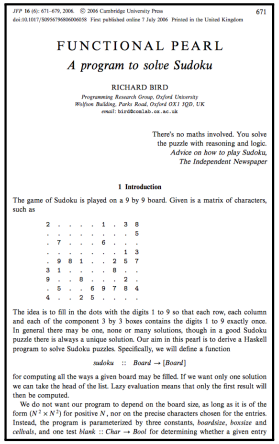
Coding dojo
A simple Sudoku solver in Haskell

B◦FP

Bucharest FP #027

Welcome



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FUNCTIONAL PEARL

A program to solve Sudoku

RICHARD BIRD
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There's no maths involved. You solve the puzzle with reasoning and logic.
Advice on how to play Sudoku,
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1 Introduction

The game of Sudoku is played on a 9 by 9 board. Given is a matrix of characters, such as

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```

The idea is to fill in the dots with the digits 1 to 9 so that each row, each column and each of the component 3 by 3 boxes contains the digits 1 to 9 exactly once. In general there may be one, none or many solutions, though in a good Sudoku puzzle there is always a unique solution. Our aim in this pearl is to derive a Haskell program to solve Sudoku puzzles. Specifically, we will define a function

sudoku :: *Board* → [*Board*]

for computing all the ways a given board may be filled. If we want only one solution we can take the head of the list. Lazy evaluation means that only the first result will then be computed.

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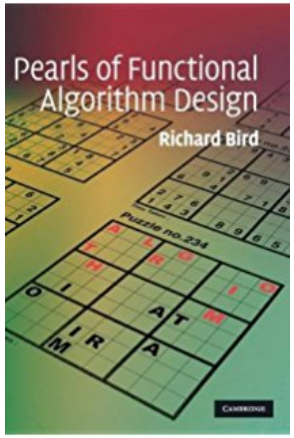

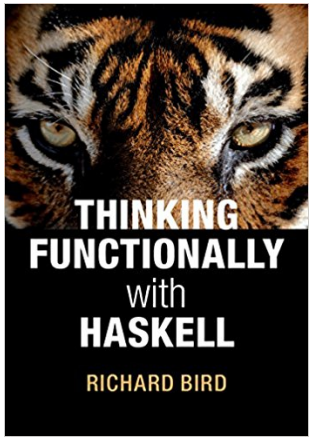

- Implement a Sudoku solver as described in (Bird, 2006, 2010, 2014)

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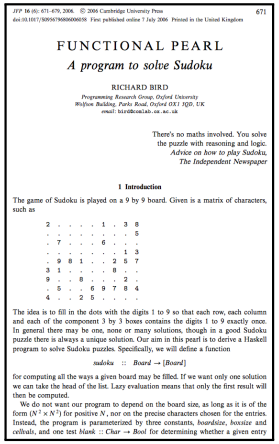

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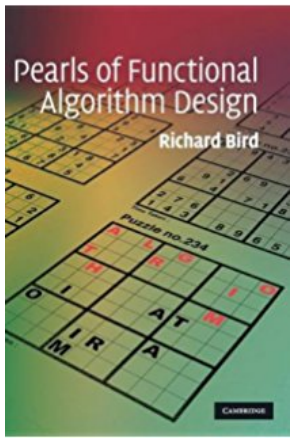

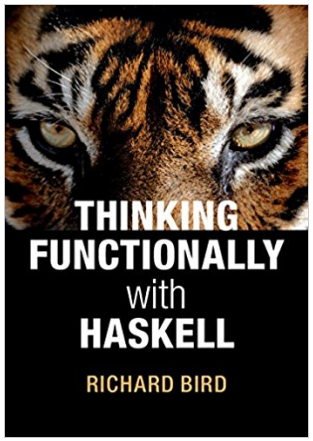

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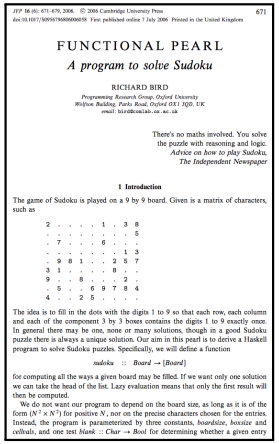

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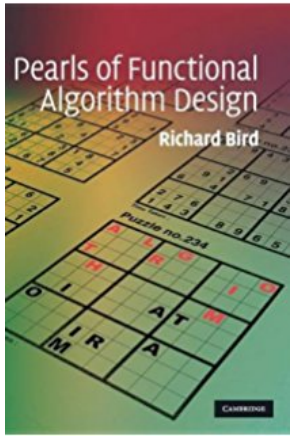

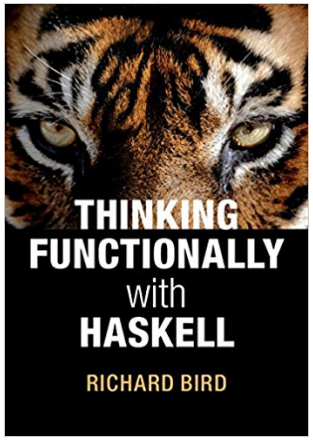

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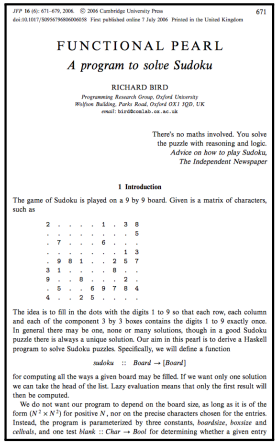

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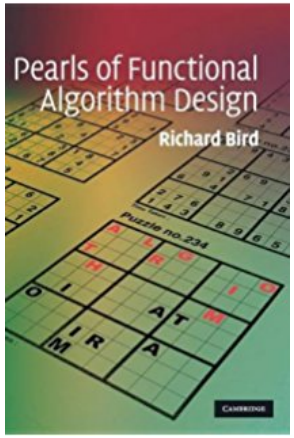

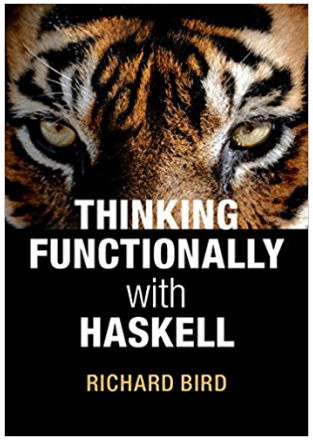

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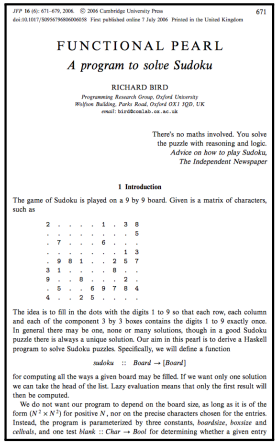

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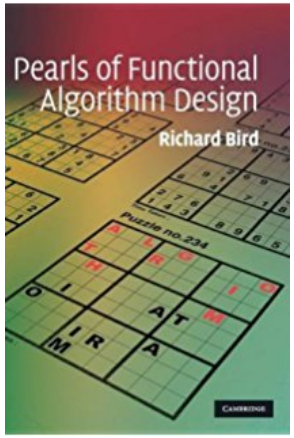

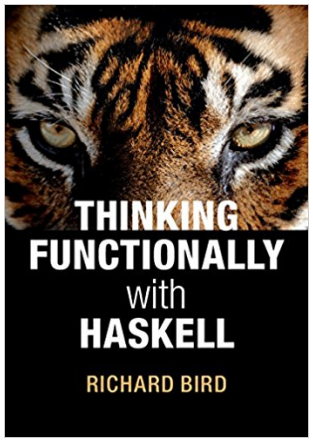

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  - Wholemeal programming (prevents a disease called “indexitis”)
  - Higher-order functions, recursion, point-free style

How to play Sudoku

	2	4	
1			3
4			2
	1	3	

$N = 2$

	2		5		1		9	
8			2		3			6
	3			6			7	
		1				6		
5	4						1	9
		2				7		
	9			3			8	
2			8		4			7
	1		9		7		6	

$N = 3$

Fill in the empty cells with digits 1 to N^2 such that every row, column and $N \times N$ box contains the digits 1 to N^2.

