = 0$ (S-wave) and $\ell = 1$ (P-wave) phase shifts are non-zero. Show that the differential cross section has the form

$$\frac{d\sigma}{d\Omega} = A + B\cos\theta + C\cos^2\theta$$

and determine A, B, and C in terms of the phase shifts. Determine the total cross section in terms of A, B, and C.

Problem 5: Determine the P-wave phase shift δ_1 for scattering from a hard sphere, for which the potential energy is

$$V(r) = \begin{cases} \infty & r < a \\ 0 & r > a \end{cases}$$

where a is a positive constant. Express your result in terms of $j_1(ka)$ and $\eta_1(ka)$. Use the leading behavior of $j_1(\rho)$ and $\eta_1(\rho)$ to show that $\delta_1 \to -(ka)^3/3$ as $ka \to 0$.