2025.09.19.

(1.) $y' = 2y + x^2$

$y(0) = 4$

$y = 4 + \int_0^x 2y(s) + s^2 \, ds$ ← Picard

$y_0(x) = 4$ és $y_{n+1}(x) = 4 + \int_0^x 2y_n(s) + s^2 \, ds$

$y_1(x) = 4 + \int_0^x 2 \cdot 4 + s^2 \, ds = 4 + \left[8s + \frac{s^3}{3} \right]_0^x$

$y_2(x) = 4 + \int_0^x 2y_1(s) + s^2 \, ds = 4 + \int_0^x 2\left(8s + \frac{s^3}{3} + 4\right) + s^2 \, ds =$

$= 4 + \left[16\frac{s^2}{2} + 2\frac{s^4}{4 \cdot 3} + \frac{s^3}{3} + 8s \right]_0^x$

$y_3(x) = 4 + \int_0^x 2\left(4 + 8s + 8s^2 + \frac{s^3}{3} + 2\frac{s^4}{3 \cdot 4}\right) + s^2$

$y_n(x) = 4 + \frac{s^3}{3} + \int_0^x 2 \cdot y_{n-1}(s) \, ds = 4 + \frac{s^3}{3} + 2\int_0^x y_{n-1}(s) \, ds$

$= 4 + \frac{s^3}{3} + 2\int_0^x \left(4 + \int_0^x 2y_{n-1}(s) + s^2 \, ds\right) =$

$= 4 + \frac{s^3}{3} + 8s + \int_0^x\int_0^x 4 + 2y_{n-1}(s) + \int_0^x\int_0^x s^2 \, ds = 4 + \frac{s^3}{3} + 8s + 2\iiint_0^x \left(\int_0^x 2y_n(s) + s^2 \, ds\right) +$

$+ \frac{x^4}{2 \cdot 3}$

5