How to play Sudoku

3	2	4	1
1	4	2	3
4	3	1	2
2	1	3	4

$N = 2$

4	2	6	5	7	1	3	9	8
8	5	7	2	9	3	1	4	6
1	3	9	4	6	8	2	7	5
9	7	1	3	8	5	6	2	4
5	4	3	7	2	6	8	1	9
6	8	2	1	4	9	7	5	3
7	9	4	6	3	2	5	8	1
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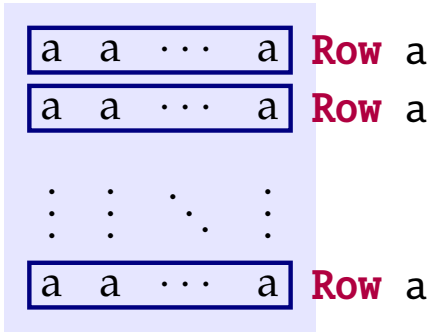


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type Matrix a = [Row a]
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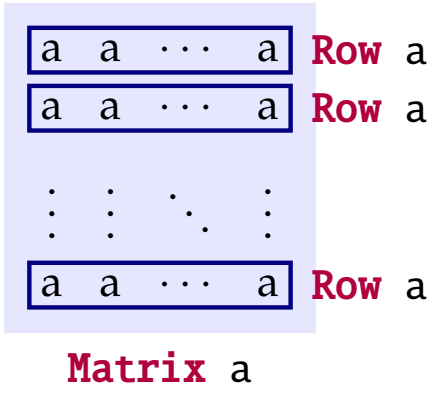

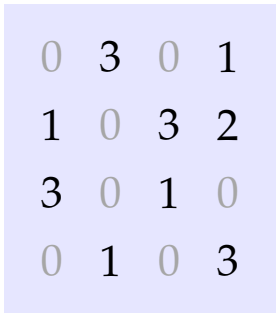


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type Digit = Int
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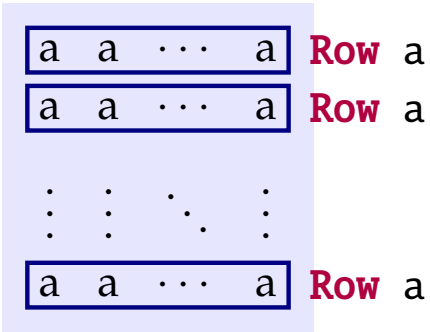

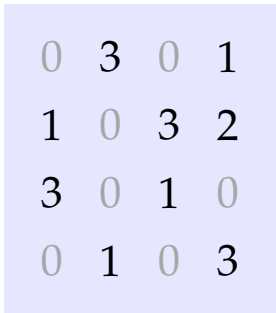


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We assume that digit zero indicates an empty cell:

```
isEmpty :: Digit -> Bool
isEmpty 0 = True
isEmpty _ = False
```

