```
In[367]:= (* This is a Mathematica notebook that accompanies the article
            on "Effective Arithmetic for Multivariate Algebraic Series"
                                                                  Reihe
            to make some of the computations more comprehensible *)

In[368]:= SetDirectory["/Users/manfredbuchacher/Documents/Universität/newtonPuiseux"]
          lege Verzeichnis fest

Out[•]= /Users/manfredbuchacher/Documents/Universität/newtonPuiseux

In[369]:= << newtonPuiseux.m

In[370]:= (* Example 1 *)

In[371]:= m = 4 x^2 y + (x^2 y + x y^2 + x y + y)^2 - Z^2
```

Out[•]= $4x^2 y + (y + x y + x^2 y + x y^2)^2 - Z^2$

```
In[372]:= vertices = NewtonPolytope[m, {x, y, Z}]

Out[•]= {{0, 0, 2}, {0, 2, 0}, {2, 1, 0}, {2, 4, 0}, {4, 2, 0}}

In[373]:= edges = Select[GetEdges[vertices], Last[#[[1]]] ≠ Last[#[[2]]] &]
                 wähle aus                  letztes Element   letztes Element

Out[•]= {{{0, 0, 2}, {0, 2, 0}}, {{0, 0, 2}, {2, 1, 0}},
         {{0, 0, 2}, {2, 4, 0}}, {{0, 0, 2}, {4, 2, 0}}}

In[374]:= edge = edges[[1]];

In[375]:= bCone = barrierCone[vertices, edge]

Out[•]= {{1, 1}, {2, -1}}

In[376]:= w = {-Sqrt[2], -1};
              Quadratwurzel

In[377]:= series[m, {x, y, Z}, edge, w, 0]

Out[•]= {{-y, {{1, 1}, {2, -1}}}, {y, {{1, 1}, {2, -1}}}}

In[378]:= (* Example 2 *)

In[379]:= m = x + y - (1 + x + y) Z

Out[•]= x + y - (1 + x + y) Z

In[380]:= vertices = NewtonPolytope[m, {x, y, Z}]

Out[•]= {{0, 0, 1}, {0, 1, 0}, {0, 1, 1}, {1, 0, 0}, {1, 0, 1}}

In[381]:= edges = Select[GetEdges[vertices], Last[#[[1]]] ≠ Last[#[[2]]] &]
                 wähle aus                  letztes Element   letztes Element

Out[•]= {{{0, 0, 1}, {0, 1, 0}}, {{0, 0, 1}, {1, 0, 0}},
         {{0, 1, 0}, {0, 1, 1}}, {{1, 0, 0}, {1, 0, 1}}}

In[382]:= bCones = barrierCone[vertices, #] & /@ edges

Out[•]= {{{0, 1}, {1, -1}}, {{1, 0}, {-1, 1}}, {{0, -1}, {1, -1}}, {{-1, 0}, {-1, 1}}}
```

```
In[383]:= w1 = {-1 + 1 / Sqrt[2], 1};
                    Quadratwurzel
          w2 = {-1 + 1 / Sqrt[2], -1};
                    Quadratwurzel
          w3 = {-1 + 1 / Sqrt[2], -2};
                    Quadratwurzel
          w4 = {-2 + 1 / Sqrt[2], -1};
                    Quadratwurzel

In[387]:= series[m, {x, y, Z}, edges[[1]], w1, 0]
          series[m, {x, y, Z}, edges[[2]], w2, 0]
          series[m, {x, y, Z}, edges[[3]], w3, 0]
          series[m, {x, y, Z}, edges[[4]], w4, 0]

Out[∘]= {{y, {{0, 1}, {1, -1}}}}

Out[∘]= {{x, {{1, 0}, {-1, 1}}}}

Out[∘]= {{1, {{0, -1}, {1, -1}}}}

Out[∘]= {{1, {{-1, 0}, {-1, 1}}}}

In[391]:= series[m, {x, y, Z}, edges[[3]], w3, 9]
Out[∘]= {{x - x^2 + x^3 - x^4 + x^5 - x^6 + y, {{0, 1}, {7, -1}}}}

In[392]:= Sort[MonomialList[x - x^2 + x^3 - x^4 + x^5 - x^6 + y], Exponents[#, {x, y}][[1]].w4 &]
          sorti⋯ Liste der Monome
Out[∘]= {-x^6, x^5, -x^4, x^3, -x^2, x, y}

          (* Example 3 *)

In[406]:= m = x + y - (1 + x + y) Z
Out[∘]= x + y - (1 + x + y) Z

In[407]:= vertices = NewtonPolytope[m, {x, y, Z}]
Out[∘]= {{0, 0, 1}, {0, 1, 0}, {0, 1, 1}, {1, 0, 0}, {1, 0, 1}}

In[408]:= edges = Select[GetEdges[vertices], Last[#[[1]]] ≠ Last[#[[2]]] &]
                  wähle aus                  letztes Element   letztes Element
Out[∘]= {{{0, 0, 1}, {0, 1, 0}}, {{0, 0, 1}, {1, 0, 0}},
          {{0, 1, 0}, {0, 1, 1}}, {{1, 0, 0}, {1, 0, 1}}}

In[409]:= edge1 = edges[[2]];
          edge2 = edges[[1]];

In[411]:= bCones = barrierCone[vertices, #] & /@ {edge1, edge2}
Out[∘]= {{{1, 0}, {-1, 1}}, {{0, 1}, {1, -1}}}

In[412]:= w1 = {-1 + 1 / Sqrt[2], -1};
                    Quadratwurzel
          w2 = {-1 + 1 / Sqrt[2], 1};
                    Quadratwurzel
```

