Suppose $G = \langle X|R \rangle$ is a hyperbolic group in the sense that all triangles in the Cayley graph of G are δ-thin.

Theorem 0.1. *Suppose $A = [a_1, \dots, a_n]$ is a tuple of elements in G. Then the centraliser of A can be computed in time $O(||A||n^n)$, or $O(||A||n)$ if a_1 is of infinite order.*

Theorem 0.2. *Suppose $A = [a_1, \dots, a_n]$ and $B = [b_1, \dots, b_n]$ are tuples of elements in G. Then one can decide if there exists some g such that $A^g = B$ and return such a g in time $O((||A|| + ||B||)n^2)$, or $O((||A|| + ||B||)n)$ if a_1 is of infinite order.*

Theorem 0.3. *There is a presentation $G = \langle Y|S \rangle$ such that all triangles in the Cayley graph of this presentation are 13-thin.*

Now suppose H is an ϵ-quasiconvex subgroup of G.

Theorem 0.4. *Triangles in the coset Cayley graph of H are $4\epsilon + 30\delta$-thin.*

Theorem 0.5. *If the Cayley graph of H has GIB($\frac{5}{2}\delta$) with constant K, then it has GIB(∞), and the constant associated to GIB(k) ($k > \frac{5}{2}\delta$) is less than or equal to $K + k$.*

Theorem 0.6. *Torsion free subgroups have GIB($\frac{5}{2}\delta$) with constant in $O(\epsilon)$.*

We now suppose H has GIB($\frac{5}{2}\delta$).

Theorem 0.7. *There exists a K such that if w labels a geodesic in the Cayley graph of G, and w labels a path lying outside of $B_K(H)$ in the coset Cayley graph of H, this path is also a geodesic.*

Corollary 0.8. *Given $\lambda \geq 1$ and $\epsilon \geq 0$, there exists a K such that if w labels a (λ, ϵ)-quasigeodesic in the Cayley graph of G, and w labels a path lying outside of $B_K(H)$ in the coset Cayley graph of H, this path is also a (λ, ϵ)-quasigeodesic.*

Corollary 0.9. *Given $g \in G$, it is possible to decide whether $g^a \in H$ for some a in G and return such an a in time $O(|g|)$.*

Corollary 0.10. *There is a number N such that if $g \in G$ and $(g^n)^a \in H$ with n a positive and minimal integer and a is any element of G, then $n \leq N$.*