

Algebra 1 gyakorlat
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1.1. ① asszoc. $b \pmod n$ inverze $n-b \pmod n$, neutr. $0 \pmod n$

② asszoc. de nincs minden inverze pl. $2 \pmod 6$ -nak nincs inverze

③ PL mod 7, ekkor $\{1, 2, 3, 4, 5, 6, 0\}$

$8 \quad 15 \quad 22 \quad 29 \quad 36 \quad 43$
 $50 \quad 57 \quad 64$

④ asszoc? , inverzi és neutr. van
igen

⑤ $\{R \hookrightarrow R, x \mapsto ax+b : 0 \neq a \in R, b \in R\}$

f_1, f_2, f_3

$$f_1(f_2 f_3) = a_1(a_2(a_3x + a_2b_3 + b_2) + b_1) = a_1a_2a_3x + a_1a_2b_3 + a_1b_2 + b_1$$

$$a_2(a_3x + b_3) + b_2 = a_2a_3x + a_2b_3 + b_2$$

$$(f_1 f_2) f_3 = a_1 a_2 (a_3 x + b_3) + a_1 b_2 + b_1 = a_1 a_2 a_3 x + a_1 a_2 b_3 + a_1 b_2 + b_1$$

⑥ R^3 , $a \times (b \times c) = (a \times b) \times c$

nincs egység (neutr.)

7.

8.

1.2. $Q_8 = \{\pm 1, \pm i, \pm j, \pm k\}$

$$\forall q \in Q_8 \quad 1q = q \quad 1 = q$$

$$(-1)q = q(-1) = -q$$

$$ij = k, jk = i, hi = j$$

$$ji = -k, kj = -i, ik = -j$$

$$i^2 = j^2 = k^2 = -1$$

$$\tau(1) = 1$$

$$\tau(i) = 4$$

$$\tau(-i) = 4$$

$$\tau(-1) = 2$$

$$\tau(j) = 4$$

$$\tau(-j) = 4$$

$$\tau(k) = 4$$

$$\tau(-k) = 4$$

$$\tau(-k) = 4$$

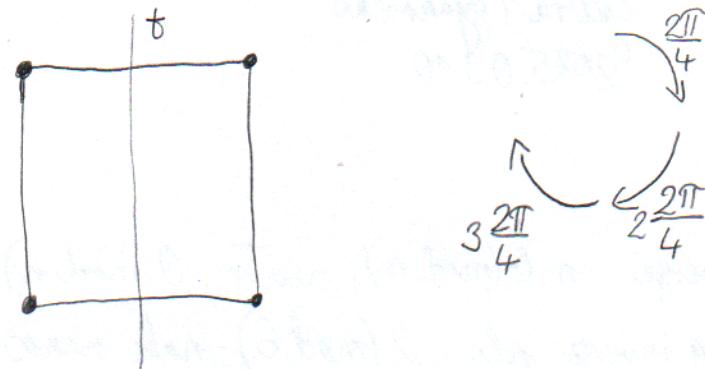
	1	i	j	k
1	1	i	j	k
i	i	-1	k	-j
j	j	-k	-1	i
k	k	j	-i	-1

$$i^2 = (-1)^2$$

$$\text{6.1) } \overline{z} \Leftrightarrow \underbrace{-1 \cdot i \cdot -i \cdots i \cdots i}_{-i} = 1 \quad /1$$

Alg 1 gyak.

1.3. $D_{2,4}$



$$D_{2,4} = \{t, f, f^2, f^3, tf, t f^2, t f^3, id\}$$

rend: $\begin{matrix} 1 & 1 & 1 & 1 \\ 2,4,2,4 & 1 & 1 & 1 \end{matrix}$

$$\begin{matrix} 1 & 2 & 3 & 4 \\ \square & 3 & 1 & 2 \\ 4 & 3 & 4 & 1 \end{matrix} \xrightarrow{t} \begin{matrix} 2 & 1 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{matrix} \xrightarrow{f} \begin{matrix} 3 & 2 & 1 & 4 \\ 4 & 1 & 2 & 3 \end{matrix} \xrightarrow{t} \begin{matrix} 2 & 3 & 1 & 4 \\ 1 & 4 & 3 & 2 \end{matrix} \xrightarrow{f} \begin{matrix} 1 & 2 & 4 & 3 \\ 4 & 3 & 2 & 1 \end{matrix}$$

