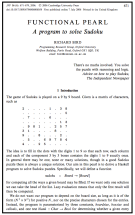
Coding dojo
A simple Sudoku solver in Haskell

B∘FP

Bucharest FP #027

Welcome



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FUNCTIONAL PEARL

A program to solve Sudoku

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There's no maths involved. You solve the puzzle with reasoning and logic.
Advice on how to play Sudoku,
The Independent Newspaper

1 Introduction

The game of Sudoku is played on a 9 by 9 board. Given is a matrix of characters, such as

```
2 . . . . 1 . 3 8
. . . . . . . . 5
. 7 . . . 6 . . .
. . . . . . . 1 3
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3 1 . . . . 8 . .
9 . . 8 . . . 2 .
. 5 . . 6 9 7 8 4
4 . . 2 5 . . . .
```
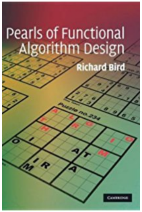
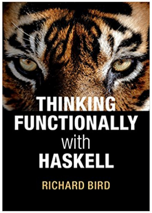

The idea is to fill in the dots with the digits 1 to 9 so that each row, each column and each of the component 3 by 3 boxes contains the digits 1 to 9 exactly once. In general there may be one, none or many solutions, though in a good Sudoku puzzle there is always a unique solution. Our aim in this pearl is to derive a Haskell program to solve Sudoku puzzles. Specifically, we will define a function

$$sudoku :: Board \to [Board]$$

for computing all the ways a given board may be filled. If we want only one solution we can take the head of the list. Lazy evaluation means that only the first result will then be computed.

We do not want our program to depend on the board size, as long as it is of the form $(N^2 \times N^2)$ for positive N, nor on the precise characters chosen for the entries. Instead, the program is parameterized by three constants, *boardsize*, *boxsize* and *cellvals*, and one test $blank :: Char \to Bool$ for determining whether a given entry





- Implement a Sudoku solver as described in (Bird, 2006, 2010, 2014)

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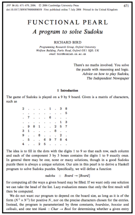
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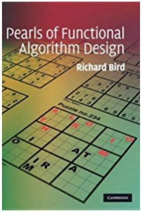

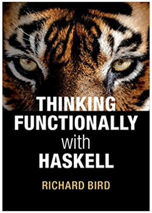

- Implement a Sudoku solver as described in (Bird, 2006, 2010, 2014)
- Twelve short functions:
  - 5 easy ($\star$)
  - 4 medium ($\star\star$)
  - 3 challenging ($\star\star\star$)

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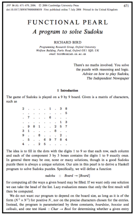

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$$\mathit{sudoku} :: \mathit{Board} \rightarrow [\mathit{Board}]$$

for computing all the ways a given board may be filled. If we want only one solution we can take the head of the list. Lazy evaluation means that only the first result will then be computed.

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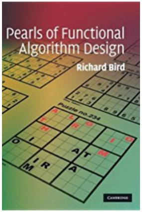
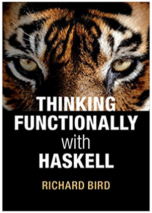




- Implement a Sudoku solver as described in (Bird, 2006, 2010, 2014)
- Twelve short functions:
  - 5 easy (⋆)
  - 4 medium (⋆⋆)
  - 3 challenging (⋆ ⋆ ⋆)
- Programming techniques:
  - Top-down programming / wishful thinking
  - Wholemeal programming (prevents a disease called “indexitis”)
  - Higher-order functions, recursion, point-free style

How to play Sudoku

	2	4	
1			3
4			2
	1	3	

$N = 2$

	2		5		1		9	
8			2		3			6
	3			6			7	
		1				6		
5	4						1	9
		2				7		
	9			3			8	
2			8		4			7
	1		9		7		6	

$N = 3$

Fill in the empty cells with digits 1 to N^2 such that every row, column and $N \times N$ box contains the digits 1 to N^2.

How to play Sudoku

3	2	4	1
1	4	2	3
4	3	1	2
2	1	3	4

$N = 2$

4	2	6	5	7	1	3	9	8
8	5	7	2	9	3	1	4	6
1	3	9	4	6	8	2	7	5
9	7	1	3	8	5	6	2	4
5	4	3	7	2	6	8	1	9
6	8	2	1	4	9	7	5	3
7	9	4	6	3	2	5	8	1
2	6	5	8	1	4	9	3	7
3	1	8	9	5	7	4	6	2

$N = 3$

Fill in the empty cells with digits 1 to N^2 such that every row, column and $N \times N$ box contains the digits 1 to N^2.

