

# A Novel Leakage-Detection Method Based on Sensitivity Matrix of Pipe Flow: Case Study of Water Distribution Systems

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**Abstract:** Urban water supply networks are important infrastructure to ensure the daily water consumption of urban residents and factories. However, the scope of leakage detection is large and inaccurate using the traditional leakage-detection method based on a sensitivity matrix of the nodal flow. Therefore, this paper proposes a novel leakage-detection method based on a sensitivity matrix of the pipe flow. The sensitivity matrices regarding the pipe flow to nodal pressure and pipe flow are deduced. Then, the least-square method based on the sensitivity of the pipe flow is used to fit the actual leakage state of the pipeline network. Moreover, the leaking pipeline is determined by using the fitting residuals of each pipeline. Finally, the proposed method is applied to fit the leakage detection of water distribution systems. Compared with the traditional leakage-detection method, the results show that the proposed method is more accurate and effective in locating the leaky pipe and improving the utilization rate of water resources. DOI: 10.1061/(ASCE)WR.1943-5452.0001025. © 2018 American Society of Civil Engineers.

**Author keywords:** Water distribution system; Hydraulic model; Leakage detection; Sensitivity matrix; Least-square method.

## Introduction

Water distribution systems are important infrastructures in industrial society, occupying an important position in economic development and normal life. In China, as the process of urbanization continues to accelerate, urban water distribution systems have rapidly expanded. Serious leakages are caused by the aging and unscientific design of the pipe network. According to Ministry of Construction of the People's Republic of China (2002), the leakage rate of the water distribution system cannot exceed 12% in China. According to statistics for 2009, 26 provinces have a leakage rate of over 12%, of which 13 provinces have a leakage rate of over 20%. The total amount of leakages in China reached 6.2 billion m<sup>3</sup> in 2010, with a leakage rate as high as 12% (Liu et al. 2013). Therefore, this paper presents a novel leakage-detection method based on a sensitivity matrix of the pipe flow to reduce the leakage rate.

This proposed method uses sensitivity matrices regarding the nodal flow to nodal pressure and pipe flow to deduce the sensitivity

matrices regarding the pipe flow to nodal pressure and pipe flow. Moreover, the least-square method based on the pipe-flow sensitivity matrix is used to fit the actual leakage state of the pipeline network. Then, the leakage pipeline is determined by using the fitting residuals of each pipeline. Finally, the proposed method is applied to fit the leakage detection of water distribution systems. Based on the leakage-detection analysis of a simple pipe network and a complex pipe network with the addition of monitoring error, the validity and applicability of the proposed method are verified by comparison with the traditional leakage-detection method based on a sensitivity matrix of the nodal flow. Meanwhile, inspection costs are reduced to improve the utilization rate of water resources.

This paper is organized as follows. Section "Related Works" introduces the current research status of leakage detection in urban water distribution networks. A leakage-detection method based on the sensitivity matrix of the pipe flow is described in Section "Leakage Detection Method Based on the Sensitivity Matrix of the Pipe Flow". Through a comparison with the method based on the sensitivity matrix of the nodal flow, the validity and the practicability of the proposed method are verified in Section "Case Study: The Leakage Detection of the Water Distribution System." Finally, a discussion and conclusion are given in Sections "Discussion" and "Conclusion", respectively.

## Related Works

Finding a leak's location using a supervisory control and data acquisition (SCADA) system to monitor changes in hydraulic pressure and flow has always been an important issue in pipe networks. It helps maintenance personnel find leaks in time, reduce inspection times, and reduce economic losses. Nowadays, studying pipeline leakage locations can be divided into two categories: (1) using abnormal changes of the monitoring data in SCADA system to detect leakages; and (2) with the aid of the constructed hydraulic model of the pipe network, comparing the calculation results of the hydraulic model and monitoring results in order to locate the leakage point.

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In the first type of approach, Poulakis et al. (2003) proposed a Bayesian system approach to estimate the technical route of the pipe network leakage location and leakage loss estimation. Lee et al. (2005) proposed a frequency response diagram for detecting leakages. The frequency response is strengthened with the appearance of a leak. Aksela et al. (2009) proposed a leakage-detection method based on the self-organizing feature map (SOM) to locate a leak according to the monitoring data of flow and leakage location. Mounce et al. (2011) proposed the support vector machine (SVM) method based on flow and pressure data at the district metering area (DMA) inlet to determine whether there is leak in this area. Mamo et al. (2014) studied the multisupport vector machine (M-SVM) to analyze leak detection and the classification of multiple DMAs. Mounce et al. (2003, 2009) proposed a leak-detection method based on the pattern recognition. The water distribution network was divided by the logic rule. Then the leakage area could be detected through statistical principles and artificial neural network.

