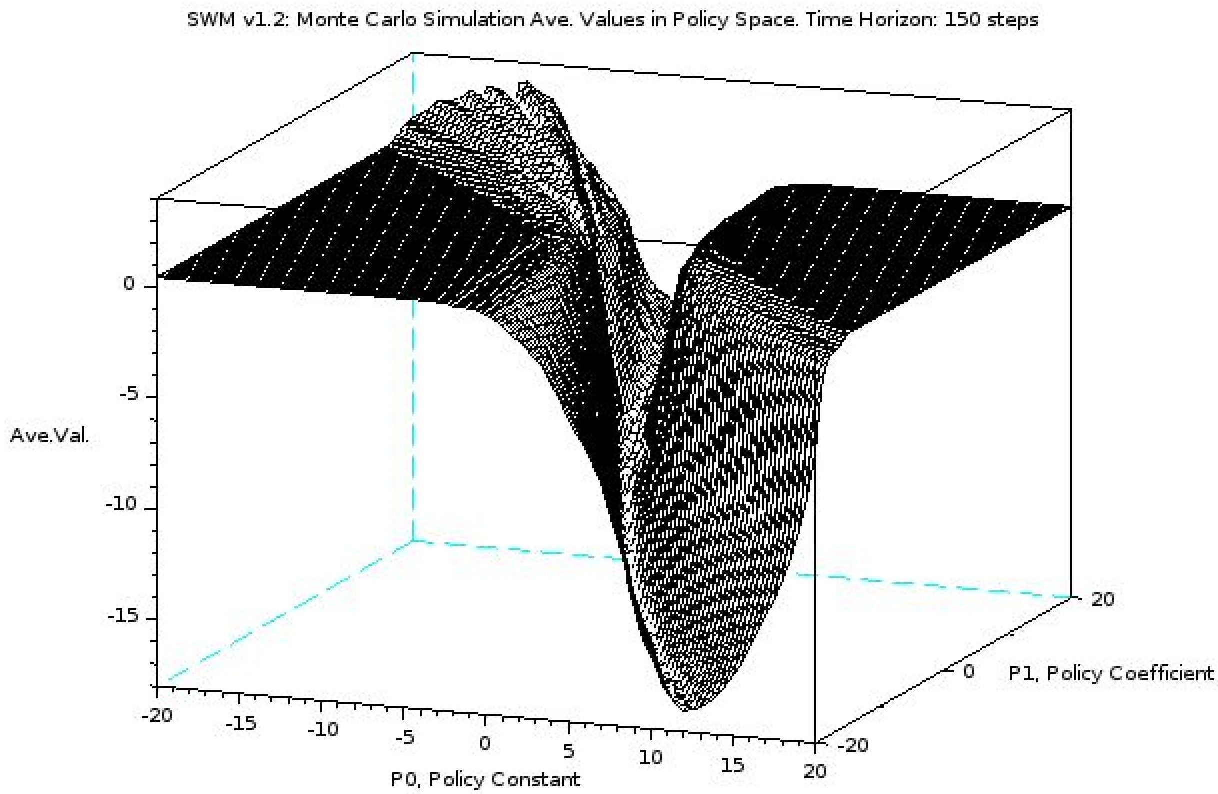
SWM v1.3

*A **S**imple **W**ildfire-inspired **M**DP Model*

Initial Draft of SWMv1.2 Documentation: Hailey K. Buckingham, 7-31-15

This Draft: Hailey K. Buckingham, 12-30-15



CONTENTS

Summary

SWM v1.3 is a Markov decision process (MDP) with five variables which evolve through time in response to the choices and events that occur in each timestep. The entire "forest" is thus modelled by only five numeric values (i.e., there is no spatial component or 'landscape'). During each timestep, a fire event occurs. The fire can be classified as either a severe, or a mild fire, depending on a stochastic and time-independent "weather severity" variable, and the current condition of the forest. At each time step (i.e. for each fire) the choice to either suppress the fire or let the fire burn must be made.