Exercise 1: `solve` [⋆]

```haskell
solve :: Grid -> [Grid]
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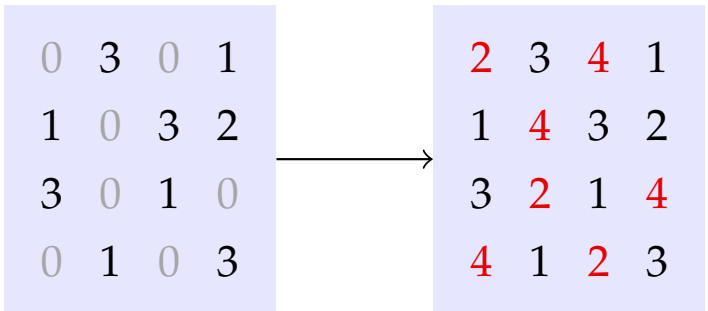

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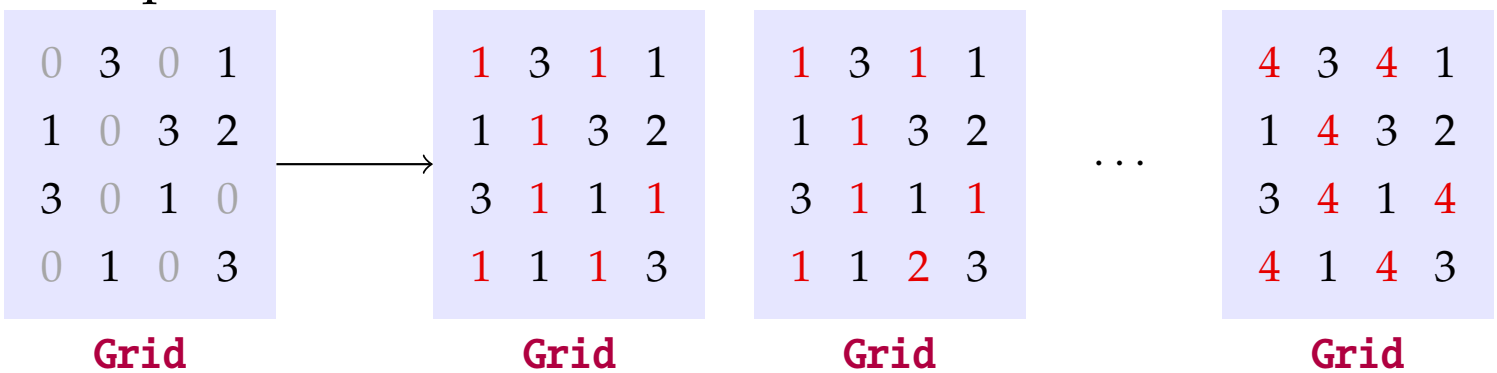

Exercise 3: `choices` $[\star]$

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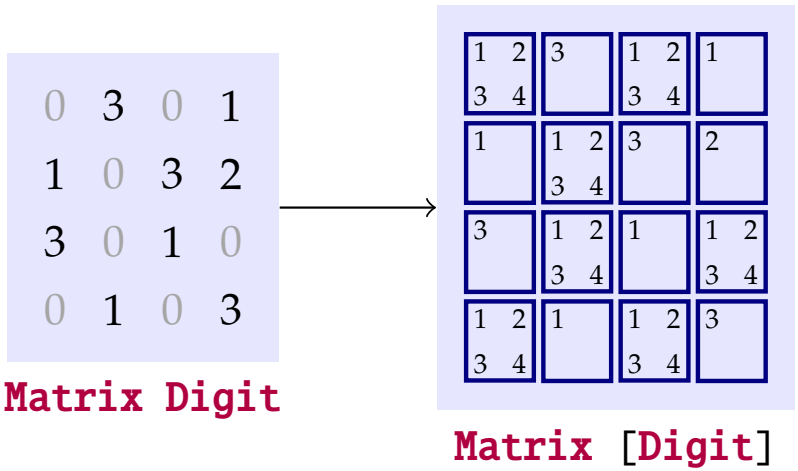

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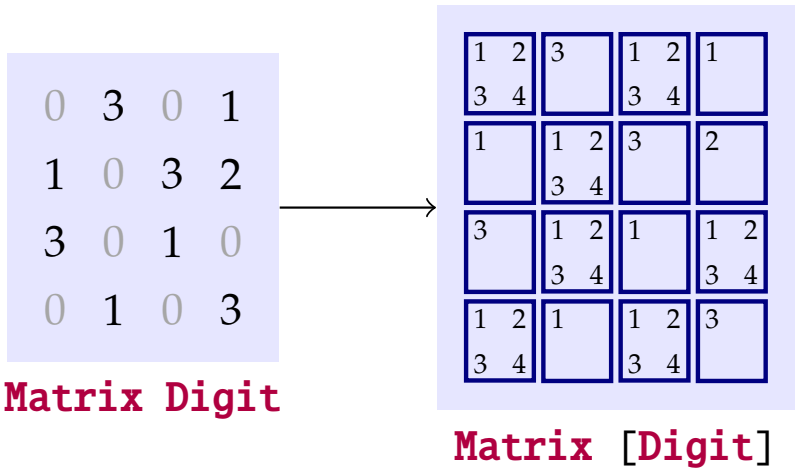

Hint:

- Define a helper function `choice :: Digit -> [Digit]`

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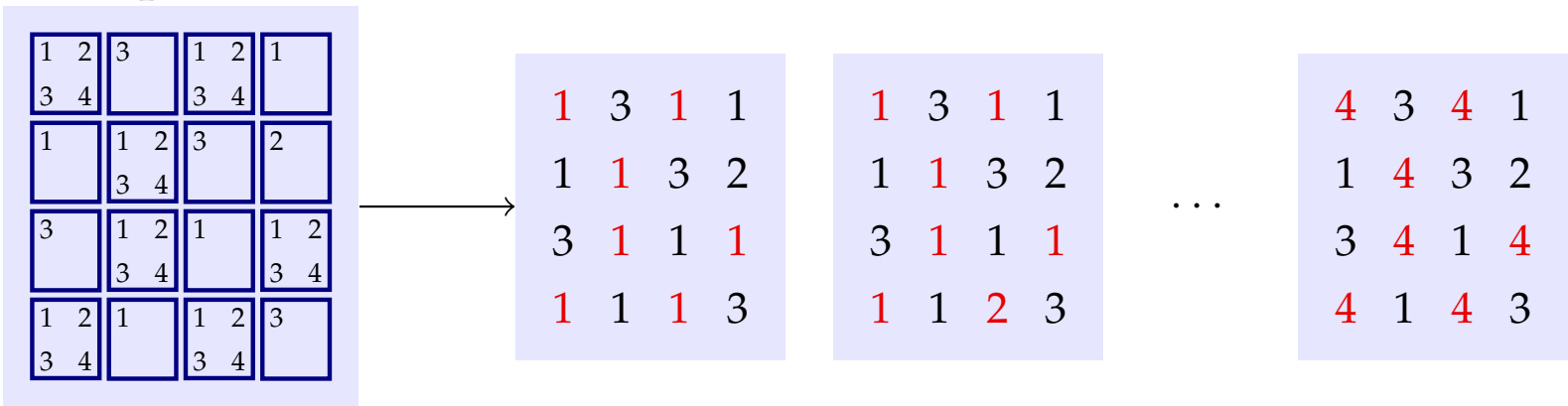

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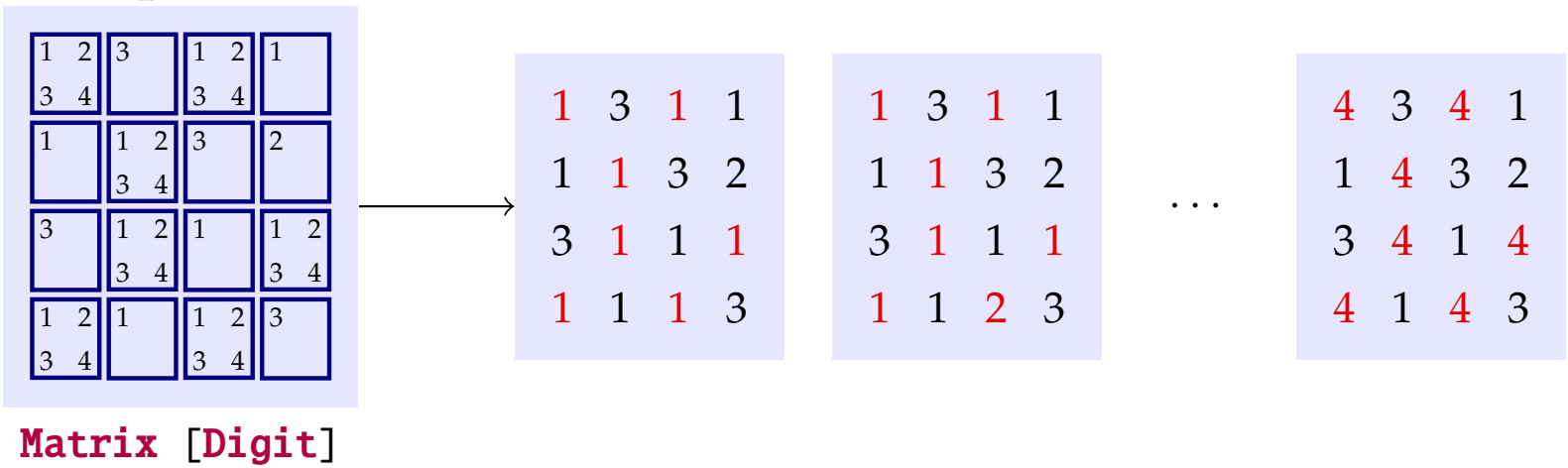

`Matrix [Digit]`

Hints:

- First generate all combinations across each row
- Then combine those generated combinations