Romano et al. (2010) proposed a neural network based on the line detection of the water distribution network leakage technology. The prediction data of the neural network were compared with the actual monitoring data, and then Bayesian theory was used to judge whether a leak exists. Ye and Fenner (2010) used adaptive Calman filtering to study the relationship among water pressure, flow changes, and leakage. The result showed that the flow data were more sensitive than pressure data when a leakage occurred. Palau et al. (2011) applied principal component analysis (PCA) of the monitoring data to improve the sensitivity of monitoring of emergencies such as leaks and squibs. Vries et al. (2016) established support vector regression (SVR) by using the adaptive orthogonal projection and clustering model to detect water supply anomalies. The results showed that when data were scarce, the potential of this technology could not be exploited. However, a large amount of leakage monitoring data is needed to establish the data model, so it is actually difficult to collect leakage monitoring data covering all the pipes.

In the second type of research, Zhang (1997) proposed a statistical leak-detection method for gas or liquid pipelines. By means of the statistical analysis, this method used the sequence probability ratio to create a hypothesis test of the two models, and the least-square method could be used to locate the leak. Mpesha et al. (2002) proposed frequency analysis of the frequency response method. This approach changed the model of the pipeline system to leak detection and location analysis in the frequency domain. Ferrante and Brunone (2003) deduced the analytic expression of the pressure head at the downstream end of the pipeline during the transient process and used the wavelet transform to detect the local singular value of a single pipeline midpressure with the leakage. Puust et al. (2008) proposed a random leak-detection method for the pipe network under a MATLAB environment combined with the EPANET hydraulic simulation software's water distribution system model. This methodology based on the shuffled complex evolution metropolis (SCEM-UA) algorithm could achieve an effective and efficient search for unknown leak areas. Pérez et al. (2009) calculated the difference between the water pressure obtained from the hydraulic model and the monitoring value to locate a leak. The water pressure calculated by the hydraulic model was relatively accurate when based on enough water pressure monitoring points. A small number of water pressure monitoring points were set in the study pipe network, and fire hydrants were used to simulate the pipeline leakage. The research showed that only the hydraulic model with high accuracy could simulate the hydraulic state of the pipeline leakage accurately and locate the leakage (Farley et al. 2010).

Vítkovský et al. (2000) applied genetic algorithms to build the hydraulic model of a pipeline system by minimizing the difference between the simulated and measured values and calculating the

water demand (leakage). When using the hydraulic model and SCADA system to locate the pipeline leakage, historical water data should be collected to quantify the fluctuation range of the flow rate of the network node under normal conditions. Otherwise, it could not judge the abnormal monitoring caused by the pipeline leakage or normal fluctuations of nodal flow (Kang and Lansey 2015). Steffebauer et al. (2014) considered the noise influence when analyzing the sensitivity matrix of the network and optimized the arrangement of monitoring points by using the sensitivity matrix analysis. Then, the differential evolution algorithm was used to calculate leakage nodes. Schwaller and Van Zyl (2014) established hydraulic models to research the determinants of the leakage. The research showed that leaks are related to the system average pressure, leakage area, and slope of the pipeline. Although a mechanism model was established to locate the leakage with a small amount of monitoring data, this method can only be located near the water supply node and cannot accurately locate the pipeline; in addition, the scope of leakage detection is very large.

Therefore, a novel method to detect a leak's location based on the sensitivity matrix of the pipe flow is proposed to further reduce the scope of the leakage detection and improve the utilization rate of water resources.

## Leakage-Detection Method Based on the Sensitivity Matrix of the Pipe Flow

### Leakage-Detection Model of Water Distribution Systems

The whole water distribution system (WDS) can be regarded as a nonlinear system, which can be abstracted into a partial flow increase when a leakage loss occurs. Therefore, the weighted least-squares method based on the sensitivity matrix of the pipe flow is used to calculate the variation of the pipe flow and make the state of the hydraulic model fit the actual leakage state of the pipe network. The best fit for the leakage condition is the leaking pipeline. The WDS can be abstracted into the following nonlinear system (Du et al. 2015):

$$g(x) = Y + e \quad (1)$$

where  $x$  is the unknown parameter vector;  $Y$  = actual monitoring value;  $g(x)$  = model calculation value; and  $e$  = error of the monitoring value and calculation value. The leakage location can be transformed into an optimization problem.

