

# The Langmuir Model

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## **Abstract**

In this document, a feature scale model is developed and used for recipe tuning. The detailed physical mechanisms and numerical algorithms are introduced and discussed.

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## **1 Introduction - Langmuir Model**

Langmuir model is a comprehensive model simulating the plasma etching processes. It consists of three sub-models, reactor model, sheath model and feature model. Reactor model simulates the plasma in the reactor scale, which is typically 30 to 50 cm.

## **2 File Structure**

Langmuir model is organized as a collection of three sub-models and designed to use solvers as shared as possible. The file structure is shown in the figure below,

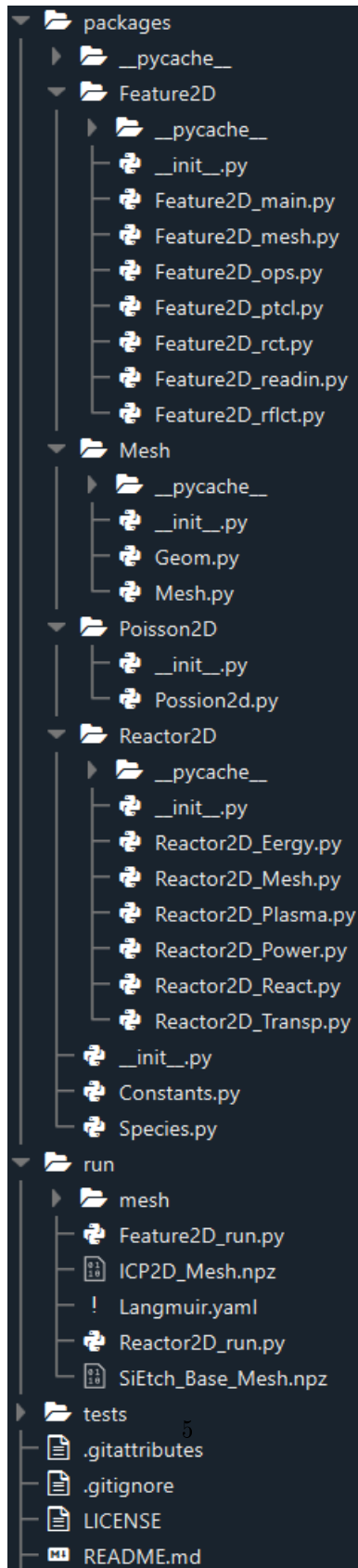


Figure 1: The directory/file tree structure for Langmuir model.

Within the root directory, there are *packages*, where all the model are placed, *run*, where applications/cases are run, and *tests*, where model tests are tested and stored. License and Readme files are placed in the root directory. Within the *packages*, *Reactor2D*, *Sheath2D* and *Feature2D* directories contains model files for each model, respectively. File naming follows the convention, *modelname+2D+module name*. *Constants.py* and *Species.py* are shared with all the three models, so they are placed in the *packages* level. As for its name, *Constants.py* defines all constats, while *Species.py* defines all species and their properties. These are predefined variables, which are not supposed to be changed or edited by users. *Mesh* directory contains *Geom.py* and *Mesh.py* to create structured mesh, which in principle can be used for any model requiring structured mesh, not limited to Langmuir model. The *Mesh* module can be either imported to each model or used as a standalone one. When using as a standalone module, *Mesh* saves all the information as *mesh.npz* file. In reactor model, both field module and transport module require a solver of Poisson-like equation, therefore *Poisson2D* is created at this level and works as a general equation solver. The Poisson solver is also not limited to Langmuir model. Within the *run* directory, model run files are placed. *Langmuir.yaml* file serves as input file as all-in-one for all three models. Within *tests* directory, test are used by developers and stored as benchmarks for the Langmuir model.

## 3 Geometry and Mesh

### 3.1 Geometry

### 3.2 Mesh

## 4 Reactor Model

### 4.1 Introduction

### 4.2 Boltzmann Equation

### 4.3 Continuity Equation

The fluid equation for plasma model is

$$\frac{\partial n}{\partial t} + \nabla \cdot \vec{\Gamma} = S$$

$n$  – density

$$\vec{\Gamma} = n\vec{u} - flux$$

$S$  – Source and loss term due to collisions

The continuity equation is used for electron, ion and neutral species. In order to solve for density  $n$ , two unknown terms are needed,  $\vec{\Gamma}$  and  $S$ . Source term,  $S$ , is discussed in Sec. Chemistry. Flux term,  $\vec{\Gamma}$ , can be solved by momentum equation,

$$mn \frac{\partial \vec{u}}{\partial t} = eqn \vec{E} - \nabla p - mn v_{coll\_em} \vec{u}$$

$\vec{u}$  – fluid velocity

$m$  – mass

$e$  – elementary charge

$q$  – charge carried by species

$\vec{E}$  – electric field

$p$  – pressure

$v_{coll\_em}$  – momentum collision frequency

The magnetic field is ignored for most applications of low-temperature plasmas and will be addressed separately in another section. In the right-hand side, the first term,  $eqn\vec{E}$ , represents the electric force. The second term,  $\nabla p$ , represents. The third term,  $mnv_{coll\_em}\vec{u}$ , is the momentum loss due to momentum collision.

#### 4.3.1 Drift-Diffusion Approximation

The momentum equation is usually difficult to solve directly. A drift-diffusion approximation is widely used in plasma model. In we are only interested in steady-state plasmas, which results in  $mn\frac{\partial}{\partial t}\vec{u} = 0$ , the momentum equation can be written as

$$eqn\vec{E} - \nabla p - mnv_{coll\_em}\vec{u} = 0$$

where we assume that the background species is at rest and that the momentum collision frequency,  $v_{coll\_em}$ , is a constant, independent of velocity. Taking an isothermal plasma, such that  $\nabla p = kT\nabla n$ , the momentum equation is written as

$$eqn\vec{E} - kT\nabla n - mnv_{coll\_em}\vec{u} = 0$$

now velocity,  $\vec{u}$ , is solvable,

$$\vec{u} = \frac{eq\vec{E}}{mv_{coll\_em}} - \frac{kT}{mv_{coll\_em}}\left(\frac{\nabla n}{n}\right)$$

$$\vec{\Gamma} = n\vec{u} = \frac{eqn\vec{E}}{mv_{coll\_em}} - \frac{kT\nabla n}{mv_{coll\_em}}$$

$$\vec{\Gamma} = \frac{q}{|q|}\mu n\vec{E} - D\nabla n$$

$$\mu = \frac{e|q|}{mv_{coll\_em}} - \text{mobility}$$

$$D = \frac{kT}{mv_{coll\_em}} - \text{diffusivity, diffusion coefficient}$$

The first term,  $e\mu n\vec{E}$ , represents the drift term; the second term,  $D\nabla n$ , represents the diffusion term. That is where the name of approximation comes from.

