Finitely long solenoid

Tuesday, September 15, 2020 1:48 PM

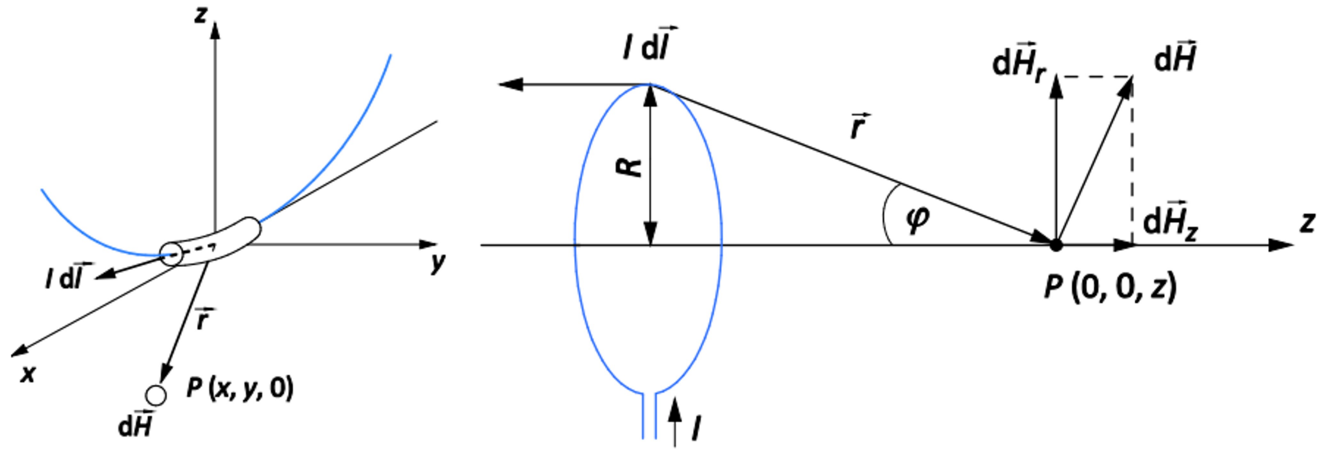

Fig. 2 Calculation of the axial component of the magnetic field of a circular conducting wire loop using Biot-Savart's law

With dH = dH_z /sinφ and sinφ =R/r one obtains

$$\mathrm{d}H_z = \frac{I}{4\pi} \frac{R\,\mathrm{d}l}{\sqrt{\left(R^2 + z^2\right)^3}} \quad , \qquad (5)$$

since $\boldsymbol{r}$ is perpendicular to d$\boldsymbol{l}$. The integration over all elements of the circular wire loop (full length 2πR) yields the equation

$$H(z)=\frac{I}{2}\frac{R^2}{\sqrt{\left(R^2+z^2\right)^3}} \quad . \tag{6}$$

If the wire loops are adjacent to form a coil, the magnetic field components are superimposed to yield a total field at the test point *P* of distance $z = a$ in Fig. 3.

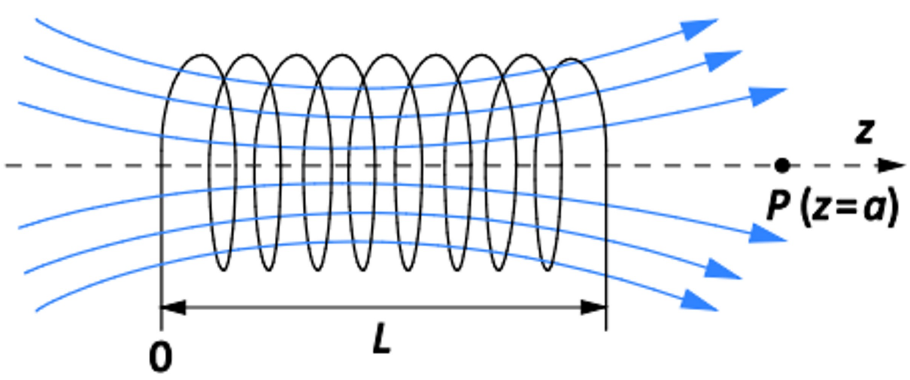

Fig. 3

Calculation of the *B*-field of a solenoid

The density of turns (number of turns per unit length) of a long, tightly wound solenoid is $n = N/L$, where *N* is the number of turns and *L* is the length of the solenoid. Thus, $n\mathrm{d}z$ wire loops are between *z* and *z*+d*z* carrying the current $I\,n\,\mathrm{d}z$. This current generates an axial component of the magnetic flux density $\mathrm{d}B_z$ in the point *P* ($z = a$) of Fig. 3

$$\mathrm{d}B_z=\frac{\mu_0}{2}I\,R^2\frac{1}{\sqrt{\left[R^2+(a-z)^2\right]^3}}n\,\mathrm{d}z \ , \tag{7}$$

where μ_0 denotes the vacuum permeability.

The integration over the total length *L* of the solenoid gives

$$B_z=B(a)=\frac{\mu_0}{2}I\,n\left[\frac{a}{\sqrt{R^2+a^2}}+\frac{L-a}{\sqrt{R^2+(L-a)^2}}\right] \quad . \tag{8}$$

It follows from Eq. (8) that the amount of magnetic flux density at the beginning ($a = 0$) and at the end $a = L$ of the solenoid is half of that in the center ($a = L/2$) of the solenoid (for $R \ll L$).