Drift-Diffusion Equation

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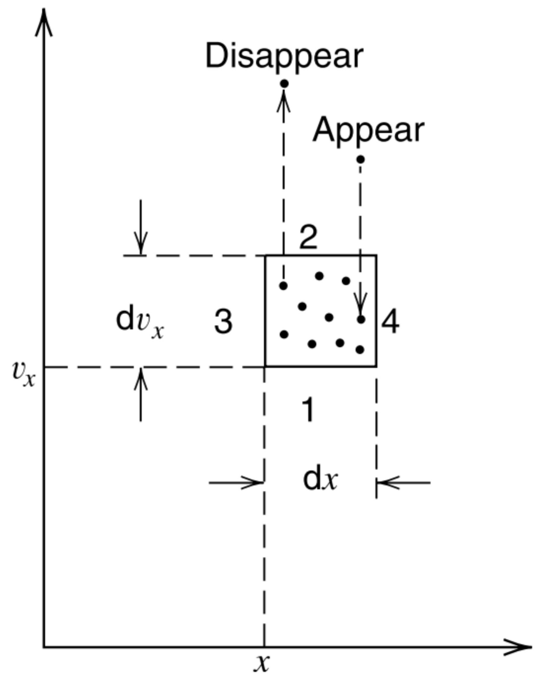
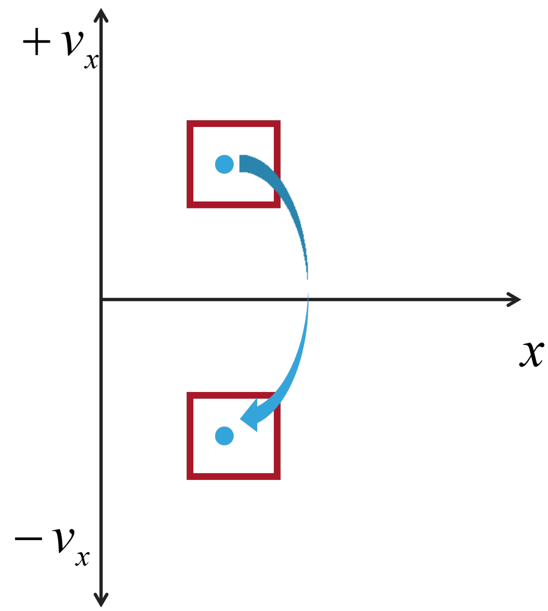
Diffusion and Mobility

$f(\mathbf{r}, \mathbf{v}, t)d^3r\, d^3v =$ number of particles inside a six-dimensional phase space volume $d^3r\, d^3v$ at $(\mathbf{r}, \mathbf{v})$ at time t





$$\frac{df}{dt} = 0$$

$$\frac{\partial f}{\partial t} + \frac{\partial \vec{r}}{\partial t} \bullet \nabla_{\vec{r}} f + \frac{\partial \vec{v}}{\partial t} \bullet \nabla_{\vec{v}} f = 0$$

$$\frac{\partial f}{\partial t} + \vec{v} \bullet \nabla_{\vec{r}} f + \vec{a} \bullet \nabla_{\vec{v}} f = 0$$

$$\frac{\partial}{\partial t} f + \vec{v} \bullet \nabla_r f + \frac{\vec{F}}{m} \bullet \nabla_v f = collision\_term$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) + etc.$$

$$dN = f(\vec{r}, \vec{v}, t) d^3 \vec{r}\, d^3 \vec{v}$$

$$n(\vec{r}, t) = \iiint f(\vec{r}, \vec{v}, t) d^3 \vec{v}$$

$$\langle v(\vec{r}, t) \rangle = \frac{\iiint \vec{v} f(\vec{r}, \vec{v}, t) d^3 \vec{v}}{n(\vec{r}, t)}$$

$$n(\vec{r}, t) \langle v(\vec{r}, t) \rangle = \iiint \vec{v} f(\vec{r}, \vec{v}, t) d^3 \vec{v}$$

$$\boxed{mn \frac{\partial}{\partial t} \vec{u} = qn \vec{E} - \nabla p - mn\nu_m \vec{u}}$$

In a cold uniform plasma with an applied electric field, gives rise to a conductivity. The friction term, arising from collisions with a background species, also leads to diffusion in a nonuniform warm plasma.

