Plasma Conductivity

Monday, October 26, 2020 9:37 PM

Consider a small-amplitude electromagnetic field, having an electric field in the x-direction of an infinite plasma. The electric field may be defined using complex numbers:

$$E = E_0 \cos(i\omega t)$$

Let's look at the electron momentum equation

$$nm\left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u}\right] = nq\mathbf{E} - \nabla p - m\mathbf{u}\left[n\nu_{\mathrm{m}} + S - L\right], \qquad (2.33)$$

Assume:

- the ions do not respond to this high-frequency perturbation
- the electron pressure gradient is not significant, effectively ignoring electron
- thermal energy
- there is no steady current in the plasma so the drift speed is zero

Assume the electron velocity follows the same oscillation, $u = u_0 \cos(i\omega t)$.
The electron momentum equation linearizes to

$$(n_e m_e i\omega)u_0 \cos(i\omega t)$$
$$= -n_e e E_0 \cos(i\omega t) - (n_e m_e v_{coll})u_0 \cos(i\omega t)$$

The magnitudes of the perturbations in velocity and electric field amplitude are therefore related by

$$u_0 = -\frac{e}{m_e(i\omega + v_{coll})}E_0$$

current is determined by

$$J_0 = -en_e u_0 = \frac{n_e e^2}{m_e(i\omega + v_{coll})}E_0$$

The electron conductivity σ_e is,

$$\sigma_e = \frac{n_e e^2}{m_e(i\omega + v_{coll})}$$

Plasma conductivity $\sigma_{plasma} = \sigma_e + \sigma_{ion} \approx \sigma_e$, since $\sigma_e \gg \sigma_{ion}$.

Notice that when collision dominates the plasma, $v_{coll} \gg \omega$,

$$\sigma_e = \frac{n_e e^2}{m_e(\upsilon_{coll})}$$

σ_e is real, E_{ext} is not impacted by the plasma, since E_{ind} is $90°$ degree out of phase with E_{ext}.

When oscillation dominates the plasma, $\omega \gg \upsilon_{coll}$,

$$\sigma_e = \frac{n_e e^2}{m_e(i\omega)}$$

σ_e is imaginary, E_{ext} is expelled/canceled by the plasma, ssince E_{ind} is $180°$ degree out of phase with E_{ext}. Within the plasma, $E_{tot} = E_{ext} + E_{ind}$, meaning no B-field penetration.