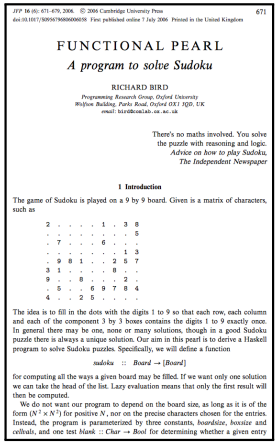
Coding dojo
A simple Sudoku solver in Haskell

B◦FP

Bucharest FP #027

Welcome



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FUNCTIONAL PEARL

A program to solve Sudoku

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There's no maths involved. You solve the puzzle with reasoning and logic.
Advice on how to play Sudoku,
The Independent Newspaper

1 Introduction

The game of Sudoku is played on a 9 by 9 board. Given is a matrix of characters, such as

```
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. 7 . . . 6 . . .
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```

The idea is to fill in the dots with the digits 1 to 9 so that each row, each column and each of the component 3 by 3 boxes contains the digits 1 to 9 exactly once. In general there may be one, none or many solutions, though in a good Sudoku puzzle there is always a unique solution. Our aim in this pearl is to derive a Haskell program to solve Sudoku puzzles. Specifically, we will define a function

sudoku :: *Board* → [*Board*]

for computing all the ways a given board may be filled. If we want only one solution we can take the head of the list. Lazy evaluation means that only the first result will then be computed.

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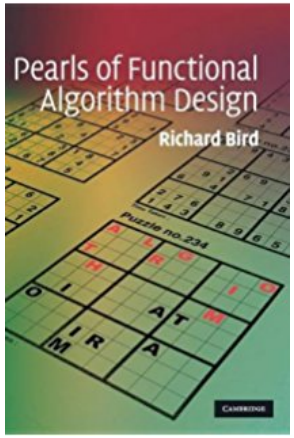

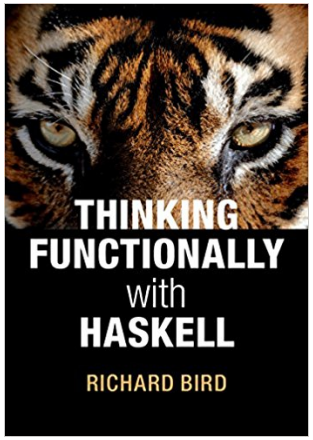

- Implement a Sudoku solver as described in (Bird, 2006, 2010, 2014)

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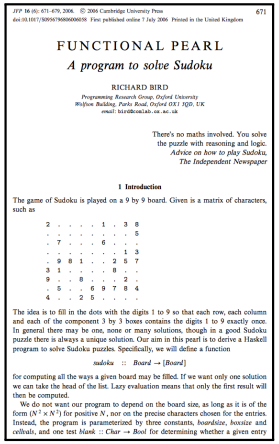

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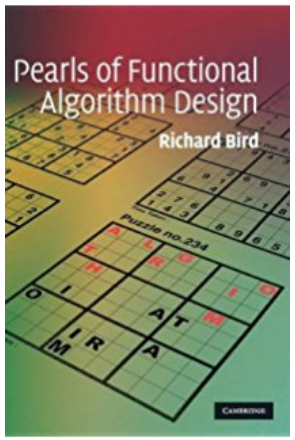

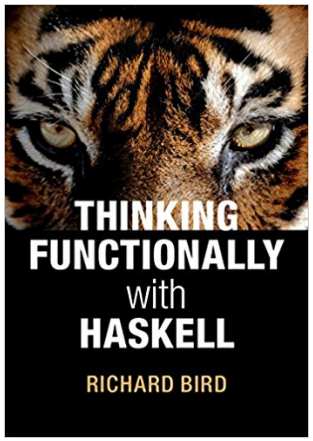

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- Twelve short functions:
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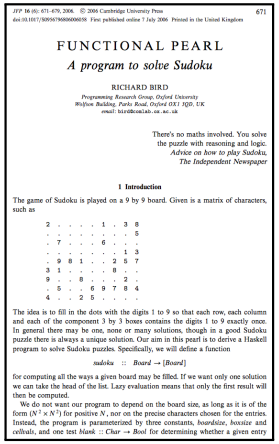

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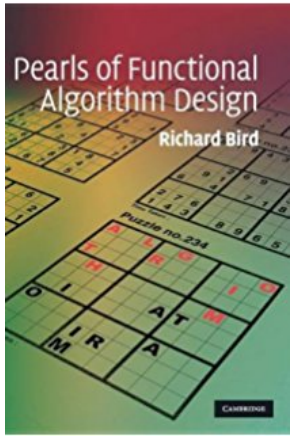

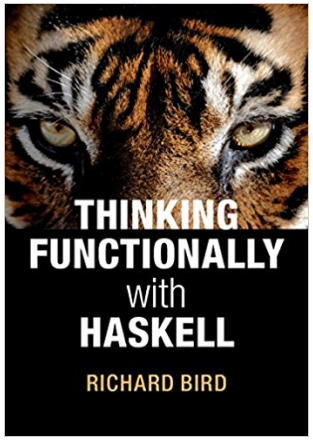

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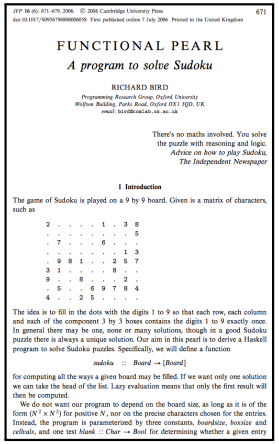

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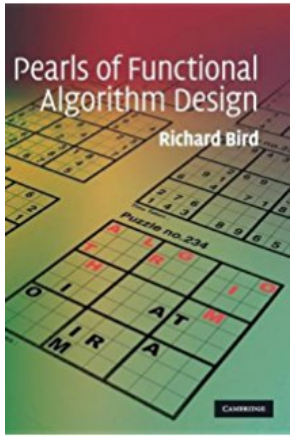

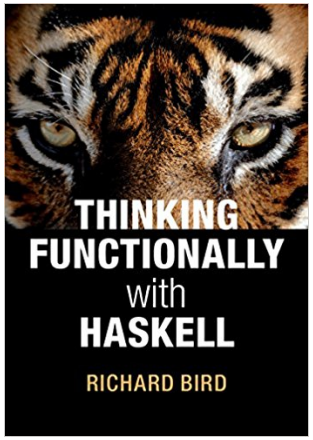

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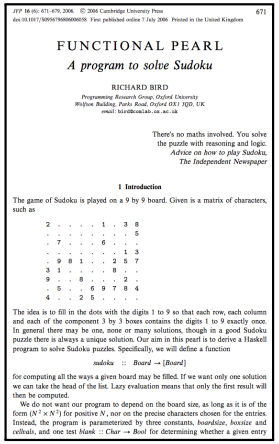

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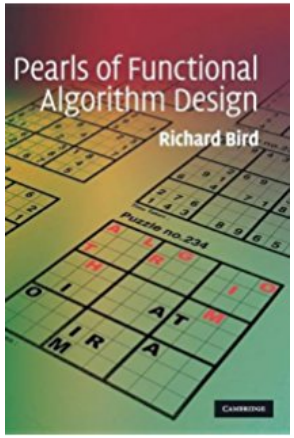

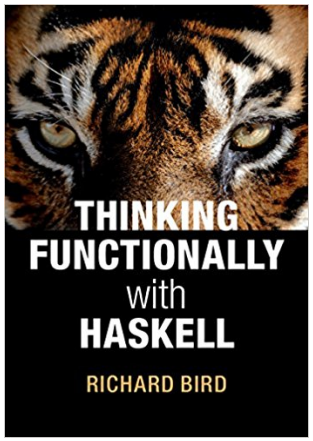

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  - Wholemeal programming (prevents a disease called “indexitis”)
  - Higher-order functions, recursion, point-free style

How to play Sudoku

	2	4	
1			3
4			2
	1	3	

$N = 2$

	2		5		1		9	
8			2		3			6
	3			6			7	
		1				6		
5	4						1	9
		2				7		
	9			3			8	
2			8		4			7
	1		9		7		6	

$N = 3$

Fill in the empty cells with digits 1 to N^2 such that every row, column and $N \times N$ box contains the digits 1 to N^2.

How to play Sudoku

3	2	4	1
1	4	2	3
4	3	1	2
2	1	3	4

$N = 2$

4	2	6	5	7	1	3	9	8
8	5	7	2	9	3	1	4	6
1	3	9	4	6	8	2	7	5
9	7	1	3	8	5	6	2	4
5	4	3	7	2	6	8	1	9
6	8	2	1	4	9	7	5	3
7	9	4	6	3	2	5	8	1
2	6	5	8	1	4	9	3	7
3	1	8	9	5	7	4	6	2

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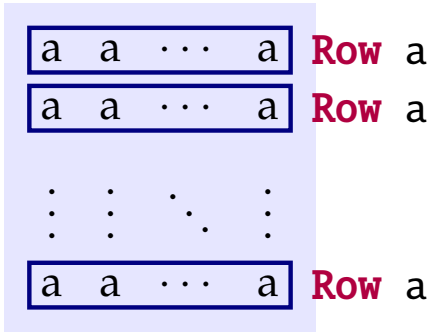
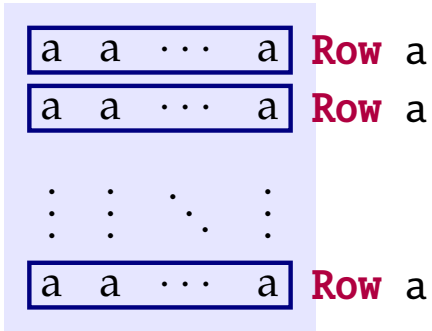
Fill in the empty cells with digits 1 to N^2 such that every row, column and $N \times N$ box contains the digits 1 to N^2.

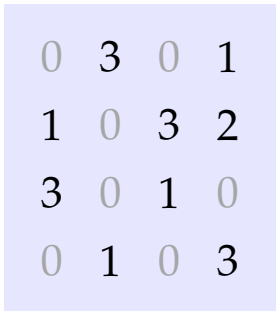
Data types