In terms of state transitions, there are essentially three cases: 1) A severe fire is allowed to burn, 2) a mild fire is allowed to burn, and 3) a fire (of either severity) is suppressed. In the first case, when a severe fire is allowed to burn, there is a benefit in the form of decreased vulnerability to future fires, and a penalty in the form of decreased timber returns for a period of time afterward. In the second case, a mild fire gives the benefit of reduced vulnerability, but does not have a penalty for future timber harvests. In the third case, when fires are suppressed, there is an immediate cost for the suppression effort itself (and can be set higher for severe fires, if desired), and also a penalty in the form of increased vulnerability (i.e. unburned fuels building up over time). Finally, to reflect recovery or forest timber assets during years when there are no fires (years in which suppression is chosen or when only mild fires burn), there is a reward in the form of increased forest timber value.

Thus, timber receipts per timestep decreases in years with severe fires, and increases in years with mild fires or suppression. Conversely, forest vulnerability increases during years with suppression, and decreases when fuels are “treated” as the result of mild or severe fires.

SWM v1.3 also tracks a separate reward for the ‘habitat quality’ of the forest. This is designed to be independent of timber values and allow a second form of policy optimization and control to be experimented with. The habitat quality of the forest is based on the time since both mild and severe fires. Boundaries are set for the minimum and maximum number of years-since-fire (for both types). If a fire occurs before it’s minimum boundary, it is “too soon” for that type of fire, and the habitat value decreases. Likewise, if it has been a very large number of years since one or the other type of fire, such that the maximum bound has been exceeded, it has been “too long” since that type of fire and habitat value decreases. If BOTH the time-since-mild-fires and time-since-severe-fires are within their respective bounds, then habitat value will increase.

Generally, the minimum bound for mild fires is set to 0 (no minimum) and the upper bound is set to something relatively small (5 to 15, or so) reflecting the need for regular mild fires on the forest to maintain an optimal ecosystem. For severe fires, the minimum is generally set to something above zero, and the maximum a decade or two later, reflecting the idea that some, but not many severe fires are important for ecosystem value.

State Space: $\mathbf{s} = [s_1, s_2, s_3, s_4, s_5]$

s_1 = Vulnerability takes on values between 0.0 and 1.0

s_2 = Timber Value takes on values between 0 and 10 (units of economic value)

s_3 = Habitat Value takes on values between 0 and 10 (units of habitat ‘goodness’)

s_4 = Time since a Mild Fire takes on non-negative integer values

s_5 = Time since a Severe Fire takes on non-negative integer values

Stochastic State Event : $\mathbf{e} = [e_1, e_2]$ Generated independently at each timestep

e_1 = Weather Severity	Randomly generated at each time step, distributed ~U(0,1)	
e_2 = Severity:	0 (MILD):	if $e_1 <= s_1$
	1 (SEVERE):	if $e_1 > s_1$

State Initialization

s_1 is initialized as a random number, drawn ~U(0.2 to 0.8)

s_2 is initialized as a random number, drawn ~U(2 to 8)

s_3 is initialized as a random number, drawn ~U(2 to 8)

s_4 is initialized at 0

s_5 is initialized at 0

e_1 is randomly generated at each timestep, drawn ~U(0,1)

Control Policy

To decide whether or not to suppress each fire during a simulation, any number of policies might be used. For this model, since we are interested in the long-term costs and benefits of fires (rather than only the immediate cost of suppression and timber losses), we've chosen a function which can allow several types of control behavior depending on what values the state variables have taken at a particular time. In particular, the function allows for either suppression or not-suppression of some types of fires, and also for certain types of fire events to be suppressed according to a probability. That is, the policy function might be parameterized such that a fire with characteristics '*a*' be suppressed 100% of the time, a fire with characteristics '*b*' to be allowed to burn 100% of the time and, with the same parameterization, a fire with characteristics '*c*' to be allowed to burn (*p*) percent of the time and suppressed (1-*p*) percent of the time

To accomplish this, the control function has been designed as a logistic function into which the dot-product of two vectors is input. The first vector is the 'feature vector, which is simply the state variables a the time of the current fire plus a constant. The second vector is a set of policy parameters of the same length.

For many of the analyses on SWMv1.3, only one state variable (weather severity) is included in the feature vector, plus a constant, making the policy 2-dimensional. This arrangement purposefully ignores important transition dynamics (in particular, forest vulnerability to fire) but allows for the simulator's dynamics under different policies to be visualized in 2 dimensions as well. As will be seen, even this simple policy function will allow for a great diversity in simulator behavior.

Formally, this 2-dimensional policy function can be defined as follows:

policy vector: $\boldsymbol{p} = [p_0 , p_1]$
feature vector $\mathbf{e} = [1, e_1]$, where e_1 is the state variable for the stochastic event's weather severity, as defined above.

