```
In[1471]:= (* This is a Mathematica notebook that accompanies the article
             on "The Newton-Puiseux Algorithm and Effective Algebraic Series"
                                                                   Reihe
             to make some of the computations more comprehensible *)

In[1772]:= Clear[series]
            lösche

In[1773]:= SetDirectory["/Users/manfredbuchacher/Documents/Uni:RICAM/newtonPuiseux/ISSAC"]
            lege Verzeichnis fest

Out[1773]= /Users/manfredbuchacher/Documents/Uni:RICAM/newtonPuiseux/ISSAC

In[1774]:= << newtonPuiseux.m

In[1749]:= (* Example 2 *)

In[1750]:= m = 4 x^2 y + (x^2 y + x y^2 + x y + y) ^2 - Z^2
```

Out[1750]= $4 x^2 y + \left(y + x y + x^2 y + x y^2\right)^2 - Z^2$

```
In[1751]:= vertices = NewtonPolytope[m, {x, y, Z}]

Out[1751]= {{0, 0, 2}, {0, 2, 0}, {2, 1, 0}, {2, 4, 0}, {4, 2, 0}}

In[1752]:= edges = Select[GetEdges[vertices], Last[#[[1]]] ≠ Last[#[[2]]] &]
                   wähle aus                        letztes Element  letztes Element

Out[1752]= {{{0, 0, 2}, {0, 2, 0}}, {{0, 0, 2}, {2, 1, 0}},
             {{0, 0, 2}, {2, 4, 0}}, {{0, 0, 2}, {4, 2, 0}}}

In[1753]:= edge = edges[[1]];

In[1754]:= bCone = barrierCone[vertices, edge]

Out[1754]= {{1, 1}, {2, -1}}

In[1755]:= w = {-Sqrt[2], -1};
                 Quadratwurzel

In[1756]:= (* check whether w is indeed compatible with bCone *)
           w.# & /@ bCone // N
                            numerischer Wert

Out[1756]= {-2.41421, -1.82843}

In[1757]:= series[m, {x, y, Z}, edge, w, 0]

Out[1757]= {{-y, {{1, 1}, {2, -1}}}, {y, {{1, 1}, {2, -1}}}}

In[1758]:= (* Example 3 *)

In[1775]:= m = x + y - (1 + x + y) Z

Out[1775]= x + y - (1 + x + y) Z

In[1776]:= vertices = NewtonPolytope[m, {x, y, Z}]

Out[1776]= {{0, 0, 1}, {0, 1, 0}, {0, 1, 1}, {1, 0, 0}, {1, 0, 1}}
```

```
In[1777]:= edges = Select[GetEdges[vertices], Last[#[[1]]] ≠ Last[#[[2]]] &]
                   wähle aus                    letztes Element  letztes Element

Out[1777]= {{{0, 0, 1}, {0, 1, 0}}, {{0, 0, 1}, {1, 0, 0}},
            {{0, 1, 0}, {0, 1, 1}}, {{1, 0, 0}, {1, 0, 1}}}

In[1778]:= bCones = barrierCone[vertices, #] & /@ edges

Out[1778]= {{{0, 1}, {1, -1}}, {{1, 0}, {-1, 1}}, {{0, -1}, {1, -1}}, {{-1, 0}, {-1, 1}}}

In[1779]:= w1 = {-2 + 1 / Sqrt[2], -1};
                          Quadratwurzel
           w2 = {-1 + 1 / Sqrt[2], -2};
                          Quadratwurzel
           w3 = {-1 + 1 / Sqrt[2], 1};
                          Quadratwurzel
           w4 = -{-1 + 1 / Sqrt[2], -1};
                           Quadratwurzel

In[1783]:= series[m, {x, y, Z}, edges[[1]], w1, 0]
           series[m, {x, y, Z}, edges[[2]], w2, 0]
           series[m, {x, y, Z}, edges[[3]], w3, 0]
           series[m, {x, y, Z}, edges[[4]], w4, 0]

Out[1783]= {{y, {{0, 1}, {1, -1}}}}

Out[1784]= {{x, {{1, 0}, {-1, 1}}}}

Out[1785]= {{1, {{0, -1}, {1, -1}}}}

Out[1786]= {{1, {{-1, 0}, {-1, 1}}}}

In[1788]:= series[m, {x, y, Z}, edges[[2]], w2, 9]
```