```
type Matrix a = [Row a]
type Row a = [a]
```

Data types



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type Grid = Matrix Digit
type Digit = Int
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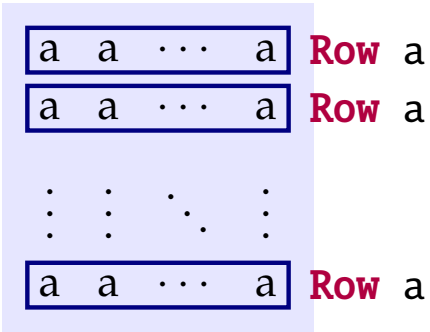

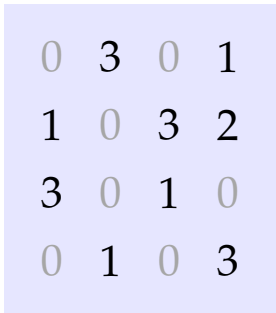


```
type Matrix a = [Row a]
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```

We assume that digit zero indicates an empty cell:

```
isEmpty :: Digit -> Bool
isEmpty 0 = True
isEmpty _ = False
```

Exercise 1: `solve` [⋆]

```haskell
solve :: Grid -> [Grid]
solve = undefined
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completions :: Grid -> [Grid]
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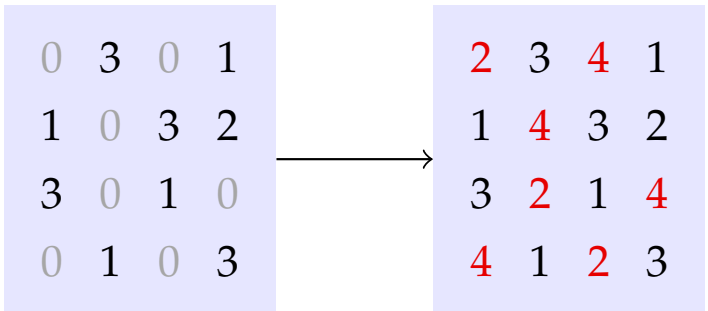

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Given:

