20598 – Finance with Big Data

Week 3 Lecture:
Capital Asset Pricing Model and APT

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Outline

Last week : Portfolio Theory

Capital Asset Pricing Model

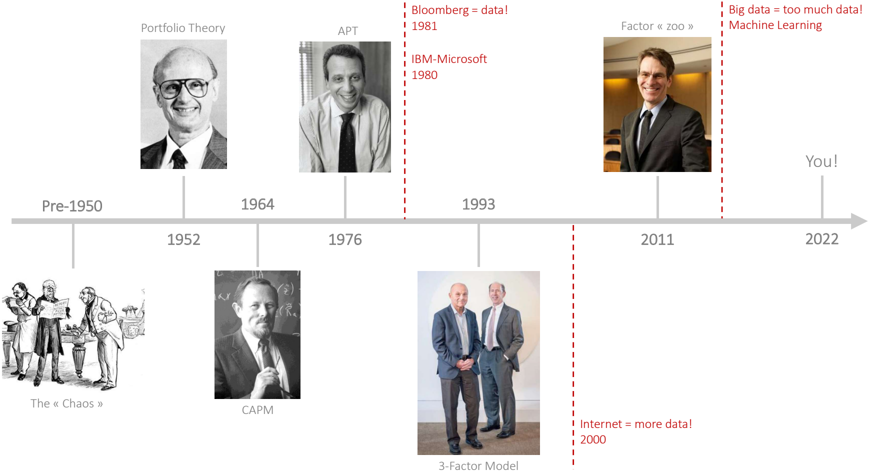
Security Market Line

CAPM vs. APT

In Practice

Outline

A long-term perspective on Asset Pricing



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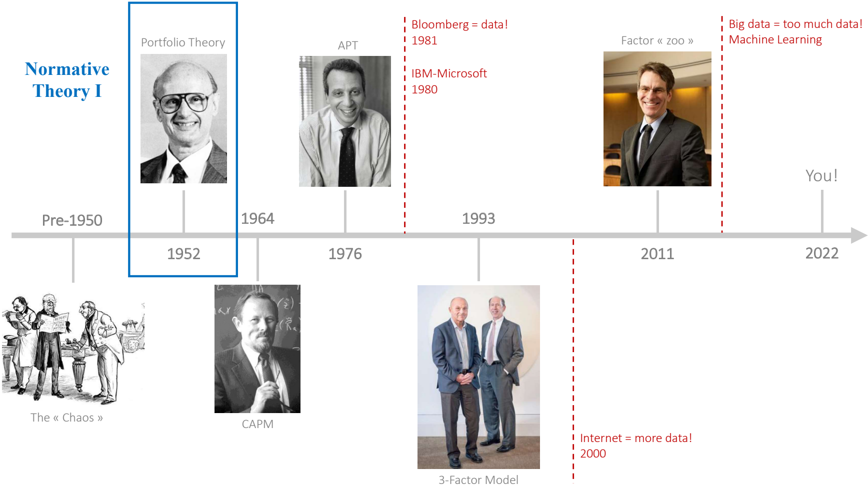

What do we learned from the Portfolio Theory ?

- Markowitz Mean-Variance trade-off:

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- Markowitz Mean-Variance trade-off:
  - Diversification reduces risk
  - Portfolio risk depends primarily on covariances
  - Hold only the risk-free asset + the tangency portfolio

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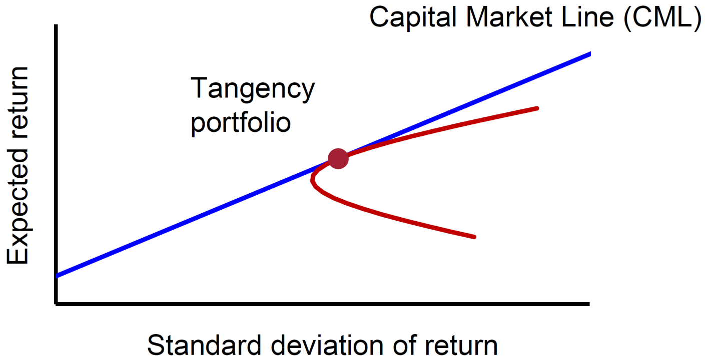

- For any portfolio p on the CML, we have :

$$\bar{r}_p = r_f + \frac{\bar{r_T} - r_f}{\sigma_T}\,\sigma_p$$

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- Last course: we used historical data and basic statistics

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$\rightarrow$ Does it work well ?

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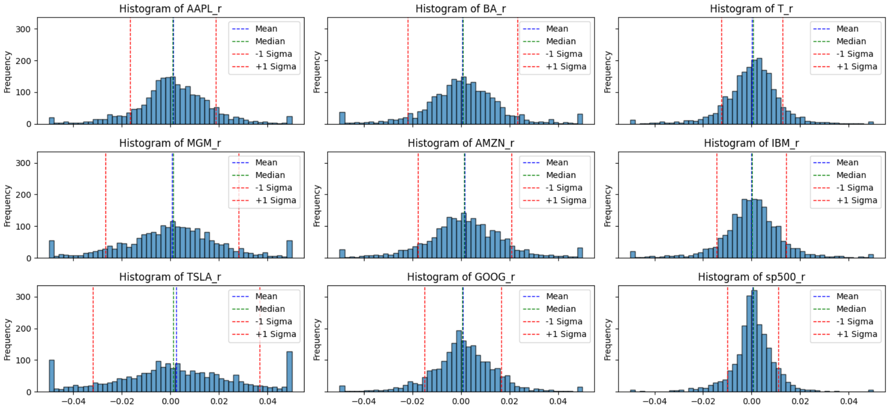
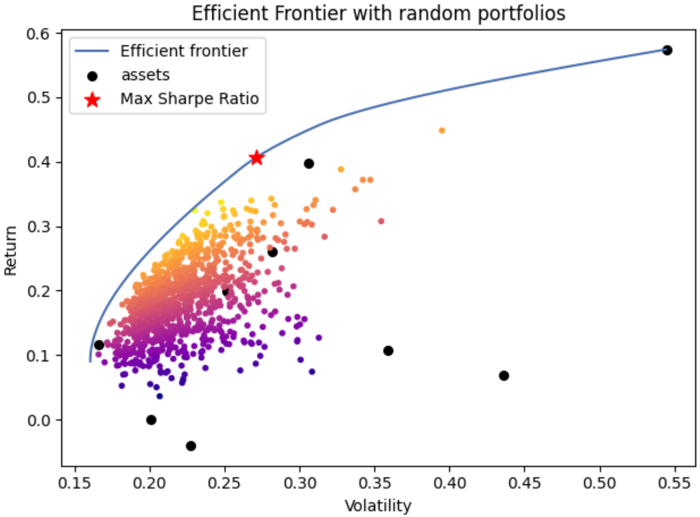
Group 2



What do we learned from applying Portfolio Theory ?