Drift-Diffusion Approximation

At this point, we are only interested in steady-state solution, which results in $\frac{\partial u}{\partial t} = 0$

$$qnE - \nabla p - mnv_m u = 0$$

where we assume that the background species is at rest and that the momentum transfer frequency nm is a constant, independent of the drift velocity.

Taking an isothermal plasma, such that $\nabla p = kT\nabla n$, and solving equation above for u, we obtain

$$u = \frac{qE}{mv_m} - \frac{kT}{mv_m}\left(\frac{\nabla \mathrm{n}}{\mathrm{n}}\right)$$

$$nu = \frac{qnE}{mv_m} - \frac{kT\nabla n}{mv_m}$$

$$\Gamma = \pm\mu n E - D\nabla n$$
$$= Drift + diffusion$$

Where

$$\mu = \frac{|q|}{m v_m}\left(\frac{m^2}{V \cdot s}\right) = mobility$$

$$D = \frac{kT}{m v_m}\left(\frac{m^2}{s}\right) = diffusivity$$

$v_m = momentum\ collision\ frequency$
$Depends\ on\ Cross\ Section\ and\ Temperature$

Einstein Relation:

$$D = \frac{kT}{|q|}\mu$$

Ambipolar Diffusion

In the steady state we make the congruence assumption that the flux of electrons and ions out of any region must be equal, $\Gamma_e = \Gamma_i$, such that charge does not build up. This is still true in the presence of ionizing collisions, which create equal numbers of both species. Since the electrons are lighter, and would tend to flow out faster (in an unmagnetized plasma), an electric field must spring up to maintain the local flux balance. That is, a few more electrons than ions initially leave the plasma region to set up a charge imbalance and consequently an electric field.

$$-\mu_e n_e E - D_e \nabla n_e = \mu_i n_i E - D_i \nabla n_i$$

Note that drift is negative for electrons and positive for ions, where E-field drags electrons down and speed ions up to balance the fluxes. Assume $n_e \simeq n_i = n$

$$E = \frac{D_i - D_e}{\mu_i + \mu_e}\left(\frac{\nabla n}{n}\right)$$

Substituting this value of E into the common flux relation we have

$$\Gamma = \mu_i \frac{D_i - D_e}{\mu_i + \mu_e} \nabla n - D_i \nabla n = -\frac{\mu_e D_i + \mu_i D_e}{\mu_i + \mu_e} \nabla n$$

You can see the coefficient is symmetric, beautiful!
Define

$$D_{ambi} = \frac{\mu_e D_i + \mu_i D_e}{\mu_i + \mu_e}$$

$$\Gamma = -D_{ambi} \nabla n$$

$$\mu_e = \frac{|q|}{v_m}\left(\frac{1}{m_e}\right) \gg \mu_i = \frac{|q|}{v_m}\left(\frac{1}{m_i}\right), sine\ m_e \ll m_i$$

$$D_{ambi} \approx D_i + \frac{\mu_i}{\mu_e} D_e = D_i(1 + \frac{T_e}{T_i})$$

Put it back to continuity equation.

$$\frac{\partial n}{\partial t} = D_{ambi} \nabla^2 n + S_e$$

Now the continuity equation becomes DIFFUSION equation, which is much easier to solve.

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When using Ambipolar Diffusion Approximation, we only calculate ion density n_i, and enforce charge neutrality, $n_e = n_i$.

Ambipolar E-field is ,

$$E = \frac{D_i - D_e}{\mu_i + \mu_e}\left(\frac{\nabla n}{n}\right)$$

Can be used for energy input for T_e.
Additionally, Poisson's Equation is avoided.

One of the main assumptions in the drift-diffusion model is that the background gas is dominant.