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-- Replaces empty entries with all possible choices
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-- Generates a list of all possible boards
-- from a given matrix of choices
expand :: Matrix [Digit] -> [Grid]
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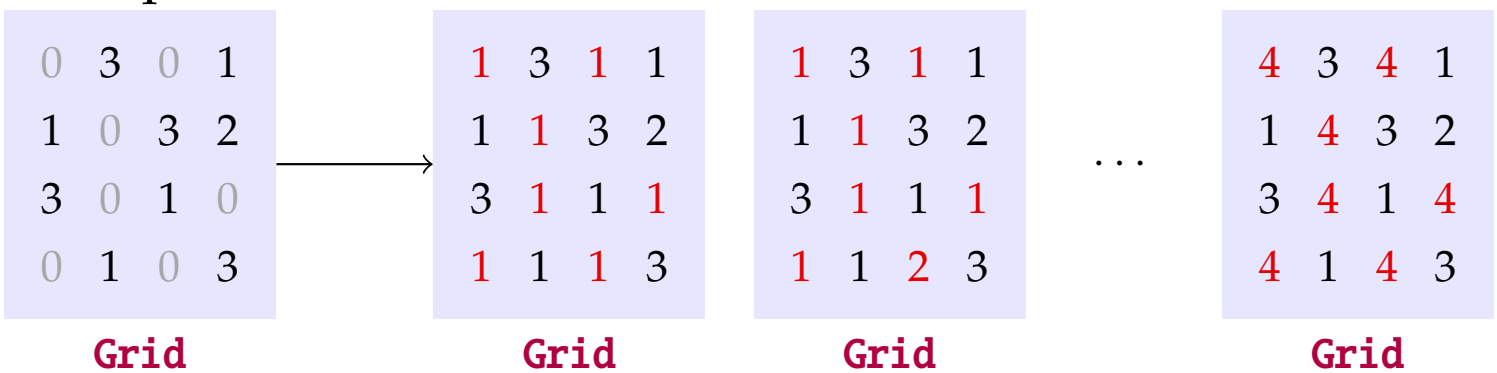

Exercise 3: `choices` $[\star]$

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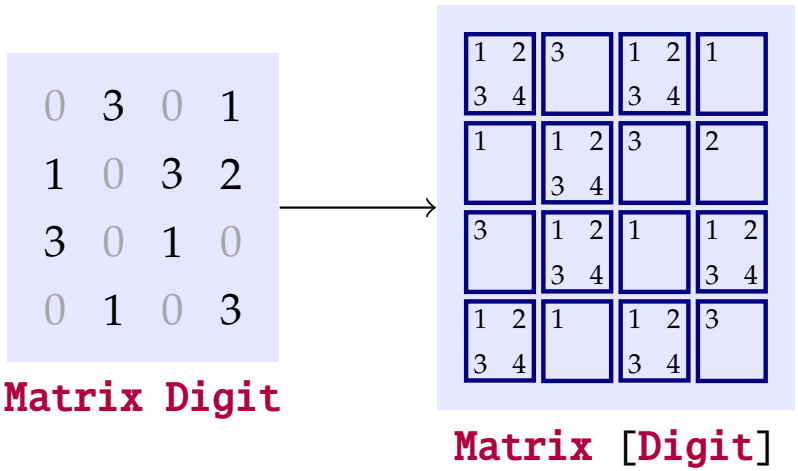

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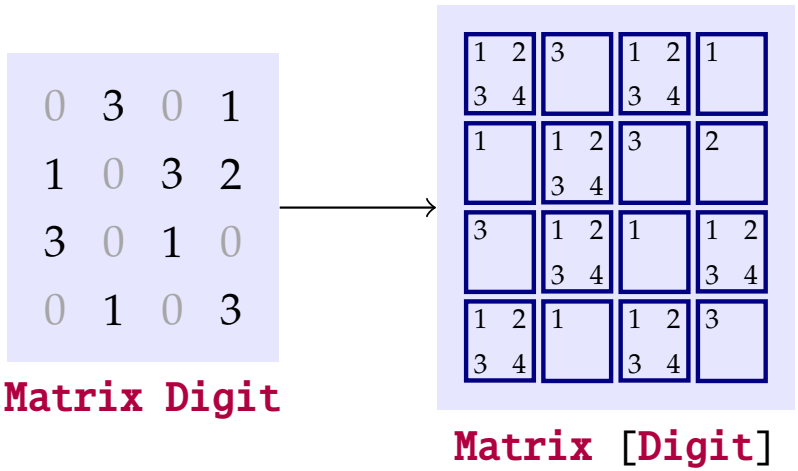

Hint:

- Define a helper function `choice :: Digit -> [Digit]`

Exercise 4: expand [⋆⋆]

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```haskell
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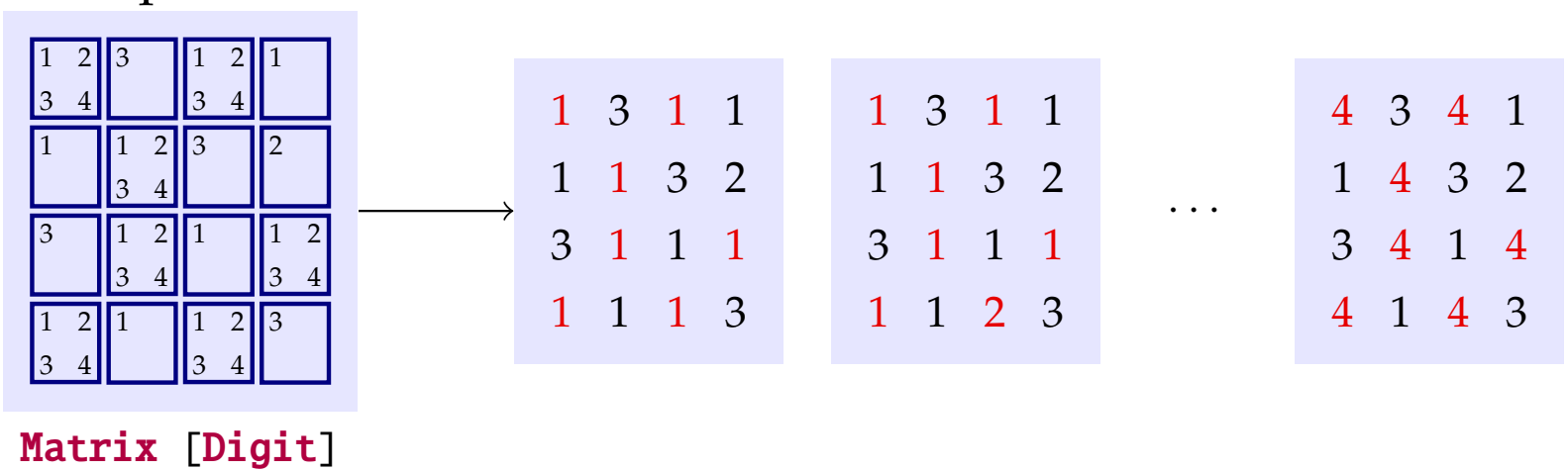

`Matrix [Digit]`

Exercise 5: cp $[\star\star\star]$