Data types

$$\begin{array}{ll}
\boxed{a \quad a \quad \cdots \quad a} & \texttt{Row a} \\
\boxed{a \quad a \quad \cdots \quad a} & \texttt{Row a} \\
\;\vdots \quad\; \vdots \quad\;\; \ddots \quad\;\; \vdots & \\
\boxed{a \quad a \quad \cdots \quad a} & \texttt{Row a} \\
\texttt{Matrix a} &
\end{array}$$

```
type Row a = [a]
type Matrix a = [Row a]
```

Data types

$$\begin{array}{ll} \boxed{a \quad a \quad \cdots \quad a} & \texttt{Row a} \\ \boxed{a \quad a \quad \cdots \quad a} & \texttt{Row a} \\ \vdots \quad \vdots \quad \ddots \quad \vdots & \\ \boxed{a \quad a \quad \cdots \quad a} & \texttt{Row a} \end{array}$$

`Matrix a`

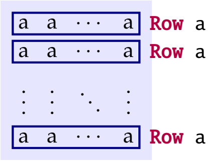
0	3	0	1
1	0	3	2
3	0	1	0
0	1	0	3

`Grid`

```
type Row a = [a]
type Matrix a = [Row a]

type Digit = Int
type Grid = Matrix Digit
```

Data types



Matrix a

0	3	0	1
1	0	3	2
3	0	1	0
0	1	0	3

Grid

```
type Row a = [a]
type Matrix a = [Row a]
```

```
type Digit = Int
type Grid = Matrix Digit
```

We assume that digit zero indicates an empty cell:

```
isEmpty :: Digit -> Bool
isEmpty 0 = True
isEmpty _ = False
```

Exercise 1: solve [$\star$]

```
solve :: Grid -> [Grid]
solve = undefined
```

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Given:

```
-- Generates grids by replacing empty entries
-- with all possible choices
completions :: Grid -> [Grid]
```

Exercise 1: solve [$\star$]

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Given:

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-- Generates grids by replacing empty entries
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completions :: Grid -> [Grid]
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```
-- Tests whether a grid is a valid solution:
-- has different entries in each row, column and box
valid :: Grid -> Bool
```

Exercise 1: solve [⋆]

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solve = undefined
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completions :: Grid -> [Grid]
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```haskell
-- Tests whether a grid is a valid solution:
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valid :: Grid -> Bool
```

Example:

$$\begin{array}{cccc} 0 & 3 & 0 & 1 \\ 1 & 0 & 3 & 2 \\ 3 & 0 & 1 & 0 \\ 0 & 1 & 0 & 3 \end{array} \longrightarrow \begin{array}{cccc} 2 & 3 & 4 & 1 \\ 1 & 4 & 3 & 2 \\ 3 & 2 & 1 & 4 \\ 4 & 1 & 2 & 3 \end{array}$$

`Grid`   `Grid`

Exercise 2: completions [$\star$]

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completions :: Grid -> [Grid]
completions = undefined
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Given:

```
-- Replaces empty entries with all possible choices
-- for that entry
choices :: Grid -> Matrix [Digit]
```

Exercise 2: completions [⋆]

```haskell
completions :: Grid -> [Grid]
completions = undefined
```

Given:

```haskell
-- Replaces empty entries with all possible choices
-- for that entry
choices :: Grid -> Matrix [Digit]

-- Generates a list of all possible boards
-- from a given matrix of choices
expand :: Matrix [Digit] -> [Grid]
```

Exercise 2: completions [⋆]

```
completions :: Grid -> [Grid]
completions = undefined
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Given:

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expand :: Matrix [Digit] -> [Grid]
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Example:

0	3	0	1
1	0	3	2
3	0	1	0
0	1	0	3

Grid

1	3	1	1
1	1	3	2
3	1	1	1
1	1	1	3

Grid

1	3	1	1
1	1	3	2
3	1	1	1
1	1	2	3

Grid

...

4	3	4	1
1	4	3	2
3	4	1	4
4	1	4	3

Grid

Exercise 3: `choices` [$\star$]

