# ICP Fluid equation

Sunday, August 30, 2020 9:08 PM

## Core Eq. in Bulk Plasma

Continuity EQ.

$$\frac{\partial n_e}{\partial t} + \nabla\cdot\vec{\Gamma}_e = S_e$$

for $\boxed{\text{steady state } \frac{\partial n_e}{\partial t}=0}$ Pulsing mode? How to process time-dependent?

$$\nabla\cdot\vec{\Gamma}_e = S_e$$ Src term is necessary here to balance lost

assume eon is produced by ionization & only lost through wall

$$S_e = k_e n_g n_e = (k_e n_g) n_e$$ determined by $T_e$, $n_g$ EEDF Flow

$$\nabla\cdot\vec{\Gamma}_e = (k_e n_g)\, n_e$$

$$(k_e n_g)\, n_e - \nabla\cdot\vec{\Gamma}_e = 0 \quad (1)$$

$\boxed{\text{Drift - Diffusion Aproximation}}$ usually valid for higher press 5mT range not good! the most imp is to occupy the seat for $\vec{\Gamma}_e$

$$\vec{\Gamma}_e = -D_e\nabla n_e - e\mu_e n_e \vec{E}$$

$$D_e\nabla n_e + e\mu_e n_e\nabla\phi + \vec{\Gamma}_e = 0 \quad (2)$$

Poisson's Eq.

$$\nabla^2\phi = -\rho/\varepsilon_0 \quad (3)$$

Since $\rho = e(\sum_i n_i - n_e)$,

$n_i$ needs to be solve simultaneously

$$\frac{\partial n_i}{\partial t} + \nabla\cdot\vec{\Gamma}_i = (k_i n_g) n_e$$

$$(k_i n_g)\, n_e - \nabla\cdot\vec{\Gamma}_i = 0$$

DD aproximation

$$\vec{\Gamma}_i = -D_i\nabla n_i + e\mu_i n_i\nabla\phi$$

$$D_i\nabla n_i - e\mu_i n_i\nabla\phi + \vec{\Gamma}_i = 0$$

Something gets wrong here!

START OVER!

## Logic thoughts

Core: Solve for $n_{e,i}$, $\vec{\Gamma}_{e,i}$, $\phi/E$ (Driving Term) (IEDF)

must have Continuity EQ.

$$\frac{\partial n_e}{\partial t} + \nabla\cdot\vec{\Gamma}_e = S_e$$

$$\frac{\partial n_e}{\partial t} = -\nabla\cdot\vec{\Gamma}_e + S_e$$ (Loss) (Src)

3 unknown, $n_e$, $\vec{\Gamma}_e$, $S_e$ → always calc from external $(k_e n_g) n_e$

Another option Momentum Eq for Eon; D-D Approximation

$$\vec{\Gamma}_e = -D_e\nabla n_e - e\mu_e n_e\nabla\phi$$

$$\frac{\partial n_e}{\partial t} = +\nabla\cdot\left(D_e\nabla n_e + e\mu_e n_e\nabla\phi\right) + S_e$$

$$+\, n_e\sum_g (k_{e,g} n_g)$$

$\boxed{D_e,\ \mu_e,\ S_e}$ are calc externally

Unknowns are $n_e$, $\phi$

Here comes the Poisson EQ. Alternatively, we can solve for ambipolar E-field instead much faster

$$\nabla\cdot(\varepsilon\nabla\phi) = -e\left(\sum_i n_i - n_e\right)$$

in order for Poisson EQ. We need all ion densities

Continuity EQ. for ions

$$\frac{\partial n_i}{\partial t} = -\nabla\cdot\vec{\Gamma}_i + S_i$$

D-D approx

$$\frac{\partial n_i}{\partial t} = +\nabla\cdot\left(D_i\nabla n_i - e\mu_i n_i\nabla\phi\right) + S_i$$

assume that all variables at $t$ are known

$$\begin{cases} \frac{\partial n_e}{\partial t} = \nabla\cdot\left(D_e\nabla n_e + e\mu_e n_e\nabla\phi\right) + S_e \\ \frac{\partial n_i}{\partial t} = \nabla\cdot\left(D_e\nabla n_i - e\mu_i n_i\nabla\phi\right) + S_i \\ \nabla\cdot(\varepsilon\nabla\phi) = -e\left(\sum_i n_i - n_e\right) \end{cases}$$

### Bulk Plasma

$$\begin{cases} n_e(t+\Delta t) = n_e(t) + \Delta t\left(-\nabla\cdot\vec{\Gamma}_e(t) + S_e(t)\right) \\ n_i(t+\Delta t) = n_i(t) + \Delta t\left(-\nabla\cdot\vec{\Gamma}_i(t) + S_i(t)\right) \\ \nabla\cdot\left(\varepsilon\nabla\phi(t+\Delta t)\right) = -e\left(\sum_i n_i(t+\Delta t) - n_e(t+\Delta t)\right) \end{cases}$$