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```
xss
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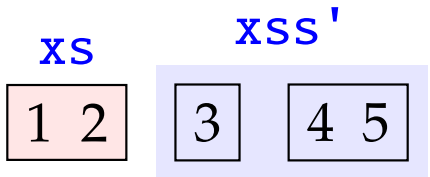

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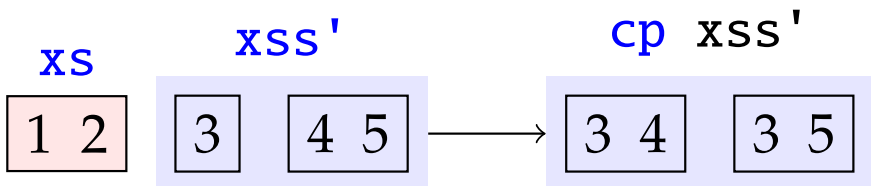

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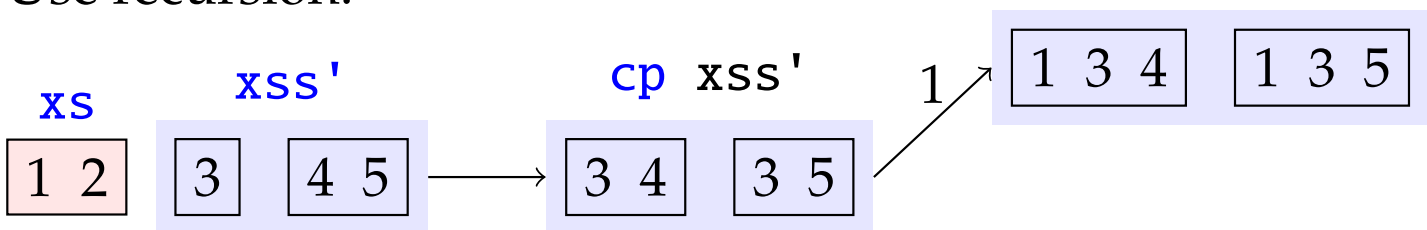

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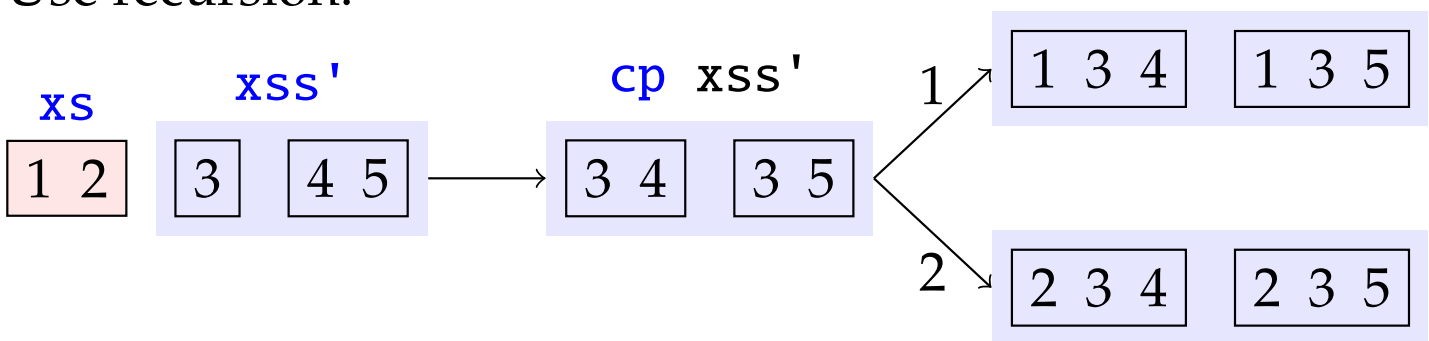

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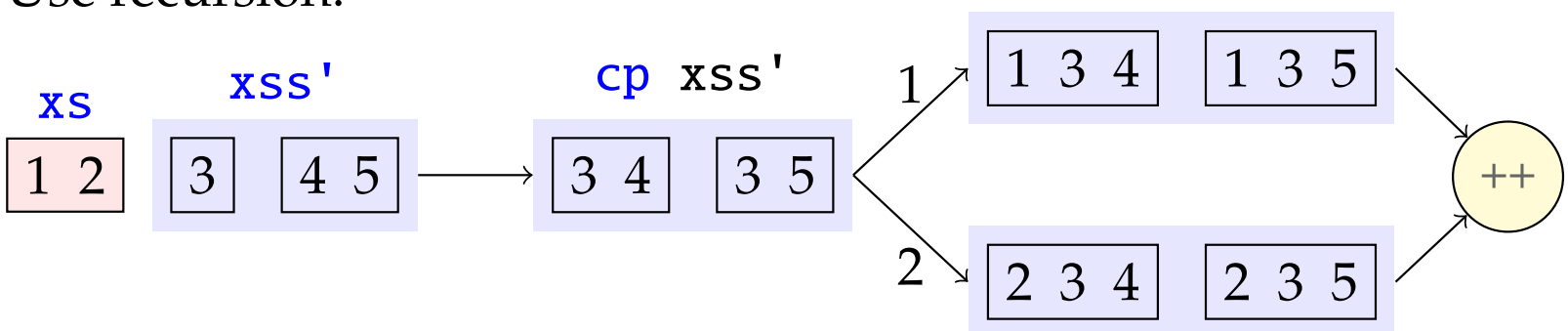
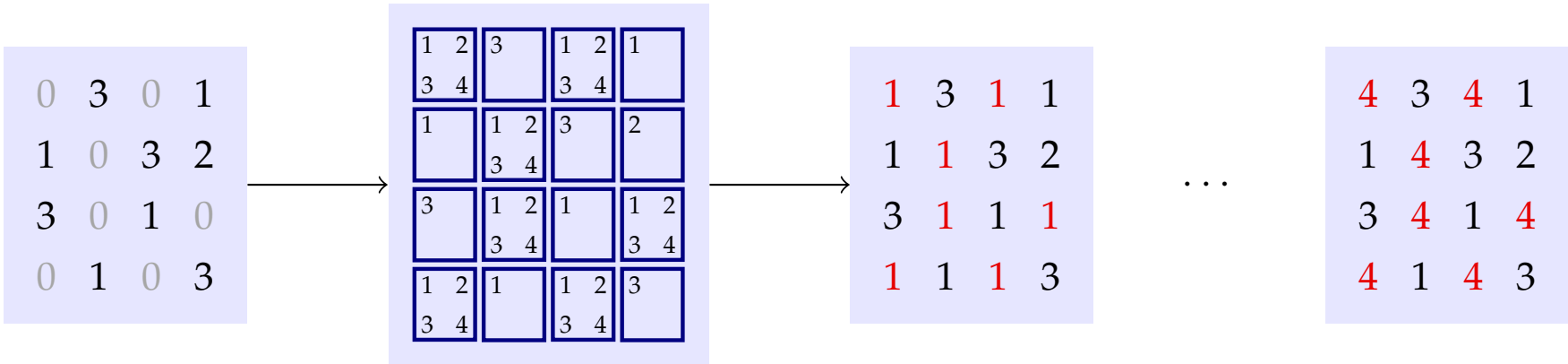


Intermezzo



- Finished implementing the `completions` function

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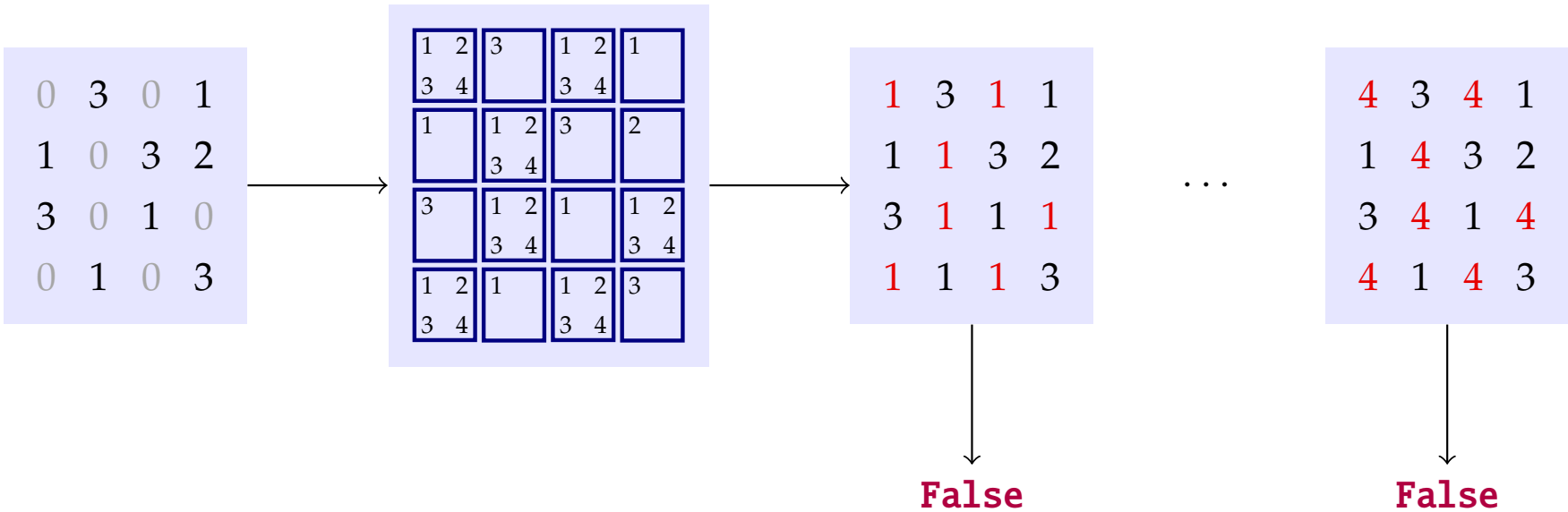

- Finished implementing the `completions` function
- Next, the `valid` function: test whether a grid is a valid solution

Exercise 6: `valid` [⋆⋆]