E-field in the continuity equation is derived by potential,  $\vec{E} = -\nabla\phi$ , continuity



equation can be written in terms of density and potential,

$$\frac{\partial n_e}{\partial t} = -\nabla \cdot (-\mu_e n_e \vec{E} - D_e \nabla n_e) + S_e = f_e(n_e(t), \phi(t))$$

$$\frac{\partial n_i}{\partial t} = -\nabla \cdot (q_i \mu_i n_i \vec{E} - D_i \nabla n_i) + S_i = f_i(n_i(t), \phi(t))$$

$$-\nabla \cdot \varepsilon \nabla \phi(t) = e(\sum_i n_i(t) - n_e(t))$$

In this form, the future density  $n_e(t1)$  is determined by current density,  $n_e(t0)$ , and current potential,  $\phi(t0)$ . The current potential,  $\phi(t0)$ , is determined by the toall charge density,  $\sum_i n_i(t) - n_e(t)$ . Explicitly, the equations above can be solved through iterative method. Given initial densities,  $n_e(t0)$  and  $n_i(t0)$ , initial potnetial  $\phi(t0)$  can be solved. By putting densities and potential into continuity equation, the densities  $n_{e,i}(t1)$  at next time,  $t1$ , can be obtained. The more detailed discussion can be found in the sec. Poisson's Equation.

### 4.3.2 Ambipolar Appoximation

In the steady state where the background gas is dominant, we make the congruence assumption that the flux of electrons and ions out of any region must be equal,  $\vec{\Gamma}_e = \vec{\Gamma}_i$ , such that charge does not build up. This is still true in the presence of ionizing collisions, which create equal numbers of both negative and positive species. Since the electrons are lighter, and would tend to flow out faster (in an unmagnetized plasma), an electric field must spring up to maintain the local flux balance. That is, a few more electrons than ions initially leave the plasma region to set up a charge imbalance and consequently an electric field. Let's expand the flux balance,  $\vec{\Gamma}_e = \vec{\Gamma}_i$ ,

$$-\mu_e n_e \vec{E} - D_e \nabla n_e = \mu_i n_i \vec{E} - D_i \nabla n_i$$

Note that the drift term is negative for electrons and positive for ions, where E-field drags electrons down and speed ions up to balance the fluxes. Assume charge neutrality in space,  $n_e = n_i$ , E-field can be solved as

$$\vec{E}_{ambi} = \vec{E} = \frac{D_i - D_e}{\mu_i + \mu_e} \left( \frac{\nabla n_e}{n_e} \right)$$

This E-field is called ambipolar E-field and substituted to the flux,

$$\vec{\Gamma}_{e,i} = \mu_i \frac{D_i - D_e}{\mu_i - \mu_e} \nabla n_e - D_i \nabla n_e = -\frac{\mu_e D_i + \mu_i D_e}{\mu_i + \mu_e} \nabla n_e$$

You can see the coefficient is symmetric, and equal to electron and ion. A new diffusion coefficient can be defined as

$$D_{ambi} = \frac{\mu_e D_i + \mu_i D_e}{\mu_i + \mu_e}$$

$$\vec{\Gamma}_{e,i} = -D_{ambi} \nabla n_e$$

$$\mu_e = \frac{|q|}{v_{coll\_em}} \left( \frac{1}{m_e} \right) \gg \mu_i = \frac{|q|}{v_{coll\_em}} \left( \frac{1}{m_i} \right), \text{ since } m_e \ll m_i$$

$$D_{ambi} \approx D_i + \frac{\mu_i}{\mu_e} D_e = D_i \left( 1 + \frac{T_e}{T_i} \right)$$

put it back to the continuity equation,

$$\frac{\partial n}{\partial t} + D_{ambi} \nabla^2 n = S$$

Now the continuity becomes a standard DIFFUSION equation with source term, and much easier to solve. When using ambipolar diffusion approximation, we only calculate ion density  $n_i$ , and enforce charge neutrality,  $n_e = n_i$ . Ambipolar E-field,  $E_{ambi}$ , can be used for electron energy equation. In this way, Poisson's equation is avoided.

## 4.4 Poisson's Equation

### 4.4.1 Explicit Poisson's equation

With explicit method, the potential in the Poisson's equation is determined by the current charge density, which is impacted by the previous potential.

$$-\nabla \cdot \varepsilon \nabla \phi(t) = e \left( \sum_{ion} n_i(t) - n_e(t) \right)$$

$$-\nabla \cdot \varepsilon \nabla \phi(t + \Delta t) = e \left( \sum_{ion} n_i(t + \Delta t) - n_e(t + \Delta t) \right)$$

$$\frac{\partial n_{e,i}}{\partial t} = D_{e,i} \nabla^2 n_{e,i}(t) \pm \nabla \cdot (\mu_{e,i} n_{e,i}(t) \nabla \phi(t)) + S_e(t)$$

$$n_{e,i}(t + \Delta t) = n_{e,i}(t) + \Delta t \times f_{e,i}(\phi(t))$$

$$-\nabla \cdot \varepsilon \nabla \phi(t + \Delta t) = F(\phi(t))$$

where

$$\phi = \text{potential}$$

$$\varepsilon = \text{permittivity}$$

$$e = \text{elementary charge}$$

$$n_{e,i} = \text{electron, ion density}$$

$$\mu_{e,i} = \text{electron, ion mobility}$$

with the derivation above, the future potential is determined by the current potential.

#### 4.4.2 Semi-implicit electron with predictor-corrector ions

The timestep is limited to as small as a few picoseconds, which is not practical for any useful simulations. Implicit method can theoretically remove the limit of the timestep, with the cost of solving for the reversed matrix. The matrix solver could be much expensive as well. A semi-implicit method is, therefore, proposed to increase the timestep and meanwhile reduce the cost of matrix solver. The principal is shown as below,

$$-\nabla \cdot \varepsilon \nabla \phi(t + \Delta t) = e \left( \sum_{ion} n_i(t + \Delta t) - n_e(t + \Delta t) \right)$$

$$\frac{\partial n_e}{\partial t} = D_e \nabla^2 n_e(t) - \nabla \cdot (\mu_e n_e(t) \nabla \phi(t + \Delta t)) + S_e(t)$$

$$n_e(t + \Delta t) = n_e(t) + \Delta t \times f_e(\phi(t + \Delta t))$$

$$\frac{\partial n_i}{\partial t} = D_i \nabla^2 n_i(t) + \nabla \cdot (\mu_i n_i(t) \nabla \phi(t))$$

$$n_i(t + \Delta t) = n_i(t) + \Delta t \times f_i(\phi(t))$$

$$-\nabla \cdot \varepsilon \nabla \phi(t + \Delta t) = e \left( \sum_{ion} [n_i(t) + f_i(\phi(t))] - [n_e(t) + f_e(\phi(t + \Delta t))] \right)$$

$$\begin{aligned}
& -\nabla \cdot \varepsilon \nabla \phi(t + \Delta t) = \\
& e(\sum_{ion} [n_i(t) + \Delta t \times f_i(\phi(t))] - [n_e(t) + \Delta t \times S_e(t) + \Delta t \times D_e \nabla^2 n_e(t) - \Delta t \times \nabla \cdot (\mu_e n_e(t) \nabla \phi(t + \Delta t))]) \\
& -\nabla \cdot \varepsilon \nabla \phi(t + \Delta t) + e \times \Delta t \times \nabla \cdot (\mu_e n_e(t) \nabla \phi(t + \Delta t)) = F(n_{e,i}(t), \phi(t)) \\
& \left[ -\nabla \varepsilon \cdot \nabla \phi(t + \Delta t) - \varepsilon \nabla^2 \phi(t + \Delta t) \right] + \left[ (e \mu_e \Delta t) \nabla n_e(t) \cdot \nabla \phi(t + \Delta t) + (e \mu_e \Delta t) n_e(t) \nabla^2 \phi(t + \Delta t) \right] \\
& = F(n_{e,i}(t), \phi(t)) \\
& [(e \mu_e \Delta t) n_e(t) - \varepsilon] \nabla^2 \phi(t + \Delta t) + [(e \mu_e \Delta t) \nabla n_e(t) - \nabla \varepsilon] \cdot \nabla \phi(t + \Delta t) = F(n_{e,i}(t), \phi(t)) \\
& \underline{A(n_e(t))} \nabla^2 \phi(t + \Delta t) + \underline{\nabla B(n_e(t))} \cdot \nabla \phi(t + \Delta t) = F(n_{e,i}(t), \phi(t)) \\
& \text{underline - emphasis}
\end{aligned}$$

