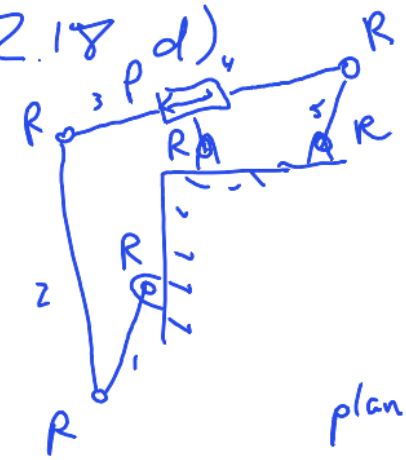
Part 1)

Assignment 1

MR

2.18 d)



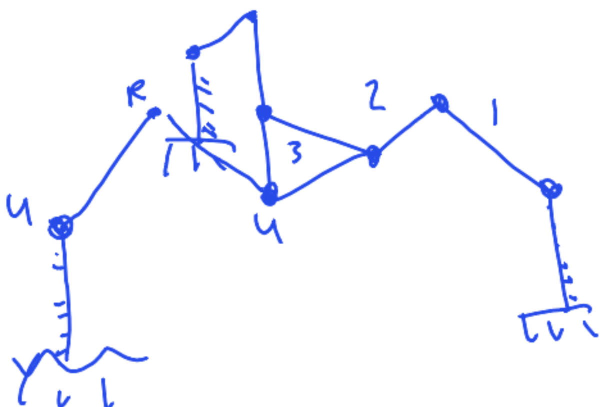
Grubler's:

$$dof = m(N-1-J) + \sum_{i=1}^{J} f_i$$

$$\left.\begin{array}{l} \text{planar} \Rightarrow m = 3 \\ N = 1 + 5 = 6 \\ J = 7 \\ \sum f_i = 6 \cdot 1 + 1 \end{array}\right\} \Rightarrow dof = 3(6-1-7) + 7 = 1$$

2.21a

$u = 2$ dof $\quad R = 1$ dof



$m = 6$

$N = 3 \cdot 2 + 1 + 1 = 8$

$J = 3 \cdot 3 = 9$

$\sum f_i = 3 \cdot (2+1+2)$
$= 15$

$\rightarrow \text{dof} = 6(8-1-9) + 15$

$\boxed{= 3}$

Part 1

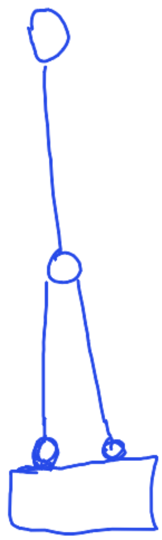
c)

assume eraser is homogeneous
$\Rightarrow$ task doesn't care about eraser orientation

$\boxed{\Rightarrow 2^d \text{ task space } \in \mathbb{R}^2}$

$\boxed{\text{topology: } \mathbb{E}^2}$

b)



if the human arm has 7 dof then with 4 free rigid bodies there are $4 \cdot 6 = 24$ total dof.

$\boxed{24 - 7 = 17 \text{ constraints imposed}}$

Part 2b)

I duplicated the section for Joint 1, because as the largest joint it must surely be the most important, and therefore deserves duplicate, albeit non-functioning, entries in the UI. In order to avoid runtime errors, the id of each new element was modified to be unique.

Simulation script "/UI_Script"