```
valid :: Grid -> Bool
valid = undefined
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```

Given:

```haskell
-- Checks that a list contains no duplicates
nodups :: [a] -> Bool
```

Exercise 6: `valid` [⋆⋆]

```haskell
valid :: Grid -> Bool
valid = undefined
```

Given:

```haskell
-- Checks that a list contains no duplicates
nodups :: [a] -> Bool

-- Re-orders the values from a matrix's rows, columns
-- or boxes to appear along the rows
rows :: Matrix a -> Matrix a
cols :: Matrix a -> Matrix a
boxs :: Matrix a -> Matrix a
```

Exercise 6: `valid` [⋆⋆]

```haskell
valid :: Grid -> Bool
valid = undefined
```

Given:

```haskell
-- Checks that a list contains no duplicates
nodups :: [a] -> Bool

-- Re-orders the values from a matrix's rows, columns
-- or boxes to appear along the rows
rows :: Matrix a -> Matrix a
cols :: Matrix a -> Matrix a
boxs :: Matrix a -> Matrix a
```

Examples:

$$\begin{matrix} 1 & 3 & 1 & 1 \\ 1 & 1 & 3 & 2 \\ 3 & 1 & 1 & 1 \\ 1 & 1 & 1 & 3 \end{matrix} \longrightarrow \texttt{False} \qquad \begin{matrix} 2 & 3 & 4 & 1 \\ 1 & 4 & 3 & 2 \\ 3 & 2 & 1 & 4 \\ 4 & 1 & 2 & 3 \end{matrix} \longrightarrow \texttt{True}$$

Exercise 7: nodups [⋆⋆]

```
nodups :: [a] -> Bool
nodups = undefined
```

Exercise 7: nodups [⋆⋆]

```
nodups :: [a] -> Bool
nodups = undefined
```

Examples:

```
nodups [] = True
nodups [1, 2, 3] = True
nodups [1, 2, 1] = False
```

Exercise 7: nodups [⋆⋆]

```
nodups :: [a] -> Bool
nodups = undefined
```

Examples:

```
nodups [] = True
nodups [1, 2, 3] = True
nodups [1, 2, 1] = False
```

Hints:

- Use recursion
- Use Hoogle to find a function of type `a -> [a] -> Bool`

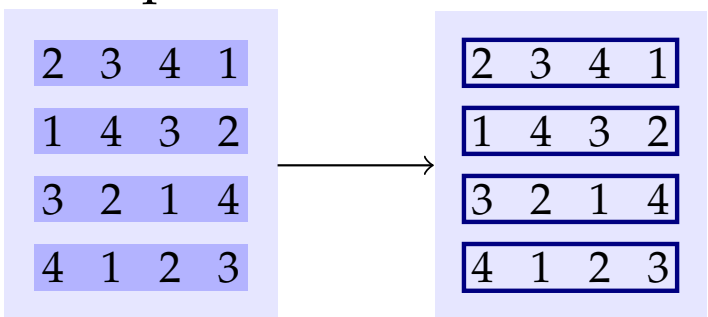
Exercise 8: `rows` $[\star]$

```
rows :: Matrix a -> Matrix a
```

Exercise 8: `rows` [⋆]

```
rows :: Matrix a -> Matrix a
```

Example:



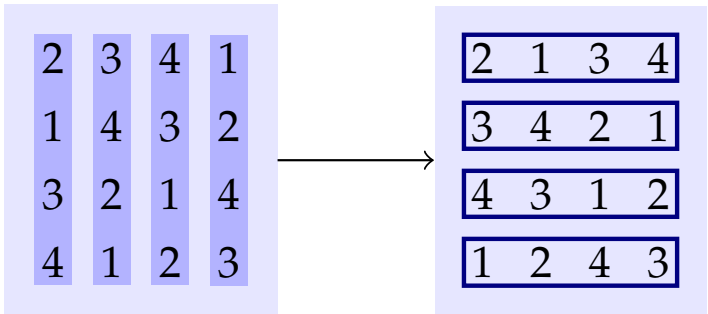
Exercise 9: `cols` $[\star\star\star]$

```
cols :: Matrix a -> Matrix a
```

Exercise 9: `cols` $[\star\star\star]$

```
cols :: Matrix a -> Matrix a
```

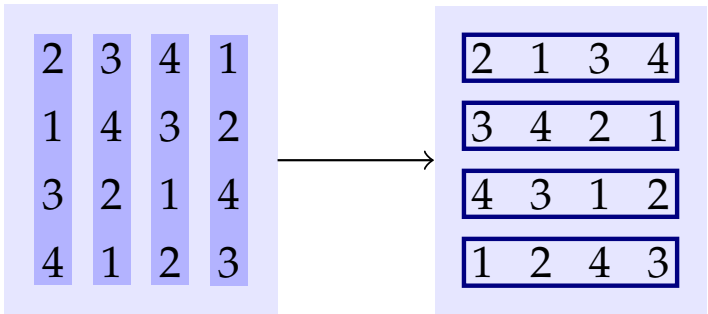
Example:



Exercise 9: `cols` [⋆⋆⋆]

```
cols :: Matrix a -> Matrix a
```

Example:



Hints:

- Use recursion
- Define a case for a one-row matrix; example:
  ```
  cols [[1,2,3,4]] = [[1],[2],[3],[4]]
  ```
- For the recursive case, use the `zipWith` function:
  ```
  zipWith :: (a -> b -> c) -> [a] -> [b] -> [c]
  ```

Exercise 10: `boxs` $[\star\star\star]$

```
boxs :: Matrix a -> Matrix a
```

Exercise 10: `boxs` $[\star\star\star]$

```haskell
boxs :: Matrix a -> Matrix a
```

Given:

```haskell
-- Groups a list into lists of length two
group :: [a] -> [[a]]
```

Exercise 10: `boxs` $[\star\star\star]$

```
boxs :: Matrix a -> Matrix a
```

Given:

```
-- Groups a list into lists of length two
group :: [a] -> [[a]]

-- Flattens a nested list of elements
ungroup :: [[a]] -> [a]
```

Exercise 10: `boxs` [⋆⋆⋆]