First, the objective function can be built based on Eq. (2)

$$\min_x f(x) = [g(x) - Y]^T \mathbf{W} [g(x) - Y] \quad (2)$$

When the objective function takes the minimum value, it shows that the hydraulic model can simulate the current leakage state.

The objective function can be solved in Eq. (3)

$$\Delta x_n = [\mathbf{J}(x_n)^T \mathbf{W} \mathbf{J}(x_n)]^{-1} \mathbf{J}(x_n)^T \mathbf{W} [Y - g(x_n)] \quad (3)$$

$$x_{n+1} = x_n + \Delta x_n \quad (4)$$

where  $\mathbf{J}(x_n)$  is the sensitivity matrix; and  $\mathbf{W}$  = weight coefficient, which is the derivative of the variance matrix of the monitoring error. The key problem of the leakage location is the derivation of the sensitivity matrix  $\mathbf{J}(x_n)$ . Nowadays, there are many ways to solve the sensitivity matrix, such as the sensitivity equation method (Yeh 1986), adjoint method (Liggett and Chen 1996),

finite-difference method (Farley et al. 2010), and analytical method (Liu et al. 2017). This paper uses the analytic method to deduce the sensitivity matrix.

### Sensitivity Matrix Analysis

The sensitivity matrix of the WDS contains the gradient information of the monitoring vector about the parameter vector (Pérez et al. 2011). There are many ways to calculate the sensitivity matrix. In this paper, the matrix analytic method is used to derive the sensitivity matrix (Liu et al. 2017).

The sensitivity matrix of the nodal flow to the nodal pressure is obtained as follows:

$$\frac{\partial H}{\partial Q} = -(\mathbf{A}\mathbf{B}\mathbf{A}^T)^{-1} \quad (5)$$

The sensitivity matrix of the nodal flow to the pipe flow is obtained as follows:

$$\frac{\partial q}{\partial Q} = \mathbf{B}\mathbf{A}^T(\mathbf{A}\mathbf{B}\mathbf{A}^T)^{-1} \quad (6)$$

where  $Q$  = nodal flow;  $q$  = pipe flow;  $H$  = nodal pressure; and  $\mathbf{A}$  is the incidence matrix with a size of  $n \times m$ , which describes the topological relationship of the pipe network, where  $n$  and  $m$  are the number of nodes and number of pipelines, respectively. The elements in the matrix  $\mathbf{A}$  are determined as follows:

$$\mathbf{A}(i, j) = \begin{cases} -1 & \text{if node } i \text{ is the starting point of edge } j \\ 0 & \text{if node } i \text{ is the end of edge } j \\ 1 & \text{if node } i \text{ is the end of edge } j \end{cases} \quad (7)$$

An improved objective function is constructed in Eq. (9)

$$\min_{\Delta Q_l^a, \Delta Q_l^b} f(\mathbf{Q}_l^a + \Delta Q_l^a, \mathbf{Q}_l^b + \Delta Q_l^b) = \begin{bmatrix} \mathbf{H} - \mathbf{H}(\mathbf{Q}_l^a + \Delta Q_l^a, \mathbf{Q}_l^b + \Delta Q_l^b) \\ \mathbf{q} - \mathbf{q}(\mathbf{Q}_l^a + \Delta Q_l^a, \mathbf{Q}_l^b + \Delta Q_l^b) \end{bmatrix}^T \mathbf{W} \begin{bmatrix} \mathbf{H} - \mathbf{H}(\mathbf{Q}_l^a + \Delta Q_l^a, \mathbf{Q}_l^b + \Delta Q_l^b) \\ \mathbf{q} - \mathbf{q}(\mathbf{Q}_l^a + \Delta Q_l^a, \mathbf{Q}_l^b + \Delta Q_l^b) \end{bmatrix} \quad (9)$$

where  $\Delta Q_l^a$  and  $\Delta Q_l^b$  = equivalent leakage amount of connection nodes of the leakage pipe;  $l$  = weight coefficients (take the reciprocal of the monitoring error variance);  $\mathbf{H}_i$  and  $q_j$  = water pressure and flow monitoring value under the condition of leakage, respectively; and  $n$  and  $m$  = number of pressure monitoring points and number of flow monitoring points, respectively. The significance of the objective function is to adjust the flow of the nodes  $a$  and  $b$  connected by pipeline  $l$  so that the hydraulic model can fit the leakage state of the pipe network.

