CSE 2331 Homework 11
Fall, 2016
Due December 6, 9:30 a.m. (in class)
NO LATE HOMEWORKS ACCEPTED.

1. Give a truth assignment which satisfies the following boolean expression:

$$\phi(x_1, x_2, x_3, x_4) = (x_1 \vee x_2 \vee \overline{x_3}) \wedge (\overline{x_1} \vee \overline{x_2} \vee x_3) \wedge (\overline{x_1} \vee \overline{x_3} \vee \overline{x_4}) \wedge (\overline{x_2} \vee x_3 \vee \overline{x_4})$$

2. The *Exact Distance* problem is:

   Given an edge weighted graph $G = (V, E)$, a vertex v_k, and a distance D, is there a simple path from v_1 to v_k with length exactly D?

   The length of a path is the sum of the weights in the path.
   An edge weighted graph is a graph which has a weight $w(e)$ assigned to each edge $e \in E(G)$.

   A simple path is a path which does not revisit any vertices or edges.

   (a) Prove that the Exact Distance problem is in NP.
   (b) Prove that the Hamiltonian Path problem reduces to the Exact Distance problem in polynomial time.

   (Note that proving a) and b) proves that the Exact Distance Problem is NP-complete.)

3. The *Team Picking* problem is:

   Given a set of n players, $\{p_1, p_2, \ldots, p_n\}$, and a list of pairs of players who don't like each other and a number $k \leq n$, is it possible to pick k players for a team such that every player geta along with every other player on the team.?

   (a) Prove that the Team Picking problem is in NP.
   (b) Prove that the Independent Set problem reduces to the Team Picking problem in polynomial time.

   (Note that proving a) and b) proves that the Team Picking Problem is NP-complete.)

4. The *Zoo* problem is:

   Given a set of n animals, $\{a_1, a_2, \ldots, a_n\}$, and a list of pairs of animals who will attack each other, is it possible to split the animals into three enclosures, E_1, E_2 and E_3, so that no animal will attack any other animal in its enclosure?

   (a) Prove that the Zoo problem is in NP.
   (b) Prove that the 3-coloring problem reduces to the Zoo problem in polynomial time.

   (Note that proving a) and b) proves that the Zoo Problem is NP-complete, assuming that 3-coloring is NP-complete.)

5. Assume that problem Q_1 reduces to problem Q_2 in polynomial time and both Q_1 and Q_2 are in NP.

   (a) If Q_1 can be solved in polynomial time, what can be concluded about Q_2? (The answer may be nothing.)
   (b) If Q_2 can be solved in polynomial time, what can be concluded about Q_1? (The answer may be nothing.)
   (c) If Q_1 is NP-complete, what can be concluded about Q_2? (The answer may be nothing.)
   (d) If Q_2 is NP-complete, what can be concluded about Q_1? (The answer may be nothing.)