4/5

CS 1510
Algorithm Design

Dynamic Programming

Problems 4 and 5

Due Friday September 12, 2014

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36/53

Problem 4

Input: Two strings, $A = a_1 a_2 ... a_m$ and $B = b_1 b_2 ... b_n$

Output: The minimum cost steps to convert A to B according (where the cost of deletion is 3, the cost of insertion is 4, and the cost of replacement is 5.

Algorithm:

//set boundries (ie, base cases from recursion)
$DIFF[m+1][n+1]$
for $0 \le a \le m$ **do**
  $DIFF[a][0] = 3 * a$ //if we get here, the "recersion" is done and we have no option but deletion
end for
for $0 \le b \le n$ **do**
  $DIFF[0][b] = 4 * a$ //if we get here, the "recursion" is done and we have no option but insertion
end for

2

//iterate in place of recursive call
for $1 \le a \le m$ **do**
  for $1 \le b \le n$ **do**
    if A[a] == B[b] **then**
      $DIFF[a][b] = DIFF[a-1][b-1] + 0$ //characters match, do nothing
    else
      $DIFF[a][b] = min(DIFF[a-1][b] + 3, DIFF[a][b-1] + 4, DIFF[a-1][b-1] + 5)$
    end if
  end for
end for

Need to explain algorithm better, e.g., what is Diff?

			a	b	c	a	b	c
		0	1	2	3	4	5	6
	0	0	3	6	9	12	15	18
a	1	4	0	3	6	9	12	15
b	2	8	4	0	3	6	9	12
a	3	12	8	4	5	3	6	9
c	4	16	12	8	4	7	10	6
a	5	20	16	12	8	4	7	10
b	6	24	20	16	12	8	4	7

To reconstruct:
Start at (m, n).
Take min value from $(m-1, n)$, $(m, n-1)$, and $(m-1, n-1)$.

If $m-1$: delete a_m from A
If $n-1$: insert b_n into A at $m+1$
If $(m-1, n-1)$: replace a_m with a_n if not equal

Continue by moving to the index chosen above.

EXAMPLE: move left. Delete

A = abcabc ⇒ A = abcab

diagonal. Insert $b_3 = a$

⇒ A = abacab ✓

Problem 5

Here are the generated tables for the given problem. Scratch-work for the non-trivial calculations are attached. A hand-drawn picture of the sample trace (explained below) for the optimal solution is also included with the scratch-work.

Table 1: Optimal Access Times

b

		1	2	3	4	5
	1	0.5	0.6	0.85	1.4	2.15
	2	-	0.05	0.2	0.55	1.05
a	3	-	-	0.1	0.4	0.9
	4	-	-	-	0.2	0.65
	5	-	-	-	-	0.25

Table 2: Optimal Roots

b

		1	2	3	4	5
	1	K_1	K_1	K_1	K_1	K_1
	2	-	K_2	K_3	K_4	K_4
a	3	-	-	K_3	K_4	K_4
	4	-	-	-	K_4	K_5
	5	-	-	-	-	K_5

Example reconstruction of optimal tree:

1) You want to find the optimal root for all nodes K_1 ... K_5, so check the Roots Table at a=1 and b=5. The optimal root is K_1 from the table. Add K_1. At this point, the left-hand side of node K_1 is null (because this is nothing less-than K_1 in the given problem) and the right-hand side is the optimal subtree for nodes K_2 ... K_5.

2) Continue in this manner. You want the optimal root for K_2 to K_5, so check the Roots Table at a=2 and b=5. The optimal root is K_4. Add K_4. The left-hand side of the node K_4 is the optimal subtree of nodes K_2 ... K_3 and the right-hand side is from K_5 ... K_5.

3) Check the table for (2, 3), get the node K_3. Add K_3 to the left-hand side of K_4. The left-hand side of our new K_3 node is the optimal subtree of K_2 ... K_2. Check the table for (5,5) – obviously it is node K_5. Add K_5 to the right-hand side of K_4. There are no subtrees for node K_5.

4) Add K_2. Verify that your answer is optimal by adding up the access times * depth and checking Table 1 at a=1 and b=5.

From these instructions, we have the optimal subtree.

