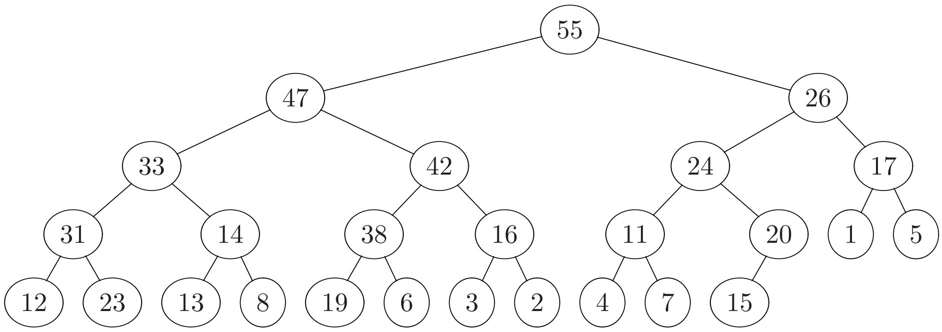
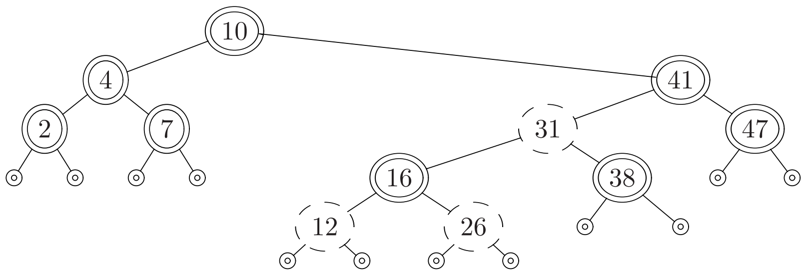
CSE 2331 Sample Midterm II
Closed book and notes. NO calculators.
(100 Points Total)

1. (10 points) Apply the ExtractMax() operation to the following heap and draw the resulting binary tree. SHOW YOUR WORK.

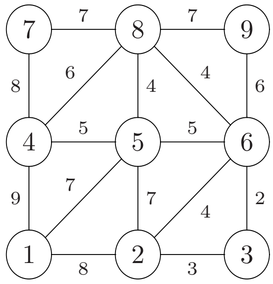


2. (12 points)
   (a) Describe the algorithm to delete a node from a binary search tree. (Describe just the five or six major steps in the algorithm.)
   (b) Give asymptotic bounds on the worst case running time of the algorithm in terms of the number of nodes and/or height of the tree. Give bounds which are as tight and precise as possible. JUSTIFY YOUR SOLUTION.

3. (12 points) Apply RBTreeInsert to insert 18 in the following red-black tree and draw the resulting binary tree. (Black nodes have double circles. Red nodes have dashed circles.) Indicate which nodes are red or black in your drawing. Be sure that the resulting tree has all the properties of a red-black tree. SHOW YOUR WORK.



4. (12 points)
   (a) Draw the MINIMUM SPANNING TREE of the following weighted graph.
   (b) Give the <u>EDGES</u> of the minimum spanning tree of the following weighted graph in the order they would be output by Prim's algorithm starting at vertex v_1.



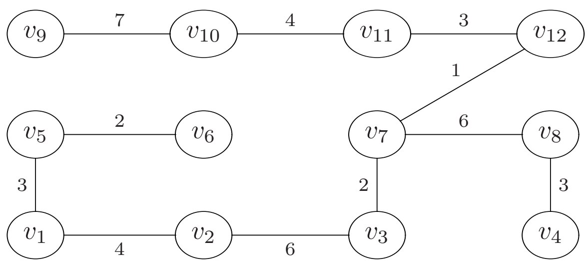
5. (20 points) Let P be a priority queue implemented by a data structure where:

   - P.Initialize() takes $\Theta(1)$ time. Initially, P has 0 elements.
   - P.Insert() takes $\Theta(s^{1/3})$ time where s is the number of elements in P. P.Insert() adds one element to P.
   - P.ExtractMax() takes $\Theta(s)$ time where s is the number of elements in P. P.ExtractMax() removes an element from P.

   Give the asymptotic running time of the following algorithm in Θ notation. Jusify your answer. Be sure to analyze all steps and operations in the algorithm.

```
Func1(A, n)
/* A is an array of n elements                                  */
P.Initialize();
for i ← 1 to n do
    for j ← 1 to 100 do
        P.Insert(A[i] * A[j]);
    end
end
s ← 0;
for i ← 1 to n do
    s ← s+ P.ExtractMax();
end
return (s);
```

6. (12 points) Let G be an edge weighted graph whose MINIMUM SPANNING TREE is:



   (The numbers on the edges are the edge weights of the edges of the MINIMUM SPANNING TREE.)

   Assume that the graph G contains the edge (v_6, v_{10}).
   Prove that the weight of (v_6, v_{10}) is NOT 5.
   (You may NOT base your proof on the correctness of any algorithm. Note that G may have many more edges in addition to the edges of the minimum spanning tree shown above.)

7. (12 points)

   (a) Give an example of an edge weighted graph with at most 6 edges whose minimum spanning tree is not the same as the shortest path tree from vertex v_1;

   (b) Draw the minimum spanning tree of your graph;

   (c) Draw the shortest path tree of your graph.

8. (10 points) Let G be an edge weighted graph with 100 vertices where the distance of the shortest path from v_1 to v_{99} is 500 and the distance of the shortest path from v_1 to v_{100} is 510.

   Prove that if G has an edge (v_{99}, v_{100}), then weight(v_{99}, v_{100}) does not equal 7.