Group 9



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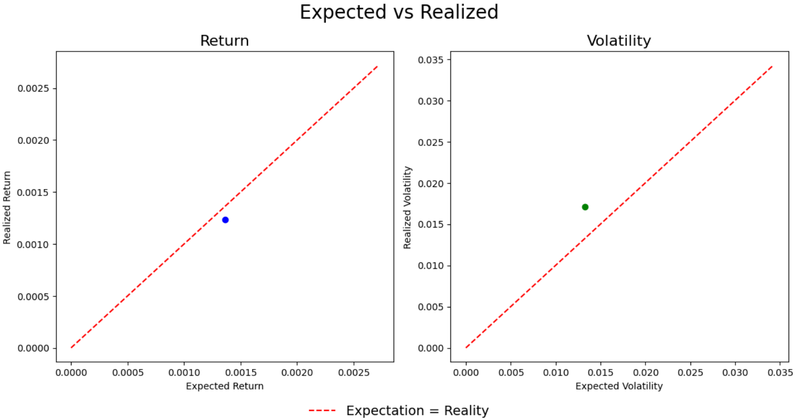
Group 11



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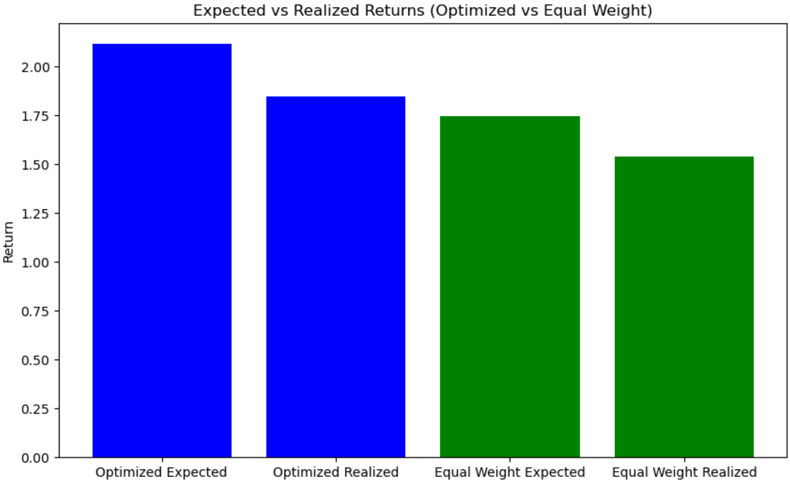

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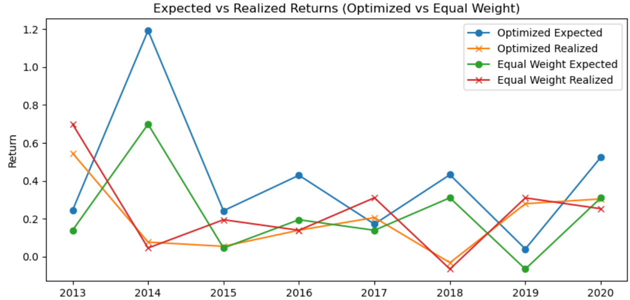
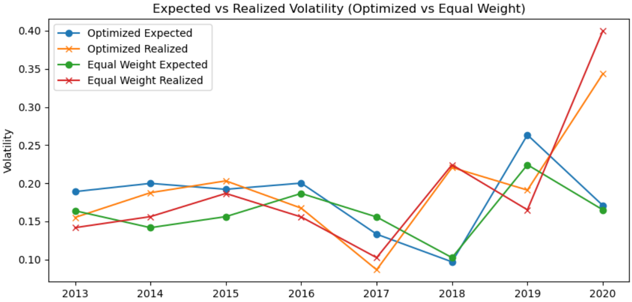
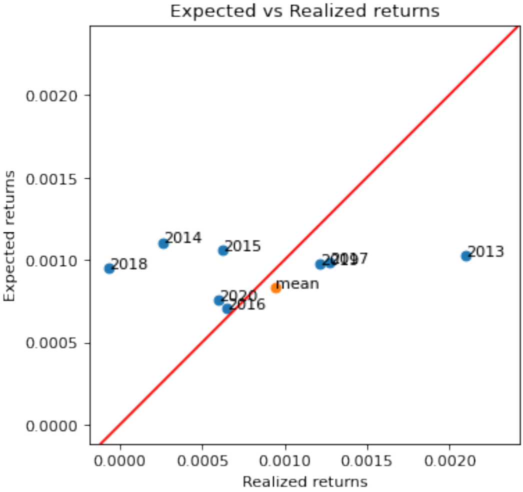




What do we learned from applying Portfolio Theory ?

Last year



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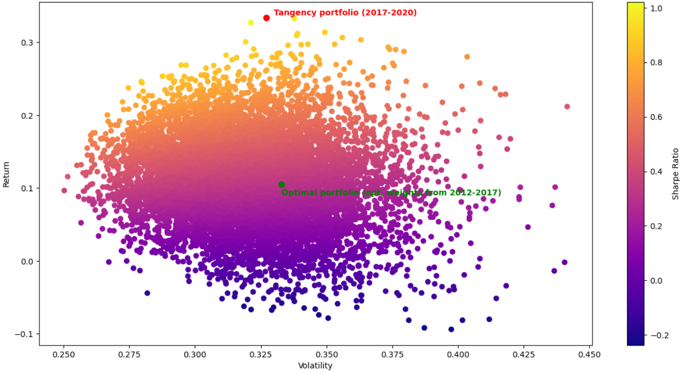

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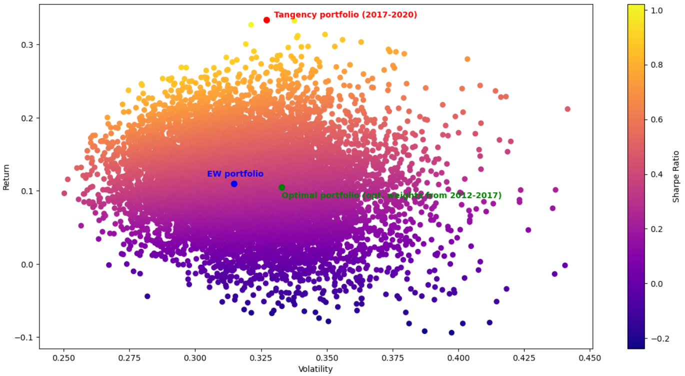