Exercise 5: cp $[\star\star\star]$

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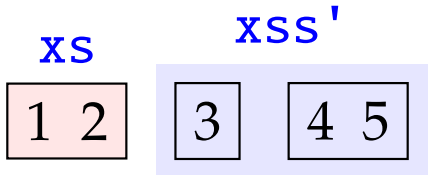

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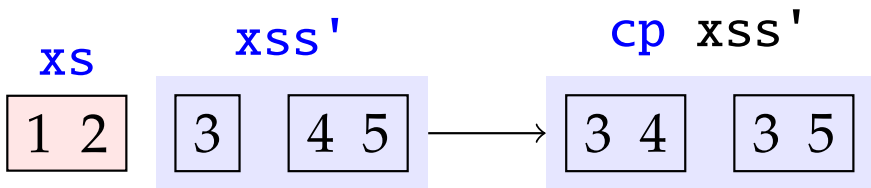

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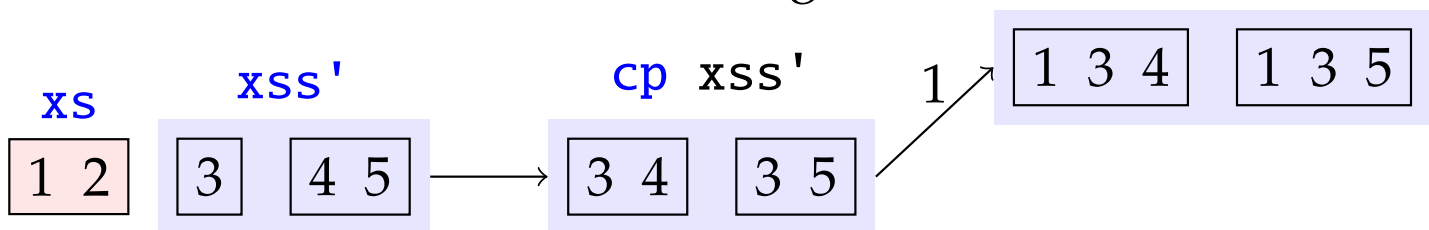

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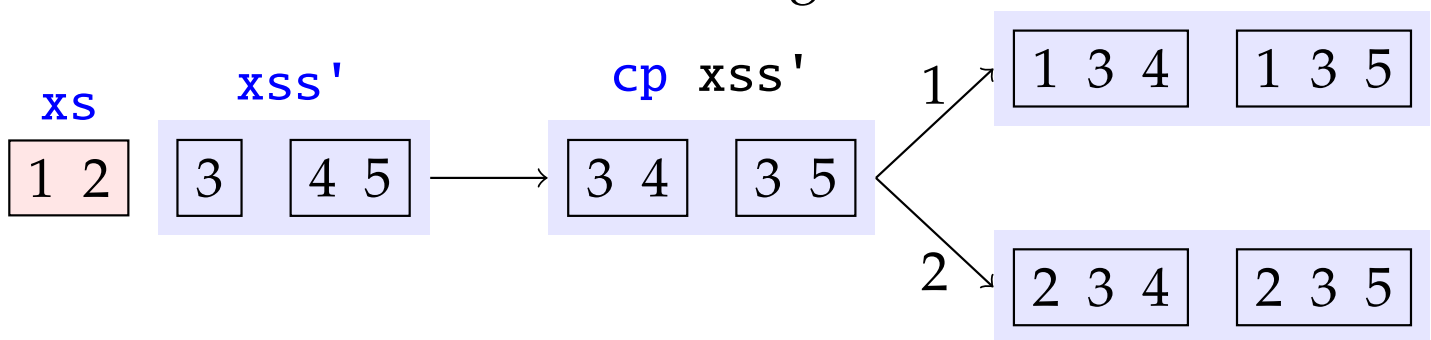

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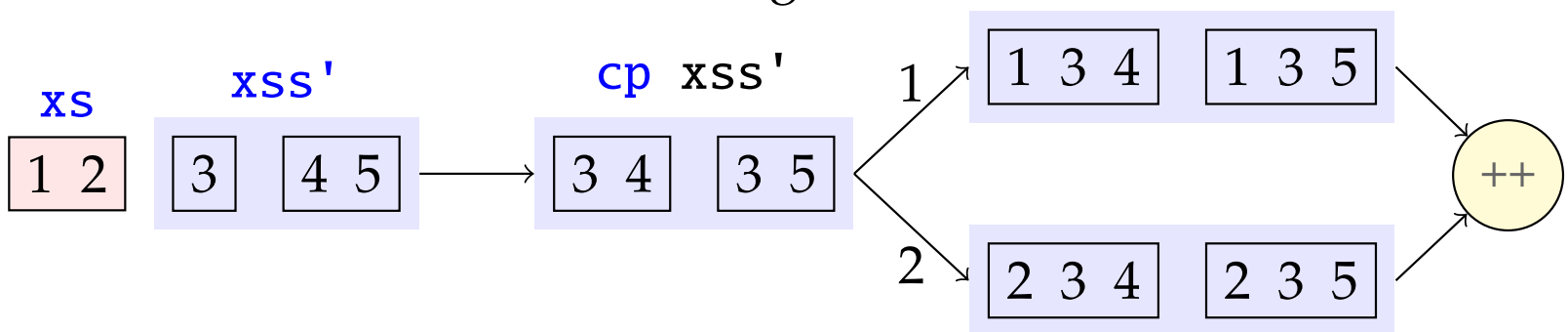
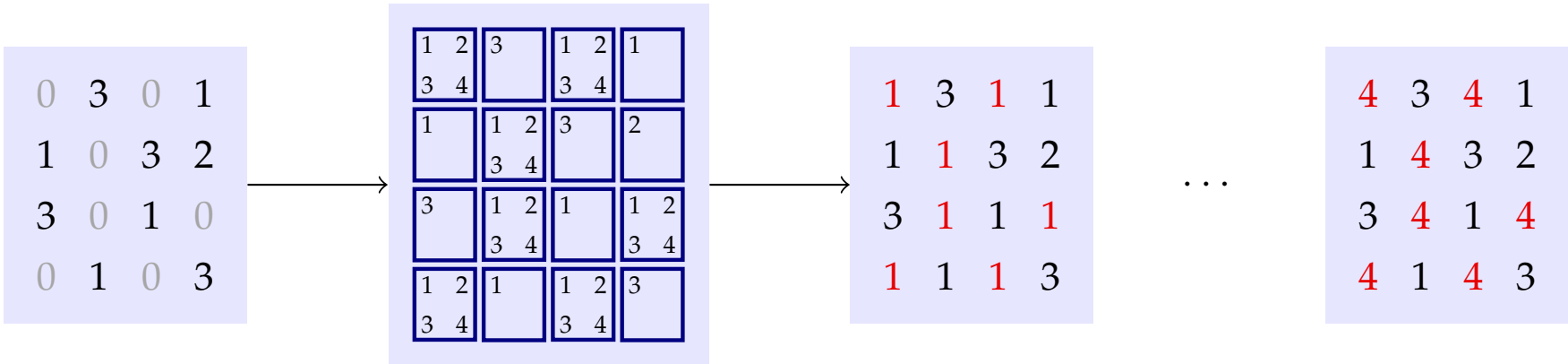
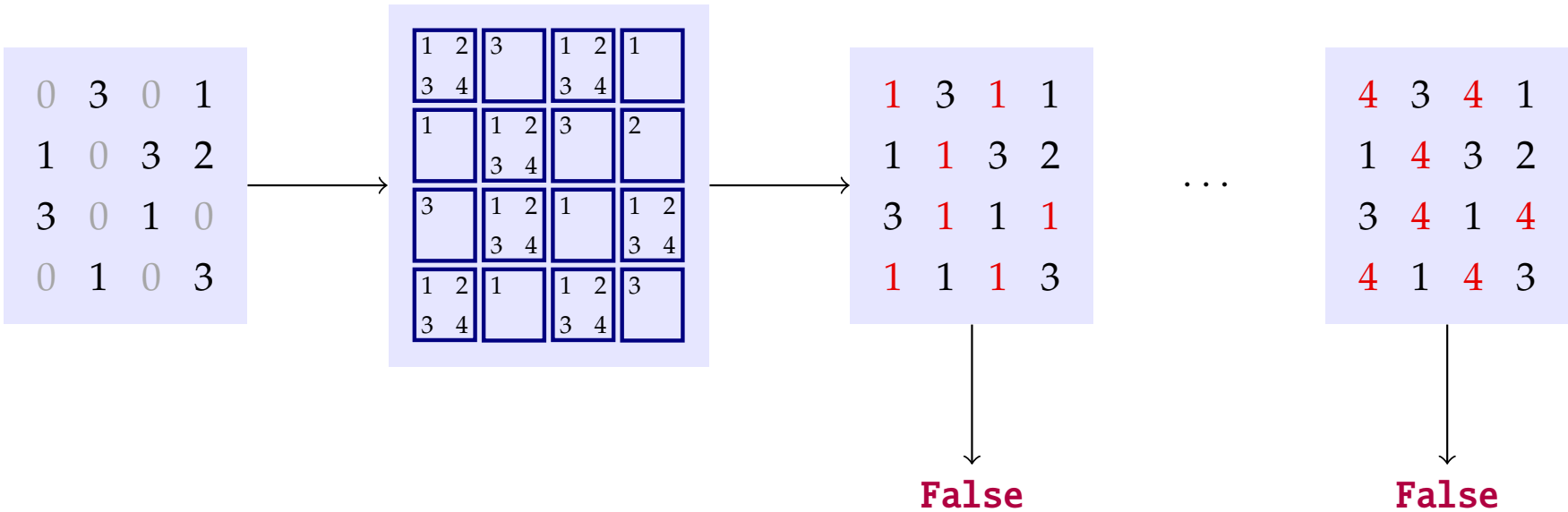


Intermezzo



- Finished implementing the `completions` function

Intermezzo



- Finished implementing the `completions` function
- Next, the `valid` function: test whether a grid is a valid solution

Exercise 6: `valid` [⋆⋆]