The policy function *policy()* is defined:
$policy(\boldsymbol{p}, \boldsymbol{e}) = logistic(\boldsymbol{p} * \boldsymbol{e}) = 1 / (1+ exp(-1 * (\boldsymbol{p} * \boldsymbol{e})))$

To make a choice for suppression or not-suppression, the value output from policy(**p**, **e**) is compared to a random number draw:

$choice_t$ = Suppression Choice at a given Timestep:

$choice_t$	=	True (SUPPRESS):	if random ~U(0 to 1) <= policy($\mathbf{p}$, $\mathbf{e}_t$)
	=	False (LET BURN):	else

TODO: This function form also works for any length of feature and policy vectors

Reward Structure for Timber Value

C_{mild} = Cost of suppressing a mild event
C_{severe} = Cost of suppressing a severe event

C_{burn} = Cost of allowing a severe fire to burn

R_{timber} = Timber Value = s_2 *(State variable for Timber Value*)

$R_{suppress}$ = Suppression Cost = C_{mild} if e_2 == 0 (MILD) AND $choice_t$ == True (SUPPRESS)
= C_{severe} if e_2 == 1 (SEVERE) AND $choice_t$ == True (SUPPRESS)
= 0 if $choice_t$ == False (LET BURN)

$R_{burn\ cost}$ = Burn Costs = C_{burn} if $choice_t$ == False (LET BURN) AND e_2 == 1 (SEVERE)

$\mathbf{Reward_t = R_{timber} - R_{suppress} - R_{burn\ cost}}$

Reward Structure for Habitat Value

The habitat value of the forest at any timestep is based upon the idea of a fire return interval. There is an “optimal” range of years-since-mild and years-since-severe fires in which habitat value increases. Outside of either of those ranges, habitat value will decrease

H_{Mild} = The loss to habitat value when there are too many or too few mild fires
H_{Severe} = The loss to habitat value when there are too many or too few severe fires
H_{Gain} = The gain to habitat value when both mild and severe fires are occurring ‘optimally
$mild_{min}$, $mild_{max}$ = the min and max timesteps bounding the optimal period for mild fires
$severe_{min}$, $severe_{max}$ = the min and max timesteps bounding the optimal period for severe fires

Let:
$time_{mild}$ be the time since a mild fire was allowed to burn (which is the state variable s_4)
$time_{severe}$ be the time since a severe fire was allowed to burn (which is the state variable s_5)

$penalty_{mild}$ = 1 if $mild_{min} \leq time_{mild} \leq mild_{max}$
and 0 otherwise

$penalty_{severe}$ = 1 if $severe_{min} \leq time_{severe} \leq severe_{max}$

and 0 otherwise

gain = 1 if $penalty_{mild} = penalty_{severe} = 0$
and 0 otherwise

Then:

$$Habitat_{t+1} = Habitat_t - (penalty_{mild} * H_{Mild}) - (penalty_{severe} * H_{Severe}) + (gain * H_{Gain})$$

Transition

Choice (*choice*$_t$)	**Event Severity (e_2)**	
	e_2 = 0 (MILD)	**e_2 = 1 (SEVERE)**
***choice*$_t$ = False (LET BURN)**	$s_{1,\,t+1} = s_{1,\,t} - 0.01$ $s_{2,\,t+1} = s_{2,\,t} + 0.5$ units	$s_{1,\,t+1} = s_{1,\,t} - 0.02$ $s_{2,\,t+1} = s_{2,\,t} - 2.0$ units
***choice*$_t$ = True (SUPPRESS)** *(ignores event severity)*	$s_{1,\,t+1} = s_{1,\,t} + 0.01$ $s_{2,\,t+1} = s_{2,\,t} + 0.5$ units	

REMINDER:

s_1 denotes forest vulnerability to fire and takes on values between 0 and 1. Higher values denote more vulnerability to large fires.

s_2 denotes timber values and takes on values between 0 and 10 'economic value units.' This is the amount of value per timestep that is received as "timber harvested."