Out[1788]= $\{\{x - x^2 + x^3 - x^4 + x^5 - x^6 + x^7 + y - 2\,x\,y,\ \{\{-1, 1\}, \{7, -1\}\}\}\}$

In[1789]:= Sort[MonomialList[$x - x^2 + x^3 - x^4 + x^5 - x^6 + y$], Exponents[#, {x, y}][[1]].w1 &]
sorti… Liste der Monome

Out[1789]= $\{-x^6, x^5, -x^4, x^3, -x^2, x, y\}$

```
           (* Example 5 *)

In[1790]:= m = x + y - (1 + x + y) Z

Out[1790]= x + y - (1 + x + y) Z

In[1791]:= vertices = NewtonPolytope[m, {x, y, Z}]

Out[1791]= {{0, 0, 1}, {0, 1, 0}, {0, 1, 1}, {1, 0, 0}, {1, 0, 1}}

In[1792]:= edges = Select[GetEdges[vertices], Last[#[[1]]] ≠ Last[#[[2]]] &]
                   wähle aus                    letztes Element  letztes Element

Out[1792]= {{{0, 0, 1}, {0, 1, 0}}, {{0, 0, 1}, {1, 0, 0}},
            {{0, 1, 0}, {0, 1, 1}}, {{1, 0, 0}, {1, 0, 1}}}

In[1793]:= edge1 = edges[[2]];
           edge2 = edges[[1]];
```

```
In[1795]:= bCones = barrierCone[vertices, #] & /@ {edge1, edge2}

Out[1795]= {{{1, 0}, {-1, 1}}, {{0, 1}, {1, -1}}}

In[1796]:= w1 = {-1 + 1 / Sqrt[2], -1};
                        Quadratwurzel
           w2 = {-1 + 1 / Sqrt[2], 1};
                        Quadratwurzel

In[1798]:= series[m, {x, y, Z}, edge1, w1, 0]
           series[m, {x, y, Z}, edge2, w2, 0]

Out[1798]= {{x, {{1, 0}, {-1, 1}}}}

Out[1799]= {{y, {{0, 1}, {1, -1}}}}

           (* Example 6: see Example 5 *)

           (* Example 7 *)

In[1800]:= m = (1 - x) ((1 - y) Z - 1)

Out[1800]= (1 - x) (-1 + (1 - y) Z)

In[1801]:= vertices = NewtonPolytope[m, {x, y, Z}]

Out[1801]= {{0, 0, 0}, {0, 0, 1}, {0, 1, 1}, {1, 0, 0}, {1, 0, 1}, {1, 1, 1}}

In[1802]:= edges = Select[GetEdges[vertices], Last[#[[1]]] ≠ Last[#[[2]]] &]
                   wähle aus                   letztes Element  letztes Element

Out[1802]= {{{0, 0, 0}, {0, 0, 1}}, {{0, 0, 0}, {0, 1, 1}},
            {{1, 0, 0}, {1, 0, 1}}, {{1, 0, 0}, {1, 1, 1}}}

In[1803]:= bCones = barrierCone[vertices, #] & /@ edges

Out[1803]= {{{1, 0}, {0, 1}}, {{1, 0}, {0, -1}}, {{0, 1}, {-1, 0}}, {{0, -1}, {-1, 0}}}

In[1804]:= orders = getDual[#] & /@ bCones
```

Out[1804]= $\left\{\left\{-1+\frac{1}{1000\sqrt{2}},-1\right\},\left\{-1+\frac{1}{1000\sqrt{2}},1\right\},\left\{1+\frac{1}{1000\sqrt{2}},-1\right\},\left\{1+\frac{1}{1000\sqrt{2}},1\right\}\right\}$

```
In[1805]:= series[m, {x, y, Z}, edges[[1]], orders[[1]], 0]

Out[1805]= {{1, {{1, 0}, {0, 1}}}}

In[1806]:= m = (1 - y) Z - 1

Out[1806]= -1 + (1 - y) Z

In[1807]:= vertices = NewtonPolytope[m, {y, Z}]

Out[1807]= {{0, 0}, {0, 1}, {1, 1}}

In[1808]:= edges = Select[GetEdges[vertices], Last[#[[1]]] ≠ Last[#[[2]]] &]
                   wähle aus                   letztes Element  letztes Element

Out[1808]= {{{0, 0}, {0, 1}}, {{0, 0}, {1, 1}}}

In[1809]:= bCones = barrierCone[vertices, #] & /@ edges

Out[1809]= {{{1}}, {{-1}}}
```

