ICP Field Eq. Derivation

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$$\nabla\cdot\mathbf{E} = \cancel{\frac{\rho}{\varepsilon_0}}$$

Charge neutrality in plasma
Charge is ignored in sheath

$$\longrightarrow \nabla\cdot\vec{E} = 0$$

Electrostatic field, E_s, is solved separately

$$\nabla\cdot\mathbf{B} = 0$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$$

$$\nabla\times\mathbf{B} = \mu_0\mathbf{J} + \cancel{\mu_0\varepsilon_0\frac{\partial\mathbf{E}}{\partial t}}$$

Displacement current is small and ignored in good conductor (plasma and metal)

$$\longrightarrow \nabla\times\vec{B} = \mu_0\vec{J}$$

Displacement can be found in dielectric and is ignored for ICP_v1.0

Introduce Vector Potential, $\vec{B} = \nabla\times\vec{A}$

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \rightarrow \nabla\times\vec{E} = -\frac{\partial}{\partial t}\cancel{\nabla}(\times\vec{A}) \rightarrow \vec{E} = -\frac{\partial\vec{A}}{\partial t}$$

$\vec{E} = \vec{E}\exp(i\omega t + \phi)$
$\vec{A} = \vec{A}\exp(i\omega t + \phi)$

$$\vec{E} = -i\omega\vec{A}$$

Solve in frequency domain

$$\nabla\times\vec{B} = \mu_0\vec{J}_{tot} \rightarrow \nabla\times\nabla\times\vec{A} = \mu_0\vec{J}_{tot}$$

use math formula $\nabla\times\nabla\times\vec{A} = \cancel{\nabla(\cdot\vec{A})}\,\nabla^2\vec{A}$

$$\cancel{\nabla(\cdot\vec{A})}\,\nabla^2\vec{A} = \mu_0\vec{J}_{tot}$$

$\nabla\cdot\vec{A} = \frac{i}{\omega}\nabla\cdot\vec{E} = 0 \leftarrow charge\ neutrality$

$$\boxed{\nabla^2\vec{A} = -\mu_0\vec{J}_{tot}}\quad where\ \vec{J}_{tot} = \vec{J}_{coil} + \vec{J}_{plasma} + \vec{J}_{mater}$$

$\nabla^2\vec{A} = -\mu_0\vec{J}_{tot}$ has an analytic solution

$$\mathbf{A} = \frac{\mu_0}{4\pi'}\int\frac{\boldsymbol{J}_{coil}(r,t') + \boldsymbol{J}_{ind}(r',t')}{|r-r'|}\,\mathrm{d}v'$$

- $\vec{J}_{coil}$ *is the external current, taken as input parameter*
- $\vec{J}_{mater} = \sigma_m\vec{E}$, *where* σ_m *is the material conductivity, specified by the material properties*
- $\vec{J}_{plasma} = \sigma_p\vec{E}$ *, assuming plasma is collissional, Ohm's law is valid in plasma.*

  σ_p *depends on the electron and ion collision frequency, which* *is determined by the cross section and energy distribution. (plasma conductivity will be discussed in details later.)*

Vector potential has only tangential component, in azimuthal direction

$$\boldsymbol{A} = (0,\, A_\theta,\, 0)$$

$$\nabla^2 A_\theta - A_\theta / r^2 = -\mu_0 (J_{coil} + J_{ind})$$

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial z^2}$$

Once A_θ is obtained, the EM field can be obtained by the following relations:

$$E_\theta = -\mathrm{i}\omega A_\theta$$

$$\mu_0 H_z = \frac{1}{r}\frac{\partial}{\partial r}(r A_\theta)$$

$$\mu_0 H_r = -\frac{\partial}{\partial r} A_\theta .$$

In the coil	**In the plasma**	**In the vaccum**
$\nabla^2 A_\theta - A_\theta / r^2 = -\mu_0 J_{coil}$ $\nabla^2 A_{\theta R} - A_{\theta R}/r^2 = -\mu_0 J_{coil}$ $\nabla^2 A_{\theta I} - A_{\theta I}/r^2 = 0.$	$\nabla^2 A_\theta - (1/r^2 + \mathrm{i}\omega\mu_0\sigma) A_\theta = 0$ $\nabla^2 A_{\theta R} - (1/r^2) A_{\theta R} + \omega\mu_0\sigma A_{\theta I} = 0$ $\nabla^2 A_{\theta I} - (1/r^2) A_{\theta I} - \omega\mu_0\sigma A_{\theta R} = 0$	$\nabla^2 A_\theta - A_\theta / r^2 = 0$

Boundary conditions:

1. $\vec{A}$ is *0* at the metal surface, where the conductivity σ is assumed to be infinite.
2. When boundary is far enough from the COIL, $\vec{A}$ is *0* ,called far-field boundary condition.
3. At the interface, continuity is enforced for $\vec{A}$.
   a. For the boundary conditions at the interfaces between

different zones we assume continuity of the vector potentials cross the interfaces without jumping. This implies that the magnitude and the direction of $A\theta R$ and $A\theta I$ remain unchanged on both sides of the interfaces. Physically this is not necessarily consistent since a change in the dielectric properties of the material across the interface would result in a corresponding difference in the normal component of the electric field, and hence of the vector potential. It is difficult, however, in the FLUENT code to set different boundary conditions on the two sides of an interface. For the interfaces shown in figure 1 the electric field and, accordingly, the vector potential are parallel to the interfaces so that the continuity in the boundary conditions can be applied.

In the coil:

$$\nabla^2 A_\theta - A_\theta/r^2 = -\mu_0 J_{coil}$$

4. or

$$\nabla^2 A_{\theta R} - A_{\theta R}/r^2 = -\mu_0 J_{coil}$$

$$\nabla^2 A_{\theta I} - A_{\theta I}/r^2 = 0.$$

Here we have assumed that the electric conductivity of the coil is zero, which means $Jind$ in the coil is zero. $Jcoil = \mathrm{I}/(\pi\ a2)$ is uniformly distributed over the cross section of the coil conductor.