The electron density is solved using implicit method, where the future electron density is determined by the future potential, while the ion density is still solved using explicit method, where the future ion density is determined by the current potential. In this scenario, electron density and potential get quickly convergent with large timestep. A further attention needs to be paid to the ion density, which could oscillate. The predictor-corrector method is used for ion density.

$$\begin{aligned}
\tilde{n}_i(t + \Delta t) &= n_i(t) + \Delta t \times f_i(n_i(t), \phi(t)) \\
n_i(t + \Delta t) &= n_i(t) + \Delta t \times \frac{1}{2} (f_i(n_i(t), \phi(t)) + f_i(\tilde{n}_i(t + \Delta t), \phi(t + \Delta t)))
\end{aligned}$$

#### 4.4.3 Semi-implicit Poisson's equation

The ion density can use implicit method as well. In this case,

$$\begin{aligned}
& -\nabla \cdot \varepsilon \nabla \phi(t + \Delta t) = e(\sum_{ion} [n_i(t) + f_i(\phi(t + \Delta t))] - [n_e(t) + f_e(\phi(t + \Delta t))]) \\
& -\nabla \cdot \varepsilon \nabla \phi(t + \Delta t) = e(\sum_{e,i} [n_{e,i}(t) + f_{e,i}(\phi(t + \Delta t))]) \\
& -\nabla \cdot \varepsilon \nabla \phi(t + \Delta t) = \\
& e(\sum_{e,i} \left[ n_{e,i}(t) + \Delta t \times S_{e,i}(t) + \Delta t \times D_{e,i} \nabla^2 n_{e,i}(t) + \Delta t \times \nabla \cdot (q \mu_{e,i} n_{e,i}(t) \nabla \phi(t + \Delta t)) \right]) \\
& -\nabla \cdot \varepsilon \nabla \phi(t + \Delta t) - \sum_{e,i} \Delta t \times \nabla \cdot (eq \mu_{e,i} n_{e,i}(t) \nabla \phi(t + \Delta t)) = F(n_{e,i}(t))
\end{aligned}$$

$$\underbrace{\left[\sum_{e,i}(eq\mu_{e,i}\Delta t)n_{e,i}(t) - \varepsilon\right]\nabla^2\phi(t+\Delta t)}_{A(n_{e,i}(t))\nabla^2\phi(t+\Delta t)} + \underbrace{\left[(e\Delta t)\sum_{e,i}\mu_{e,i}\nabla n_{e,i}(t) - \nabla\varepsilon\right]\cdot\nabla\phi(t+\Delta t)}_{\nabla B(n_{e,i}(t))\cdot\nabla\phi(t+\Delta t)} = F(n_{e,i}(t))$$

The future potential is solved directly, which means that the future potential does not depends on the current potential anymore. All coefficients in this equation are densities at current time. The second term in the LHS is probably (not sure, need additional check) not easy to deal with. It could be rewritten in explicit form and moved to the RHS.

$$\begin{aligned} \underline{A(n_{e,i}(t))\nabla^2\phi(t+\Delta t)} + \underline{\nabla B(n_{e,i}(t))\cdot\nabla\phi(t+\Delta t)} &= F(n_{e,i}(t)) \\ \underline{A(n_{e,i}(t))\nabla^2\phi(t+\Delta t)} &= F(n_{e,i}(t)) + \underline{\nabla B(n_{e,i}(t))\cdot\nabla\phi(t)} \\ \underline{A(n_{e,i}(t))\nabla^2\phi(t+\Delta t)} &= F(n_{e,i}(t), \phi(t)) \end{aligned}$$

Now the Poisson's equation retains its Lapalacian form, with the source term depending on current potential. The same technique can be applied to Sec. 3.2 as well.

## 4.5 Momentum Equation

### 4.5.1 Plasma Conductivity

Consider a small-amplitude perturbation of electromagnetic field, having an electric field in the x-direction of an infinite plasma. The electric field may be defined using complex numbers:  $E = E_0 \exp(i\omega t)$ . In a typical plasma etching application,  $\omega = 2\pi f$ , where  $f = 13.56$  MHz. Let's look at the electron momentum equation,

$$m_e n_e \frac{\partial}{\partial t} u_e = -en_e E_0 \exp(i\omega t) - \nabla p_e - m_e n_e v_{coll\_em} u_e$$

With the assumption that :

1. the ions do not respond to this high-frequency perturbation
2. the electron pressure gradient is not significant, effectively ignoring electron thermal energy
3. there is no steady current in the plasma so the drift speed is zero

4. electron velocity follows the same oscillation,  $u = u_0 \exp(i\omega t)$ .

The electron momentum equation linearizes to

$$(n_e m_e i\omega) u_0 \exp(i\omega t) = -n_e e E_0 \exp(i\omega t) - (n_e m_e v_{coll\_em}) u_0 \exp(i\omega t)$$

The magnitudes of the perturbations in velocity and electric field amplitude are therefore related by

$$u_0 = -\frac{e}{m_e(i\omega + v_{coll\_em})} E_0$$

current is determined by

$$J_0 = -en_e u_0 = \frac{e^2 n_e}{m_e(i\omega + v_{coll\_em})} E_0$$

The electron conductivity  $\sigma_e$  is,

$$\sigma_e = \frac{e^2 n_e}{m_e(i\omega + v_{coll\_em})}$$

Plasma conductivity  $\sigma_{plasma} = \sigma_e + \sigma_{ion} \approx \sigma_e$ , since  $\sigma_e \gg \sigma_{ion}$ .

Notice that when collision dominates the plasma,  $v_{coll\_em} \gg \omega$ ,  $\sigma_e = \frac{e^2 n_e}{m_e v_{coll\_em}}$ ,  $\sigma_e$  is real. Current is synchronized in phase with E-field and power is absorbed by the electron. When oscillation dominates the plasma,  $\omega \gg v_{coll\_em}$ ,  $\sigma_e = -i \frac{e^2 n_e}{m_e \omega}$ ,  $\sigma_e$  is imaginary. Current is 90° lagged in phase with E-field so that power is reflected by the electron.

