

The Halting Problem is:

INPUT: A string P and a string I . We will think of P as a program.

OUTPUT: 1, if P halts on I , and 0 if P goes into an infinite loop on I .

Theorem (Turing circa 1940): There is no program to solve the Halting Problem.

Proof: Assume to reach a contradiction that there exists a program $\text{Halt}(P, I)$ that solves the halting problem, $\text{Halt}(P, I)$ returns True if and only if P halts on I . The given this program for the Halting Problem, we could construct the following string/code Z :

```
Program (String x)
```

```
  If Halt(x, x) then
```

```
    Loop Forever
```

```
  Else Halt.
```

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End.
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Consider what happens when the program Z is run with input Z

Case 1: Program Z halts on input Z . Hence, by the correctness of the Halt program, Halt returns true on input Z, Z . Hence, program Z loops forever on input Z . Contradiction.

Case 2: Program Z loops forever on input Z . Hence, by the correctness of the Halt program, Halt returns false on input Z, Z . Hence, program Z halts on input Z . Contradiction.

End Proof.

One can now show that there is no program for some new problem problem U by showing that $\text{Halting} \leq U$, i.e. Halting is reducible to U .