Binomiális eloszlás várható értéke

$$k = (t^k)'\Big|_{t=1}$$

$$\Psi \sim Binom(n, p)$$

$$\mathbb{E}(\Psi) = \sum_{k=0}^{n} k\,\mathbb{P}(\Psi(k)) = \sum_{k=1}^{n} k \binom{n}{k} p^k (1-p)^{n-k} =$$

$$= \left(\sum_{k=1}^{n} t^k \binom{n}{k} (1-p)^{n-k}\right)'\Bigg|_{t=1} = \left(\sum_{k=0}^{n} \binom{n}{k} (tp)^k (1-p)^{n-k}\right)'\Bigg|_{t=1} =$$

$$= n(tp + 1 - p)^{n-1} \cdot p\Big|_{t=1} = n \cdot p$$

nice!

Geometriai eloszlás

$$W \sim Geom(p)$$

$$\mathbb{E}(W) = \sum_{k=1}^{\infty} k p q^{k-1} = p \sum_{k=1}^{\infty} k q^{k-1} = p\left(\sum_{k=0}^{\infty} q^k\right)' =$$

$$= p \frac{1}{(1-q)^2} = \frac{1}{p}$$

nice! (but not as much