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- Non normality of the returns distribution is an issue!
- In practice, what is the right sample to estimate mean and variance?
- Most Markowitz's portfolios exclude many stocks in practice
- Above all, **risk = variance** rather than **downside risk** is criticized:
  - Two portfolios that have the same level of variance and returns are considered equally desirable
  - Frequent small losses vs. rare but spectacular declines

To go further

ELSEVIER

European Journal of Operational Research

Volume 234, Issue 2, 16 April 2014, Pages 356-371

60 Years of portfolio optimization: Practical challenges and current trends

Petter N. Kolm [a], Reha Tütüncü [b], Frank J. Fabozzi [c]

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- review approaches for implementing Markowitz mean–variance analysis in practice
- Inclusion of transaction costs, constraints, sensitivity to inputs
- Highlights new trends and developments

To go further (Chen et al., 2021)

Mean–variance portfolio optimization using machine learning-based stock price prediction

Wei Chen [a], Haoyu Zhang [a], Mukesh Kumar Mehlawat [b], Lifen Jia [a]

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https://doi.org/10.1016/j.asoc.2020.106943

Highlights

- Stock prediction is integrated into portfolio selection to capture future features.
- A hybrid model named IFAXGBoost is developed for stock price prediction.
- The prediction model IFAXGBoost is incorporated into the mean-variance model.
- Extensive experiments demonstrate the effectiveness of the proposed method.

What do we learned from applying Portfolio Theory ?

- What is missing?

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- What is missing?

When I studied microeconomics forty years ago, I was first taught how optimizing firms and consumers would behave, and then taught the nature of the economic equilibrium which would result from such behavior. Let me refer to this as part one and part two of my microeconomics course. My work on portfolio theory considers how an optimizing investor would behave, whereas the work by Sharpe and Lintner on the Capital Asset Pricing Model (CAPM for short) is concerned with economic equilibrium assuming all investors optimize in the particular manner I proposed. Thus, my work on the one hand, and that of Sharpe and Lintner on the other, provide part one and part two of a microeconomics of capital markets.

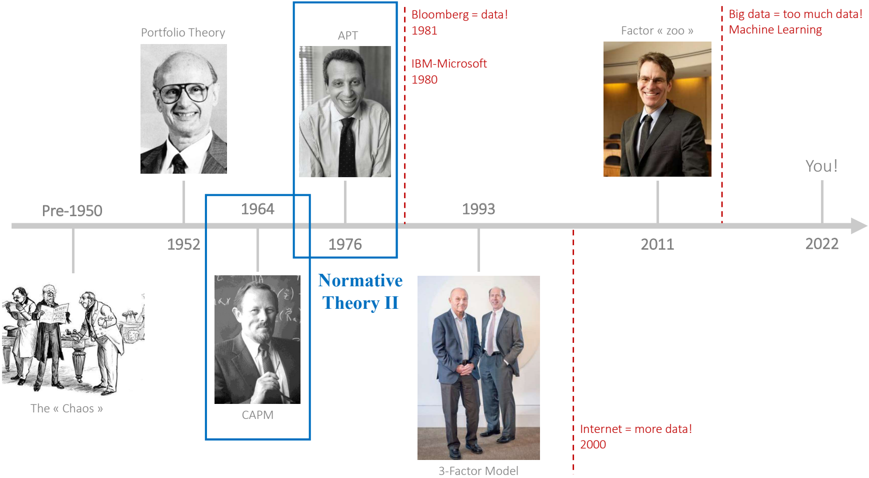
Source : Markowitz Nobel Prize Lecture

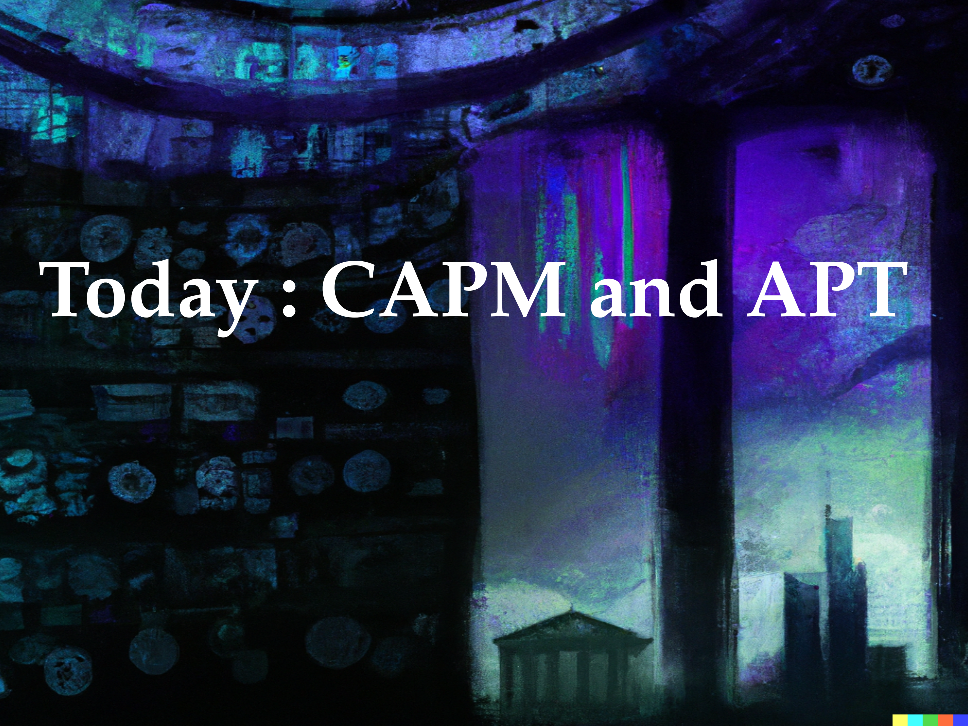
What do we learned from applying Portfolio Theory ?