$$\begin{matrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{matrix} \xrightarrow{t} \begin{matrix} 2 & 1 & 4 & 3 \\ 3 & 4 & 1 & 2 \end{matrix} \xrightarrow{f^2} \begin{matrix} 4 & 3 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{matrix} \xrightarrow{t} \begin{matrix} 3 & 4 & 1 & 2 \\ 2 & 1 & 4 & 3 \end{matrix} \xrightarrow{f^2} \begin{matrix} 1 & 2 & 4 & 3 \\ 4 & 3 & 1 & 2 \end{matrix}$$

$$\begin{matrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{matrix} \xrightarrow{t} \begin{matrix} 2 & 1 & 3 & 4 \\ 1 & 4 & 2 & 3 \end{matrix} \xrightarrow{f^3} \begin{matrix} 1 & 4 & 3 & 2 \\ 2 & 3 & 1 & 4 \end{matrix} \xrightarrow{t} \begin{matrix} 4 & 1 & 2 & 3 \\ 3 & 2 & 4 & 1 \end{matrix} \xrightarrow{f^3} \begin{matrix} 1 & 2 & 4 & 3 \\ 4 & 3 & 2 & 1 \end{matrix}$$

$$D_{2n} = \{id, t, f, \dots, f^{n-1}, tf, \dots, t f^{n-1}\}$$

rend: $\begin{matrix} 1, 2, \dots, n \\ 1, 2, \dots, n \end{matrix}$

f^k esetén ha $k|n$, akkor $r(f^k) = \frac{n}{k}$

ha $k \nmid n$, akkor $r(f^k) = n$

$$tf^k t f^k \quad r(tf^k) = 2$$

megfordítja a forgatás irányát

$$\textcircled{1.4} \quad |G| = 2n$$

$$G = \{g_1, \dots, g_{2n}\}$$

$\underset{\substack{\parallel \\ e}}{e}$

$$r(e) = 1 \quad \text{ha } i \in 2\mathbb{N} \quad g_i = :g$$

$$g \neq e \quad \exists g' \in G \quad gg' = e = gg'$$

$$\text{ha } g' = e \quad gg' = g \rightarrow g = e \quad \not\rightarrow g' \neq e$$

olyan elemet hentesünk, ami nem e és inverze önmaga, ~~az inverz~~
így inverz-párok lesznek, így $2n-1$ nélkül számítás elém, így

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$$\textcircled{2.1} / 1. \quad g^m = e \rightarrow \sigma(g) \mid m$$

$$\text{ha } \sigma(g) \nmid m \rightarrow m = q \cdot \sigma(g) + r \quad r < \sigma(g)$$

$$g^m = g^{q \cdot \sigma(g)} \cdot g^r \rightarrow g^r = e \quad \not\rightarrow (\sigma(g)) \text{ függja miatt}$$

$$\textcircled{2.3} / 1. \quad \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix} = (1243)$$

$$/ 2. \quad \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 6 & 1 & 3 & 5 \end{pmatrix} = (124)(365)$$

$$/ 3. \quad (12345)^{-1}(123)(45)(12345) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 1 & 2 & 3 & 4 \end{pmatrix}^{-1} (123)(45)(12345) =$$

$$= \underbrace{\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \end{pmatrix}}_{(15)(234)} (123)(45)(12345) = (15)(234)$$

$$/ 4. \quad (12)(13) = (123)$$

$$(12)(13)(14) = (1234) \quad \text{nincs teljes indukció}$$

$$\textcircled{2.5} / 1. \quad \sigma((123)(4567)(89)) = \text{elhelytés 1,3,4,2} \quad / 3. \quad \sigma((123)(4567)(567)) =$$

$$/ 2. \quad \sigma((123)(234)) = (13)(24) = 2 \quad = (1246543) = 7$$

	$(\)^2$	$(\)^3$	$(\)^{-1}$
2.4/1.	(12345678)	$(1357)(2468)$	(14725836)
	$(1234)(567)$	$(13)(24)(576)$	(1432) id