```
valid :: Grid -> Bool
valid = undefined
```

Exercise 6: `valid` [⋆⋆]

```haskell
valid :: Grid -> Bool
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```

Given:

```haskell
-- Checks that a list contains no duplicates
nodups :: [a] -> Bool
```

Exercise 6: `valid` [⋆⋆]

```haskell
valid :: Grid -> Bool
valid = undefined
```

Given:

```haskell
-- Checks that a list contains no duplicates
nodups :: [a] -> Bool

-- Re-orders the values from a matrix's rows, columns
-- or boxes to appear along the rows
rows :: Matrix a -> Matrix a
cols :: Matrix a -> Matrix a
boxs :: Matrix a -> Matrix a
```

Exercise 6: `valid` [⋆⋆]

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-- Checks that a list contains no duplicates
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-- Re-orders the values from a matrix's rows, columns
-- or boxes to appear along the rows
rows :: Matrix a -> Matrix a
cols :: Matrix a -> Matrix a
boxs :: Matrix a -> Matrix a
```

Examples:

$$\begin{matrix} 1 & 3 & 1 & 1 \\ 1 & 1 & 3 & 2 \\ 3 & 1 & 1 & 1 \\ 1 & 1 & 1 & 3 \end{matrix} \longrightarrow \texttt{False} \qquad \begin{matrix} 2 & 3 & 4 & 1 \\ 1 & 4 & 3 & 2 \\ 3 & 2 & 1 & 4 \\ 4 & 1 & 2 & 3 \end{matrix} \longrightarrow \texttt{True}$$

Exercise 7: nodups [⋆⋆]

```
nodups :: [a] -> Bool
nodups = undefined
```

Exercise 7: nodups [⋆⋆]

```
nodups :: [a] -> Bool
nodups = undefined
```

Examples:

```
nodups [] = True
nodups [1, 2, 3] = True
nodups [1, 2, 1] = False
```

Exercise 7: nodups [⋆⋆]

```
nodups :: [a] -> Bool
nodups = undefined
```

Examples:

```
nodups [] = True
nodups [1, 2, 3] = True
nodups [1, 2, 1] = False
```

Hints:

- Use recursion
- Use Hoogle to find a function of type `a -> [a] -> Bool`

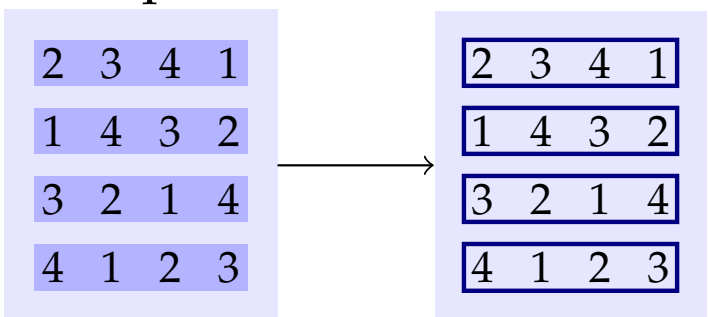
Exercise 8: `rows` $[\star]$

```
rows :: Matrix a -> Matrix a
```

Exercise 8: `rows` [⋆]

```
rows :: Matrix a -> Matrix a
```

Example:



Exercise 9: `cols` $[\star\star\star]$

```
cols :: Matrix a -> Matrix a
```

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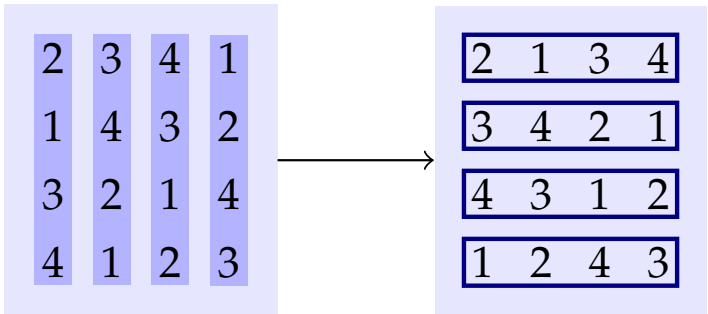
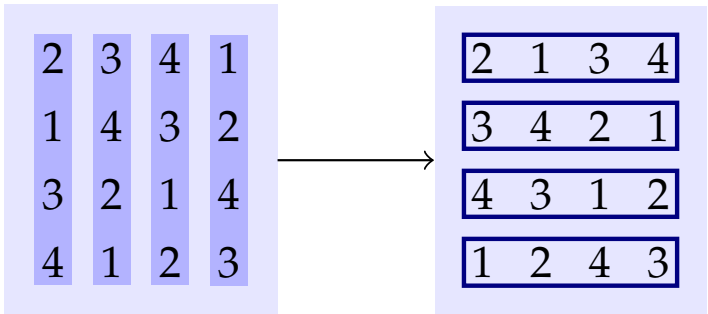
Example:



Exercise 9: `cols` [⋆⋆⋆]