Eq. (9) is expanded by using Taylor's expansion of the first order

$$f(\mathbf{Q}_l^a + \Delta Q_l^a, \mathbf{Q}_l^b + \Delta Q_l^b) \approx \begin{bmatrix} \Delta \mathbf{H}_0 - \mathbf{J}_H(\mathbf{Q}_l^a) \Delta Q_l^a - \mathbf{J}_H(\mathbf{Q}_l^b) \Delta Q_l^b \\ \Delta \mathbf{q}_0 - \mathbf{J}_q(\mathbf{Q}_l^a) \Delta Q_l^a - \mathbf{J}_q(\mathbf{Q}_l^b) \Delta Q_l^b \end{bmatrix}^T \mathbf{W} \begin{bmatrix} \Delta \mathbf{H}_0 - \mathbf{J}_H(\mathbf{Q}_l^a) \Delta Q_l^a - \mathbf{J}_H(\mathbf{Q}_l^b) \Delta Q_l^b \\ \Delta \mathbf{q}_0 - \mathbf{J}_q(\mathbf{Q}_l^a) \Delta Q_l^a - \mathbf{J}_q(\mathbf{Q}_l^b) \Delta Q_l^b \end{bmatrix} \quad (10)$$

where  $\Delta \mathbf{H}_0 = \mathbf{H} - \mathbf{H}(\mathbf{Q}_a, \mathbf{Q}_b)$ ;  $\Delta \mathbf{q}_0 = \mathbf{q} - \mathbf{q}(\mathbf{Q}_a, \mathbf{Q}_b)$ ;  $\mathbf{H}$  and  $\mathbf{q}$  are the monitoring vectors of the water pressure and flow under the condition of the leakage, respectively; and  $\Delta \mathbf{Q}_l$  = leakage of pipeline. It is assumed that the equivalent increment of the upstream and downstream nodes is the same, that is  $\Delta Q_l^a = \Delta Q_l^b = 0.5 \Delta Q_l$

Eq. (10) can be expressed

$$f(\Delta Q_l) \approx \begin{bmatrix} \Delta \mathbf{H}_0 - 0.5(\mathbf{J}_H(\mathbf{Q}_l^a) + \mathbf{J}_H(\mathbf{Q}_l^b)) \Delta Q_l \\ \Delta \mathbf{q}_0 - 0.5(\mathbf{J}_q(\mathbf{Q}_l^a) + \mathbf{J}_q(\mathbf{Q}_l^b)) \Delta Q_l \end{bmatrix}^T \mathbf{W} \begin{bmatrix} \Delta \mathbf{H}_0 - 0.5(\mathbf{J}_H(\mathbf{Q}_l^a) + \mathbf{J}_H(\mathbf{Q}_l^b)) \Delta Q_l \\ \Delta \mathbf{q}_0 - 0.5(\mathbf{J}_q(\mathbf{Q}_l^a) + \mathbf{J}_q(\mathbf{Q}_l^b)) \Delta Q_l \end{bmatrix} \quad (11)$$

The diagonal matrix  $\mathbf{B}$  is described as follows:

$$\mathbf{B} = \begin{bmatrix} \frac{\partial q_1}{\partial h_1} & 0 & \cdots & 0 \\ 0 & \frac{\partial q_2}{\partial h_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{\partial q_m}{\partial h_m} \end{bmatrix} = \begin{bmatrix} \frac{q_1}{1.852h_1} & 0 & \cdots & 0 \\ 0 & \frac{q_2}{1.852h_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{q_m}{1.852h_m} \end{bmatrix} \quad (8)$$

where  $h$  = head loss.

### Leakage Detection

In previous research on leakage detection in a pipe network, a leak is usually regarded as an abnormal increase of a single nodal flow. Therefore, the problem can be converted to checking the nodal flow to make the state of hydraulic model consistent with actual leakage state of the pipeline network. The method based on the sensitivity matrix of the nodal flow can determine a node connecting the leaky pipeline (Andersen and Powell 2000). However, a node connects at least two pipelines, and the scope of leakage detection is still very large. Therefore, the leakage amount is equivalent to the increment of the flow rate of the upper and lower nodes of the leaking pipeline. Moreover, the sensitivity matrix of the pipe flow is deduced to determine the pipeline in which a leakage occurs.

