Axioms of Quantum Mechanics

(Underlined terms are linear algebra concepts whose definitions you need to know.)

Axioms, version 1:

I. The *state* of a system is a vector, $|\psi\rangle$, in a Hilbert space (a complex vector space with a positive definite inner product), and is normalized: $\langle\psi|\psi\rangle = 1$.

II. An *observable* (allowed measurement) is a choice of an orthonormal basis, $\{|\phi_n\rangle, n = 1, \ldots, d\}$, of the Hilbert space.

III. The only possible *outcomes* of measuring this observable are one of the states, $|\phi_n\rangle$, in the orthonormal basis. I denote this outcome by "$|\psi\rangle \Rightarrow |\phi_n\rangle$".

IV. The *probability*, of observing a given possible outcome of such a measurement is $\mathcal{P}(|\psi\rangle \Rightarrow |\phi_n\rangle) = |\langle\phi_n|\psi\rangle|^2$.

V. Once we observe the outcome $|\psi\rangle \Rightarrow |\phi_n\rangle$, the *state changes as a result of the measurement* to $|\psi\rangle \to |\phi_n\rangle$.

VI. The *time evolution* of the state of a system when it is not being measured is given by $|\psi(t)\rangle = \widehat{U}(t)|\psi(0)\rangle$, where the unitary time evolution operator is given by $\widehat{U}(t) = \exp\{-it\widehat{H}/\hbar\}$ where $\widehat{H}$ is the hermitean energy operator (also known as the Hamiltonian operator).

Axioms, version 2:

I. The *state* of a system is a vector, $|\psi\rangle$, in a Hilbert space (a complex vector space with a positive definite inner product), and is normalized: $\langle\psi|\psi\rangle = 1$.

II. An *observable* (allowed measurement) is a choice of a hermitean operator, $\widehat{M}$. By the spectral theorem, $\widehat{M} = \sum_i \mu_i \widehat{P}_i$, where μ_i are its eigenvalues and $\widehat{P}_i$ are the orthogonal projection operators onto their corresponding eigenspaces.

III. The only possible *outcomes* of measuring $\widehat{M}$ are one of its eigenvalues. I denote this outcome of this measurement by "$M = \mu_i$".

IV. The *probability* of observing a given possible outcome of such a measurement is $\mathcal{P}(M = \mu_i) = \langle\psi|\widehat{P}_i|\psi\rangle$.

V. Once we observe the outcome $M = \mu_i$, the *state changes as a result of the measurement* to $|\psi\rangle \to \widehat{P}_i|\psi\rangle/\sqrt{\mathcal{P}(M=\mu_i)}$.

VI. The *time evolution* of the state of a system when it is not being measured is given by $|\psi(t)\rangle = \widehat{U}(t)|\psi(0)\rangle$, where the unitary time evolution operator is given by $\widehat{U}(t) = \exp\{-it\widehat{H}/\hbar\}$ where $\widehat{H}$ is the hermitean energy operator (also known as the Hamiltonian operator).