```
cols :: Matrix a -> Matrix a
```

Example:



Hints:

- Use recursion
- Define a case for a one-row matrix; example:
  ```
  cols [[1,2,3,4]] = [[1],[2],[3],[4]]
  ```
- For the recursive case, use the `zipWith` function:
  ```
  zipWith :: (a -> b -> c) -> [a] -> [b] -> [c]
  ```

Exercise 10: `boxs` $[\star\star\star]$

```
boxs :: Matrix a -> Matrix a
```

Exercise 10: `boxs` $[\star\star\star]$

```haskell
boxs :: Matrix a -> Matrix a
```

Given:

```haskell
-- Groups a list into lists of length two
group :: [a] -> [[a]]
```

Exercise 10: `boxs` $[\star\star\star]$

```
boxs :: Matrix a -> Matrix a
```

Given:

```
-- Groups a list into lists of length two
group :: [a] -> [[a]]

-- Flattens a nested list of elements
ungroup :: [[a]] -> [a]
```

Exercise 10: `boxs` [⋆⋆⋆]

```
boxs :: Matrix a -> Matrix a
```

Given:

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-- Groups a list into lists of length two
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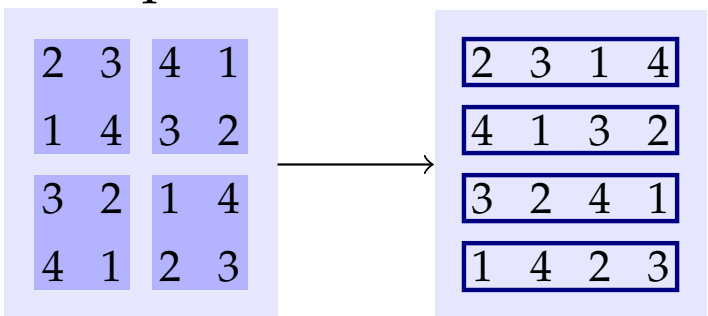

Exercise 10: `boxs` [⋆⋆⋆]

```
boxs :: Matrix a -> Matrix a
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Given:

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-- Groups a list into lists of length two
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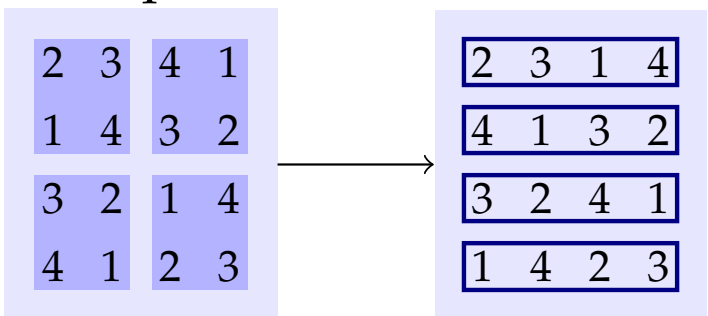
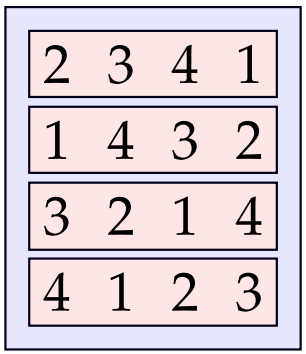
Example:



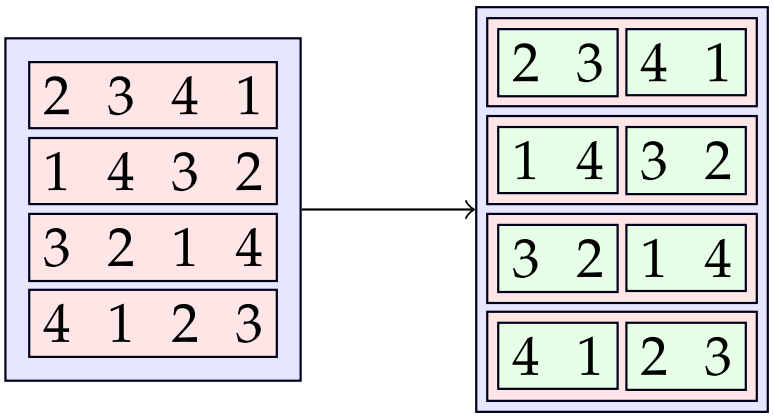
Hints:

- Use the previously defined `cols` function
- Chain five transformations (see next slide)