## 4.6 Electron Energy Equation

The energy conservation equation is obtained by multiplying the Boltzmann equation by kinetic energy,  $\frac{1}{2}mv^2$ , and integrating over velocity,

$$\frac{\partial}{\partial t} \left( \frac{3}{2} n_e k T_e \right) + \nabla \cdot Q_e = P_{in} - P_{loss}$$

$$\frac{3}{2} n_e k T_e - \text{thermal energy density (J/m}^3\text{)}$$

$k$  – Boltzmann Constant

$$Q_e - \text{energy flux (W/m}^2\text{)}, Q_e = \frac{5}{2} k T_e \Gamma_e - k_e \nabla T_e$$

$k_e$  – thermal conductivity,  $k_e = 3kD_en_e/2$

$D_e$  – electron diffusion coefficient

$P_{in}$  – Power Input,  $P_{loss}$  – Power Loss

The form of energy equation, where energy density is solved, is similar to that of continuity equation, where species density is solved. Energy flux,  $Q_e$ , consists of two terms, in which  $\frac{5}{2}kT_e\Gamma_e$  represents the thermal convection and  $k_e\nabla T_e$  represents the thermal conduction. The input power includes both external and internal energy flowing into the electrons. The source can come from electric field, electron beam, or photons. Under the assumption of resistive plasma where Ohm heating (also called Joule heating/resistive heating) dominates,

$$P_{in} = \sigma E^2$$

$\sigma$  – Plasma Conductivity, (S/m)

$E$  – Electric Field, including both external and internal field

External electric field is obtained from the the Field Solver and internal electric field is obtained from the Transport Solver. In plasma, power gets lost mainly through energy flow to the walls and electron-impact collisions. The former loss is included in the  $\nabla \cdot Q_e$  term. The collision loss can be seen as below,

$$P_{loss} = 3\frac{m_e}{M}n_e k v_{coll\_em}(T_e - T_g) + n_e N \sum_j k_j(T_e)\Delta\varepsilon_j$$

The first term,  $3\frac{m_e}{M}n_e k v_{coll\_em}(T_e - T_g)$ , is due to electron elastic collision (also called momentum collision), in which no reaction occurs. For elastic collision, the electron energy loss can be derived from momentum and energy conservation equations for 2-body collision. The second term,  $n_e N \sum_j k_j(T_e)\Delta\varepsilon_j$ , counts for the inelastic collisions, in which reactions, such as excitation and ionization, occur.

The boundary conditions of the electron energy towards the wall were taken as

$$Q_e(at\ b.c.) = \frac{5}{2}kT_e\Gamma_e(at\ b.c.) \cdot \hat{n}$$

$\hat{n}$  – surface normal vector

At the wall surface, energy which is normal with respect to the surface is lost

completely.

## 4.7 Electron Energy Distribution Function

### 4.7.1 Null-collision technique

The electron energy and collision frequency can change during the free flight between collisions. There is an ambiguity in choosing the time between collisions, seen in the figure below.

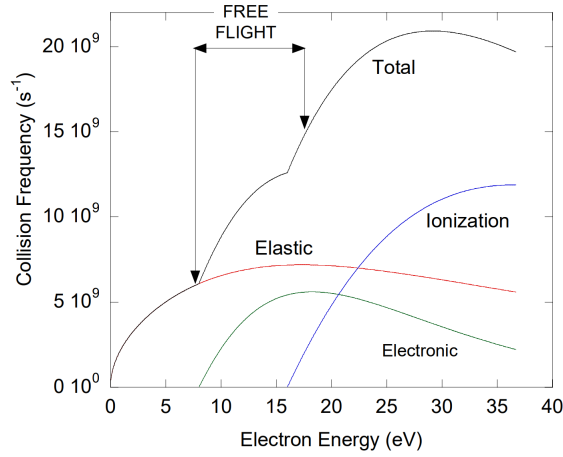


Figure 2: Free flight time changes during the flight.

The ambiguity is eliminated by the “null collision frequency” (NCF). The null-collision concept is introduced into the direct-simulation Monte Carlo method in the rarefied gas dynamics. The null-collision technique overcomes the principle fault in the time-counter technique and the difficulties in the collision-frequency technique. The computation time required for the null-collision technique is comparable to that for the time-counter technique. Therefore, it is concluded that the null-collision technique is superior to any other existing techniques in the direct-simulation Monte Carlo method.

The NCF is a fictitious process used to make it appear that all energies have the same collision frequency.

$$v_{null}(\varepsilon) = \max_{\varepsilon}[v_{total}(\varepsilon)] - v_{total}(\varepsilon)$$



$$xs_{null}(\varepsilon) = \max_{\varepsilon}[xs_{total}(\varepsilon)] - xs_{total}(\varepsilon)$$

*v – collision frequency*

*xs – cross section*

When collision falls to the “Null Collision”, there is no impact on the electron, which proceeds to the next free flight time without changing its velocity. The “Null Collision” frequency and cross section can be seen as below,

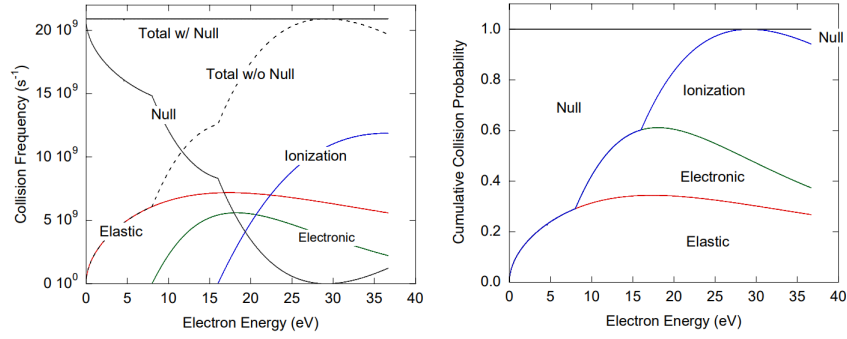


Figure 3: The collision frequency and cross section of “Null Collision”.

The free flight time between collisions is determined by the maximum total collision frequency.

$$\Delta t = \frac{-1}{\max_{\varepsilon}[v_{total}(\varepsilon)]} \log(1 - r_1)$$

The collision which occurs is that which satisfies

$$p_{j-1}(\varepsilon) < r_2 \leq p_j(\varepsilon)$$

*p<sub>j</sub> – the pth collision process*

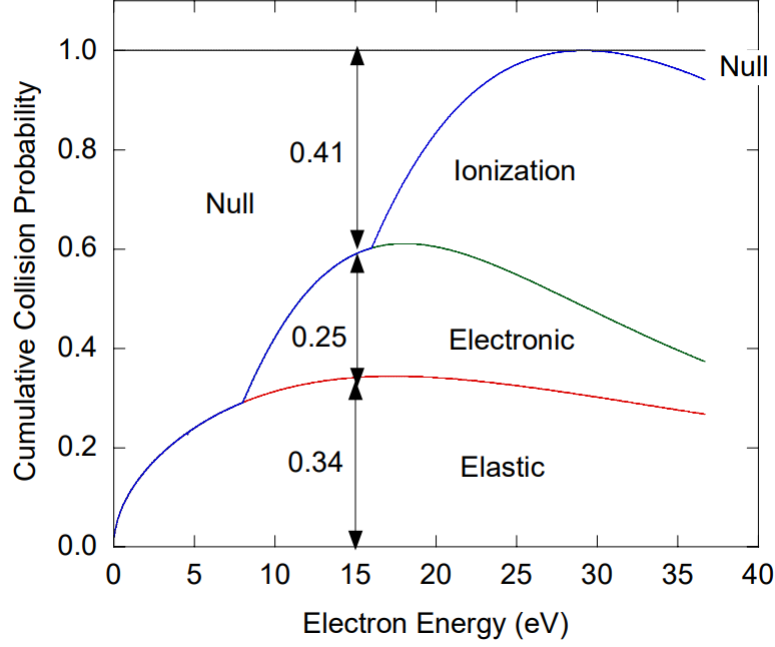


Figure 4: Collision process chosen according to the cumulative probability.