In[1810]:=
```
orders = getDual[#] & /@ bCones
```

Out[1810]= $\left\{\left\{-1+\frac{1}{1000\sqrt{2}}\right\},\left\{1+\frac{1}{1000\sqrt{2}}\right\}\right\}$

In[1811]:=
```
series[m, {y, Z}, edges[[1]], orders[[1]], 0]
```

Out[1811]= {{1, {{1}}}}

In[1812]:=
```
(* Example 8 and Example 9 *)
```

In[1813]:=
```
m = 1 + x + y + (1 + x y + 2 y) Z + y Z^2
```

Out[1813]= $1+x+y+(1+2y+xy)Z+yZ^2$

In[1814]:=
```
vertices = NewtonPolytope[m, {x, y, Z}]
```

Out[1814]= {{0, 0, 0}, {0, 0, 1}, {0, 1, 0}, {0, 1, 2}, {1, 0, 0}, {1, 1, 1}}

In[1815]:=
```
edges = Select[GetEdges[vertices], Last[#[[1]]] ≠ Last[#[[2]]] &]
```
wähle aus — letztes Element — letztes Element

Out[1815]= {{{0, 0, 0}, {0, 0, 1}}, {{0, 0, 1}, {0, 1, 2}},
{{0, 0, 1}, {1, 0, 0}}, {{0, 1, 0}, {0, 1, 2}},
{{0, 1, 0}, {1, 1, 1}}, {{0, 1, 2}, {1, 1, 1}}, {{1, 0, 0}, {1, 1, 1}}}

In[1816]:=
```
bCones = barrierCone[vertices, #] & /@ edges
```

Out[1816]= {{{1, 0}, {0, 1}}, {{0, 1}, {1, 1}}, {{-1, 0}, {1, 1}}, {{1, 0}, {0, -1}},
{{-1, 0}, {1, -1}}, {{-1, 0}, {-1, -1}}, {{-1, 1}, {-1, -1}}}

In[1817]:=
```
w = {-1 + 1 / Sqrt[2], -1}
```
Quadratwurzel

Out[1817]= $\left\{-1+\frac{1}{\sqrt{2}},-1\right\}$

In[1818]:=
```
series[m, {x, y, Z}, edges[[1]], w, 0]
```

Out[1818]= {{-1, {{1, 0}, {0, 1}}}}

In[1819]:=
```
series[m, {x, y, Z}, edges[[1]], w, 2]
```

Out[1819]= {{-1 - x, {{0, 1}, {1, 1}}}}

In[1820]:=
```
series[m, {x, y, Z}, edges[[1]], w, 3]
```

Out[1820]= {{-1 - x + x y, {{1, 1}, {1, 2}}}}

In[1821]:=
```
series[m, {x, y, Z}, edges[[1]], w, 5]
```

Out[1821]= $\{\{-1-x+xy+x^2y^2-x^2y^3, \{\{1,0\},\{1,2\}\}\}\}$

```
(* Example 12 *)
```

In[1823]:=
```
m1 = (1 + x + x^2 - Y) (x^2 - (1 - x) Y)
```

Out[1823]= $(1+x+x^2-Y)(x^2-(1-x)Y)$

In[1824]:=
```
vertices1 = NewtonPolytope[m1, {x, Y}]
```

Out[1824]= {{0, 1}, {0, 2}, {1, 2}, {2, 0}, {3, 1}, {4, 0}}

In[1825]:=
```
edges1 = Select[GetEdges[vertices1], Last[#[[1]]] ≠ Last[#[[2]]] &]
```
wähle aus — letztes Element — letztes Element

Out[1825]= {{{0, 1}, {0, 2}}, {{0, 1}, {2, 0}}, {{1, 2}, {3, 1}}, {{3, 1}, {4, 0}}}

