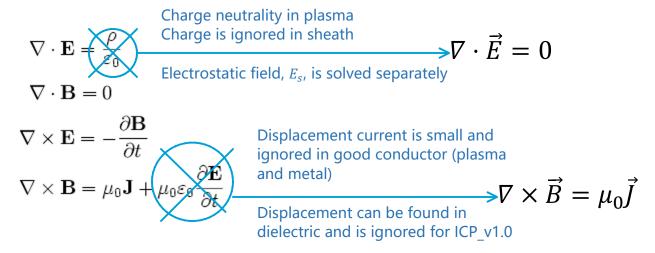
ICP Field Eq. Derivation

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Introduce Vector Potential, $\vec{B} = \nabla \times \vec{A}$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \rightarrow \nabla \times \vec{E} = -\frac{\partial}{\partial t} \nabla \times \vec{A} \rightarrow \vec{E} = -\frac{\partial \vec{A}}{\partial t}$$

$$\vec{E} = \vec{E} \exp(i\omega t + \phi)$$

$$\vec{E} = -i\omega \vec{A} \leftarrow \vec{E} = \vec{E} \exp(i\omega t + \phi)$$
Solve in frequency domain

$$abla imes \vec{B} = \mu_0 \vec{J}_{tot} \rightarrow \nabla \times \nabla \times \vec{A} = \mu_0 \vec{J}_{tot}$$
use math formula $\nabla \times \nabla \times \vec{A} = W(\cdot \vec{A}) \nabla^2 \vec{A}$

$$\overline{W}(\cdot \overrightarrow{A}) \nabla^2 \overrightarrow{A} = \mu_0 \overrightarrow{J}_{tot}$$

$$\nabla^2 \overrightarrow{A} = -\mu_0 \overrightarrow{J}_{tot} \text{ has an analytic solution}$$

$$A = \frac{i}{\omega} \nabla \cdot \overrightarrow{E} = 0 \leftarrow \text{charge neutrality}$$

$$A = \frac{\mu_0}{4\pi'} \int \frac{J_{coil}(r, t') + J_{ind}(r', t')}{|r - r'|} dv'$$

$$\nabla^2 \vec{A} = -\mu_0 \vec{J}_{tot}$$
 where $\vec{J}_{tot} = \vec{J}_{coil} + \vec{J}_{plasma} + \vec{J}_{mater}$

- ullet $ec{J}_{coil}$ is the external current, taken as input parameter
- $\vec{J}_{mater} = \sigma_m \vec{E}$, where σ_m is the material conductivity, specified by the material properties
- $\vec{J}_{plasma} = \sigma_p \vec{E}$, assuming plasma is collissional, Ohm's law is valid in plasma.

 σ_p depends on the electron and ion collision frequency, which

is determined by the cross section and energy distribution. (plasma conductivity will be discussed in details later.)

Vector potential has only tangential component, in azimuthal direction

$$A = (0, A_{\theta}, 0)$$

$$\nabla^2 A_{\theta} - A_{\theta}/r^2 = -\mu_0 (J_{coil} + J_{ind})$$

$$\nabla^2 = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{\partial^2}{\partial z^2}$$

Once A_{θ} is obtained, the EM field can be obtained by the following relations:

$$E_{\theta} = -\mathrm{i}\omega A_{\theta}$$

$$\mu_{0}H_{z} = \frac{1}{r}\frac{\partial}{\partial r}(rA_{\theta})$$

$$\mu_{0}H_{r} = -\frac{\partial}{\partial r}A_{\theta}.$$

In the coil	In the plasma	In the vaccum
$\nabla^2 A_\theta - A_\theta / r^2 = -\mu_0 J_{coil}$	$\nabla^2 A_\theta - (1/r^2 + i\omega\mu_0\sigma)A_\theta = 0$	$\nabla^2 A_\theta - A_\theta / r^2 = 0$
$\nabla^2 A_{\theta R} - A_{\theta R}/r^2 = -\mu_0 J_{coil}$ $\nabla^2 A_{\theta I} - A_{\theta I}/r^2 = 0.$	$\nabla^2 A_{\theta R} - (1/r^2) A_{\theta R} + \omega \mu_0 \sigma A_{\theta I} = 0$ $\nabla^2 A_{\theta I} - (1/r^2) A_{\theta I} - \omega \mu_0 \sigma A_{\theta R} = 0$	

Boundary conditions:

- 1. \vec{A} is 0 at the metal surface, where the conductivity σ is assumed to be infinite.
- 2. When boundary is far enough from the COIL, \vec{A} is θ ,called far-field boundary condition.
- 3. At the interface, continuity is enforced for \vec{A} .
 - a. For the boundary conditions at the interfaces between

different zones we assume continuity of the vector potentials cross the interfaces without jumping. This implies that the magnitude and the direction of $A\theta R$ and $A\theta I$ remain unchanged on both sides of the interfaces. Physically this is not necessarily consistent since a change in the dielectric properties of the material across the interface would result in a corresponding difference in the normal component of the electric field, and hence of the vector potential. It is difficult, however, in the FLUENT code to set different boundary conditions on the two sides of an interface. For the interfaces shown in figure 1 the electric field and, accordingly, the vector potential are parallel to the interfaces so that the continuity in the boundary conditions can be applied.

In the coil:

$$\nabla^2 A_\theta - A_\theta / r^2 = -\mu_0 J_{coil}$$

$$\nabla^2 A_{\theta R} - A_{\theta R}/r^2 = -\mu_0 J_{coil}$$
$$\nabla^2 A_{\theta I} - A_{\theta I}/r^2 = 0.$$

Here we have assumed that the electric conductivity of the coil is zero, which means Jind in the coil is zero. $\mathit{Jcoil} = I/(\pi \ a2)$ is uniformly distributed over the cross section of the coil conductor.