In[414]:=
```
series[m, {x, y, Z}, edge1, w1, 0]
series[m, {x, y, Z}, edge2, w2, 0]
```
Out[∘]= {{x, {{1, 0}, {-1, 1}}}}

Out[∘]= {{y, {{0, 1}, {1, -1}}}}

```
(* Example 4: see Example 3 *)

(* Example 5 *)
```

In[418]:= `m = (1 - x) ((1 - y) Z - 1)`

Out[∘]= $(1-x)(-1+(1-y)Z)$

In[419]:= `vertices = NewtonPolytope[m, {x, y, Z}]`

Out[∘]= {{0, 0, 0}, {0, 0, 1}, {0, 1, 1}, {1, 0, 0}, {1, 0, 1}, {1, 1, 1}}

In[420]:= `edges = Select[GetEdges[vertices], Last[#[[1]]] ≠ Last[#[[2]]] &]`
wähle aus | letztes Element | letztes Element

Out[∘]= {{{0, 0, 0}, {0, 0, 1}}, {{0, 0, 0}, {0, 1, 1}}, {{1, 0, 0}, {1, 0, 1}}, {{1, 0, 0}, {1, 1, 1}}}

In[421]:= `bCones = barrierCone[vertices, #] & /@ edges`

Out[∘]= {{{1, 0}, {0, 1}}, {{1, 0}, {0, -1}}, {{0, 1}, {-1, 0}}, {{0, -1}, {-1, 0}}}

In[422]:= `orders = getDual[#] & /@ bCones`

Out[∘]= $\left\{\left\{-1+\frac{1}{1000\sqrt{2}}, -1\right\}, \left\{-1+\frac{1}{1000\sqrt{2}}, 1\right\}, \left\{1+\frac{1}{1000\sqrt{2}}, -1\right\}, \left\{1+\frac{1}{1000\sqrt{2}}, 1\right\}\right\}$

In[423]:= `series[m, {x, y, Z}, edges[[1]], orders[[1]], 0]`

Out[∘]= {{1, {{1, 0}, {0, 1}}}}

In[424]:= `m = (1 - y) Z - 1`

Out[∘]= $-1+(1-y)Z$

In[425]:= `vertices = NewtonPolytope[m, {y, Z}]`

Out[∘]= {{0, 0}, {0, 1}, {1, 1}}

In[426]:= `edges = Select[GetEdges[vertices], Last[#[[1]]] ≠ Last[#[[2]]] &]`
wähle aus | letztes Element | letztes Element