The sensitivity matrix of the pipes flow can be approximated as follows:

$$\begin{bmatrix} J_H(Q_1) \\ J_q(Q_1) \end{bmatrix} = \begin{bmatrix} 0.5(J_H(Q_1^a) + J_H(Q_1^b)) \\ 0.5(J_q(Q_1^a) + J_q(Q_1^b)) \end{bmatrix} \quad (12)$$

It is known from the Section “Leakage Detection Model of Water Distribution Systems” that the leakage of the pipe can be obtained as follows:

$$\Delta Q_1 = \left( \begin{bmatrix} J_H(Q_1) \\ J_q(Q_1) \end{bmatrix}^T W \begin{bmatrix} J_H(Q_1) \\ J_q(Q_1) \end{bmatrix} \right)^{-1} \begin{bmatrix} J_H(Q_1) \\ J_q(Q_1) \end{bmatrix}^T W \begin{bmatrix} \Delta H_0 \\ \Delta q_0 \end{bmatrix} \quad (13)$$

The variation of pressure and flow can be obtained from hydraulic model where the amount of leakage is  $\Delta Q_1$

$$\begin{bmatrix} \Delta H \\ \Delta q \end{bmatrix} \approx \Delta Q_1 \begin{bmatrix} J_H(Q_1) \\ J_q(Q_1) \end{bmatrix} \quad (14)$$

Based on Eqs. (13) and (14), changes in the water pressure and flow values can be obtained

$$\begin{bmatrix} \Delta H \\ \Delta q \end{bmatrix} = \left( \begin{bmatrix} J_H(Q_1) \\ J_q(Q_1) \end{bmatrix}^T W \begin{bmatrix} J_H(Q_1) \\ J_q(Q_1) \end{bmatrix} \right) \begin{bmatrix} J_H(Q_1) \\ J_q(Q_1) \end{bmatrix}^{-1} W \begin{bmatrix} \Delta H_0 \\ \Delta q_0 \end{bmatrix} \begin{bmatrix} J_H(Q_1) \\ J_q(Q_1) \end{bmatrix} \quad (15)$$

The residual of the objective function can be calculated

$$\begin{bmatrix} \Delta H_r \\ \Delta q_r \end{bmatrix} = \begin{bmatrix} \Delta H_0 \\ \Delta q_0 \end{bmatrix} - \left( \begin{bmatrix} J_H(Q_1) \\ J_q(Q_1) \end{bmatrix}^T W \begin{bmatrix} J_H(Q_1) \\ J_q(Q_1) \end{bmatrix} \right) \begin{bmatrix} J_H(Q_1) \\ J_q(Q_1) \end{bmatrix}^{-1} W \begin{bmatrix} \Delta H_0 \\ \Delta q_0 \end{bmatrix} \begin{bmatrix} J_H(Q_1) \\ J_q(Q_1) \end{bmatrix} \quad (16)$$

The smallest residual pipe is a leaky pipe.

MATLAB version 2014a is used to call the EPANET version 2.0 dynamic link library to get the sensitivity matrix of the pipe flow. The details for getting the sensitivity matrix of the pipe flow are as in the following steps:

1. The dynamic link library is used to get the number of the upstream and downstream nodes of each pipe. Then, the connecting direction and flow direction of the pipe based are compared using Eq. (7), where the same direction is positive and the opposite direction is negative. The incidence matrix **A** can be obtained.
2. The hydraulic model is run by using the dynamic link library to get the flow of each pipe and the head loss. Then, the matrix **B** can be calculated based on Eq. (8).
3. The sensitivity matrices regarding the nodal flow to nodal pressure and pipe flow can be calculated based on Eqs. (5)–(6).
4. The sensitivity matrices regarding the pipe flow to nodal pressure and pipe flow can be calculated based on Eq. (12).

A flowchart of the leakage location based on the network traffic sensitivity matrix is shown in Fig. 1.

### Case Study: Leakage Detection in a Water Distribution System

In order to illustrate the proposed method, two cases are given in this section. Compared with the method based on the sensitivity matrix of the nodal flow, a simple pipe network and a complex pipe network with the addition of random noise produced by a Monte Carlo simulation are chosen to verify the validity and engineering applicability of the proposed method.