```haskell
choices :: Grid -> Matrix [Digit]
choices = undefined
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choices :: Grid -> Matrix [Digit]
choices = undefined
```

Example:

0	3	0	1
1	0	3	2
3	0	1	0
0	1	0	3

Matrix Digit

$\longrightarrow$

1 2 3 4	3	1 2 3 4	1
1	1 2 3 4	3	2
3	1 2 3 4	1	1 2 3 4
1 2 3 4	1	1 2 3 4	3

Matrix [Digit]

Exercise 3: `choices` [$\star$]

```haskell
choices :: Grid -> Matrix [Digit]
choices = undefined
```

Example:

0	3	0	1
1	0	3	2
3	0	1	0
0	1	0	3

`Matrix Digit`

$\longrightarrow$

1 2 3 4	3	1 2 3 4	1
1	1 2 3 4	3	2
3	1 2 3 4	1	1 2 3 4
1 2 3 4	1	1 2 3 4	3

`Matrix [Digit]`

Hint:

- Define a helper function `choice :: Digit -> [Digit]`

Exercise 4: `expand` [$\star\star$]

```haskell
expand :: Matrix [Digit] -> [Grid]
expand = undefined
```

Exercise 4: expand [$\star\star$]

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expand :: Matrix [Digit] -> [Grid]
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Given:

```haskell
-- Computes the cartesian product of a list of lists
cp :: [[a]] -> [[a]]
```

Exercise 4: expand [⋆⋆]

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expand :: Matrix [Digit] -> [Grid]
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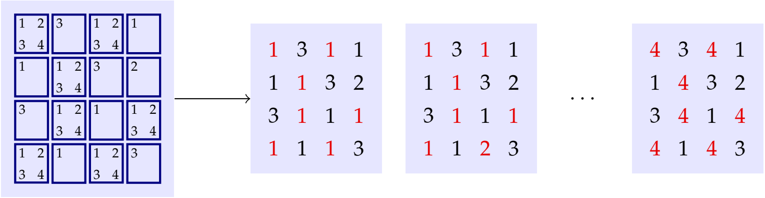

`Matrix [Digit]`

Exercise 5: cp [⋆ ⋆ ⋆]

```haskell
cp :: [[a]] -> [[a]]
cp = undefined
```

Exercise 5: cp $[\star\star\star]$

```
cp :: [[a]] -> [[a]]
cp = undefined
```

Example:

```
cp [[1, 2], [3, 4]] = [[1, 3], [1, 4], [2, 3], [2, 4]]
```

Exercise 5: cp $[\star\star\star]$

```
cp :: [[a]] -> [[a]]
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Example:

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```

Hint:

- Use recursion:

Exercise 5: cp $[\star\star\star]$

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cp :: [[a]] -> [[a]]
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Example:

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cp [[1, 2], [3, 4]] = [[1, 3], [1, 4], [2, 3], [2, 4]]
```

Hint:

- Use recursion:

xss

1 2	3	4 5

Exercise 5: cp [⋆⋆⋆]

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cp :: [[a]] -> [[a]]
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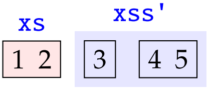

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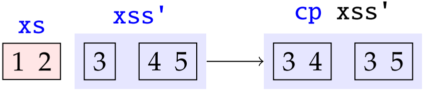

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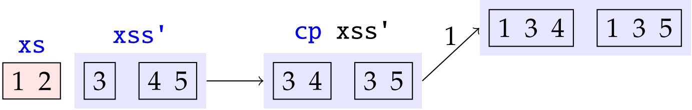

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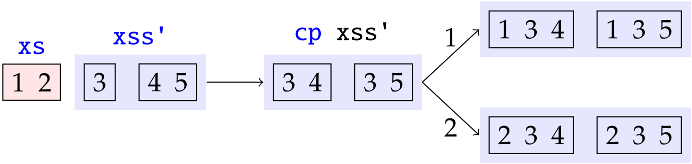

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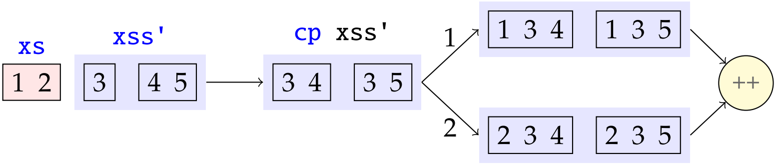
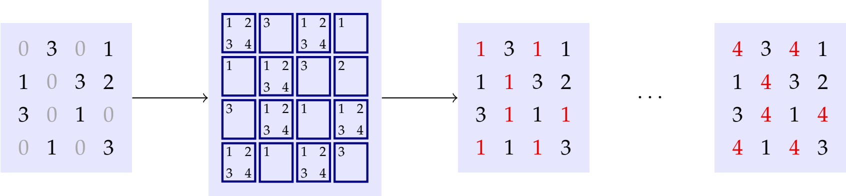
Hint:

- Use recursion:

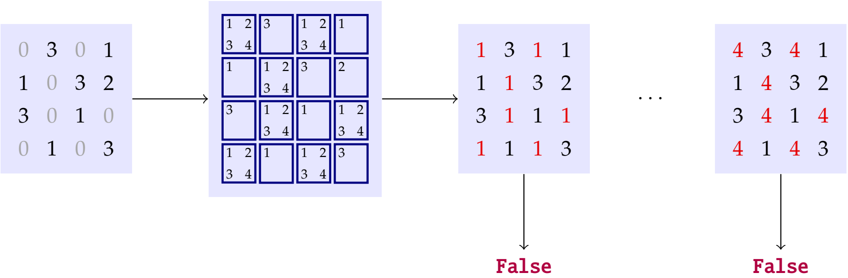


Intermezzo



- Finished implementing the `completions` function

Intermezzo



- Finished implementing the `completions` function
- Next, the `valid` function: test whether a grid is a valid solution

Exercise 6: `valid` [$\star\star$]