Out[∘]= {{{0, 0}, {0, 1}}, {{0, 0}, {1, 1}}}

In[427]:= `bCones = barrierCone[vertices, #] & /@ edges`

Out[∘]= {{{1}}, {{-1}}}

In[428]:= `orders = getDual[#] & /@ bCones`

Out[∘]= $\left\{\left\{-1+\frac{1}{1000\sqrt{2}}\right\}, \left\{1+\frac{1}{1000\sqrt{2}}\right\}\right\}$

In[429]:= `series[m, {y, Z}, edges[[1]], orders[[1]], 0]`

Out[∘]= {{1, {{1}}}}

```
(* Example 6 and Example 7 *)

In[431]:= m = 1 + x + y + (1 + x y + 2 y) Z + y Z^2
Out[•]= 1 + x + y + (1 + 2 y + x y) Z + y Z^2

In[432]:= vertices = NewtonPolytope[m, {x, y, Z}]
Out[•]= {{0, 0, 0}, {0, 0, 1}, {0, 1, 0}, {0, 1, 2}, {1, 0, 0}, {1, 1, 1}}

In[433]:= edges = Select[GetEdges[vertices], Last[#[[1]]] ≠ Last[#[[2]]] &]
                   wähle aus                       letztes Element  letztes Element
Out[•]= {{{0, 0, 0}, {0, 0, 1}}, {{0, 0, 1}, {0, 1, 2}},
         {{0, 0, 1}, {1, 0, 0}}, {{0, 1, 0}, {0, 1, 2}},
         {{0, 1, 0}, {1, 1, 1}}, {{0, 1, 2}, {1, 1, 1}}, {{1, 0, 0}, {1, 1, 1}}}

In[434]:= bCones = barrierCone[vertices, #] & /@ edges
Out[•]= {{{1, 0}, {0, 1}}, {{0, 1}, {1, 1}}, {{-1, 0}, {1, 1}}, {{1, 0}, {0, -1}},
         {{-1, 0}, {1, -1}}, {{-1, 0}, {-1, -1}}, {{-1, 1}, {-1, -1}}}

In[435]:= w = {-1 + 1 / Sqrt[2], -1}
                        Quadratwurzel
```

