Vector-Rotation to Euler Angles

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Some calculations were verified using Python.

1 Answer

$R_{BA} = R_y(\alpha)R_x(\beta)R_y(\gamma)$

1.1 Solving for α, β, γ

1.1.1 β

Using the atan method, one can solve for β using the middle column of the matrix. The middle value, $r_{22} = cos(\beta)$. Using $r_{12}^2 + r_{32}^2 = sin^2(\beta)(sin^2(\gamma) + cos^2(\gamma)) = sin^2(\beta)$, $sin(\beta) = \sqrt{r_{12}^2 + r_{32}^2}$. Therefore, $\beta = atan2(\sqrt{r_{12}^2 + r_{32}^2}, r_{22})$.

1.1.2 γ

Looking at two elements in the middle row of the matrix, one can solve for γ using the atan method. $r_{21} = sin(\beta)sin(\gamma)$, so $sin(\gamma) = r_{21}/sin(\beta)$. $r_{23} = -sin(\beta)cos(\gamma)$, so $cos(\gamma) = -r_{23}/sin(\beta)$. Therefore, $\gamma = atan2(r_{21}/sin(\beta), -r_{23}/sin(\beta))$.

1.1.3 α

$r_{12} = sin(\alpha)sin(\beta)$, $sin(\alpha) = r_{12}/sin(\beta)$
$r_{32} = cos(\alpha)sin(\beta)$, $cos(\alpha) = r_{32}/sin(\beta)$
$\alpha = atan2(r_{12}/sin(\beta), r_{32}/sin(\beta))$

1.2 Conclusion

Substituting the correct values from R_{BA} calculated by Rodriguez Formula into the spaces allocated by r_{ij} that were solved into an $atan2$ formula using R_{BA} calculated by the rotation matrices gives

the solution for the equivalent rotations in radians (rounded to 3 decimal places).
$\alpha = 1.833, \beta = 1.231, \gamma = -2.880$