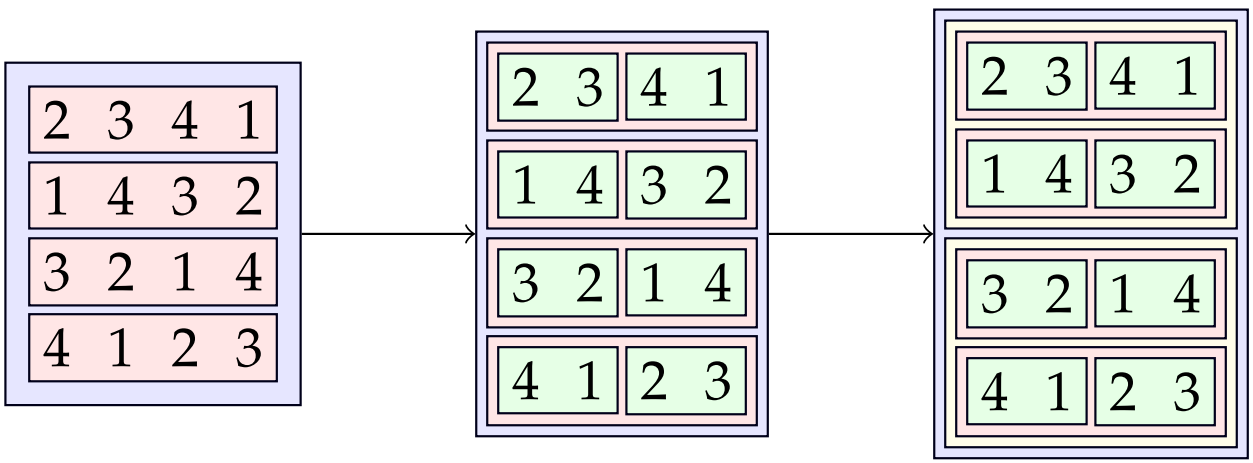
Exercise 10: `boxs` $[\star\star\star]$



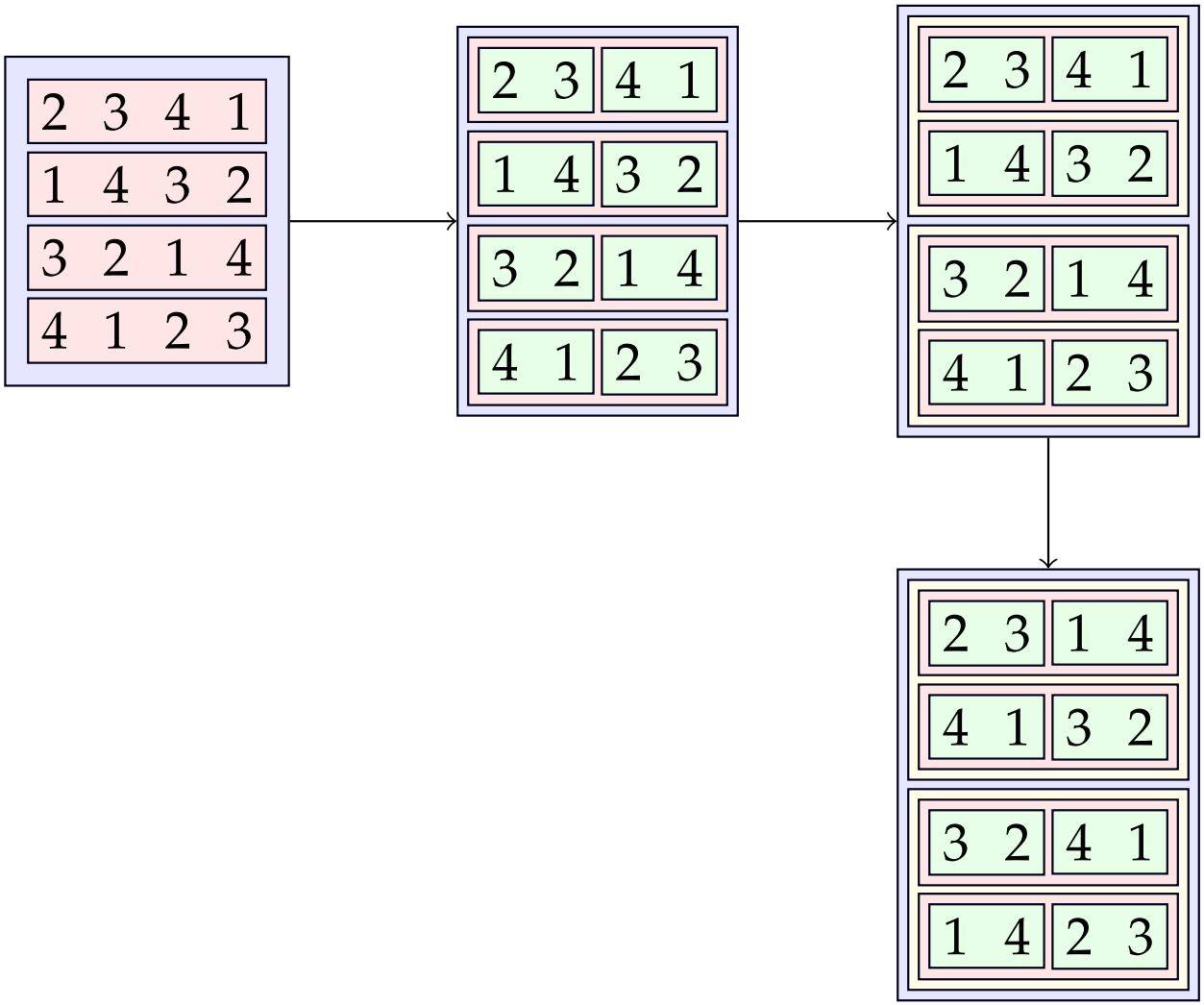
Exercise 10: `boxs` $[\star\star\star]$



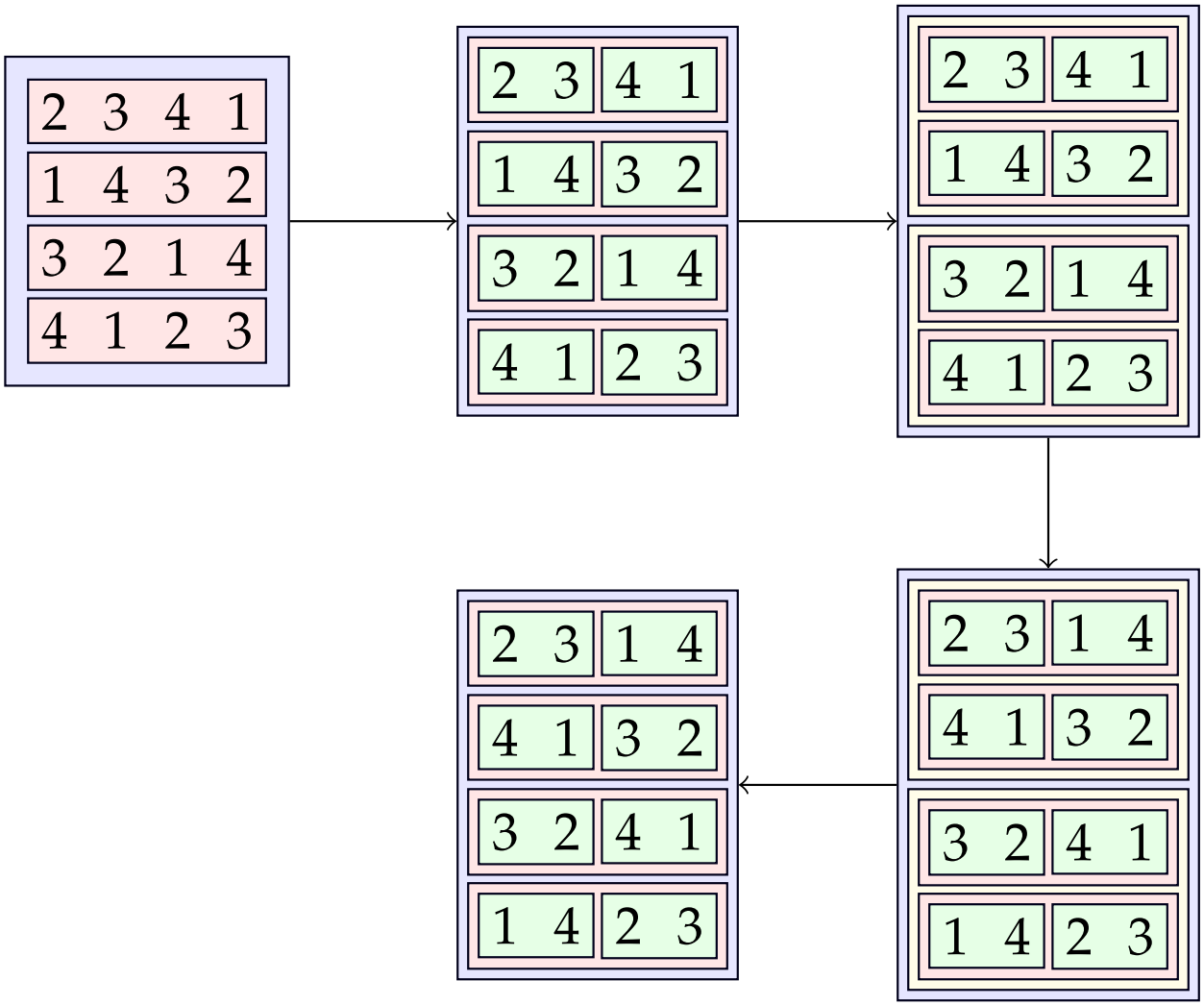
Exercise 10: `boxs` $[\star\star\star]$



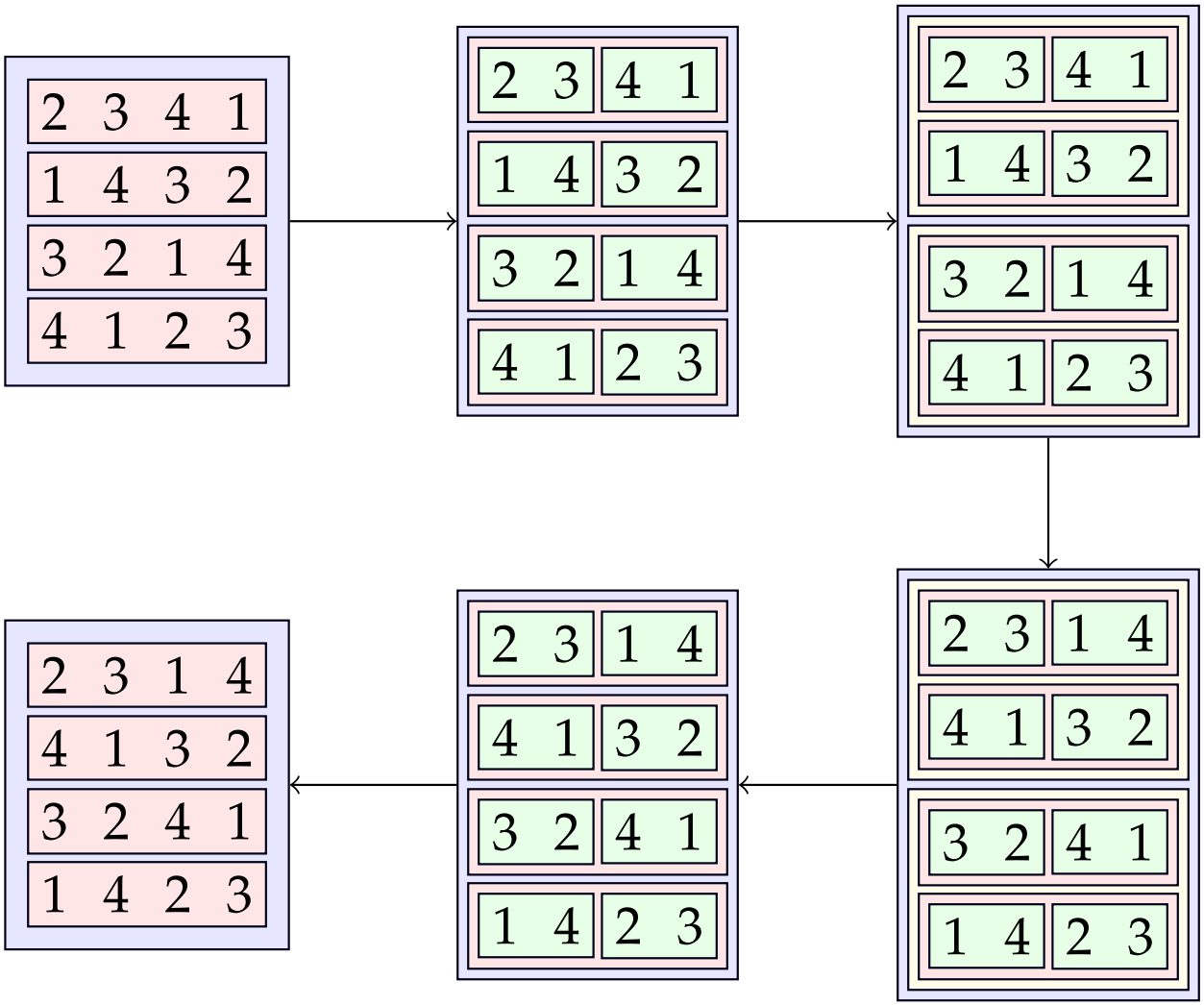
Exercise 10: `boxs` [⋆⋆⋆]



Exercise 10: `boxs` $[\star\star\star]$



Exercise 10: `boxs` $[\star\star\star]$



Exercise 11: `group` [⋆⋆]

```
group :: [a] -> [[a]]
```

Exercise 11: `group` [⋆⋆]

```
group :: [a] -> [[a]]
```

Example:

```
group [1,2,3,4] = [[1,2],[3,4]]
```

Exercise 12: ungroup $[\star]$

```
ungroup :: [[a]] -> [a]
```

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```
ungroup :: [[a]] -> [a]
```

Example:

```
ungroup [[1,2],[3,4]] = [1,2,3,4]
```

Exercise 12: ungroup [⋆]

```
ungroup :: [[a]] -> [a]
```

Example:

```
ungroup [[1,2],[3,4]] = [1,2,3,4]
```

Hints:

- Use Hoogle

That's all folks

- Time to solve some Sudokus

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  - Define a function `prune` that eliminates early invalid solutions

```
filter valid . expand = filter valid . expand . prune
```

That's all folks

- Time to solve some Sudokus
- The current approach is inefficient, but correct
- Equational reasoning to improve performance
  - Define a function `prune` that eliminates early invalid solutions

    ```
    filter valid . expand = filter valid . expand . prune
    ```
  - It is not hard to define a function to prune a row (exercise):

    ```
    pruneRow [[4],[1,2],[1],[1,3]] = [[4],[2],[1],[3]]
    ```

That's all folks

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  - Equational reasoning to define `prune` in terms of `pruneRow`

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- The current approach is inefficient, but correct
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    pruneRow [[4],[1,2],[1],[1,3]] = [[4],[2],[1],[3]]
    ```