```haskell
valid :: Grid -> Bool
valid = undefined
```

Exercise 6: valid [★★]

```haskell
valid :: Grid -> Bool
valid = undefined
```

Given:

```haskell
-- Checks that a list contains no duplicates
nodups :: [a] -> Bool
```

Exercise 6: `valid` [⋆⋆]

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valid :: Grid -> Bool
valid = undefined
```

Given:

```haskell
-- Checks that a list contains no duplicates
nodups :: [a] -> Bool
```

```haskell
-- Re-orders the values from a matrix's rows, columns
-- or boxes to appear along the rows
rows :: Matrix a -> Matrix a
cols :: Matrix a -> Matrix a
boxs :: Matrix a -> Matrix a
```

Exercise 6: valid [⋆⋆]

```
valid :: Grid -> Bool
valid = undefined
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Given:

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-- Checks that a list contains no duplicates
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```

```
-- Re-orders the values from a matrix's rows, columns
-- or boxes to appear along the rows
rows :: Matrix a -> Matrix a
cols :: Matrix a -> Matrix a
boxs :: Matrix a -> Matrix a
```

Examples:

$$\begin{matrix} 1 & 3 & 1 & 1 \\ 1 & 1 & 3 & 2 \\ 3 & 1 & 1 & 1 \\ 1 & 1 & 1 & 3 \end{matrix} \longrightarrow \texttt{False}$$

$$\begin{matrix} 2 & 3 & 4 & 1 \\ 1 & 4 & 3 & 2 \\ 3 & 2 & 1 & 4 \\ 4 & 1 & 2 & 3 \end{matrix} \longrightarrow \texttt{True}$$

Exercise 7: nodups [⋆⋆]

```haskell
nodups :: [a] -> Bool
nodups = undefined
```

Exercise 7: nodups [$\star\star$]

```haskell
nodups :: [a] -> Bool
nodups = undefined
```

Examples:

```haskell
nodups [] = True
nodups [1, 2, 3] = True
nodups [1, 2, 1] = False
```

Exercise 7: nodups [⋆⋆]

```haskell
nodups :: [a] -> Bool
nodups = undefined
```

Examples:

```haskell
nodups [] = True
nodups [1, 2, 3] = True
nodups [1, 2, 1] = False
```

Hints:

- Use recursion
- Use Hoogle to find a function of type `a -> [a] -> Bool`

Exercise 8: `rows` [$\star$]

```haskell
rows :: Matrix a -> Matrix a
```

Exercise 8: `rows` [$\star$]

```haskell
rows :: Matrix a -> Matrix a
```

Example:

$$\begin{array}{cccc} 2 & 3 & 4 & 1 \\ 1 & 4 & 3 & 2 \\ 3 & 2 & 1 & 4 \\ 4 & 1 & 2 & 3 \end{array} \longrightarrow \begin{array}{c} \boxed{2 \quad 3 \quad 4 \quad 1} \\ \boxed{1 \quad 4 \quad 3 \quad 2} \\ \boxed{3 \quad 2 \quad 1 \quad 4} \\ \boxed{4 \quad 1 \quad 2 \quad 3} \end{array}$$

Exercise 9: `cols` [$\star\star\star$]

```
cols :: Matrix a -> Matrix a
```

Exercise 9: `cols` [⋆⋆⋆]