## 4.8 Chemistry

### 4.8.1 Introduction

In the continuity equation, there is a source term in the right hand side,

$$\frac{\partial n}{\partial t} + \nabla \cdot \Gamma = S$$

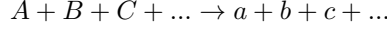
$n$  – density of any species, such as electron, ion and neutrals

$\Gamma$  – flux of any species, such as electron, ion and neutrals

$S$  – source term of species due to reactions

The flux term has been discussed in the previous section. The source term,  $S$ , here refers to the volume production and sink of a given species. The source term due to surface reaction will be discussed later.

Usually, reactions can be written as following form,



*left – reactants, right – products*

For any species appeared in the left hand side, the reaction serves as a sink for that species; for any species appeared in the right hand side, the reaction serves as a production for that species. The reaction rate can be calculated as,

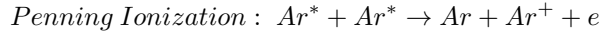
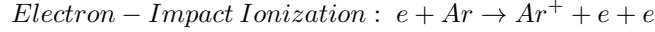
$$R = k \times n_A \times n_B \times n_C \times \dots$$

*R – reaction rate*

*k – reaction rate coefficient*

*n<sub>A</sub> – density of species A*

Let us take an example. In Ar discharge, a basic reactions set includes electron-impact excitation, ionization and Penning ionization,



*Ar\* – excited Ar state*

For electron, the source term will include first and third reactions. The second reaction loses an electron in the left side and produce one in the right side, resulting net gain or loss. In the electron-impact ionization, one electron is consumed while two electrons are produced, resulting in net gain of one electron. Penning ionization produces one electron. The source term for electron is

$$S_e = k_{ioniz} \times n_e \times n_{Ar} + k_{pen} \times n_{Ar^*}^2$$

Similarly, the source terms for ion,  $S_{Ar^+}$ , and for excited state,  $S_{Ar^*}$ , can be calculated as

$$S_{Ar^+} = k_{ioniz} \times n_e \times n_{Ar} + k_{pen} \times n_{Ar^*}^2$$

$$S_{Ar^*} = k_{exc} \times n_e \times n_{Ar} - 2 \times k_{pen} \times n_{Ar^*}^2$$

In this simple reaction set, electron-ion pair is always created simultaneously so that  $S_{Ar^+} = S_e$  everywhere.

When the source term is added to plasma transport equation, it could result in different coupling depending on the way to process it. For example, using the Ar discharge defined above,

$$\frac{\partial n_e}{\partial t} + \nabla \cdot \Gamma_e(n_e) = S_e$$

$$\frac{\partial n_e}{\partial t} + \nabla \cdot \Gamma_e(n_e) = k_{ioniz} n_e n_{Ar} + k_{pen} n_{Ar^*}^2$$

In the first equation,  $S_e$  can be used as a constant source which is computed from a fixed electron density. In the second equation,  $S_e$  is treated as a function of  $n_e$ , which means that  $n_e$  is needed to be solved at the same time as left-hand side. Since the source term,  $S_e$ , is a linear function of  $n_e$ , the complexity does not increase much. However, when it comes to the transport equation for ions, coupling effect has to be counted.

$$\frac{\partial n_i}{\partial t} + \nabla \cdot \Gamma_i(n_i) = k_{ioniz} n_e n_{Ar} + k_{pen} n_{Ar^*}^2$$

The left-hand side is a function of  $n_i$  while the right-hand side is a function of  $n_e$ . It means that  $n_e$  and  $n_i$  need to be solved simultaneously. Furthermore, all species,  $n_e$ ,  $n_i$  and  $n_{Ar^*}$  need to be solved at the same time.

$$\frac{\partial n_e}{\partial t} + \nabla \cdot \Gamma_e(n_e) = k_{ioniz} n_e n_{Ar} + k_{pen} n_{Ar^*}^2$$

$$\frac{\partial n_i}{\partial t} + \nabla \cdot \Gamma_i(n_i) = k_{ioniz} n_e n_{Ar} + k_{pen} n_{Ar^*}^2$$

$$\frac{\partial n_{Ar^*}}{\partial t} + \nabla \cdot \Gamma_{Ar^*}(n_{Ar^*}) = k_{exc} n_e n_{Ar} - 2k_{pen} n_{Ar^*}^2$$

The three equations are coupled together and all densities need to be solved simultaneously.

### 4.8.2 Electron Impact Reaction

Assuming electron follows the Maxwellian distribution, reaction rate coefficients are modeled using Arrhenius form, which is

$$k_e(T_e) = A \times T_e^n \times \exp(-\frac{E_{act}}{T_e})$$

$k_e$  – rate coefficient

$A$  – Arrhenius coefficient, 1st order : in  $1/s$ ; 2nd order : in  $m^3/s$

$T_e$  – electron temperature, in eV

$E_{act}$  – activation energy, in eV

When electron energy distribution cannot be assumed Maxwellian distribution, reaction rate coefficients have to be computed by electron energy module.

### 4.8.3 Heavy Particle Reaction

For heavy particle reactions with no electron in reactants, the Arrhenius form can also be used for rate coefficients.

$$Rate\ coeff = A \times (\frac{T}{298})^n \times \exp(-\frac{E_{ref}}{T})$$

$A$  – Arrhenius coefficient, 1st order : in  $1/s$ ; 2nd order : in  $m^3/s$

$T$  – gas temperature, in K

$E_{ref}$  – reference energy, in K

## 5 Sheath Model

### 5.1 Introduction

When plasmas with quasi-neutrality ( $n_e \approx n_i$ ) are joined to wall surfaces, a positively charged layer called *sheath* is required in physics to maintain the balance of electrons and ions. The electron thermal velocity  $(eT_e/m_e)^{1/2}$  is at least 100 times the ion thermal velocity  $(eT_i/m_i)^{1/2}$ , as  $T_e \geq T_i$  and  $m_e \ll m_i$ . Let us assume an initial plasma with zero electric potential and E-field

everywhere, since  $n_e = n_i$  at  $t = 0$ . The electrons are not confined by any field or potential and hence move faster to the walls than ions. On a short timescale, some electrons near the walls are lost, leading to net positive space charges near the walls. This positively charged space, which is SHEATH, creates an E-field pointing to the walls, reducing the electron speed and increasing the ion speed to the walls. Eventually, the loss of electrons and ions balance each other and plasma remains quasi-neutral. Sheath plays an important role in the plasma etching. As positive ions flowing out of the bulk plasma enter the sheath, they get accelerated by the sheath fields and pick up high energies as they traverse across the sheath. The ions carry these high energies and deliver to the materials surface, such as Si surface. The ion etch rates, selectivity and damage are also impacted by the energies, which are determined by the sheath. A diagram of sheath can be seen in the figure below.