- What is missing? $\rightarrow$ **A model to price risky assets**

$$\mathbb{E}[\tilde{r}_i] = ?$$

A long-term perspective on Asset Pricing




Today : CAPM and APT

Today: the Capital Asset Pricing Model

- Introduce by W. Sharpe (1964) to answer the question : $\mathbb{E}[\tilde{r}_i] = ?$

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- Introduce by W. Sharpe (1964) to answer the question : $\mathbb{E}[\tilde{r}_i] =?$
- Builds on the Portfolio Theory of Markowitz (1952) + market equilibrium

$\rightarrow$ Read the paper here (+30K citations)

The Journal of FINANCE

VOL. XIX SEPTEMBER 1964 No. 3

CAPITAL ASSET PRICES: A THEORY OF MARKET EQUILIBRIUM UNDER CONDITIONS OF RISK*

WILLIAM F. SHARPE†

I. INTRODUCTION

ONE OF THE PROBLEMS which has plagued those attempting to predict the behavior of capital markets is the absence of a body of positive microeconomic theory dealing with conditions of risk. Although many useful insights can be obtained from the traditional models of investment under conditions of certainty, the pervasive influence of risk in financial transactions has forced those working in this area to adopt models of price behavior which are little more than assertions. A typical classroom ex-

The market portfolio
Definition

- The market portfolio is the portfolio of all risky assets traded in the market.
- A total of n risky assets, market capitalization of asset i is :

$$\text{Market Cap.}_i = (\text{price per share})_i \times (\#\text{ of shares outstanding})_i$$

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- The market portfolio has the following portfolio weights :

$$w_i = \frac{\text{Market Cap.}_i}{\sum_{i=j}^{n} \text{Market Cap.}_j} = \frac{\text{Market Cap.}_i}{\text{Market Cap.}_M}$$

- We denote the market portfolio by w_m

Derivation of CAPM

Assumptions

1. Investors agree on the distribution of asset returns

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3. There is a risk-free asset
   - paying interest rate r_f
   - in zero net supply

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3. There is a risk-free asset
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4. Demand of assets equals supply in equilibrium

Derivation of CAPM

Implications

- Every investor puts their money into 2 pots :
  - the risk-free asset
  - a single portfolio of risky assets, the tangency portfolio
- i.e., all investors hold the risky assets in same proportions

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$\Rightarrow$ The tangent portfolio $\equiv$ **the market portfolio !**

Example with 3 risky assets

Market clearing

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- There are only three investors, 1, 2, 3, with total wealth of 500, 1000, 1500 $billion, respectively. Their asset holdings are :

Investor	Riskless	A	B	C
1	100	100	200	100
2	200	200	400	200
3	-300	450	900	450
Total	0	750	1500	750

Example with 3 risky assets

Equilibrium

- In equilibrium, the total dollar holding of each asset = its market value :

$$\text{Market capitalization of A} = \$750 \text{ billion}$$
$$\text{Market capitalization of B} = \$1,500 \text{ billion}$$
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- What is the total market capitalization ?

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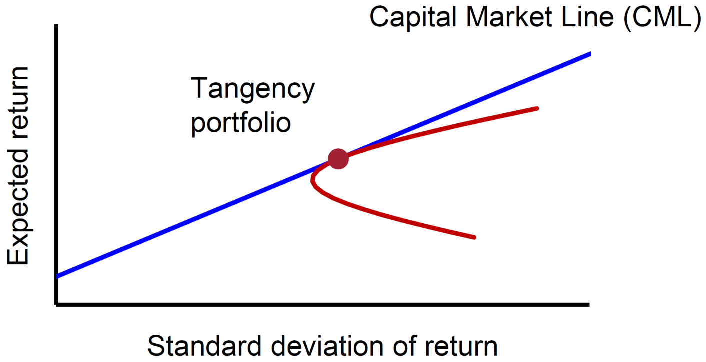
- What are the market portfolio weights ? $w_M = (\frac{750}{3000}, \frac{1500}{3000}, \frac{750}{3000})$

- The **market portfolio is the tangent portfolio !**

$$w_M = (\frac{750}{3000}, \frac{1500}{3000}, \frac{750}{3000}) = (0.25, 0.50, 0.25) = w_T$$

- Implications ?

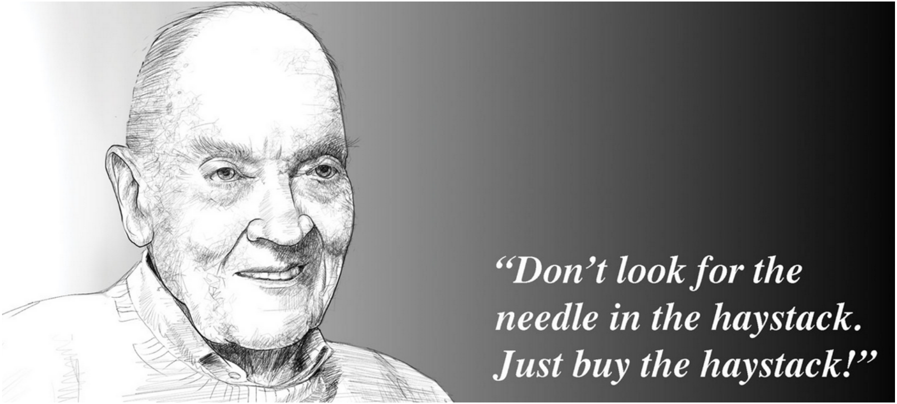
Tangency portfolio = Market portfolio



- The tangency is the market portfolio
- For any portfolio p on the CML, we have:

$$\bar{r}_p = r_f + \frac{(\bar{r_M} - r_f)}{\sigma_M}\sigma_p = r_f + \frac{\sigma_p}{\sigma_M} \cdot (\bar{r_M} - r_f)$$

