1. Consider the vertex cover problem, that is, given a graph G find a minimal cardinality collection S of vertices with the property that every edge in G is incident to a vertex in S.

   Consider the following algorithm:

   (a) Pick an arbitrary edge $e = (v, w)$ from G
   (b) Add v and w to S
   (c) Remove v and w and all incident edges from G
   (d) Go to step a.

   Show that the performance ratio of this algorithm is at most 2.

   HINT: First consider why the size of any matching (a collection of edges such that no pair are incident on a common vertex) is a lower bound on the size of the vertex cover.

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2. Give an example instance of the vertex cover problem where the above algorithm does not give an optimal solution.

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3. In this problem you have n boxes $b_1, \ldots, b_n$ that you wish to load onto k trucks. You know the integral weight in kilograms w_i of each box b_i. You goal is to minimize the weight on more heavily loaded truck.

   Consider the obvious greedy algorithm that considers the boxes in an arbitrary order, and always places the box under consideration into the least heavily loaded truck. Show that this algorithm guarantees that the weight on the more heavily loaded truck is at most 2 times optimal. That is this algorithm has performance ratio of at most 2.

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4. In this problem you have n boxes $b_1, \ldots, b_n$ that you wish to load onto two trucks. You know the integral weight in kilograms w_i of each box b_i. You goal is to minimize the weight on more heavily loaded truck.

   Consider the obvious greedy algorithm that considers the boxes in an arbitrary order, and always places the box under consideration into the least heavily loaded truck. Show that this algorithms guarantees that the weight on the more heavily loaded truck is at most $\frac{3}{2}$ times optimal. That is this algorithm has performance ratio $\frac{3}{2}$.

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5. Give an example of an instance to the above problem where the load on the more heavily loaded truck is fifty percent larger than the load on the more heavily loaded truck in the optimal solution.

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6. In this problem you have n boxes $b_1, \ldots, b_n$ that you wish to load onto two trucks. You know the integral weight in kilograms w_i of each box b_i. You goal is to minimize the weight on more heavily loaded truck.

   Consider generalizations of the obvious greedy algorithm that considers the boxes in an arbitrary order, and always places the box under consideration into one of the trucks based on some rule that relies only on the load of the two trucks at that time. Show no such algorithm can have performance ratio strictly better than $\frac{3}{2}$.

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7. In this problem you have n chickens with weights $b_1, \ldots, b_n$ grams that you wish to pack into packages. Each package must contain at least L grams of chicken so you don't get sued for false advertising. The goal is to do this by maximizie the number of packages that you fill to L or more grams.

   Consider the obvious greedy algorithm that considers the chickens in an arbitrary order, and adds the chickens to the same package until that package is full. Show that the performance ratio of this algorithm is 2. That is, that this algorithm fills at least 1/2 as many bags as optimal.