Please check the attached sheet for a drawn picture representing the above process.

non-trivial Computations for the Optimal Access Time's Table

a=3
b=5

$$\min\left[(\emptyset + .65),\ (.1 + .25),^{\bigstar\,\text{root}=4}\ (.4 + \emptyset)\right] + \sum_{k=a}^{b} w_k$$

$$= .35 + .1 + .2 + .25 = .9$$

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a=2
b=4

$$\min\left[(\emptyset + .4),\ (.05 + .2),\ \text{root}=4\bigstar\,(.2 + \emptyset)\right] + \sum_{k=a}^{b} w_k$$

$$= .2 + .05 + .1 + .2 = .55$$

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a=1
b=3

$$\min\left[(\emptyset + .2),^{\bigstar\,\text{root}=1}\ (.5 + .1),\ (.6 + \emptyset)\right] + \sum_{k=a}^{b} w_k$$

$$= .2 + .5 + .05 + .1 = .85$$

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a=2
b=5

$$\min\left[(\emptyset + .9),\ (.05 + .65),\ (.2 + .25),^{\bigstar\,\text{root}=4}\ (.55 + \emptyset)\right] + \sum_{k=a}^{b} w_k$$

$$= .45 + .05 + .1 + .2 + .25 = 1.05$$

$= 1$
$b = 4$

$$\min\big[(\varnothing + .55)^{*\,\text{root}=1},\ (.5 + .4),\ (.6 + .2),\ (.85 + \varnothing)\big] + \sum_{k=a}^{b} w_k$$

$$= .55 + .5 + .05 + .1 + .2 = 1.4$$

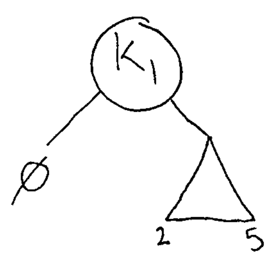
$a = 1$
$b = 5$

$$\min\big[(\varnothing + 1.05)^{*\,\text{root}=1},\ (.5 + .9),\ (.6 + .65),\ (.85 + .25),\ (1.4 + \varnothing)\big] + \sum_{k=a}^{b}$$

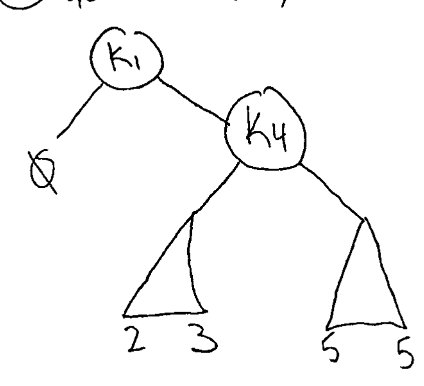
$$= 1.05 + .5 + .05 + .1 + .2 + .25 = 2.15$$

SAMPLE TRACE OF $a=1$ $b=5$

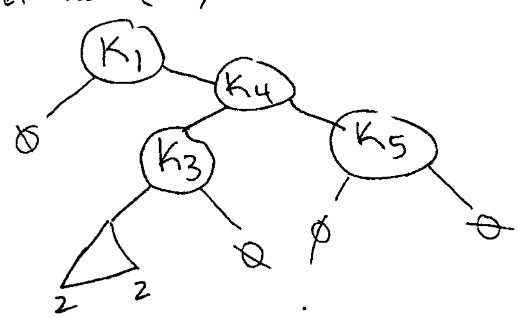
① Get root (1,5) from Root Table



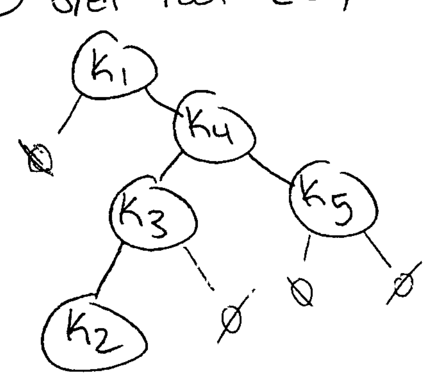
② Get root (2,5)



③ Get root (2,3) and (5,5)



④ Get root (2,2)



$1 \times .5$
$2 \times .2$
$3 \times .1 + 3 \times .25$
$4 \times .05$

2.15 ✓