Vector-Rotation to Euler Angles

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Some calculations were verified using Python.

1 Answer

$R_{BA} = R_y(\alpha)R_x(\beta)R_y(\gamma)$

$$R_y(\alpha) = \begin{bmatrix} \cos(\alpha) & 0 & \sin(\alpha) \\ 0 & 1 & 0 \\ -\sin(\alpha) & 0 & \cos(\alpha) \end{bmatrix}$$

$$R_x(\beta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\beta) & -\sin(\beta) \\ 0 & \sin(\beta) & \cos(\beta) \end{bmatrix}$$

$$R_y(\gamma) = \begin{bmatrix} \cos(\gamma) & 0 & \sin(\gamma) \\ 0 & 1 & 0 \\ -\sin(\gamma) & 0 & \cos(\gamma) \end{bmatrix}$$

$$R_{BA} = \begin{bmatrix} \cos(\alpha) & 0 & \sin(\alpha) \\ 0 & 1 & 0 \\ -\sin(\alpha) & 0 & \cos(\alpha) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\beta) & -\sin(\beta) \\ 0 & \sin(\beta) & \cos(\beta) \end{bmatrix} \begin{bmatrix} \cos(\gamma) & 0 & \sin(\gamma) \\ 0 & 1 & 0 \\ -\sin(\gamma) & 0 & \cos(\gamma) \end{bmatrix}$$

$$R_{BA} = \begin{bmatrix} -\sin(\alpha)\sin(\gamma)\cos(\beta) + \cos(\alpha)\cos(\gamma) & \sin(\alpha)\sin(\beta) & \sin(\alpha)\cos(\beta)\cos(\gamma) + \sin(\gamma)\cos(\alpha) \\ \sin(\beta)\sin(\gamma) & \cos(\beta) & -\sin(\beta)\cos(\gamma) \\ -\sin(\alpha)\cos(\gamma) - \sin(\gamma)\cos(\alpha)\cos(\beta) & \sin(\beta)\cos(\alpha) & -\sin(\alpha)\sin(\gamma) + \cos(\alpha)\cos(\beta)\cos(\gamma) \end{bmatrix}$$

The equivalent rotation to $R(\begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}, -90°)$ using Rodriguez Formula:

$e^{\hat{\omega}\theta} = I + \hat{\omega} sin(\theta) + (1 - cos(\theta))\hat{\omega}^2$

For $\theta = -90$:

$e^{\hat{\omega}\theta} = I - \hat{\omega} + \hat{\omega}^2$

$$\hat{\omega} = \begin{bmatrix} 0 & -\frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} & 0 & -\frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & 0 \end{bmatrix}$$

$$\hat{\omega}^2 = \begin{bmatrix} -\frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & -\frac{2}{3} \end{bmatrix}$$

$$e^{\hat{\omega}\theta} = I - \begin{bmatrix} 0 & -\frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} & 0 & -\frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & 0 \end{bmatrix} + \begin{bmatrix} -\frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & -\frac{2}{3} \end{bmatrix}$$

$$e^{\hat{\omega}\theta} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} + \frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} + \frac{1}{3} \\ -\frac{\sqrt{3}}{3} + \frac{1}{3} & \frac{1}{3} & \frac{1}{3} + \frac{\sqrt{3}}{3} \\ \frac{1}{3} + \frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} + \frac{1}{3} & \frac{1}{3} \end{bmatrix} = R_{BA} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

$$\begin{bmatrix} -\sin(\alpha)\sin(\gamma)\cos(\beta)+\cos(\alpha)\cos(\gamma) & \sin(\alpha)\sin(\beta) & \sin(\alpha)\cos(\beta)\cos(\gamma)+\sin(\gamma)\cos(\alpha) \\ \sin(\beta)\sin(\gamma) & \cos(\beta) & -\sin(\beta)\cos(\gamma) \\ -\sin(\alpha)\cos(\gamma)-\sin(\gamma)\cos(\alpha)\cos(\beta) & \sin(\beta)\cos(\alpha) & -\sin(\alpha)\sin(\gamma)+\cos(\alpha)\cos(\beta)\cos(\gamma) \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

1.1 Solving for α, β, γ

1.1.1 β

Using the atan method, one can solve for β using the middle column of the matrix. The middle value, $r_{22} = cos(\beta)$. Using $r_{12}^2 + r_{32}^2 = sin^2(\beta)(sin^2(\gamma) + cos^2(\gamma)) = sin^2(\beta)$, $sin(\beta) = \sqrt{r_{12}^2 + r_{32}^2}$. Therefore, $\beta = atan2(\sqrt{r_{12}^2 + r_{32}^2}, r_{22})$.

1.1.2 γ

Looking at two elements in the middle row of the matrix, one can solve for γ using the atan method. $r_{21} = sin(\beta)sin(\gamma)$, so $sin(\gamma) = r_{21}/sin(\beta)$. $r_{23} = -sin(\beta)cos(\gamma)$, so $cos(\gamma) = -r_{23}/sin(\beta)$. Therefore, $\gamma = atan2(r_{21}/sin(\beta), -r_{23}/sin(\beta))$.

1.1.3 α

$r_{12} = sin(\alpha)sin(\beta)$, $sin(\alpha) = r_{12}/sin(\beta)$

$r_{32} = cos(\alpha)sin(\beta)$, $cos(\alpha) = r_{32}/sin(\beta)$

$\alpha = atan2(r_{12}/sin(\beta), r_{32}/sin(\beta))$

1.2 Conclusion

Substituting the correct values from R_{BA} calculated by Rodriguez Formula into the spaces allocated by r_{ij} that were solved into an $atan2$ formula using R_{BA} calculated by the rotation matrices gives the solution for the equivalent rotations in radians (rounded to 3 decimal places).

$\alpha = 1.833, \beta = 1.231, \gamma = -2.880$