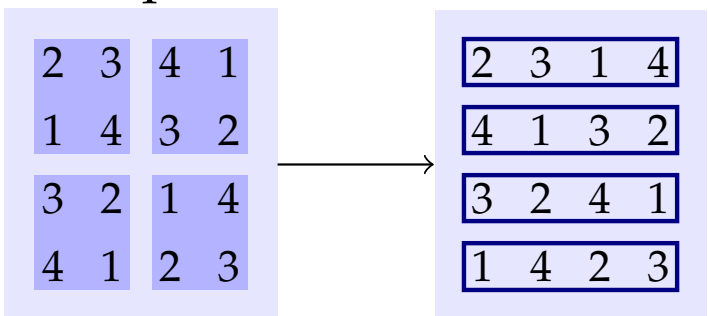
```
boxs :: Matrix a -> Matrix a
```

Given:

```
-- Groups a list into lists of length two
group :: [a] -> [[a]]

-- Flattens a nested list of elements
ungroup :: [[a]] -> [a]
```

Example:



Exercise 10: `boxs` [⋆⋆⋆]

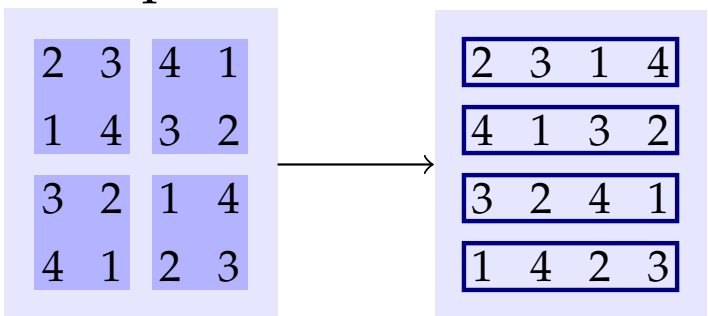
```
boxs :: Matrix a -> Matrix a
```

Given:

```
-- Groups a list into lists of length two
group :: [a] -> [[a]]

-- Flattens a nested list of elements
ungroup :: [[a]] -> [a]
```

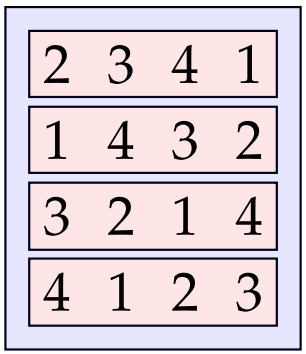
Example:



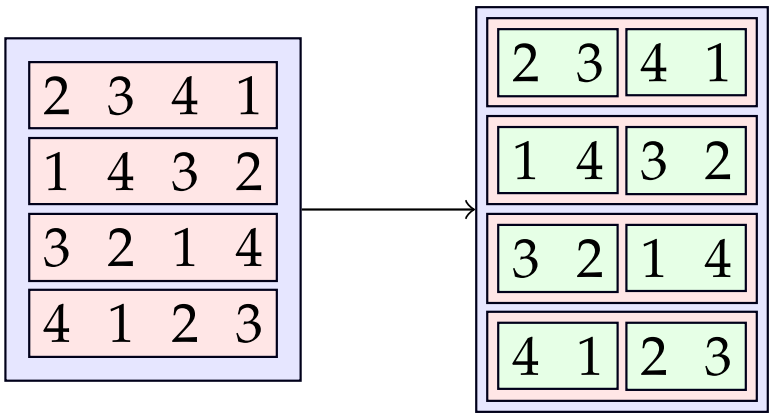
Hints:

- Use the previously defined `cols` function
- Chain five transformations (see next slide)