### Boundary Condition

Continuity EQ. Cartesian

i) $n_e, n_i = 0 \,\big|_{\text{at all surf}}$

ii) $(\vec{\Gamma}_e, \vec{\Gamma}_i)\cdot\hat{n} = 0\,\big|_{\text{at all surf}}$ Not Correct

Cylindrical Reflective at $r=0$

i) $(\vec{\Gamma}_e, \vec{\Gamma}_i)\cdot\hat{n} = -(\vec{\Gamma}_e, \vec{\Gamma}_i)\cdot\hat{n}\,\big|_{\text{at } r=0}$

$\Gamma = 0$

Poisson's EQ. Cartesian

i) w/o surf charging

$$-\nabla\cdot(\varepsilon\nabla\phi) = 0$$

D : M; $\varepsilon_0$, $\varepsilon_m$; $n_s(t)=0$

ii) w/ surf charging

$$-\nabla\cdot(\varepsilon\nabla\phi) = e\, n_s(t)$$

$$\frac{d\, n_s(t)}{dt} = -\nabla\left(\sum_i q_i\vec{\Gamma}_i - \sigma_m\nabla\phi_s\right)$$ How to solve surf charging?

Cylindrical

$$\nabla\phi = 0\,\big|_{r=0}$$

→

initial Condition $n_e(t)$, $n_i(t)$

↓

Solve for $\phi(t)$ ←

↓

Calc Increment $\frac{\partial n_e}{\partial t}(t)$, $\frac{\partial n_i}{\partial t}(t)$

↓

update at $t+\Delta t$,
$n_e(t+\Delta t) = n_e(t) + \Delta t\frac{\partial n_e}{\partial t}(t)$
$n_i(t+\Delta t) = n_i(t) + \Delta t\frac{\partial n_i}{\partial t}(t)$

→ Update $\phi(t+\Delta t)$ → back to Solve for $\phi(t)$

Since at surf/bc, timestep $\Delta t$ is limited by the dielectric relaxation time $\Delta t < \varepsilon_0/\sigma_m$, an implicit Coupling method is needed.

Continuity Eq.

$$\frac{\partial n_e}{\partial t} = -\nabla\cdot\vec{\Gamma}_e + S_e$$

Two unknown var

$S_e$
- $\boxed{S_e = 0,\ \text{Plasma Decay}}$ ICP_v1.0
- $S_e = (k_e(T_e) n_g) n_e$ depends on $T_e$
- $S_e = (\int f(\varepsilon)\sigma(\varepsilon)d\varepsilon \cdot n_g) n_e$ depends on EEDF $f(\varepsilon)$

$\vec{\Gamma}_e$
- Drift-Diffusion Approximation
  - $\boxed{\text{No B-field}: \vec{\Gamma}_e = -D_e\nabla n_e - e\mu_e n_e \nabla\phi}$
  - With B-field: $\vec{\Gamma}_e = \overline{\overline{\mu}}_e \cdot \left(-\frac{kT_e}{e}\nabla n_e - e n_e \nabla\phi\right)$
  - Schaffeter-Gummel Numerical method
- Momentum Equation
  - Only $\frac{\partial \vec{\Gamma}_e}{\partial t} = -\frac{1}{m_e}\nabla(k_B T_e n_e) - \nabla\cdot(n_e \vec{v}_e\vec{v}_e) - \frac{e n_e}{m_e}(\nabla\phi + \vec{v}_e\times\vec{B})$
    $- \nabla\cdot\overline{\overline{\nu}}_e - \sum_j \frac{m_e}{m_j} n_e n_j (\vec{v}_e - \vec{v}_j)\nu_{ej}$
  - Momentum + Energy Eq.

$$\frac{\partial n_e}{\partial t} = -\nabla\cdot\vec{\Gamma}_e + S_e$$

$$\frac{\partial n_e}{\partial t} = +\nabla\cdot\left(D_e\nabla n_e + e\mu_e n_e\nabla\phi\right)$$

$\phi = \phi(n_e, n_i)$

$$D_e = \int_0^\infty \frac{2e\,\varepsilon^{5/2}}{3 m_e \nu_{en}\langle\varepsilon\rangle} f(\varepsilon)\,d\varepsilon$$

Maxwell Distribution for ICP_v1.0

$$\mu_e = \int_0^\infty \frac{2e\,\varepsilon^{5/2}}{3 m_e \nu_{en}\langle\varepsilon\rangle}\frac{\partial f(\varepsilon)}{\partial\varepsilon}\,d\varepsilon$$

Poisson's EQ.