Out[•]= $\left\{-1+\frac{1}{\sqrt{2}}, -1\right\}$

```
In[436]:= series[m, {x, y, Z}, edges[[1]], w, 0]
Out[•]= {{-1, {{1, 0}, {0, 1}}}}

In[437]:= series[m, {x, y, Z}, edges[[1]], w, 2]
Out[•]= {{-1 - x, {{0, 1}, {1, 1}}}}

In[438]:= series[m, {x, y, Z}, edges[[1]], w, 3]
Out[•]= {{-1 - x + x y, {{1, 1}, {1, 2}}}}

In[439]:= series[m, {x, y, Z}, edges[[1]], w, 5]
Out[•]= {{-1 - x + x y + x^2 y^2 - x^2 y^3, {{1, 0}, {1, 2}}}}

In[440]:= (* Example 10 *)

In[441]:= m1 = (1 + x + x^2 - Y) (x^2 - (1 - x) Y)
Out[•]= (1 + x + x^2 - Y) (x^2 - (1 - x) Y)

In[442]:= vertices1 = NewtonPolytope[m1, {x, Y}]
Out[•]= {{0, 1}, {0, 2}, {1, 2}, {2, 0}, {3, 1}, {4, 0}}

In[443]:= edges1 = Select[GetEdges[vertices1], Last[#[[1]]] ≠ Last[#[[2]]] &]
                    wähle aus                        letztes Element  letztes Element
Out[•]= {{{0, 1}, {0, 2}}, {{0, 1}, {2, 0}}, {{1, 2}, {3, 1}}, {{3, 1}, {4, 0}}}

In[444]:= bCones1 = barrierCone[vertices1, #] & /@ edges1
Out[•]= {{{1}}, {{1}}, {{-1}}, {{-1}}}
```

```
In[445]:= w1 = {-Sqrt[2]};
                Quadratwurzel

In[446]:= series[m1, {x, Y}, edges1[[2]], w1, 0]

Out[•]= {{x^2, {{1}}}}

In[447]:= m2 = Y (x^2 + (1 - x) Y)

Out[•]= Y (x^2 + (1 - x) Y)

In[448]:= vertices2 = NewtonPolytope[m2, {x, Y}]

Out[•]= {{0, 2}, {1, 2}, {2, 1}}

In[449]:= edges2 = Select[GetEdges[vertices2], Last[#[[1]]] ≠ Last[#[[2]]] &]
                   wähle aus                     letztes Element    letztes Element

Out[•]= {{{0, 2}, {2, 1}}, {{1, 2}, {2, 1}}}

In[450]:= bCones2 = barrierCone[vertices2, #] & /@ edges2

Out[•]= {{{1}}, {{-1}}}

In[451]:= w2 = {Sqrt[2]};
               Quadratwurzel

In[452]:= series[m2, {x, Y}, edges2[[2]], w2, 0]

Out[•]= {{x, {{-1}}}}

In[453]:= m = GroebnerBasis[{m1 /. Y → X, m2, Z - X - Y}, {x, X, Y, Z}, {X, Y}][[1]]
              Gröbnerbasis

Out[•]= -x^2 Z - x^3 Z + 2 x^5 Z + 2 x^6 Z + x^7 Z + Z^2 + x^2 Z^2 - 2 x^3 Z^2 -
         x^4 Z^2 - x^5 Z^2 + x^6 Z^2 - 2 Z^3 + 2 x Z^3 + 2 x^3 Z^3 - 2 x^4 Z^3 + Z^4 - 2 x Z^4 + x^2 Z^4

In[454]:= m = Factor[m]
              faktorisiere

Out[•]= (1 + x + x^2 - Z) Z (-1 + x^2 + x^3 + Z - x Z) (x^2 - Z + x Z)

In[455]:= vertices = NewtonPolytope[m, {x, Z}]

Out[•]= {{0, 2}, {0, 4}, {2, 1}, {2, 4}, {6, 2}, {7, 1}}

In[456]:= edges = Select[GetEdges[vertices], Last[#[[1]]] ≠ Last[#[[2]]] &]
                  wähle aus                    letztes Element    letztes Element

Out[•]= {{{0, 2}, {0, 4}}, {{0, 2}, {2, 1}}, {{2, 4}, {6, 2}}, {{6, 2}, {7, 1}}}

In[457]:= bCones = barrierCone[vertices, #] & /@ edges

Out[•]= {{{1}}, {{1}}, {{-1}}, {{-1}}}
```

In[458]:=
```
series[m, {x, Z}, edges⟦1⟧, {-Sqrt[2]}, 0]
                                 Quadratwurzel
series[m, {x, Z}, edges⟦2⟧, {-Sqrt[2]}, 0]
                                 Quadratwurzel
series[m, {x, Z}, edges⟦3⟧, {Sqrt[2]}, 0]
                                Quadratwurzel
series[m, {x, Z}, edges⟦4⟧, {Sqrt[2]}, 0]
                                Quadratwurzel
```

Out[∘]= $\{\{1 + x - x^{5/2}, \{\{1\}, \{-1\}\}\}, \{1 + x + x^{5/2}, \{\{1\}, \{-1\}\}\}\}$

Out[∘]= $\{\{x^2, \{\{1\}\}\}\}$

Out[∘]= $\{\{x + x^2, \{\{-1\}\}\}, \{2\,x + x^2, \{\{-1\}\}\}\}$

Out[∘]= $\{\{-x, \{\{-1\}\}\}\}$