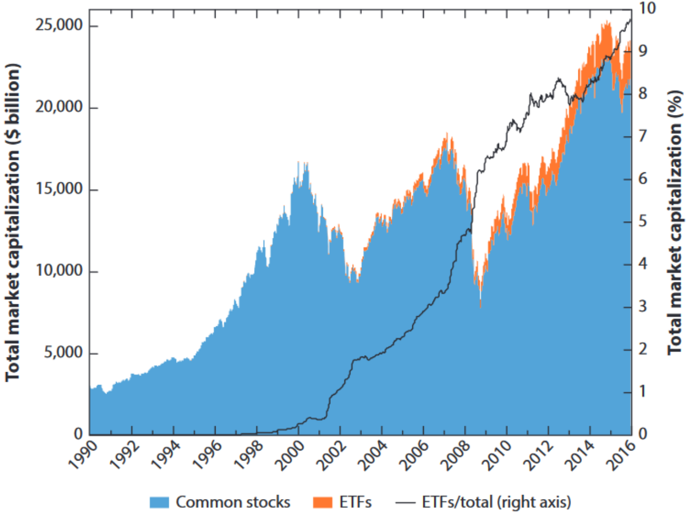
Tangency portfolio = Market portfolio



- John Bogle, founder of Vanguard: advocate of passive investment

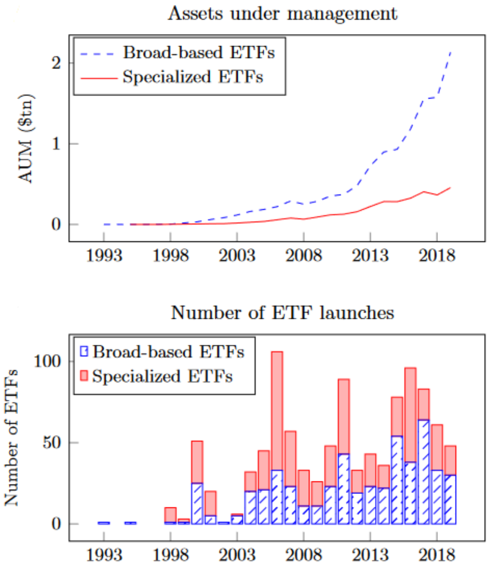
Tangency portfolio = Market portfolio

Ben David et al., 2017



Tangency portfolio = Market portfolio

Ben David et al., 2023



Derivation of the CAPM

Reminder: Learning the maths here is **optional**

This derivation is for those who like equations better than words

but the important remains the **economic intuitions**

Derivation of the CAPM

- We now have a linear relationship between any portfolio p on the CML and the market portfolio.

$$\mathbb{E}[\tilde{r}_p] = r_f + \frac{\sigma_p}{\sigma_M} \cdot (\mathbb{E}[\tilde{r}_M] - r_f)$$

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- That's *great*, but remember: our goal was to answer the question $\mathbb{E}[\tilde{r}_i] = ?$

$\rightarrow$ What is the contribution of asset i to the market portfolio?

Derivation of the CAPM

Contribution of asset i to the market portfolio

- n risky assets compose the market
- In the presence of a risk-free asset, the return of the market portfolio is :

$$\tilde{r}_M = (1 - \sum_{i=1}^{n} w_i)\, r_f + \sum_{i=1}^{n} w_i\, \tilde{r}_i$$

- Each asset i contributes to the market portfolio in 2 dimensions:
  - expected return (desirable)
  - risk measured by return volatility (bad)
- We will consider these 2 aspects separately

Derivation of the CAPM

Contribution of asset i to **the expected return**

- Expected return of the market portfolio writes :

$$\bar{r}_M = (1 - \sum_{i=1}^{n} w_i)\, r_f + \sum_{i=1}^{n} w_i\, \bar{r}_i$$
$$= r_f + \sum_{i=1}^{n} w_i\, (\bar{r}_i - r_f)$$

- The marginal contribution of risky asset i to the expected portfolio return is its risk premium:

$$\frac{\partial \bar{r}_M}{\partial w_i} = \bar{r}_i - r_f$$

- Note: *marginal contribution of x to A* means the incremental changes of A when x changes by a small amount

Derivation of the CAPM

Contribution of asset i to **the variance of the return**

- Portfolio return variance = sum of all elements of the covariance matrix :

	1	$\cdots$	n
1	$w_1^2\sigma_1^2$	$\cdots$	$w_1 w_n \sigma_{1n}$
$\vdots$	$\vdots$	$\ddots$	$\vdots$
n	$w_n w_1 \sigma_{n1}$	$\cdots$	$w_n^2\sigma_n^2$

Derivation of the CAPM

Contribution of asset i to **the variance of the return**

- Portfolio return variance = sum of all elements of the covariance matrix :
- The contribution of asset i alone is the sum of the element of the i-th row and the i-th column :

$$w_i^2\,\sigma_i^2 + 2\sum_{i\neq j}^{n} w_i w_j \sigma_{ij}$$

- The marginal contribution of asset i to market portfolio variance is :

$$\frac{\partial \sigma_M^2}{\partial w_i} = 2w_i\sigma_i^2 + 2\sum_{i\neq j}^{n} w_j\sigma_{ij} = 2\sum_{j=1}^{n} w_j\sigma_{ij} = 2\ \mathrm{Cov}(r_i, r_M)$$

Derivation of the CAPM

Return to Risk Ratio of asset i

- We summarize the marginal contribution of the risky asset i by the return to risk ratio (RRR) :

$$RRR_{iM} = \frac{\bar{r}_i - r_f}{\sigma_{iM}}$$

Derivation of the CAPM

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- For the market (tangency) portfolio to be optimal, the return-to-risk ratio of all risky assets should also be optimal – and must be the same :

$$RRR_i = \frac{\bar{r}_i - r_f}{\sigma_{iM}} = RRR_M = \frac{\bar{r}_M - r_f}{\sigma_M^2}$$

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- Rewriting this equation gives :

$$\bar{r}_i = r_f + \frac{\sigma_{iM}}{\sigma_M^2}(\bar{r}_M - r_f)$$

The CAPM

Main result

Capital Asset Pricing Model :

$$\bar{r}_i - r_f = \beta_i \left(\bar{r}_M - r_f\right)$$

$$\text{with } \beta_i = \frac{\sigma_{iM}}{\sigma_M^2}$$

- risk-reward relation is linear

The CAPM

Main result

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- risk-reward relation is linear
- β gives a measure of asset i's systematic risk
- $\mathbb{E}[\tilde{r}_M] - r_f$ gives the premium per unit of systematic risk

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Main result

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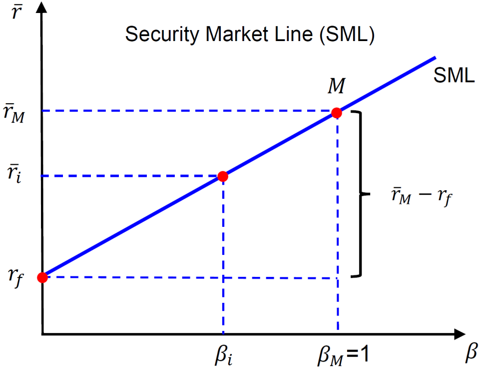
- risk-reward relation is linear
- β gives a measure of asset i's systematic risk
- $\mathbb{E}[\tilde{r}_M] - r_f$ gives the premium per unit of systematic risk