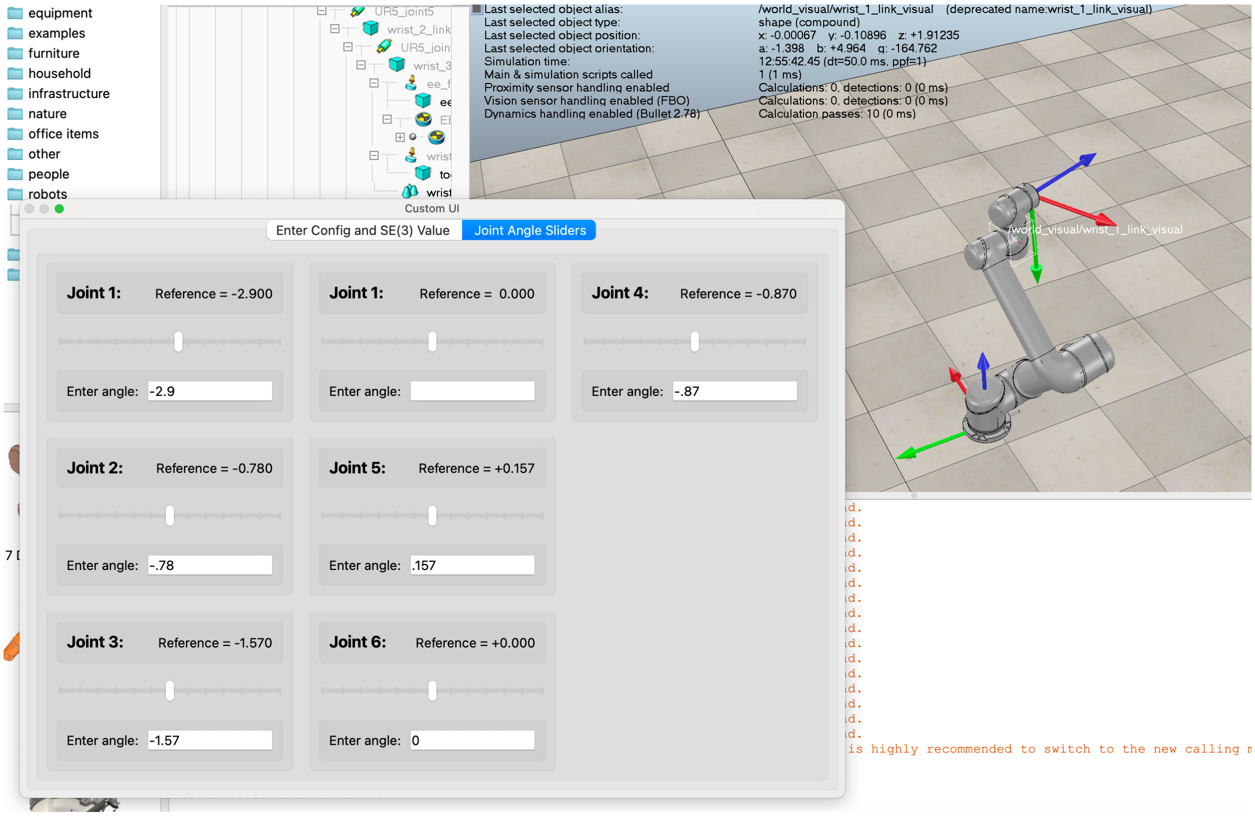
```
        <tab title="Joint Angle Sliders">
          <group layout="grid" >
            <group>
              <group layout="grid">
                <label text="<big> Joint 1:</big>" id="6000" wordwrap="false" style="font-weight: b
                <label text="Reference =  0.000" id="3000" wordwrap="false" />
                <!-- <label text="Actual =  0.000" id="5000" wordwrap="false" /> -->
              </group>
              <hslider id="4000" tick-position="above" tick-interval="1000" minimum="-6280" maximum
              <group layout="grid">
                <label text="Enter angle:" />
                <edit value="" id="7000" oneditingfinished="jointEntry" />
              </group>
            </group>

            <group>
              <group layout="grid">
                <label text="<big> Joint 1:</big>" id="6010" wordwrap="false" style="font-weight: b
                <label text="Reference =  0.000" id="3010" wordwrap="false" />
                <!-- <label text="Actual =  0.000" id="5010" wordwrap="false" /> -->
              </group>
              <hslider id="4010" tick-position="above" tick-interval="1000" minimum="-6280" maximum
              <group layout="grid">
                <label text="Enter angle:" />
                <edit value="" id="7010" oneditingfinished="jointEntry" />
              </group>
            </group>

            <group>
              <group layout="grid">
                <label text="<big> Joint 4:</big>" id="6003" wordwrap="false" style="font-weight: b
                <label text="Reference =  0.000" id="3003" wordwrap="false" />
                <!-- <label text="Actual =  0.000" id="5003" wordwrap="false" /> -->
              </group>
              <hslider id="4003" tick-position="above" tick-interval="1000" minimum="-6280" maximum
              <group layout="grid">
                <label text="Enter angle:" />
                <edit value="" id="7003" oneditingfinished="jointEntry" />
              </group>
            </group>

            <br/>
b_init then ... end
. end
```

And here's the new UI:



Part 3) Derivation

I first isolated single joint rotation matrices by using the subscript cancellation rule. I then used the matrix logarithm to convert each rotation matrix into so(3) skew-symmetric matrices. I extracted the ω vector from each so(3) matrix and finally multiplied by the transpose of the unit rotation vector to get the rotation angle.