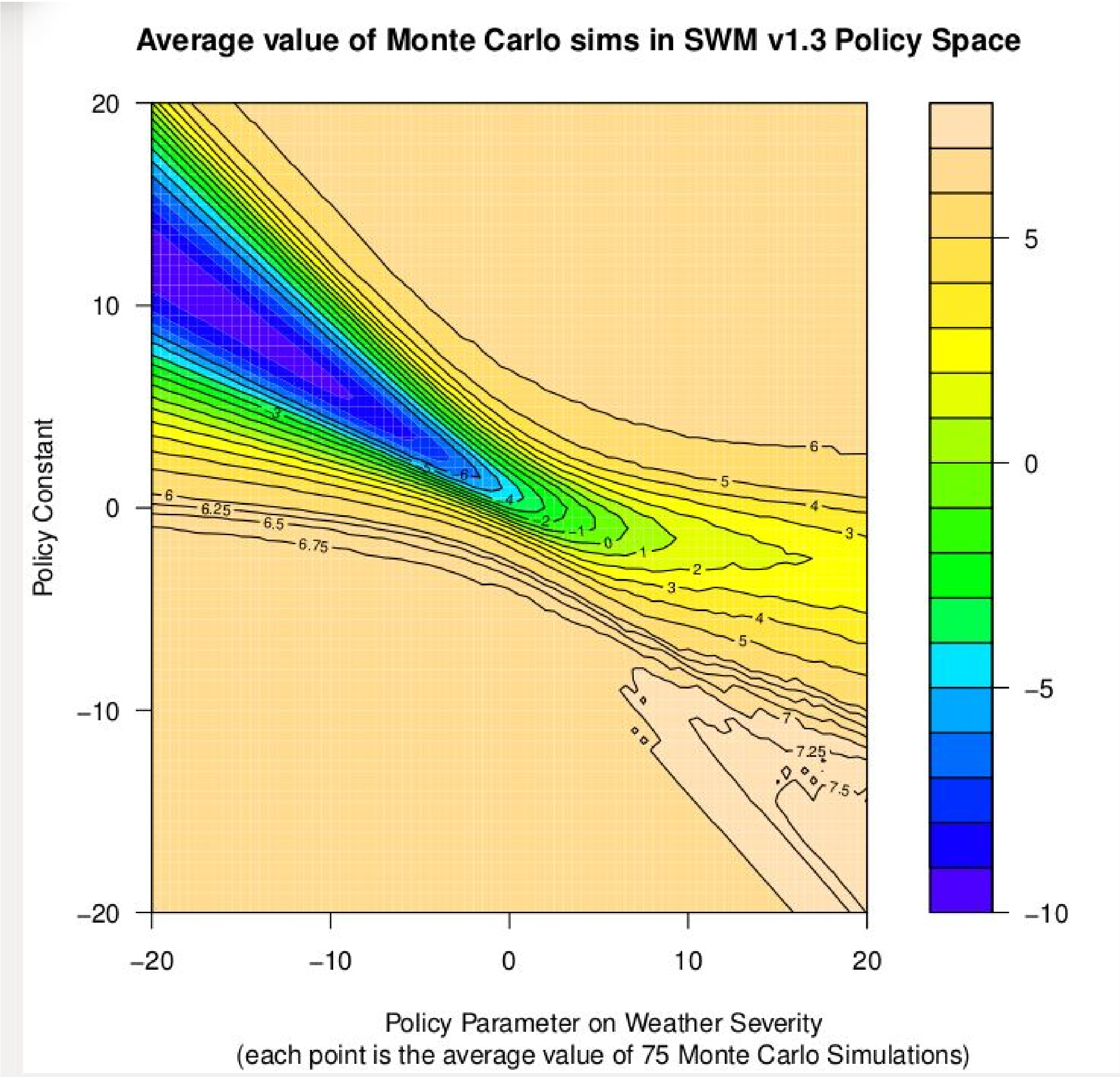
NOTES:

1) *Values listed in blue are model parameters which can be modified as needed to control transition dynamics*
2) *State variables are always subject to their maximum and minimum values, so some additions and subtractions may be ignored when those values are already at their upper or lower bound, respectively.*