Within the Langmuir model, Sheath model serves as a connector between Reactor model and Feature model. It takes the E-field and species from the Reactor model and compute the angular and energy distribution of electrons and ions, which is fed into the Feature model as input. Sheath model uses particle tracing algorithm and basically traces particles under varying E-field. Particle collisions are taken into account and they widen the distribution of angle. In the code structure, modules from Feature model, such as particle and move, can be shared with Sheath model.

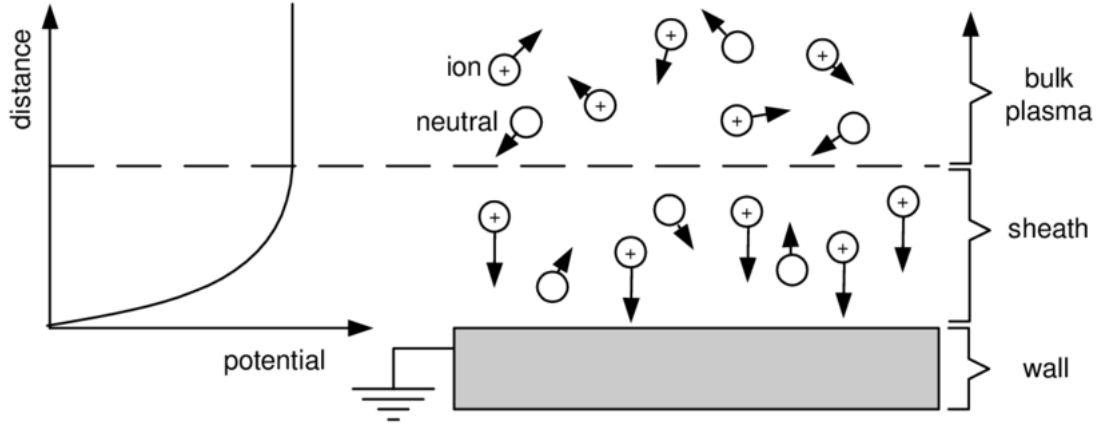


Figure 5: The plasma sheath. Ions in the plasma happen upon the sheath, where they are accelerated to the wall. At the wall, ions are neutralized by electrons from the ground and return to the bulk.

## 5.2 Collisionless Sheath

### 5.2.1 What is collisionless sheath

When the ion mean free path is much larger than the sheath thickness, the sheath is called a collisionless sheath. Within a collisionless sheath, the velocities of ions are only determined by the sheath field and ions are continuously accelerated by the sheath field. Ions pick up energies as they enter the plasma-sheath edge and exit the sheath with an energy distribution of a bimodal shape, seen the figure below. At low frequencies ( $\tau_{ion}/\tau_{rf} \ll 1$ , transverse time of ion is much smaller than the RF period), the ions traverse the sheath within a small fraction of an RF cycle. The phase of the RF cycle at which ions enter the sheath determines their energies at the exit. In this case, the IED (Ion Energy Distribution) is broad and bimodal, with the two peaks corresponding to the minimum and maximum of the sheath drops. At high frequency ( $\tau_{ion}/\tau_{rf} \gg 1$ , transverse time of ion is much larger than the RF period), it takes the ions many RF cycles to cross the sheath. In such a scenario, the net energy gained by the ions is determined by the DC component, which is the time-averaged sheath voltage. The effect of phase at which they enter the sheath is significantly reduced. The IED is still bimodal, but much narrower.

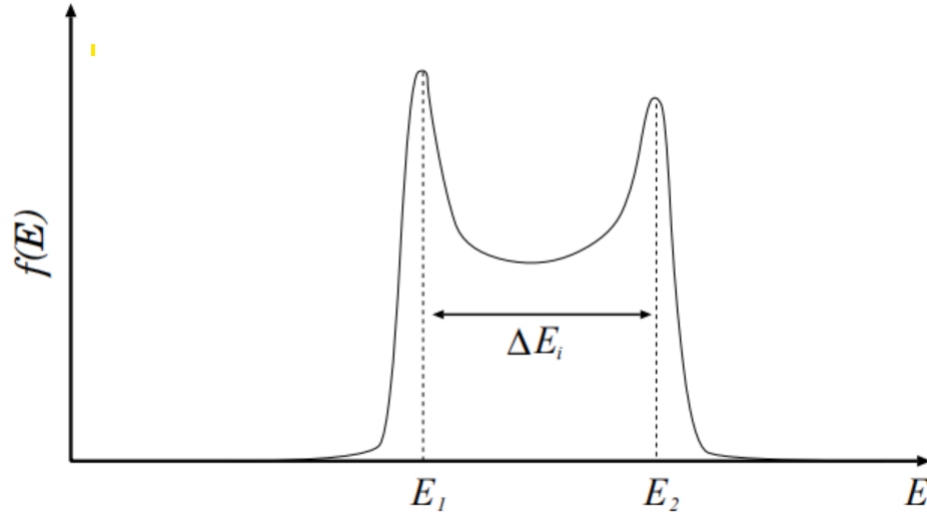


Figure 6: A bimodal ion energy distribution.

### 5.2.2 Analytic Collisionless Sheath Model

Benoit-Cattin et al[] obtained an analytic solution for IED at the high-frequency regime ( $\tau_{ion}/\tau_{rf} \gg 1$ , transverse time of ion is much larger than the RF period), assuming

1. a constant sheath thickness,  $\bar{s}$
2. a uniform sheath electric field,  $\vec{E}$  is independent of position  $x$
3. a sinusoidal sheath voltage  $V_{sh}(t) = V_{dc} + V_s \sin(\omega t)$
4. zero initial ion velocity at the plasma-sheath boundary,  $v_{ion}(x = \bar{s}) = 0$

The resulting expressions for  $\Delta E_i$  and the IED are

$$\Delta E_i = \frac{2eV_s}{\bar{s}\omega} \left( \frac{2eV_{dc}}{m_i} \right)^{1/2} = \frac{3eV_s}{\pi} \left( \frac{\tau_{rf}}{\tau_{ion}} \right)$$

$$f(E) = \frac{dn}{dE} = \frac{2n_t}{\omega \Delta E_i} \left[ 1 - \frac{4}{\Delta E_i^2} (E - eV_{dc})^2 \right]^{-1/2}$$

where  $n_t$  is the number of ions entering the sheath per unit time.

The calculations yield a bimodal IED with two peaks symmetric about  $eV_{dc}$  and  $\Delta E_i$  proportional to  $\frac{\tau_{rf}}{\tau_{ion}}$ , seen the figure below.