$\nabla\cdot(\varepsilon\nabla\phi) =$
- $\boxed{\text{No Surf Charging} \quad -\left(\sum_i n_i - n_e\right)}$
- With Surf Charging $\quad -\left(\sum_i n_i - n_e\right) + n_{surf}(t)$

$$\frac{dn_{surf}(t)}{dt} = -\nabla\cdot\left(\sum_i \Gamma_{i\text{-to-surf}} - \sigma_M\nabla\phi\right)$$

$n_i$ is needed.

assume $n_i$ follows the same routine as $n_e$

$$\frac{\partial n_i}{\partial t} = \nabla\left(D_i\nabla n_i - e\mu_i n_i\nabla\phi\right)$$

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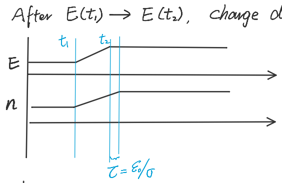
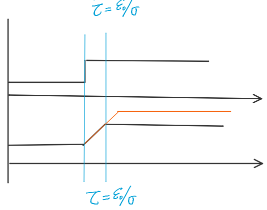
Dielectric Relaxation Time

Changing E-field

Physics: After $E(t_1) \to E(t_2)$, charge density needs a characteristic time to response.




if $\Delta t > \tau$, $\Delta t = \tau + \delta t$,

Charge density saturate/reach SS after $\tau$.

prediction error $= \Delta n(\tau + \delta t) - \Delta n(\tau)$ due to $\delta t$

Mathematics: $\nabla\times\vec{B} = \mu_0\left(\vec{J} + \varepsilon_0\frac{\partial\vec{E}}{\partial t}\right)$

$$\nabla\cdot(\nabla\times\vec{B}) = 0 \longrightarrow \nabla\cdot\left(\vec{J} + \varepsilon_0\frac{\partial\vec{E}}{\partial t}\right)$$

Assume $\vec{J} /\!/ \vec{E}$, to be precise, $\vec{J}$ & $\vec{E}$ anti-parallel

$$\frac{\partial E}{\partial t} = -J/\varepsilon_0 \quad , \text{ assume const } C=0$$

Consider only conduction current, $J = \sigma E$

$$\frac{\partial E}{\partial t} = -\frac{\sigma}{\varepsilon_0} E$$

$$\frac{E(t_2)-E(t_1)}{\Delta t} = -\frac{\sigma}{\varepsilon_0} E(t_1)$$

$$E(t_2) = \left(1-\frac{\sigma}{\varepsilon_0}\Delta t\right) E(t_1)$$

if $\frac{\sigma}{\varepsilon_0}\Delta t > 1$, $E(t_2)$ always flips sign w.r.t. $E(t_1)$

$E$ is sinusoidal, the solution is not stable, or could not resolve the sin form

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(1) $$\frac{\partial n_e}{\partial t} = +\nabla\cdot\left(D_e \nabla n_e + e\mu_e n_e \nabla\phi\right)$$

$$\downarrow$$

$$n_e(t+\Delta t) = n_e(t) + \Delta t \cdot \nabla\cdot\left(D_e(t)\nabla n_e(t) + e\mu_e(t)\, n_e(t)\nabla\phi(t)\right) + \Delta t\, S_e(t)$$

$$n_i(t+\Delta t) = n_i(t) + \Delta t \cdot \nabla\cdot\left(D_i(t)\nabla n_i(t) - e\mu_i(t)\, n_i(t)\nabla\phi(t)\right) + \Delta t\, S_i(t)$$

$$\nabla\cdot(\varepsilon\nabla\phi(t+\Delta t)) = -e\left(\sum_i n_i(t+\Delta t) - n_e(t+\Delta t)\right)$$ [NO Surf charging]

(2) Explicit method, time step is limited by Dielectric Relaxation Time

$$\Delta t < \varepsilon_0/\sigma$$ (typically in order of pico-sec, $10^{-12}$s)

where, $\sigma = e(n_e\mu_e + n_i\mu_i)$

in plasma, $\mu_e \gg \mu_i$ since electron is much lighter

$$\sigma_{plasma} \approx \sigma_e = e n_e \mu_e$$

(3) Semi-Implicit

$$n_e(t+\Delta t) = G_e\left(n_e(t), \phi(t)\right) \longrightarrow n_e(t+\Delta t) = G_e\left(n_e(t), \phi(t+\Delta t)\right)$$

$$n_i(t+\Delta t) = G_i\left(n_i(t), \phi(t)\right) \longrightarrow n_i(t+\Delta t) = G_i\left(n_i(t), \phi(t+\Delta t)\right)$$

$$\nabla\cdot(\varepsilon\nabla\phi(t+\Delta t)) = -e\left(\sum_i n_i(t+\Delta t) - n_e(t+\Delta t)\right)$$

$$\downarrow$$

$$\nabla\cdot(\varepsilon\nabla\phi(t+\Delta t)) = -e\Bigg(\sum_i\Big(n_i(t) - \Delta t\nabla\cdot\big(\underbrace{D_i(t)\nabla n_i(t) + e\mu_i(t)\, n_i(t)\nabla\phi(t+\Delta t)}_{\Gamma_i}\big) + S_i(t)\Big)$$

$$n_e(t) - \Delta t\nabla\cdot\big(\underbrace{D_e(t)\nabla n_e(t) + e\mu_e(t)\, n_e(t)\nabla\phi(t+\Delta t)}_{\Gamma_e}\big) + S_e(t)\Bigg)$$

$$\boxed{\nabla\cdot\left(\varepsilon\nabla\phi(t+\Delta t)\right)=G_\phi\left(\sum_i n_i(t),\ n_e(t),\ \phi(t+\Delta t)\right)}$$

This is the semi-implicit poisson equation we are going for!