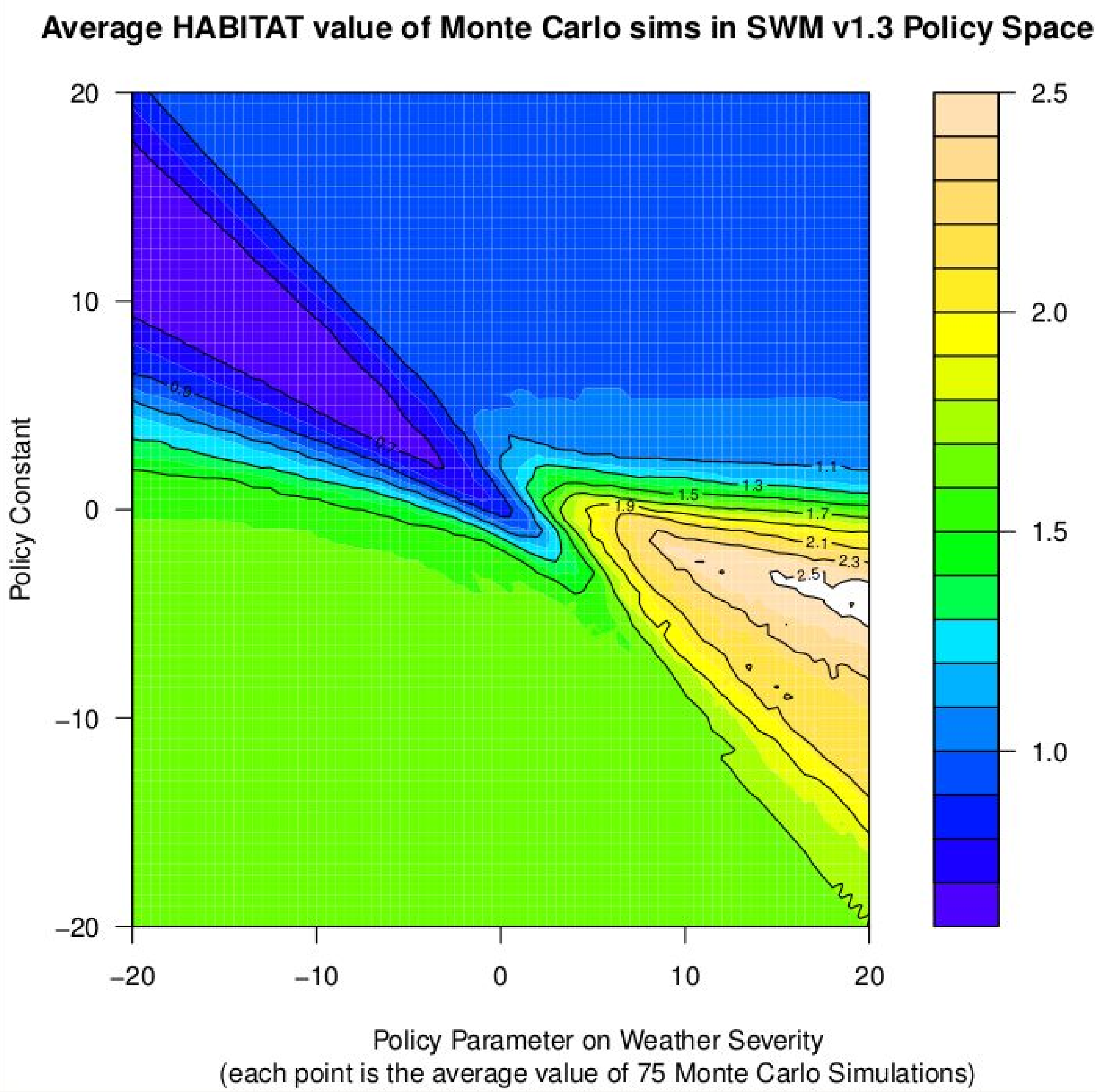
Model Behavior

~optima for timber value: policy = [-15, 18] with suppression rate ~0.30

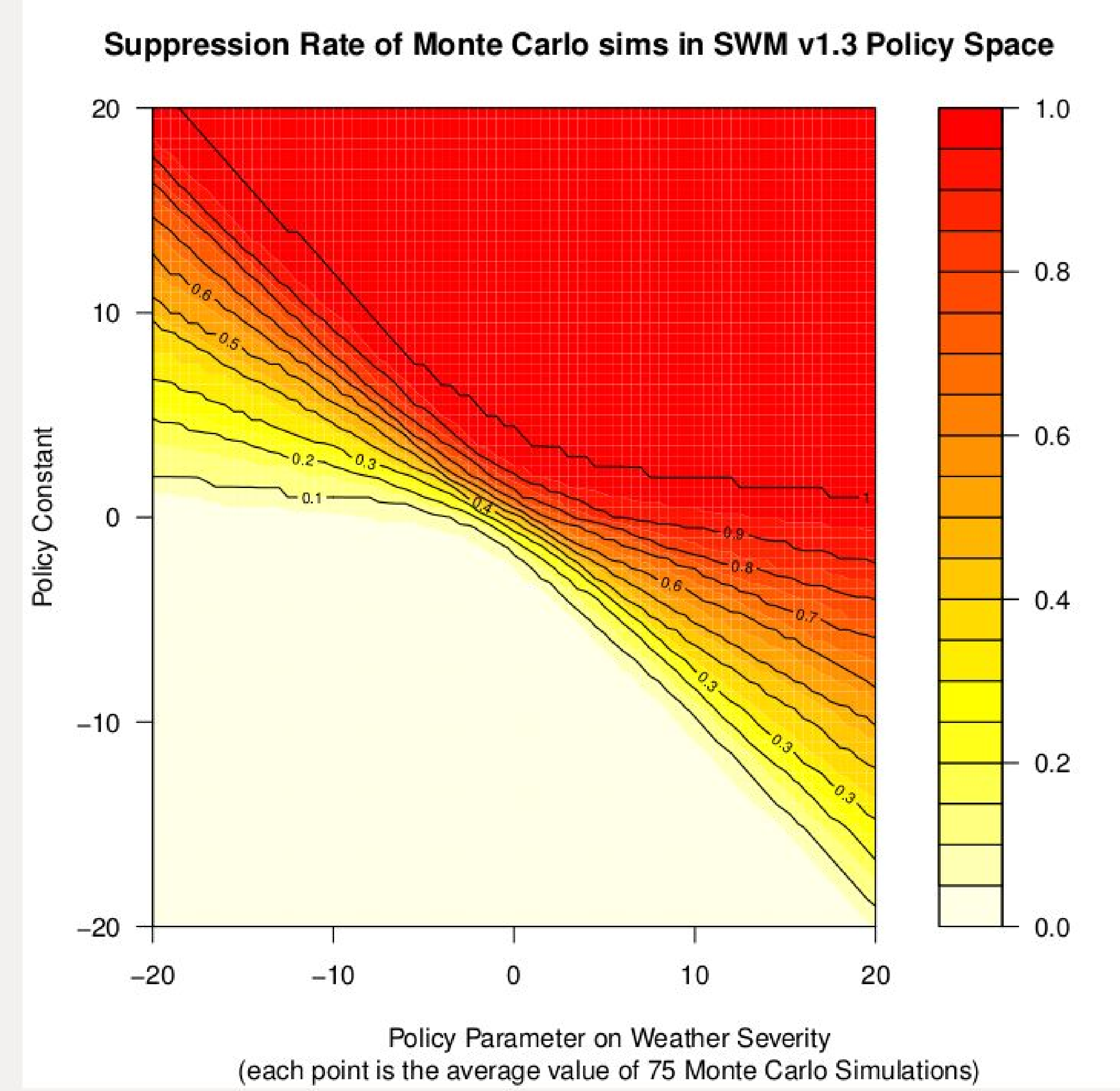
~optima for habitat value: policy = [-4, 19] with suppression rate ~0.75



Average value of Monte Carlo sims in SWM v1.3 Policy Space
Policy Constant
Policy Parameter on Weather Severity
(each point is the average value of 75 Monte Carlo Simulations)
20
10
0
−10
−20
−20
−10
0
10
20
5
0
−5
−10
6
6.25
6.5
6.75
5
4
3
2
1
0
7
7.25
7.5



Average HABITAT value of Monte Carlo sims in SWM v1.3 Policy Space
20
10
0
−10
−20
Policy Constant
−20
−10
0
10
20
Policy Parameter on Weather Severity
(each point is the average value of 75 Monte Carlo Simulations)
1.1
1.3
1.5
1.7
1.9
2.1
2.3
2.5
2.5
2.0
1.5
1.0


Suppression Rate of Monte Carlo sims in SWM v1.3 Policy Space
Policy Constant
Policy Parameter on Weather Severity
(each point is the average value of 75 Monte Carlo Simulations)
20
10
0
−10
−20
−20
−10
0
10
20
1.0
0.8
0.6
0.4
0.2
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1