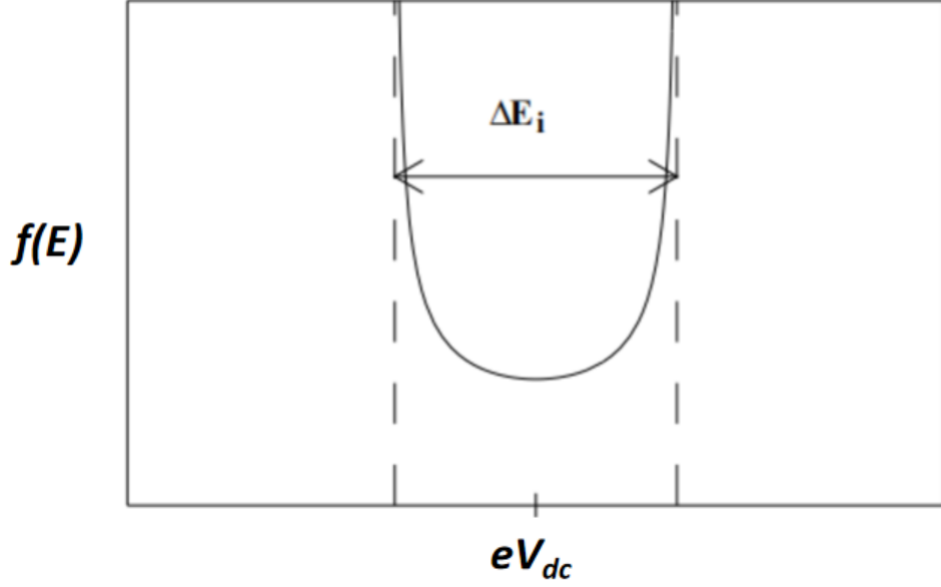


Figure 7: The plot of analytic solution for IED at the regime of high frequency ( $\tau_{ion}/\tau_{rf} \gg 1$ ). The singular peaks are due to the assumption of a mono-energetic initial ion velocity distribution.

### 5.2.3 Collisionless Sheath Model

In the collisionless sheath model, we only need to solve the Newton's equation,

$$\frac{d}{dt}\vec{x} = \vec{v}$$

$$\frac{d}{dt}\vec{v} = \frac{eq}{m_i}\vec{E}(t)$$

$$\vec{E}(\vec{x}, t) = f(V_{sh}(x, t), s(t))$$

$\vec{x}, \vec{v}$  – position, velocity

$e$  – elementary charge

$q$  – # of charges carried by ion

$\vec{E}(\vec{x}, t)$  – electric field within sheath

$V_{sh}(\vec{x}, t)$  – sheath potential

$s(t)$  – sheath thickness

### 5.3 Collisional Sheath

When the mean free path of ions,  $\lambda_{ion}$ , is much smaller than the sheath thickness, the ions entering the sheath will experience collisions before exit. Collisions can alter the velocity, both speed and angle. The probability of a collision event occurring depends on the ion-neutral collision frequency,  $v_{in}$ , which is defined as:

$$v_{in} = N_d \sigma |\vec{v}_i - \vec{v}_g|$$

$N_d$  – background number density

$\sigma$  – ion – neutral charge exchange collision cross section

$\vec{v}_i, \vec{v}_g$  – ion velocity, background gas velocity

The collision probability defined as

$$P = 1 - \exp(-v_{in}\Delta t) = 1 - \exp(-\frac{\Delta x}{\lambda_{ion}})$$

### 5.4 Analytic Sheath Model

## 6 Feature Model

### 6.1 Introduction

The Langmuir Feature Model uses particle-based Monte Carlo methods to simulate the evolution of etch features when exposed to plasma discharges. The model uses pseudo-particles to represent the incoming species, including electron, ions and neutral particles. All these pseudo-particles are tracked for their trajectories and interactions with materials. The materials in the model are represented by a structured mesh of voxels or cubes. Each voxel or cube represents a macro solid material, which consists of hundreds of atoms or molecules. The mesh can be initialized in an arbitrary shape with surface conditions, which may include multiple materials and features within the each domain. This allows the simulation of complex structures and steps in the fabrication process, such as finFET structure.

A pseudo-particle is also a macro particle, which consists of atoms or molecules with the same number as in the materials. The pseudo-particles are launched with specified flux, angle and energy, which are often derived from a reactor

scale model, which is, within the Langmuir Model, the Langmuir Reactor Model. Without the reactor model, the Langmuir Feature Model can also self generate generic functions of flux, angle and energy. The coupling of feature scale model to reactor scale model allows the Langmuir Model to explore the process recipe with the etch result, or to be used to study fundamental physics. This versatility makes the Langmuir Model a strong tool for recipe tuning and optimization, as well as new physics investigation.

## 6.2 Mesh - 2D

The mesh in Langmuir Feature Model is constructed in 2D space, in which  $(x, z)$  are used to represent the 2D coordinates and infinity is assumed in  $y$  direction. The model discretizes the 2D space into a rectangular computational cells. The cell center is marked as a node, which determines the location  $(x, z)$  of the cell. Each cell has a volume of  $\Delta x \times \Delta z$ , where  $\Delta x$  and  $\Delta z$  are the resolutions in  $x$  and  $z$  directions, respectively. Usually, square cells, where  $\Delta x = \Delta z$ , are used. Non-square cells, which are used for high aspect ratio domain for memory saving, need future test and validation. The computational complexity increases with reducing resolution as approximately  $O(n^3)$ , where  $n$  is the number of cells per side in the simulation domain. The choice of resolution also affects the time weighting of each pseudo-particle, which will be described later.

Each cell, representing a solid material, is assigned a material property. The most commonly used materials, Such as  $Si$  and  $SiO_2$ , in the Langmuir Feature Model are pre-defined in the database coming along with the model. If not found in the database, the materials can be defined by the user. The materials are used for the chemical reactions. It is important to respect stoichiometry, and most materials are defined as elements or compounds to facilitate this, but there is no inherent limitation on the material properties in the model. It means that arbitrary materials can be built upon simulation request. In that case, the user is responsible for the validity of the physics and chemistry represented by the model. An example of an arbitrary material definition is “*Photo Resist*”, which is commonly used as a etching mask. “*Photo Resist*” is a polymer with multiple elements and complicated structures, where a long chain of molecules could surpass the cell size. It is okay for these resists to have varying chemistries and properties, and it is not always possible or necessary to capture their stoichiometry accurately.

The basic element in the mesh is a cell, which is assigned a single material and not dividable. Each solid cell in the mesh is assumed to have the same atomic density,  $\rho \text{ cm}^{-3}$ , which is typically  $5.0 \times 10^{22} \text{ cm}^{-3}$  for *Si* and  $2.3 \times 10^{22} \text{ cm}^{-3}$  for *SiO<sub>2</sub>*. This density is a user input and used to calculate number of atoms per cell,

$$N_{cell} = \Delta x \times \Delta z \times \rho$$

Because all cells contain a single (usually stoichiometric) material, but are represented as having the same volume and density, it is important to keep in mind that all materials in the Langmuir Feature Model represent average behaviors of their respective compounds. The Langmuir Feature Model is designed to address the nano-scale feature during the fabrication process, but not able to resolve the inter-atomic interactions. To apply the model within the valid window, the user should make sure that  $N_{cell} \gg 1$ , which will not be automatically checked in the model.