$\rightarrow$ The risk premium of an asset equals its systematic risk times the premium per unit of risk

The CAPM

Security Market Line (SML)

- The relationship between an asset's premium and its market beta is called the Security Market Line



- Given an asset's beta, we can determine its expected return

The CAPM

Example

- We are in a world where CAPM holds.
- The expected market return is 8% and T-bill rate is 2%

1. What should be the expected return on a stock with $\beta = 0$?

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$$2\%$$

2. What should be the expected return on a stock with $\beta = 1$?

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$$2\%$$

2. What should be the expected return on a stock with $\beta = 1$?

$$8\%$$

3. What should be the expected return on a stock with $\beta = -0.6$?

The CAPM

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$$2\%$$

2. What should be the expected return on a stock with $\beta = 1$?

$$8\%$$

3. What should be the expected return on a stock with $\beta = -0.6$?

$$\bar{r} = 2\% + -0.6 \times (8\% - 2\%) = -1.6\%$$

The CAPM
Empirically

- We can bring this equation to the data :

The CAPM
Empirically

- We can bring this equation to the data :

$$\tilde{r}_i - r_f = \alpha_i + \beta_i(\tilde{r}_M - r_f) + \epsilon_i$$

The CAPM
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- We can isolate 3 characteristics of an asset:
  - α should be zero, according to the CAPM
  - β measures an asset's systemic risk
  - $\sigma = \mathrm{SD}(\epsilon_i)$ measures non-systematic risk

The CAPM

Empirically: $\alpha_i = 0$

- We can bring this equation to the data :

$$\tilde{r}_i - r_f = \alpha_i + \beta_i(\tilde{r}_M - r_f) + \epsilon_i$$

- According to the CAPM, alpha should be 0 for all assets
- Alpha measures an asset's return in excess of its risk-adjust award
- What to do with an asset with positive alpha ?

What should you do with alpha $\neq 0$?

- Suppose that asset i violates the CAPM ($\alpha_i > 0$) :

$$\tilde{r}_i - r_f = \alpha_{\mathbf{i}} + \beta_i(\tilde{r}_M - r_f) + \epsilon_i$$

- You are a mean-variance optimizing investor with initial wealth = \$1
  - How would your portfolio deviate from the market portfolio ?

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- You are a mean-variance optimizing investor with initial wealth = \$1
  - How would your portfolio deviate from the market portfolio ?
- Answer. Construct a portfolio P :
  - long \$1 of asset i
  - short \$ β_i units of the market portfolio
  - invest \$ β_i in the risk-free asset
- What is your portfolio return :

$$\begin{aligned} r_p &= 1 + \tilde{r}_i - \beta_i \tilde{r}_M + \beta_i r_f \\ &= 1 + r_f + \alpha_i + \epsilon_i \end{aligned}$$

- No risk + uncorrelated with market return !
  - If you mix M and P, how will be your Sharpe ratio ?

Arbitrage Pricing Theory

CAPM vs. Arbitrage Pricing Theory (APT)

- APT = Arbitrage Pricing Theory = alternative theory that generalizes the CAPM
- Main intuition : they are multiple sources (= factors F) of systematic risk

The APT model with n factors

$$\tilde{r}_i - r_f = \alpha_i + \beta_{i1}\,\mathbf{F_1} + \beta_{i2}\,\mathbf{F_2} + ... + \beta_{in}\,\mathbf{F_n} + \epsilon_i$$

- Here, α_i is the expected return of i

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- Strengths of the APT:
  - Derivation does not require market equilibrium
  - Allows for multiple sources of systematic risk
- Weaknesses of the APT:
  - No theory about what the factors should be
  - Assumption of linearity is quite restrictive

→ The same fundamental insight in both models: The only component of total risk relevant for pricing is the systematic risk

Arbitrage Pricing Theory (APT)

Example f a 2-factor model

- Suppose that the only two systematic sources of risk are:
  - Unanticipated changes in economic growth ; and
  - Unanticipated changes in energy prices.
- The return on any stock respond to both sources of macro shocks and to firm-specific shocks:

$$\tilde{r}_i - r_f = \alpha_i + \beta_{i1}\,\mathbf{F_{GR}} + \beta_{i2}\,\mathbf{F_{EN}} + \epsilon_i$$

Arbitrage Pricing Theory (APT)

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$$\tilde{r}_i - r_f = \alpha_i + \beta_{i1}\,\mathbf{F_{GR}} + \beta_{i2}\,\mathbf{F_{EN}} + \epsilon_i$$

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- What about a long-distance trucking firm ?

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- What about a solar panel installer ?
  - cash-flows have moderate exposure to economic growth $\rightarrow \beta_1 \geq 0$
  - strong exposure to rising energy costs $\rightarrow \beta_2 > 0$ and large
- What about a long-distance trucking firm ?

Arbitrage Pricing Theory (APT)

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$$\tilde{r}_i - r_f = \alpha_i + \beta_{i1}\,\mathbf{F_{GR}} + \beta_{i2}\,\mathbf{F_{EN}} + \epsilon_i$$

- What about a solar panel installer ?
- What about a long-distance trucking firm ?
  - cash flows are very sensitive to economic activity $\rightarrow \beta_1 > 0$
  - Sensitive to energy costs $\rightarrow \beta_2 < 0$ and large

In Practice

Application of factor models

Estimate the expected return on Dell and IBM with the CAPM

1. Use the value-weighted stock portfolio (S&P 500) to measure $\bar{r}_m$

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5. Apply the CAPM (suppose r_f today is 2%) :

$$\begin{aligned} \bar{r}_{IBM} &= r_f + \beta_{IBM} \left(\bar{r}_M - r_f \right) \\ &= 2\% + 0.8 \times 6\% = 6.8\% \end{aligned}$$

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6. The expected rate of return on IBM is 6.8%. For Dell, it is 9.8%