$$(1234)^2 = (13)(24) \quad [567]^2 = (576)$$

$$(1234)^3 = (1432) \quad [567]^3 = (\cancel{567}) \text{ id}$$

2.5 1. $((123)(4567)(89)) = 12$

2. $(123)(234) = (13)(24) \quad \sigma(\pi_2) = 2$

3. $\pi_3(123)(34567)(567) = (1246573) \quad \sigma(\pi_3) = 7$

2.7 S_n osztes permutacioja (M0)

2.9. A. Legyen $H \subset \mathbb{Z}$

- * $H \cap \mathbb{Z}^+ = \emptyset \rightarrow H = \{\emptyset\}$

- * $H \cap \mathbb{Z}^+ \neq \emptyset$ legkisebb $n \in H \cap \mathbb{Z}^+$ a legkisebb ilyen $\xrightarrow{H \text{ osztó}} n$ többegysége is benne van

$$\exists k \in H - n\mathbb{Z} \rightarrow k = q \cdot n + r \quad r < n$$

$$k - q \cdot n = r \rightarrow r \in H \quad \begin{matrix} \uparrow \\ \text{emiat} \end{matrix}$$

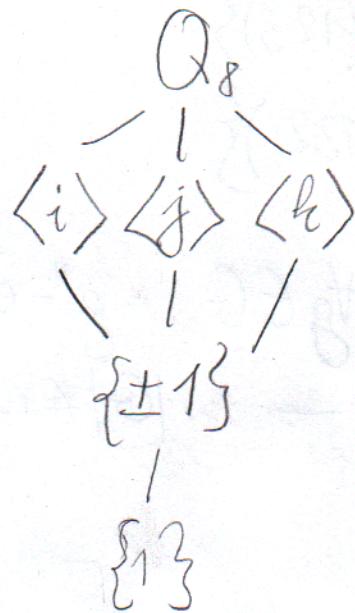
Tehát csak $n\mathbb{Z}$ -h részegyütjái $(\mathbb{Z}, +) \rightarrow \forall k \in n\mathbb{Z}$

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$$|G| = |H| \cdot |H : G|$$

3.1 Lagrange alapján

$$|H| \in \{1, 2, 4, 8\}$$



részben rendezett
halmazrendszerek

3.2 $|G| = \{\mathbb{Z}, +\}$, $H = n\mathbb{Z}$

$$H \text{ rés.} \leftrightarrow n\mathbb{Z}$$

$$\text{Bal melléhoztályok: } 3+H = \{3+x \cdot n : x \in \mathbb{Z}\} =$$

$$= (2n+3) + H = \{2n+3 + h : h \in H\} = n \cdot \mathbb{Z}$$

1/2 $G = Q_8$, $H = \langle -1 \rangle = \{1, -1\}$

$$1H = \{1, -1\}$$

$$iH = \{i, -i\} = -iH$$

$$jH = \{j, -j\}$$

$$kH = \{k, -k\}$$

1/3 $G = S_3$, $H = \{(1), (12)\}$ $|G| = 3! = 6$ $H = 2$

$$|G : H| = \frac{|G|}{|H|} = 3$$

$$(1)H = \{(1), (12)\}$$

$$(13)H = \{(13), (13)(12)\} = \{(13), (132)\}$$

$$(23)H = \{(23), (123)\}$$

/5

$$H(1) = \{(1), (12)\}$$

$$H(3) = \{(13), (123)\}$$

$$H(23) = \{(23), (132)\}$$

$$\textcircled{3.4} \quad n = |G| \in \mathbb{N} \rightarrow \exists g \in G : g^n = e$$

$$\exists g \in G : g^n \neq e \rightarrow |G| \neq n \in \mathbb{N}$$

máshol $|G|=n$ $\text{o}(g)|=|G|$ $\exists h \in \mathbb{N} : h \text{o}(g)=|G|=n$

$$g^n = g^{h \text{o}(g)} = (g^{\text{o}(g)})^h = e^h = e$$

\textcircled{3.5} Ciklikus csoportban $\{\text{osztók}\} \leftrightarrow \{\text{számolók}\}$

\textcircled{3.7} első rész trivi

után
eset $\langle g^a \rangle = \langle g^b \rangle$ $\langle g^a \rangle = \{g^0, g^{-a}, g^{-a}, \dots\}$ $a \neq 0$

$$\langle g^a \rangle = \langle g^b \rangle \leftrightarrow \exists n \in \mathbb{Z}, \exists k \in \mathbb{Z} \quad nb = ka \rightarrow$$

$$\rightarrow k = n \frac{b}{a} \rightarrow$$