Intro to QM due: February 27, 2019

Problem Set 14

All problems in this problem set will be worth 3 points instead of the usual 1 point.

Problem 1: Consider a perturbation $\widehat{H}_1 = \lambda\widehat{x}^4$ to the simple harmonic oscillator $\widehat{H}_0 = \widehat{p}^2/(2m) + \frac{1}{2}m\omega^2\widehat{x}^2$.

(a) Show that the first order shift in the energy eigenvalues are

$$E_n^{(1)} = \frac{3\hbar^2\lambda}{4m^2\omega^2}(2n^2 + 2n + 1).$$

(b) Explain why no matter how small λ is, the perturbation expansion will break down for sufficiently large n.

Problem 2: Consider a spin-$\frac{1}{2}$ particle in a constant uniform magnetic field $\vec{B} = B_0\,\widehat{k} + B\,\widehat{\imath}$. Its Hamiltonian, $\widehat{H} = -\vec{\mu}\cdot\vec{B}$, is

$$\widehat{H} = \widehat{H}_0 + \widehat{H}_1 \qquad \text{with} \qquad \widehat{H}_0 := -\gamma B_0\widehat{S}_z, \quad \text{and} \quad \widehat{H}_1 := -\gamma B\widehat{S}_x,$$

where g is the gyromagnetic ratio of the particle.

(a) For $B \ll B_0$, treat $\widehat{H}_1$ as a perturbation, and compute the first and second-order corrections to the energy eigenvalues, and the first order corrections to the energy eigenstates.

(b) Compare these results to the exact answer. (It may be useful to recall from problem set 1 or from problem 3.2 of the text the expression for the spin-$\frac{1}{2}$ eigenstates of $\widehat{S}_n$, the component of the spin angular momentum in the direction of a unit vector $\widehat{n} := \sin\theta\cos\phi\,\widehat{\imath} + \sin\theta\sin\phi\,\widehat{\jmath} + \cos\theta\,\widehat{k}$.)

Problem 3: Consider a spin-1 particle with Hamiltonian

$$\widehat{H} = A\widehat{S}_z^2 + B(\widehat{S}_x^2 - \widehat{S}_y^2). \tag{1}$$

Assume $B \ll A$, treat the second term as a perturbation, and calculate the unperturbed energies and their first-order corrections using perturbation theory. Be careful of the degeneracy. Compare your perturbative results with the exact eigenvalues.