

$$y^{(3)} = 2y'' + 2 \xrightarrow{x=0} y^{(3)}(0) = 34$$

$$y^{(4)} = 2y^{(3)} \xrightarrow{x=0} y^{(4)}(0) = 2 \cdot 34 = 2^2 \cdot 17$$

$$y^{(n)} = 2y^{(n-1)} \rightarrow y^{(n)}(0) = 2y^{(n-1)}(0) = \dots = 2^{n-3}y'''(0) = 2^{n-2} \cdot 17$$

Taylor-sor képlete

$$y(x) = \sum_{k=0}^{\infty} \frac{y^{(k)}(0)}{k!} (x-x_0)^k = y(0) + y'(0)x + \frac{y''(0)}{2}x^2 + \sum_{k=3}^{\infty} \frac{y^{(k)}(0)}{k!} x^k = 4 + 8x + 8x^2 + \sum_{k=3}^{\infty} \frac{2^{k-2} \cdot 17}{k!} x^k = \dots =$$

$$= -\frac{1}{2}x^2 - \frac{1}{2}x - \frac{1}{4} + \frac{17}{4}e^{2x}$$

Határozatlan együtthatós módszer

$$y = C_0 + C_1(x-x_0) + C_2(x-x_0)^2 + \dots + C_k(x-x_0)^k + \dots \quad \text{alakin megoldást keresünk}$$

$$\rightarrow y' = C_1 + 2C_2(x-x_0) + \dots + kC_k(x-x_0)^{k-1} + \dots$$

// konvergenciasugaron belül lehet deriválni

$$y(x_0) = C_0, \text{ ha } x_0 = 0, \text{ akkor } C_0 = y(0) = 4 \text{ és a DE}$$

$$C_1 + 2C_2x + \dots + kC_kx^{k-1} + \dots = (C_0 + 2C_1x + \dots + 2C_kx^2 + \dots) + x^2$$

$$C_1 = 2C_0$$

$$2C_2 = 2C_1 = 16$$

$$3C_3 = 2C_2 + 1 = 17$$

$$4C_4 = 2C_3 \rightarrow C_4 = \frac{2 \cdot 17}{3 \cdot 4}$$

$$\vdots$$

$$nC_n = 2C_{n-1} \rightarrow C_n = 2^{n-2} \cdot \frac{17}{n!}$$