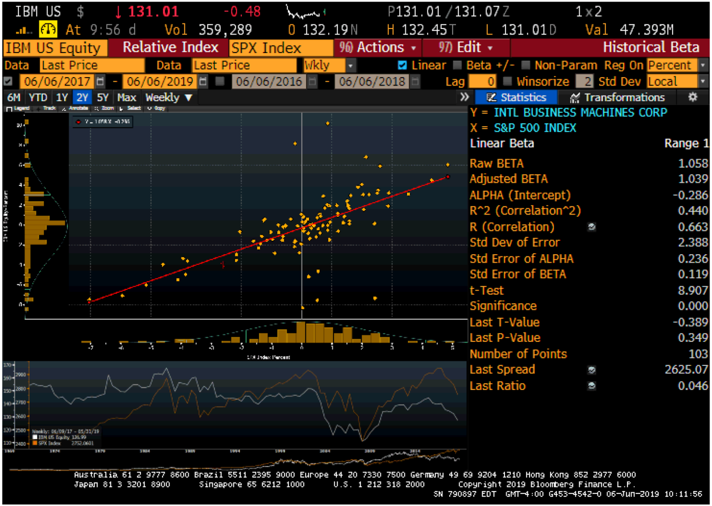
Application of factor models

Estimation of betas

1. OLS regression: regress excess returns of an asset (stock) on market returns
2. Regression estimates are noisy – additional statistical adjustments (shrink estimates towards their average value, ~ 1) $\rightarrow$ Adjusted betas
3. Certain firm characteristics help forecasting betas: size, leverage, industry, P/E ratios, etc.
4. Betas are time-varying: incorporate time-variation into regression estimates ; possibly use current data on firm characteristics to capture time-variation.

Application of factor models

Estimation of betas: an illustration



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- Historical averages of returns on individual stocks are poor estimates of expected returns going forward: Short samples; time-variation in risk and expected returns
- Can estimate betas more precisely than average returns.
- CAPM and APT allow us to use firm-level beta estimates (relatively precise) and market or factor risk premia (long sample)
- Estimates of future expected returns on an asset are based on its systematic risk, not on the average of its past returns

Empirical properties of CAPM betas

Data from Kogan and Wang (2021)

CAPM Assumptions	2015	2019
Risk Free Rate	0.00%	2.50%
Risk Premium	10.86%	4.30%
Total E[R]	10.86%	6.82%

Value Line Betas for Individual Stocks in 2015 versus 2019 Equilibrium CAPM Returns Using the Market-Based Capital Asset Pricing Model

	Value Line Beta	Value Line Beta	E[R] CAPM	E[R] CAPM		Value Line Beta	Value Line Beta	E[R] CAPM	E[R] CAPM
	2015	2019	2015	2019		2015	2019	2015	2019
AIG	1.04	0.93	11.25%	6.49%	Equifax	0.82	0.84	8.89%	6.1%
American Express	1.02	1.02	11.03%	6.88%	Exxon Mobil	1.06	0.81	11.46%	5.98%
Amgen	1.31	0.97	14.24%	6.68%	Fair Isaac	1.19	1.42	12.92%	8.6%
Apple	1.02	1.5	11.04%	8.93%	Fedex	0.96	1.38	10.42%	8.43%
Bank of America	1.17	1.01	12.67%	6.82%	Ford Motor	1.02	0.94	11.08%	6.54%
BB&T	0.95	0.67	10.36%	5.38%	Gap	0.83	0.65	9.06%	5.29%
Barnes and Noble	1.33	0.97	14.44%	6.66%	General Electrric	0.93	0.76	10.06%	5.75%
Best Buy	1.29	0.98	14.01%	6.69%	General Mills	0.62	0.21	6.78%	3.42%
Boeing	0.91	1.3	9.9%	8.08%	Goldman Sachs	1.2	1.08	13.01%	7.14%
Caterpillar	1.2	1.62	13.02%	9.46%	Google	0.98	1.32	10.61%	8.19%
CEMEX ADS	1.59	1.03	17.26%	6.91%	Harley Davidson	1.21	1.07	13.09%	7.08%
Cheesecake Factory	0.53	0.53	5.8%	4.79%	Hershey	0.7	0.15	7.65%	3.13%
Chevron Oil	1.03	0.76	11.22%	5.77%	Home Depot	1.13	0.81	12.26%	5.97%
China Fund	0.16	0.84	1.74%	6.13%	IBM	0.92	0.98	10.03%	6.7%
Citigroup	1.28	1.2	13.91%	7.67%	JP Morgan Chase	1.25	0.91	13.61%	6.4%
Coca Cola	0.49	0.36	5.35%	4.05%	Johnson & Johnson	0.96	0.64	10.43%	5.23%
Consolidated Edison	0.52	-0.03	5.6%	2.37%	Microsoft	1.24	1.41	13.42%	8.57%
Delta Airlines	1.35	1.04	14.66%	6.95%	Morgan Stanley	1.36	1.19	14.75%	7.6%
Disney	0.91	0.8	9.92%	5.95%	Procter & Gamble	0.65	0.35	7.09%	4%
Duke Energy	0.48	-0.02	5.16%	2.41%	Walmart	0.66	0.5	7.21%	4.67%
Etrade	1.38	1.2	14.97%	7.64%	Wells Fargo	1.05	0.75	11.44%	5.73%

Empirical properties of CAPM betas

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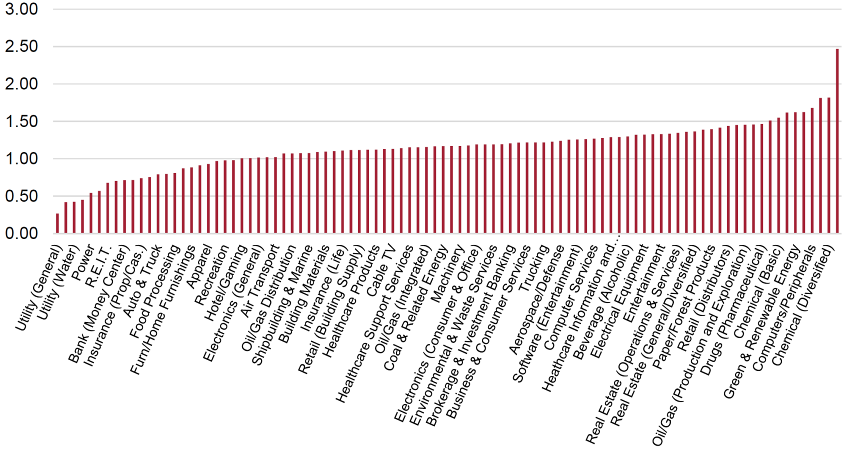


Empirical properties of CAPM betas

Why do industry betas differ?

- Fundamental differences?

Empirical properties of CAPM betas

Why do industry betas differ?

