1 Basic Definitions

A *optimization problem* has the following form: output a best solution S satisfying some property P. Best usually means least cost, a minimization problem, or most benefit, a maximization problem. A best solution is called an *optimal solution*. Note that for many problems there may be many different optimal solutions. A *feasible solution* is a solution that satisfies the property P. Most of the problems that we consider can be viewed as optimization problems.

An algorithm A for a minimization problem P has *performance ratio* c if for every input I, the cost of the output of A on input I has cost at most c time the cost of the least cost feasible solution to I.

An algorithm A for a maximization problem P has *performance ratio* c if for every input I, the benefit of most beneficial feasible solution to I is at most c times the benefit of the output of A on input I.

2 MST doubling algorithm for TSP with triangle inequality

See section 9.5.1

3 Christofides algorithm for TSP with triangle inequality

See section 9.5.1

4 Without triangle inequality, there is no algorithm with constant performance ratio

Theorem For all c, if there is a polynomial time algorithm for TSP with a constant performance ratio c then every NP-complete problem has a polynomial time algorithm.

Proof: We use a many-to-one reduction from the Hamiltonian cycle problem (i.e. the problem of deciding whether a graph contains a simple spanning cycle). Let H a weighted complete graph constructed from an unweighted graph G with n vertices in the folloring manner:

- each edge in G is given weight 1, and
- a edge (x, y) with weight cn is added between each pair of nonadjacent vertices in G.

Note that if G has a Hamiltonian cycle then H has a tour of weight n, and if G does not have a Hamiltonian cycle then every tour in H has weight greater than cn. Hence, if the approximiation algorithm for TSP finds a tour with length at most cn then we may be sure the G has a Hamiltonian cycle, and if the approximiation algorithm for TSP finds a tour with length more than cn then we may be sure the G does not have a Hamiltonian cycle.

5 Analysis of First Fit for Bin Packing

6 Analysis of First Fit Decreasing for Bin Packing

See section 9.5.2