#### Leakage Detection in a Simple Pipe Network

The simple pipe network consists of six nodes, one pump, one water tower, one high water tank. Water towers and pumps supply water to

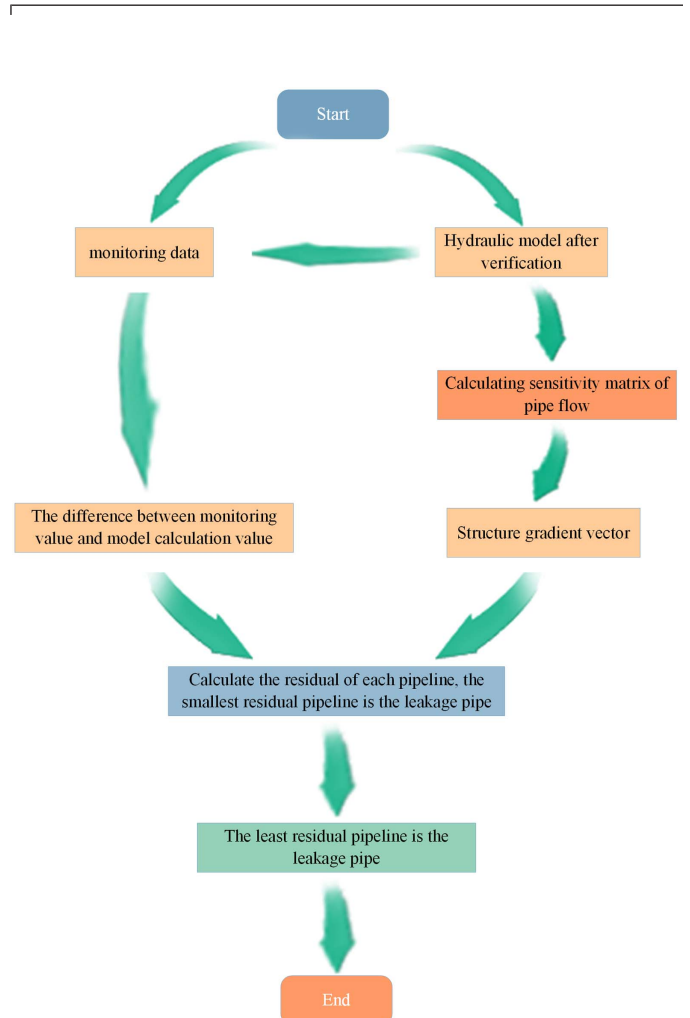


Fig. 1. Process of leakage detection.