- Fundamental differences?
  - Cyclicality of demand
  - Exposure to credit market risk
  - Technological shocks

Does it work?

Empirical test of the CAPM

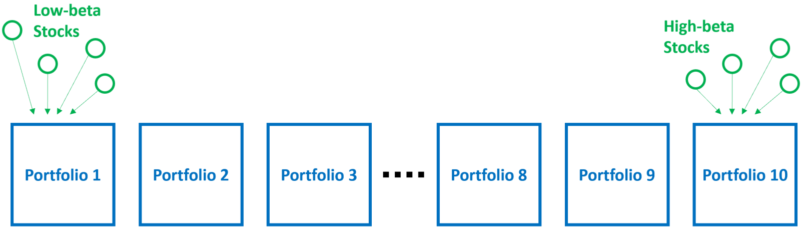
Step 1: Create 10 portfolios sorted by β at $y-1$

- For each asset i and month t, we defined excess return $x_{it} = r_{it} - r_{ft}^{1m}$
  - r_{it} is the return over the month t
  - r_{ft}^{1m} is the 1-month T-bill rate at the start of t (1-month risk-free rate)
- For every pair $i \times t$, estimate beta on a 60-month window, conditional on 24 non-missing month observations
- On January 1^{st} of every year y, create ten portfolios by sorting on betas estimates recorded in December of the previous year $y-1$

Empirical test of the CAPM

Step 1: Create 10 portfolios sorted by β at $y-1$

$$\tilde{r}_i - r_f = \alpha_i + \beta_i(\tilde{r}_M - r_f) + \epsilon_i$$



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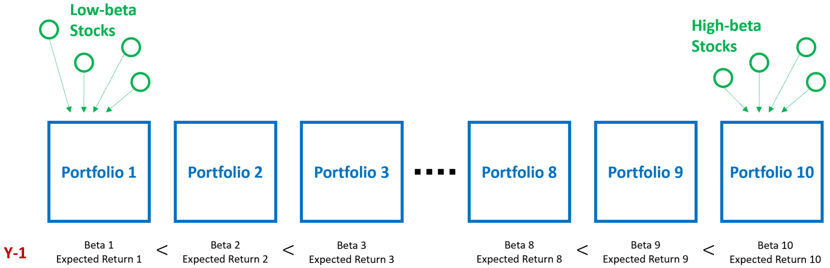


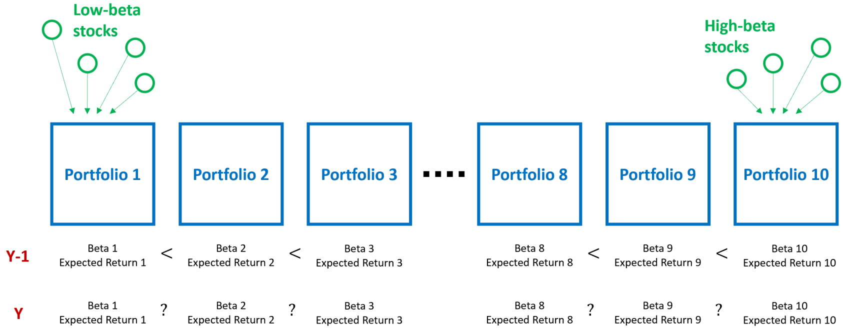
Empirical test of the CAPM

Step 2: Estimate the CAPM parameters of those 10 portfolios on year y

- For each portfolio p and month t of year y compute :
  - monthly excess returns
  - estimate portfolio beta
- For each portfolio, compute time-series average of beta and excess return

Empirical test of the CAPM

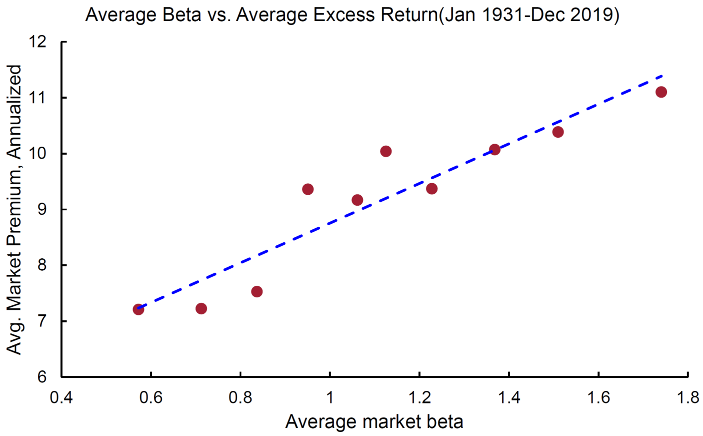
Step 2: Estimate the CAPM parameters of those 10 portfolios on year y



Empirical test of the CAPM

Step 3: Plot β against x for 10 portfolios

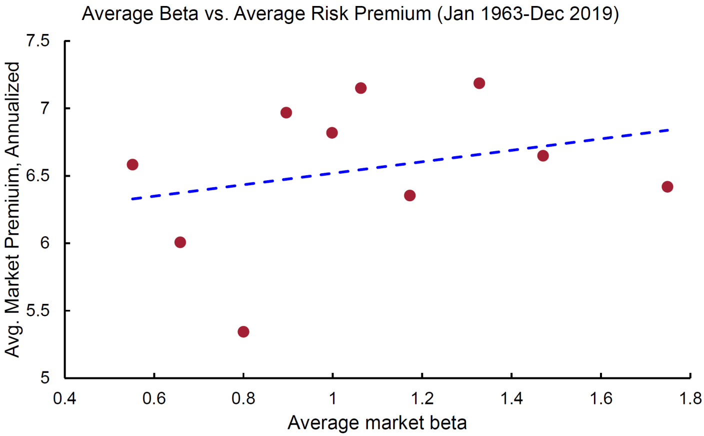
- Estimated SML is too flat: the CAPM is not performing well at predicting returns



Empirical test of the CAPM

Step 3: Plot β against x for 10 portfolios

- Estimated SML is really too flat !



What do we learned ?

- Sharpe (1964): The **tangency portfolio is the market portfolio**

- The CAPM model writes :

$$\tilde{r}_i - r_f = \beta_i(\tilde{r}_M - r_f)$$

- β is the **right measure of risk**

- Empirical research is mixed – Let's see tomorrow !

- but Graham and Harvey (2000): 74% of firms use the CAPM to estimate the cost of capital and Asset management industry uses it massively for performance attribution

Discussion