  - Equational reasoning to define `prune` in terms of `pruneRow`
    - The function `pruneRow` satisfies the equation

      ```
      filter nodups . cp = filter nodups . cp . pruneRow
      ```

That's all folks

- Time to solve some Sudokus
- The current approach is inefficient, but correct
- Equational reasoning to improve performance
  - Define a function `prune` that eliminates early invalid solutions

    ```
    filter valid . expand = filter valid . expand . prune
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  - It is not hard to define a function to prune a row (exercise):

    ```
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  - Equational reasoning to define `prune` in terms of `pruneRow`
    - The function `pruneRow` satisfies the equation

      ```
      filter nodups . cp = filter nodups . cp . pruneRow
      ```
    - Expand the expression `filter valid . expand`

That's all folks

- Time to solve some Sudokus
- The current approach is inefficient, but correct
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  - Define a function `prune` that eliminates early invalid solutions

    ```
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    ```
  - Equational reasoning to define `prune` in terms of `pruneRow`
    - The function `pruneRow` satisfies the equation

      ```
      filter nodups . cp = filter nodups . cp . pruneRow
      ```
    - Expand the expression `filter valid . expand`
    - Use the above equation and compress back the formula

Further references

- Richard Bird's papers and books (Bird, 2006, 2010, 2014)
- Conor McBride's Sudoku solver using applicative and traversable: https://stackoverflow.com/a/10242673/474311

References

Bird, R. (2010). *Pearls of Functional Algorithm Design*. Cambridge University Press.

Bird, R. (2014). *Thinking Functionally with Haskell*. Cambridge University Press.

Bird, R. S. (2006). A program to solve Sudoku. *Journal of Functional Programming*, 16(6):671–679.