In[1826]:=
```
bCones1 = barrierCone[vertices1, #] & /@ edges1
```

Out[1826]= {{{1}}, {{1}}, {{-1}}, {{-1}}}

In[1827]:=
```
w1 = {-Sqrt[2]};
```
Quadratwurzel

In[1828]:=
```
series[m1, {x, Y}, edges1[[2]], w1, 0]
```

Out[1828]= $\{\{x^2, \{\{1\}\}\}\}$

In[1829]:=
```
m2 = Y (x^2 + (1 - x) Y)
```

Out[1829]= $Y\left(x^2 + (1 - x)\, Y\right)$

In[1830]:=
```
vertices2 = NewtonPolytope[m2, {x, Y}]
```

Out[1830]= {{0, 2}, {1, 2}, {2, 1}}

In[1831]:=
```
edges2 = Select[GetEdges[vertices2], Last[#[[1]]] ≠ Last[#[[2]]] &]
```
wähle aus — letztes Element — letztes Element

Out[1831]= {{{0, 2}, {2, 1}}, {{1, 2}, {2, 1}}}

In[1832]:=
```
bCones2 = barrierCone[vertices2, #] & /@ edges2
```

Out[1832]= {{{1}}, {{-1}}}

In[1833]:=
```
w2 = {Sqrt[2]};
```
Quadratwurzel

In[1834]:=
```
series[m2, {x, Y}, edges2[[2]], w2, 0]
```

Out[1834]= {{x, {{-1}}}}

In[1835]:=
```
m = GroebnerBasis[{m1 /. Y → X, m2, Z - X - Y}, {x, X, Y, Z}, {X, Y}][[1]]
```
Gröbnerbasis

Out[1835]=
$$-x^2 Z - x^3 Z + 2 x^5 Z + 2 x^6 Z + x^7 Z + Z^2 + x^2 Z^2 - 2 x^3 Z^2 - x^4 Z^2 - x^5 Z^2 + x^6 Z^2 - 2 Z^3 + 2 x Z^3 + 2 x^3 Z^3 - 2 x^4 Z^3 + Z^4 - 2 x Z^4 + x^2 Z^4$$

In[1836]:=
```
m = Factor[m]
```
faktorisiere

Out[1836]= $\left(1 + x + x^2 - Z\right) Z \left(-1 + x^2 + x^3 + Z - x Z\right) \left(x^2 - Z + x Z\right)$

In[1837]:=
```
vertices = NewtonPolytope[m, {x, Z}]
```

Out[1837]= {{0, 2}, {0, 4}, {2, 1}, {2, 4}, {6, 2}, {7, 1}}

In[1838]:=
```
edges = Select[GetEdges[vertices], Last[#[[1]]] ≠ Last[#[[2]]] &]
```
wähle aus — letztes Element — letztes Element

Out[1838]= {{{0, 2}, {0, 4}}, {{0, 2}, {2, 1}}, {{2, 4}, {6, 2}}, {{6, 2}, {7, 1}}}

In[1839]:=
```
bCones = barrierCone[vertices, #] & /@ edges
```

Out[1839]= {{{1}}, {{1}}, {{-1}}, {{-1}}}

In[1840]:=
```
series[m, {x, Z}, edges[[1]], {-Sqrt[2]}, 0]
                                Quadratwurzel
series[m, {x, Z}, edges[[2]], {-Sqrt[2]}, 0]
                                Quadratwurzel
series[m, {x, Z}, edges[[3]], {Sqrt[2]}, 0]
                               Quadratwurzel
series[m, {x, Z}, edges[[4]], {Sqrt[2]}, 0]
                               Quadratwurzel
```

Out[1840]= $\{\{1 + x - x^{5/2}, \{\{1\}, \{-1\}\}\}, \{1 + x + x^{5/2}, \{\{1\}, \{-1\}\}\}\}$

Out[1841]= $\{\{x^2, \{\{1\}\}\}\}$

Out[1842]= $\{\{x + x^2, \{\{-1\}\}\}, \{2\,x + x^2, \{\{-1\}\}\}\}$

Out[1843]= $\{\{-x, \{\{-1\}\}\}\}$