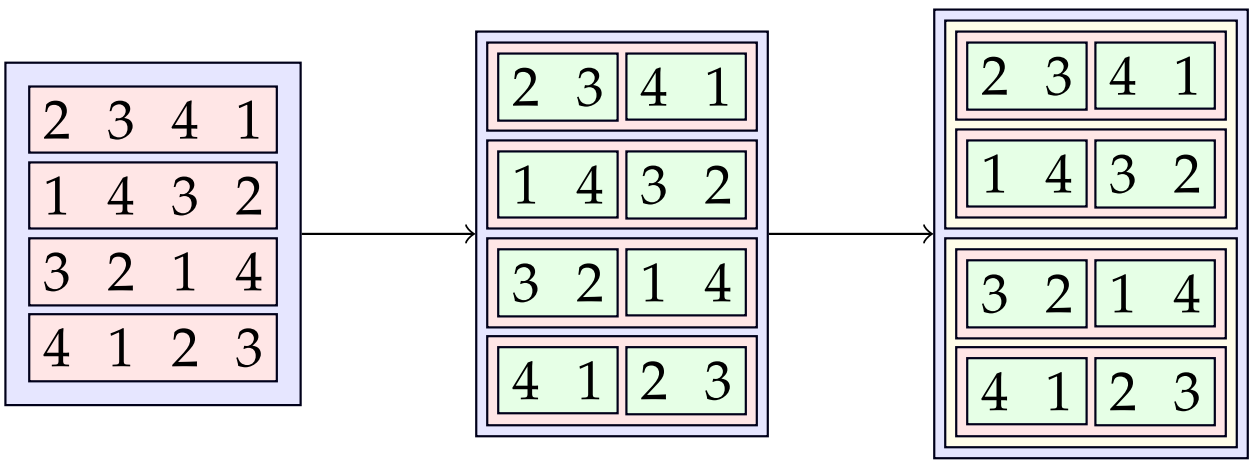
Exercise 10: `boxs` $[\star\star\star]$



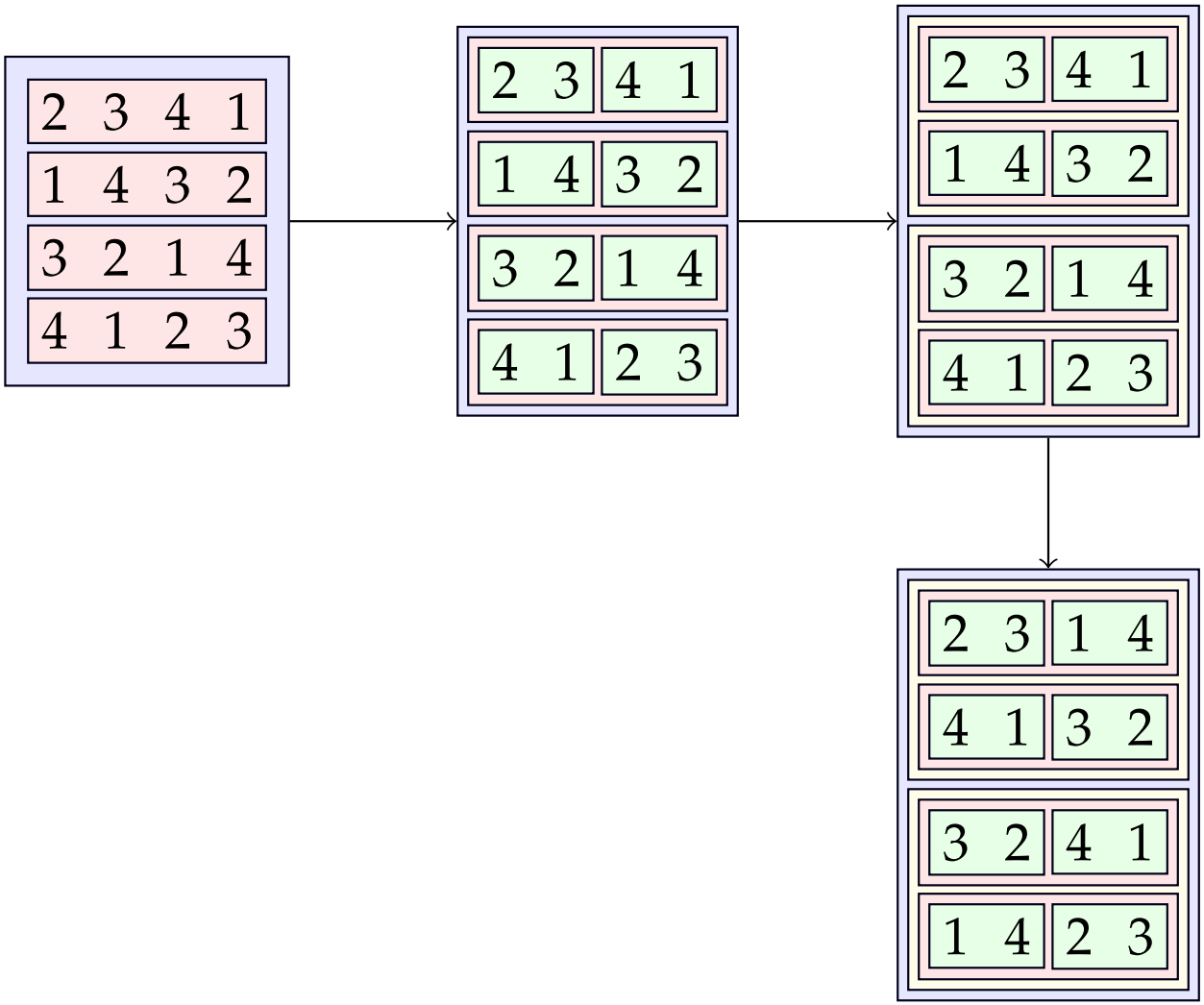
Exercise 10: `boxs` $[\star\star\star]$



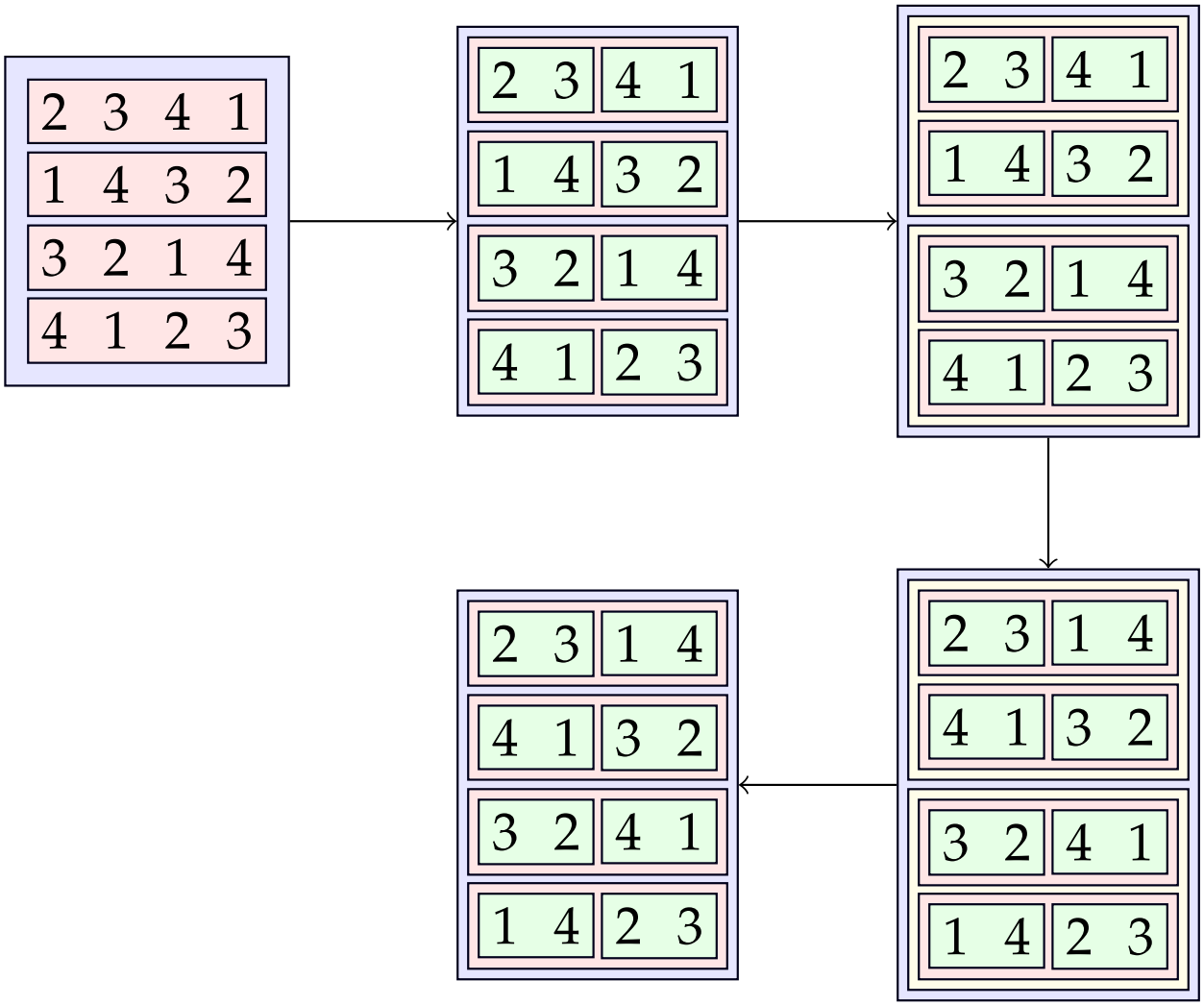
Exercise 10: `boxs` $[\star\star\star]$



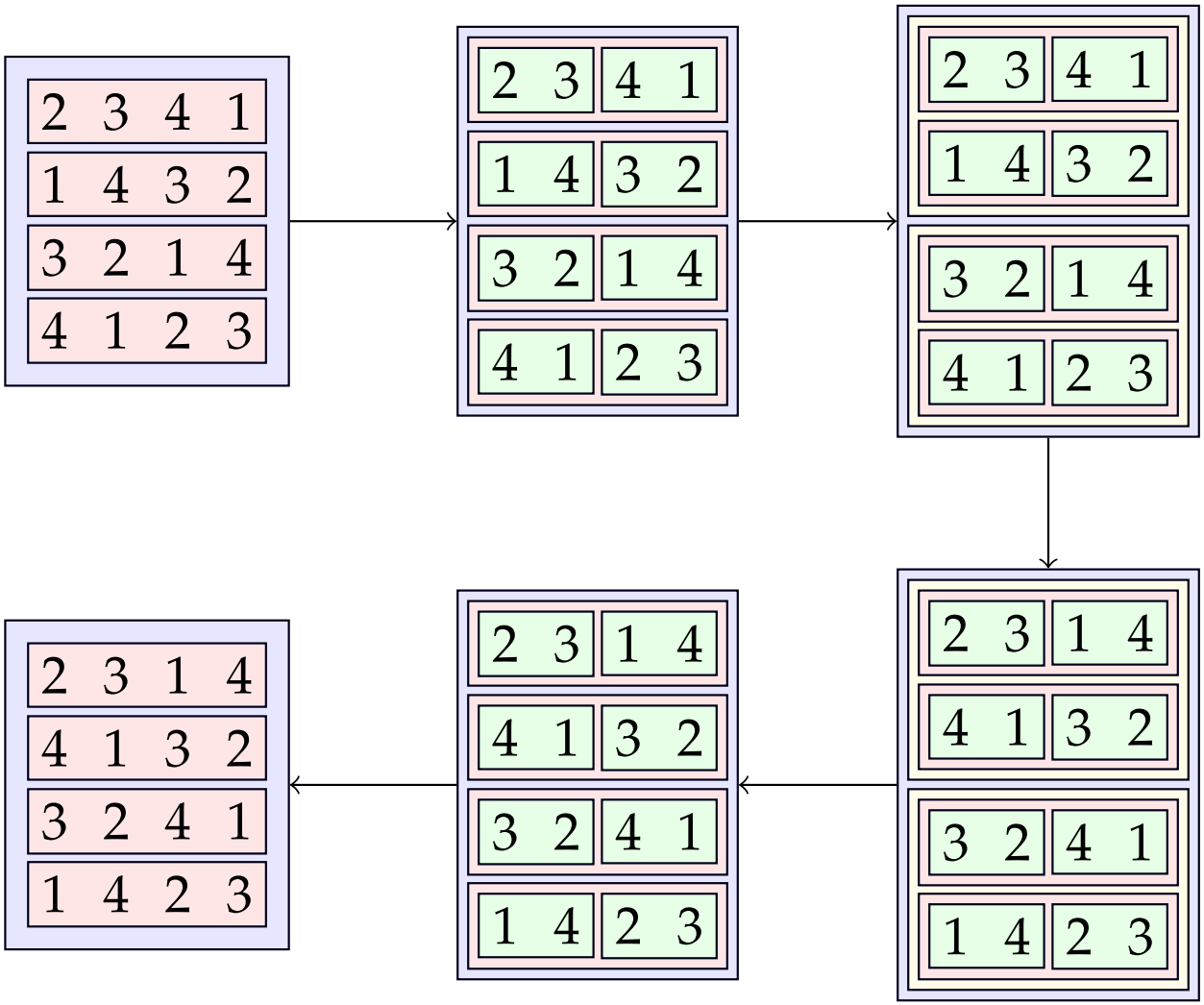
Exercise 10: `boxs` [⋆⋆⋆]



Exercise 10: `boxs` $[\star\star\star]$



Exercise 10: `boxs` $[\star\star\star]$



Exercise 11: `group` [⋆⋆]

```
group :: [a] -> [[a]]
```

Exercise 11: `group` [⋆⋆]

```
group :: [a] -> [[a]]
```

Example:

```
group [1,2,3,4] = [[1,2],[3,4]]
```

Exercise 12: ungroup $[\star]$

```
ungroup :: [[a]] -> [a]
```

Exercise 12: ungroup $[\star]$

```
ungroup :: [[a]] -> [a]
```

Example:

```
ungroup [[1,2],[3,4]] = [1,2,3,4]
```

Exercise 12: ungroup [⋆]

```
ungroup :: [[a]] -> [a]
```

Example:

```
ungroup [[1,2],[3,4]] = [1,2,3,4]
```

Hints:

- Use Hoogle

That's all folks

- Time to solve some Sudokus

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- The current approach is inefficient, but correct

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- Time to solve some Sudokus
- The current approach is inefficient, but correct
- Equational reasoning to improve performance
  - Define a function `prune` that eliminates early invalid solutions

```
filter valid . expand = filter valid . expand . prune
```

That's all folks

- Time to solve some Sudokus
- The current approach is inefficient, but correct
- Equational reasoning to improve performance
  - Define a function `prune` that eliminates early invalid solutions

    ```
    filter valid . expand = filter valid . expand . prune
    ```
  - It is not hard to define a function to prune a row (exercise):

    ```
    pruneRow [[4],[1,2],[1],[1,3]] = [[4],[2],[1],[3]]
    ```

That's all folks

- Time to solve some Sudokus
- The current approach is inefficient, but correct
- Equational reasoning to improve performance
  - Define a function `prune` that eliminates early invalid solutions

    ```
    filter valid . expand = filter valid . expand . prune
    ```
  - It is not hard to define a function to prune a row (exercise):

    ```
    pruneRow [[4],[1,2],[1],[1,3]] = [[4],[2],[1],[3]]
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  - Equational reasoning to define `prune` in terms of `pruneRow`

That's all folks

- Time to solve some Sudokus
- The current approach is inefficient, but correct