```haskell
cols :: Matrix a -> Matrix a
```

Example:

$$\begin{matrix} 2 & 3 & 4 & 1 \\ 1 & 4 & 3 & 2 \\ 3 & 2 & 1 & 4 \\ 4 & 1 & 2 & 3 \end{matrix} \longrightarrow \begin{matrix} 2 & 1 & 3 & 4 \\ 3 & 4 & 2 & 1 \\ 4 & 3 & 1 & 2 \\ 1 & 2 & 4 & 3 \end{matrix}$$

Exercise 9: `cols` [⋆⋆⋆]

```haskell
cols :: Matrix a -> Matrix a
```

Example:

$$\begin{matrix} 2 & 3 & 4 & 1 \\ 1 & 4 & 3 & 2 \\ 3 & 2 & 1 & 4 \\ 4 & 1 & 2 & 3 \end{matrix} \longrightarrow \begin{matrix} \boxed{2 \quad 1 \quad 3 \quad 4} \\ \boxed{3 \quad 4 \quad 2 \quad 1} \\ \boxed{4 \quad 3 \quad 1 \quad 2} \\ \boxed{1 \quad 2 \quad 4 \quad 3} \end{matrix}$$

Hints:

- Use recursion
- Use the `zipWith` function

Exercise 10: `boxs` [$\star\star\star$]

```haskell
boxs :: Matrix a -> Matrix a
```

Exercise 10: boxs [⋆⋆⋆]

```haskell
boxs :: Matrix a -> Matrix a
```

Given:

```haskell
-- Groups a list into lists of length two
group :: [a] -> [[a]]
```

Exercise 10: boxs [$\star\star\star$]

```haskell
boxs :: Matrix a -> Matrix a
```

Given:

```haskell
-- Groups a list into lists of length two
group :: [a] -> [[a]]

-- Flattens a nested list of elements
ungroup :: [[a]] -> [a]
```

Exercise 10: boxs [⋆⋆⋆]

```haskell
boxs :: Matrix a -> Matrix a
```

Given:

```haskell
-- Groups a list into lists of length two
group :: [a] -> [[a]]

-- Flattens a nested list of elements
ungroup :: [[a]] -> [a]
```

Example:

$$\begin{matrix}2&3&4&1\\1&4&3&2\\3&2&1&4\\4&1&2&3\end{matrix} \longrightarrow \begin{matrix}\boxed{2\quad 3\quad 1\quad 4}\\\boxed{4\quad 1\quad 3\quad 2}\\\boxed{3\quad 2\quad 4\quad 1}\\\boxed{1\quad 4\quad 2\quad 3}\end{matrix}$$

Exercise 10: boxs [⋆⋆⋆]

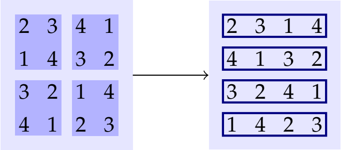
```
boxs :: Matrix a -> Matrix a
```

Given:

```
-- Groups a list into lists of length two
group :: [a] -> [[a]]

