0.0.1 Ambipolar Appoximation

In the steady state where the background gas is dominant, we make the congruence assumption that the flux of electrons and ions out of any region must be equal, $\vec{\Gamma}_e = \vec{\Gamma}_i$, such that charge does not build up. This is still true in the presence of ionizing collisions, which create equal numbers of both negative and positive species. Since the electrons are ligher, and would tend to flow out faster (in an unmagnetized plasma), an electric field must spring up to maintain the local flux balance. That is, a few more electrons than ions initially leave the plasma region to set up a charge imbalance and consequently an electric field. Let's expand the flux balance, $\vec{\Gamma}_e = \vec{\Gamma}_i$,

$$-\mu_e n_e \vec{E} - D_e \nabla n_e = \mu_i n_i \vec{E} - D_i \nabla n_i$$

Note that the drift term is negative for electrons and positive for ions, where E-field drags electrons down and speed ions up to balance the fluxes. Assume charge neutrality in space, $n_e = n_i$, E-field can be solved as

$$\vec{E}_{ambi} = \vec{E} = \frac{D_i - D_e}{\mu_i + \mu_e}(\frac{\nabla n_i}{n_i})$$

$$\vec{E}_{ambi} = \vec{E} \approx -\frac{D_e}{\mu_e}(\frac{\nabla n_i}{n_i})$$

This E-field is called ambipolar E-field and substituted to the flux,

$$\vec{\Gamma}_{e,i} = \mu_i \frac{D_i - D_e}{\mu_i - \mu_e} \nabla n_i - D_i \nabla n_i = -\frac{\mu_e D_i + \mu_i D_e}{\mu_i + \mu_e} \nabla n_i$$

You can see the coefficient is symmetric, and equal to electron and ion. A new diffusion coefficient can be defined as

$$D_{ambi} = \frac{\mu_e D_i + \mu_i D_e}{\mu_i + \mu_e}$$

$$\vec{\Gamma}_{e,i} = -D_{ambi} \nabla n_i$$

$$\mu_e = \frac{|q|}{v_{coll\_em}}(\frac{1}{m_e}) \gg \mu_i = \frac{|q|}{v_{coll\_em}}(\frac{1}{m_i}),\ since\, m_e \ll m_i$$

$$D_{ambi} \approx D_i + \frac{\mu_i}{\mu_e} D_e = D_i(1 + \frac{T_e}{T_i})$$

put it back to the continuity equation,

$$\frac{\partial n_i}{\partial t} + D_{ambi} \nabla^2 n_i = S_i$$

Now the continuity becomes a standard DIFFUSION equation with source term, and much easier to solve. When using ambipolar diffusion appoximation, we only calculate ion density n_i, and enforce charge neutrality, $n_e = n_i$. Ambipolar E-field, E_{ambi}, can be used for electron energy equation. In this way, Poisson's equation is avoided.

Computatinally, ion density, n_i, is first solved from the continuity equation,

$$\frac{\partial n_i}{\partial t} + D_{ambi} \nabla^2 n_i = S_i$$

After that, it is simply to put n_e equal to n_i. And electric field is obtained by,

$$\vec{E}_{internal} = \vec{E}_{ambi} = \frac{D_i - D_e}{\mu_i + \mu_e} (\frac{\nabla n_e}{n_e}) \approx -\frac{D_e}{\mu_e} (\frac{\nabla n_e}{n_e})$$

0.0.2 Multiple Ions

When there are more than one ions in plasmas, ambipolar assumption might result in complex solution. Another trick could be imposed for simplicity. Assume that each ion gets equilibrium only with electrons, indicating that ion-ion interaction is ignored, we can have ambipolar equations for each ion. Taking a simple case that contains two ions, the original ambiplor assumption is,

$$n_e = n_{i1} + n_{i2}$$

$$\vec{\Gamma}_e = \vec{\Gamma}_{i1} + \vec{\Gamma}_{i2}$$

The stronger ambipolar assumption becomes,

$$n_e = n_{e1} + n_{e2},\ n_{e1} = n_{i1},\ and\ n_{e2} = n_{i2}$$

$$\vec{\Gamma}_e = \vec{\Gamma}_{e1} + \vec{\Gamma}_{e2},\ \vec{\Gamma}_{e1} = \vec{\Gamma}_{i1},\ and\ \vec{\Gamma}_{e2} = \vec{\Gamma}_{i2}$$

The additional assumption, in some degree, divides the whole plasma into two independent plasmas. Ions interact only with electrons, instead of other ions. Computationally, the densities of the two ions are computed separately and independently. For example,

$$D_{ambi-i1} = D_{i1}(1 + \frac{T_e}{T_{i1}}),\ \frac{\partial n_{i1}}{\partial t} + D_{ambi-i1} \nabla^2 n_{i1} = S_{i1}$$

$$D_{ambi-i2} = D_{i2}(1 + \frac{T_e}{T_{i2}}),\ \frac{\partial n_{i2}}{\partial t} + D_{ambi-i2} \nabla^2 n_{i2} = S_{i2}$$

$$n_e = n_{i1} + n_{i2}$$

$$\vec{E}_{internal} = \vec{E}_{ambi-i1} + \vec{E}_{ambi-i2} = -\frac{D_e}{\mu_e} (\frac{\nabla n_{i1}}{n_{i1}}) - -\frac{D_e}{\mu_e} (\frac{\nabla n_{i2}}{n_{i2}}) = -\frac{D_e}{\mu_e} (\frac{\nabla n_{i1}}{n_{i1}} + \frac{\nabla n_{i2}}{n_{i2}})$$

0.0.3 Multiple Charges

If an ion has charges more than one, the ambipolar equation needs to be modified slightly. A simple case is used to demonstrate the changes.

$$q_i - charges\ of\ an\ ion$$

Now the ambipolar assumption becomes, $n_e = q_i \times n_i$ and $\Gamma_e = q_i \times \Gamma_i$,

$$-\mu_e n_e \vec{E} - D_e \nabla n_e = q_i \mu_i n_i \vec{E} - q_i D_i \nabla n_i$$

substitute $n_e = q_i \times n_i$ to LHS,

$$-\mu_e q_i n_i \vec{E} - D_e q_i \nabla n_i = q_i \mu_i n_i \vec{E} - q_i D_i \nabla n_i$$

actually q_i is canceled from both sides, leaving the flux balance as the same in the orginal form,

$$-\mu_e n_i \vec{E} - D_e \nabla n_l = \mu_i n_i \vec{E} - D_i \nabla n_i$$

$$D_{ambi} = \frac{\mu_e D_i + \mu_i D_e}{\mu_i + \mu_e}$$

$$\frac{\partial n_i}{\partial t} + D_{ambi} \nabla^2 n_i = S_i$$