

## Solutions to Exercises 1

- 1A** The GCD of 6 and 9 is 3; the GCD of 12 and 18 is 6; the GCD of 15 and 21 is 3; the GCD of 11 and 15 is 1.
- 1B** omitted
- 1C** Using Newton's algorithm to calculate the square root of a number accurate to two decimal places, the square root of 4 is 2.00; the square root of 6 is 2.45; the square root of 8 is 2.83; and the square root of 9 is 3.00.

Method to calculate the square root of  $a$  to 2 decimal places:

```
static float squareRoot (float a) {  
    float r = (1 + a) / 2;  
    while (Math.abs(r*r - a) > 0.01)  
        r = (r + a/r) / 2;  
    return r;  
}
```

If step 2 of the algorithm continued while  $r^2 \neq a$ , the algorithm would be unlikely to terminate, since two approximately-computed numbers are unlikely to be exactly equal.

- 1E** To find the (real) roots of the general quadratic equation  $ax^2 + bx + c = 0$ :

1. Let  $d$  be  $b^2 - 4ac$ .
2. If  $d > 0$ :
  - 2.1. Let  $r$  be the square root of  $d$ .
  - 2.2. Terminate with answers  $(-b + r) / 2a$  and  $(-b - r) / 2a$ .
3. Else, if  $d = 0$ :
  - 3.1. Terminate with answer  $-b / 2a$ .
4. Else, if  $d < 0$ :
  - 4.1. Terminate with no answer.

- 1F** The highest power of 2 that is smaller than 73 is 64, which is  $2^6$ . So  $\log(73)=6$ . Similarly, since the highest power of 2 that is smaller than 12 is 9, which is  $2^3$ . So  $\log(12)=3$ .

Now since  $876 = 17 \times 9$ ,  $\log(876) = \log(17 \times 9) = \log(17) + \log(9) = 6 + 3 = 9$