## 6.3 Pseudo-Particle

### 6.3.1 Definition of the pseudo-particle

Simulating a single ion or radical coming to the feature is not practical due to the huge computational cost. During a typical etching process, the ion flux coming to the wafer is of  $10^{16} \text{ cm}^{-2}\text{s}^{-1}$ , while radical flux of  $10^{18} \text{ cm}^{-2}\text{s}^{-1}$ . In a feature domain of  $100\text{nm} \times 100\text{nm}$  for a process of  $10\text{s}$ , there are  $10^7$  ions and  $10^9$  radicals needed to be launched and tracked, which is clearly beyond the capability of any existing computer or cluster. Instead of a single particle, a macro-particle called pseudo-particle and designed like the material cell, is used in the Langmuir Feature Model. A pseudo-particle consists of  $N_{cell}$  identical particles, which could be electrons, ions, neutrals or even photons. Each pseudo-particle is assigned the properties of a single species and not dividable. The number density of a pseudo-particle matches the material cell so that any reactions occur between them are balanced and act as single-particle reactions.

### 6.3.2 Particle Launch

In a time period  $T$ , the total launched pseudo-particles,  $N_{particle} \times N_{cell}$ , needs to match the total fluence,  $Flux \times Area \times T$ , into the domain. In average, each

pseudo-particle occupies a time duration of

$$\Delta t = \frac{N_{cell}}{Flux \times Area}$$

By considering the average velocity of the pseudo-particle and the domain of the feature, the life time of a pseudo-particle is about

$$t_{life} = \frac{L \times N_{reflect}}{V_{particle}}$$

where  $L$  is the characteristic length of the domain,  $L < \sqrt{width^2 + height^2}$ ,  $N_{reflect}$  is the number of reflections experienced by the pseudo-particle,  $N_{reflect} < 10$  for most scenarios,  $V_{particle}$  is the average speed of the pseudo-particle,  $t_{life}$  is the lifetime of a pseudo-particle. Let us put those numbers into an example case.  $L = 100 \text{ nm}$  for a domain of  $100 \text{ nm} \times 100 \text{ nm}$ ,  $N_{reflect} = 10$ , and  $V_{particle} = 500 \text{ m/s} = 500 \text{ nm/ns}$  at room temperature. The resulting  $t_{life} = 2 \text{ ns}$  is far smaller than typical  $\Delta t = 100 \text{ ns}$ . It means that a pseudo-particle is launched and gets dead before next pseudo-particle is launched, which indicates that no interaction of pseudo-particles is necessary to be taken into account. The only interacting object of a pseudo-particle is the material cell. The summary of the important assumptions can be seen as below:

- Pseudo-particles uniformly enter the feature. This indicates that each pseudo-particle occupies exact  $\Delta t$ , defined as above.
- A pseudo-particle entering the feature is a rare event. This implies that each pseudo-particle event is instantaneous compared to the time between incoming pseudo-particle. Pseudo-particles do not interact with each other.
- The number of total pseudo-particle entering the feature per area and per time is exactly the same as the flux. This ensures that the overall effects of total pseudo-particles well align with the physics requirements.

The first assumption can be argued as the real entering events could follow Poisson's distribution more than uniform distribution. Even under the assumption of Poisson's distribution, you could find the event of two pseudo-particles entering the feature with overlap in time is rare. Compared to millions of pseudo-particles launched in a simulation, the first two assumptions together still hold.

### 6.3.3 Particle Tracking without E-field

In the serial version of Langmuir Feature Model, all pseudo-particles are launched in sequence and particle tracking is only applied to a single particle each time. In the terms of memory management, a memory space is created initially for a single pseudo-particle. When a pseudo-particle dies, a new pseudo-particle can reuse the memory space by updating the particle properties, such as position and velocity. The pseudo-particles are by default launched from the top boundary of the domain. The initial position of pseudo-particle is randomly picked. The velocity vector, consisting of speed and angle w.r.t.  $x = 0^+$ , is chosen randomly from the given distribution. The given distribution could be either generated from the feature model itself, or from the IAEDF generated from the Langmuir Reactor Model.

Without E-field, a pseudo-particle is not accelerated during the flight. It means that the pseudo-particle follows the line-of-sight trajectory. Although the geometry is meshed to grids, the pseudo-particle advances in continuous space. Newton's equations are solved for the trajectory,

$$\vec{r} = \vec{r} + \vec{v} \times dt$$

where  $\vec{v}$  and  $\vec{r}$  are the velocity and position of the pseudo-particle, respectively.  $dt$  is the flight timestep, which is far smaller than the simulation timestep. Instead of integrating over flight timestep  $dt$ , an advancing step,  $\Delta L$ , is used in Langmuir Feature Model,

$$\vec{r} = \vec{r} + \vec{v}_{unit} \times \Delta L$$

where  $\vec{v}_{unit}$  is the normalized unit vector of the velocity and  $\Delta L$  is the advance step. Typical  $\Delta L$  is constant and set to be about the resolution of the mesh. Larger  $\Delta L$  definitely reduces the computing time. A varying  $\Delta L$ , which is determined by the position, can be used and will be discussed separately.

### 6.3.4 Particle Tracking with E-field

When E-field is taken into account, velocity for charged particles is not constant anymore. Full Newton's equations need to be solved,

$$\vec{v}(t + dt) = \vec{v}(t) + \frac{q\vec{E}}{m} dt$$

$$\vec{r} = \vec{r} + \vec{v} \times dt$$

where  $\vec{E}$  is the E-field,  $q$  is the particle charge, and  $m$  is the particle mass. In the feature model, E-field is a function of position and changes with deposited surface charges. Within the tracking of a single particle, E-field does not change with time. Therefore, Newton's equations can be solved using spacestep instead of timestep,

$$\vec{v}(\vec{r} + d\vec{r}) = \vec{v}(\vec{r}) + \frac{q\vec{E}}{m} \times \frac{\Delta L}{abs(\vec{v})}$$

$$\vec{r} = \vec{r} + d\vec{r}$$

$$d\vec{r} = \vec{v}_{unit} \times \Delta L$$

The chosen of  $\Delta L$  depends on the spatial variation of E-field. In general, if the gradient of E-field is small,  $\Delta L$  can be increased; vice versa. In most cases,  $\Delta L$  should not be larger than the resolution of the mesh.

### 6.3.5 Ray Tracing

To be added.

## 6.4 Particle-Materials Interactions

### 6.4.1 Hit Check

When the particle is tracked by step advance algorithm, particle-material collision needs to be checked. In Langmuir Feature Model, there is no volume assigned to the pseudo-particle, which means that particle trajectory is an 1D line. At a fixed time, the pseudo-particle is a point without any dimensions. When the pseudo-particle gets inside a material cell, the model flags a "Hit". "Inside a cell" means that the position  $(x, z)$  of the particle lies within the boundary

of the cell,

$$C_{left} < x < C_{right}$$

$$C_{bottom} < z < C_{top}$$

where  $C_{left}$ ,  $C_{right}$ ,  $C_{bottom}$  and  $C_{top}$  are the four boundaries for a cell. In the program, instead of checking the four boundaries, the particle is mapped onto the computational mesh of materials using,

$$i = \text{int}(x - 0.5\Delta x)$$

$$j = \text{int}(z - 0.5\Delta z)$$

It means that the particle is now within the cell  $C(i, j)$ . If  $C(i, j)$  is vacuum, then the particles continues to advance. Otherwise, the particle is considered to hit the material cell,  $C(i, j)$ .