- Equational reasoning to improve performance
  - Define a function `prune` that eliminates early invalid solutions

    ```
    filter valid . expand = filter valid . expand . prune
    ```
  - It is not hard to define a function to prune a row (exercise):

    ```
    pruneRow [[4],[1,2],[1],[1,3]] = [[4],[2],[1],[3]]
    ```
  - Equational reasoning to define `prune` in terms of `pruneRow`
    - The function `pruneRow` satisfies the equation

      ```
      filter nodups . cp = filter nodups . cp . pruneRow
      ```

That's all folks

- Time to solve some Sudokus
- The current approach is inefficient, but correct
- Equational reasoning to improve performance
  - Define a function `prune` that eliminates early invalid solutions

    ```
    filter valid . expand = filter valid . expand . prune
    ```
  - It is not hard to define a function to prune a row (exercise):

    ```
    pruneRow [[4],[1,2],[1],[1,3]] = [[4],[2],[1],[3]]
    ```
  - Equational reasoning to define `prune` in terms of `pruneRow`
    - The function `pruneRow` satisfies the equation

      ```
      filter nodups . cp = filter nodups . cp . pruneRow
      ```
    - Expand the expression `filter valid . expand`

That's all folks

- Time to solve some Sudokus
- The current approach is inefficient, but correct
- Equational reasoning to improve performance
  - Define a function `prune` that eliminates early invalid solutions

    ```
    filter valid . expand = filter valid . expand . prune
    ```
  - It is not hard to define a function to prune a row (exercise):

    ```
    pruneRow [[4],[1,2],[1],[1,3]] = [[4],[2],[1],[3]]
    ```
  - Equational reasoning to define `prune` in terms of `pruneRow`
    - The function `pruneRow` satisfies the equation

      ```
      filter nodups . cp = filter nodups . cp . pruneRow
      ```
    - Expand the expression `filter valid . expand`
    - Use the above equation and compress back the formula

Further references

- Richard Bird's papers and books (Bird, 2006, 2010, 2014)
- Conor McBride's Sudoku solver using applicative and traversable: https://stackoverflow.com/a/10242673/474311

References

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Bird, R. (2014). *Thinking Functionally with Haskell*. Cambridge University Press.

Bird, R. S. (2006). A program to solve Sudoku. *Journal of Functional Programming*, 16(6):671–679.