$$rot(\hat{\omega}, \theta) = e^{[\omega]\theta}$$

$$R_{12} = e^{[\omega_1]\theta}$$

$$\log(R_{12}) = [\omega_2]\theta_2$$

$\to \theta_1 = 0.78$?

$$\log(R_{s1}) = [\omega_1]\theta_1$$

$$R_{13} = R_{s1} R_{12} R_{23}$$

$$= e^{\boxed{[\omega_1]}\theta_1} e^{[\omega_2]\theta_2} e^{[\omega_3]\theta_3} = e^{[\omega_1]\theta_1 + [\omega_2]\theta_2 + [\omega_3]\theta_3}$$

$$\log(R_{13}) = \cancel{[\omega_1]\theta_1 +} [\omega_2]\theta_2 + [\omega_3]\theta_3$$

$$\log R_{s2} = [\omega_1]\theta_1 + [\omega_2]\theta_2$$

$$\underset{3\times3}{A} = \underset{3\times3}{B}\,\theta$$

$$B^{-1}A = \underset{3\times3}{I}\,\theta$$

Current configuration:

(-2.900, -0.780, -1.570, -0.870, 0.157, 0.000)

Messages:

Current SE(3):

T(?) =

```
| -0.919 -0.076 +0.388 +0.039 |
| -0.387 -0.019 -0.922 -0.186 |
| +0.077 -0.997 -0.012 +0.760 |
| +0.000 +0.000 +0.000 +1.000 |
```

Code:

```
w1 =np.array( [0,0,1])
w2 =np.array( [0, 1, 0])
w3 =np.array( w2)
w4 =np.array( w2)
w5 =np.array( [0, 0, -1])
w6 =np.array( w2)

R13 = np.array([[-0.7071, 0, -0.7071], [0, 1, 0], [0.7071, 0, -0.7071]])
Rs2 = np.array([[-0.6964, 0.1736, 0.6964], [-0.1228, -0.9848, 0.1228], [0.7071, 0,
0.7071]])
R25 = np.array([[-0.7566, -0.1198, -0.6428], [-0.1564, 0.9877, 0], [0.6348, 0.1005,
-0.7661]])
R12 = np.array([[0.7071, 0, -0.7071], [0, 1, 0], [0.7071, 0, 0.7071]])
R34 = np.array([[0.6428, 0, -0.7660], [0, 1, 0], [0.7660, 0, 0.6428]])
Rs6 = np.array([[0.9418, 0.3249, -0.0859], [0.3249, -0.9456, -0.0151], [-0.0861, -
0.0136, -0.9962]])
R6b = np.array([[-1, 0, 0], [0, 0, 1], [0, 1, 0]])
```

```python
so3_1 = mr.VecToso3(w1)
so3_2 = mr.VecToso3(w2)
so3_3 = mr.VecToso3(w3)
so3_4 = mr.VecToso3(w4)
so3_5 = mr.VecToso3(w5)
so3_6 = mr.VecToso3(w6)

log12 = mr.MatrixLog3(R12)
v12 = mr.so3ToVec(log12)
a2_test = v12 @ w2.T

Rs1 = Rs2 @ R12.T
R23 = R12.T @ R13
R45 =  R34.T @ R23.T @ R25
R56 = (Rs1 @ R12 @ R23 @ R34 @ R45).T @ Rs6

a1 = mr.so3ToVec(mr.MatrixLog3(Rs1)) @ w1.T
a2 = mr.so3ToVec(mr.MatrixLog3(R12)) @ w2.T
a3 = mr.so3ToVec(mr.MatrixLog3(R23)) @ w3.T
a4 = mr.so3ToVec(mr.MatrixLog3(R34)) @ w4.T
a5 = mr.so3ToVec(mr.MatrixLog3(R45)) @ w5.T
a6 = mr.so3ToVec(mr.MatrixLog3(R56)) @ w6.T

angles = [a1, a2, a3, a4, a5, a6]

for i in range(len(angles)):
    print("angle {}: {}".format(i+1, angles[i]))

Rsb = Rs1 @ R12 @ R23 @ R34 @ R45 @ R56 @ R6b
print("Rsb", Rsb)

pass
```

Output:

```
angle 1: -2.969482157066879
angle 2: -0.7853926894212007
angle 3: -1.5707661989213484
angle 4: -0.8726096667837093
angle 5: 0.15702980014757423
angle 6: 9.050365593053613e-07
Rsb [[-0.94155819 -0.08587249  0.32478673]
 [-0.32492508 -0.01511086 -0.94558895]
 [ 0.08609033 -0.9960651  -0.01360784]]
```