-- Flattens a nested list of elements
ungroup :: [[a]] -> [a]
```

Example:



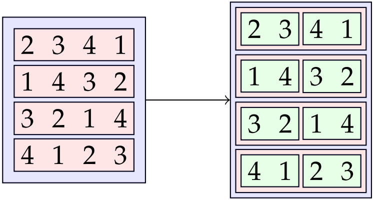
Hints:

- Use the previously defined `cols` function
- Chain five transformations (see next slide)

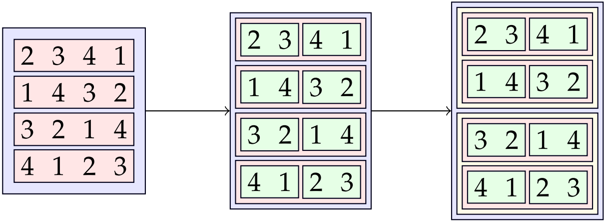
Exercise 10: `boxs` [⋆ ⋆ ⋆]

2	3	4	1
1	4	3	2
3	2	1	4
4	1	2	3

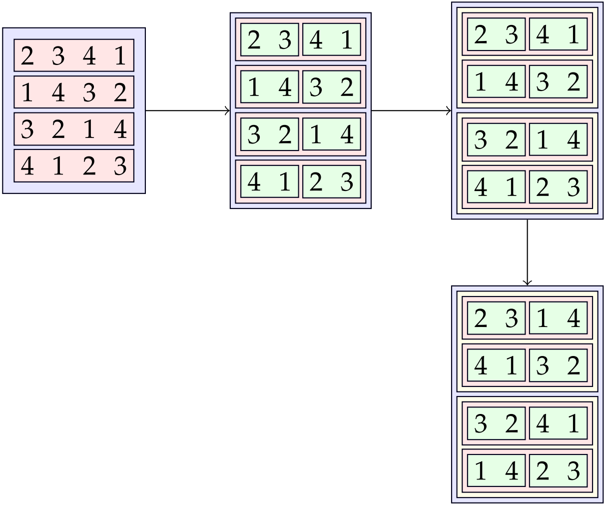
Exercise 10: `boxs` [$\star\star\star$]



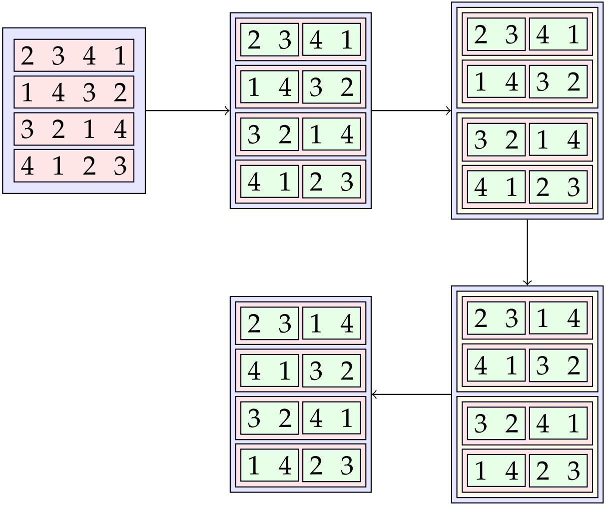
Exercise 10: `boxs` [⋆ ⋆ ⋆]



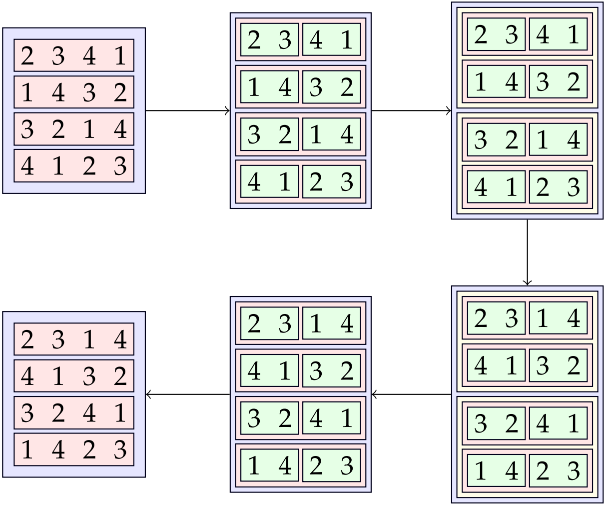
Exercise 10: boxs [⋆⋆⋆]



Exercise 10: `boxs` [⋆⋆⋆]



Exercise 10: boxs [⋆ ⋆ ⋆]



Exercise 11: `group` [$\star\star$]

```haskell
group :: [a] -> [[a]]
```

Exercise 11: group [⋆⋆]

```
group :: [a] -> [[a]]
```

Example:

```
group [1,2,3,4] = [[1,2],[3,4]]
```

Exercise 12: `ungroup` $[\star]$

```haskell
ungroup :: [[a]] -> [a]
```

Exercise 12: `ungroup` $[\star]$

```
ungroup :: [[a]] -> [a]
```

Example:

```
ungroup [[1,2],[3,4]] = [1,2,3,4]
```

Exercise 12: `ungroup` [$\star$]

```haskell
ungroup :: [[a]] -> [a]
```

Example:

```haskell
ungroup [[1,2],[3,4]] = [1,2,3,4]
```

Hints:

- Use Hoogle

That's all folks

- Time to solve some Sudokus

That's all folks

- Time to solve some Sudokus
- The current approach is inefficient, but correct

That's all folks

- Time to solve some Sudokus
- The current approach is inefficient, but correct
- Equational reasoning to improve performance

References

Bird, R. (2010). *Pearls of Functional Algorithm Design*. Cambridge University Press.

Bird, R. (2014). *Thinking Functionally with Haskell*. Cambridge University Press.

Bird, R. S. (2006). A program to solve Sudoku. *Journal of Functional Programming*, 16(6):671–679.