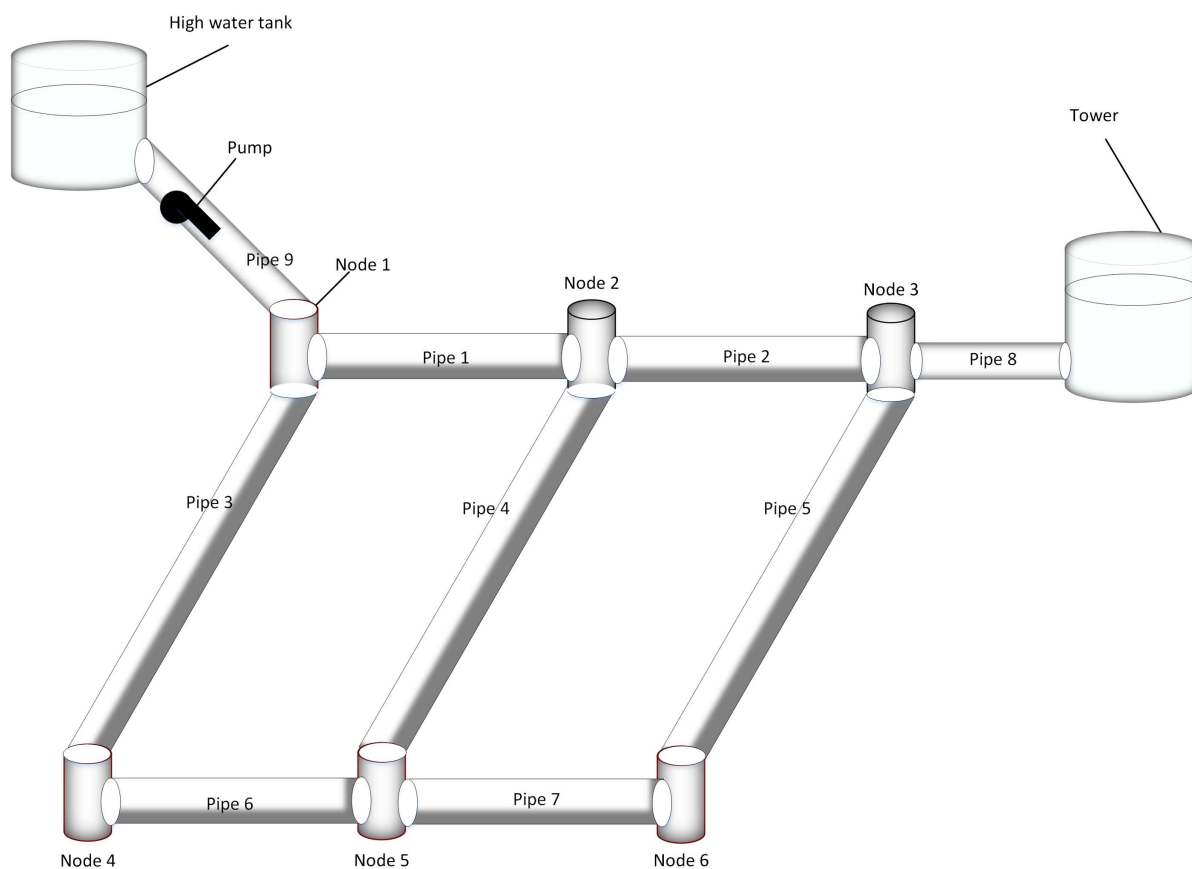


the pipe network. Three monitoring points are arranged in the pipe network, and the Pressure monitoring point 1 is arranged at Node 4. Meanwhile, Pressure monitoring point 2 is arranged at Node 6 and Flow monitoring point is arranged on Pipe 9 as shown in Fig. 2.

The network incidence matrix **A** based on Eq. (7) and diagonal matrix **B** based on Eq. (8) are obtained as demonstrated in Tables 1 and 2, respectively.

The sensitivity matrices regarding the nodal flow to nodal pressure and pipe flow based on Eqs. (5) and (6) are obtained as shown in Figs. 3 and 4, respectively.

Based on Eq. (12), the sensitivity matrices regarding the pipe flow to the nodal pressure of the pressure monitoring point and pipe flow of the flow monitoring point can be obtained as presented in Table 3.



**Fig. 2.** Sample pipe network structure.

**Table 1.** Incidence matrix **A**

Node number	Pipe 1	Pipe 2	Pipe 3	Pipe 4	Pipe 5	Pipe 6	Pipe 7	Pipe 8	Pump
1	-1	0	-1	0	0	0	0	0	1
2	1	-1	0	-1	0	0	0	0	0
3	0	1	0	0	-1	0	0	-1	0
4	0	0	1	0	0	-1	0	0	0
5	0	0	0	1	0	1	-1	0	0
6	0	0	0	0	1	0	1	0	0

**Table 2.** Diagonal matrix **B**

Pipe number	Pipe 1	Pipe 2	Pipe 3	Pipe 4	Pipe 5	Pipe 6	Pipe 7	Pipe 8	Pump
Pipe 1	15.634	0	0	0	0	0	0	0	0
Pipe 2	0	5.6115	0	0	0	0	0	0	0
Pipe 3	0	0	20.922	0	0	0	0	0	0
Pipe 4	0	0	0	7.5426	0	0	0	0	0
Pipe 5	0	0	0	0	6.4862	0	0	0	0
Pipe 6	0	0	0	0	0	18.083	0	0	0
Pipe 7	0	0	0	0	0	0	1.1417	0	0
Pipe 8	0	0	0	0	0	0	0	100.77	0
Pump	0	0	0	0	0	0	0	0	5.6233

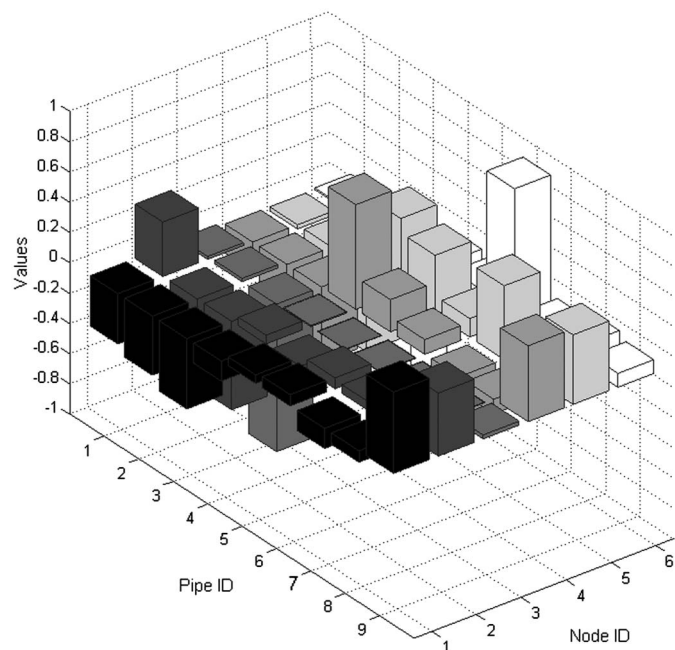


Fig. 3. Sensitivity matrices regarding the nodal flow to nodal pressure.

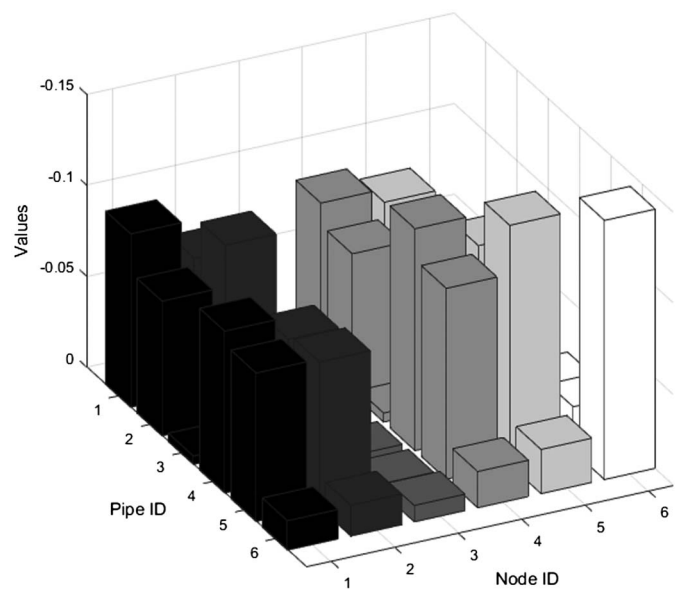


Fig. 4. Sensitivity matrices regarding the nodal flow to pipe flow.

Table 3. Sensitivity matrix of pipe flows

Monitoring point number	Pipe 1	Pipe 2	Pipe 3	Pipe 4	Pipe 5	Pipe 6	Pipe 7
Pressure monitoring point 1	-0.0828	-0.0409	-0.1055	-0.0908	-0.0124	-0.1134	-0.0624
Pressure monitoring point 2	-0.0165	-0.0130	-0.0180	-0.0206	-0.0757	-0.0221	-0.0833
Flow monitoring point 1	0.4760	0.2211	0.5179	0.4377	0.0583	0.4796	0.2749

Table 4. Residuals of each pipe

Monitoring point number	Pipe 1	Pipe 2	Pipe 3	Pipe 4	Pipe 5	Pipe 6	Pipe 7
Pressure monitoring point 1	-0.0055	0.0279	0.0882	0.0999	-0.3025	0.1893	0.1072
Pressure monitoring point 2	0.0145	0.0927	0.0142	0.0541	1.5893	0.0499	0.8179
Flow monitoring point 1	-0.0005	0.0106	0.0184	0.0233	1.9986	0.0471	0.2723
Target function residuals	<b>0.0205</b>	0.1312	0.1208	0.1773	3.8904	0.2862	1.1974

EPANET is used to simulate Pipeline 1's leakage; leakage is  $\Delta Q_1 = 6.8 \text{ L/s}$ . The pressure change at Pressure monitoring point 1 is  $\Delta H_1 = -0.58 \times m$ . The pressure change at Pressure monitoring point 2 is  $\Delta H_2 = -0.1 \text{ m}$ . The flow monitoring point 1 of the flow rate of change is  $\Delta q_1 = 3.3 \text{ L/s}$ .

Therefore, the change vector of monitoring value is constructed as follows:

$$\begin{bmatrix} \Delta H_1^0 \\ \Delta H_2^0 \\ \Delta q_1^0 \end{bmatrix} = \begin{bmatrix} -0.58 \\ -0.1 \\ 3.3 \end{bmatrix}$$

Suppose that leakage occurs in Pipeline 1, and the gradient vector is constructed based on Table 3

$$\begin{bmatrix} J_H(Q_1) \\ J_q(Q_1) \end{bmatrix} = \begin{bmatrix} -0.0828 \\ -0.0165 \\ 0.4760 \end{bmatrix}$$

The leakage of pipeline is calculated based on the weighted least-squares regression

$$\Delta Q_1 = \left( \begin{bmatrix} J_H(Q_1) \\ J_q(Q_1) \end{bmatrix}^T W \begin{bmatrix} J_H(Q_1) \\ J_q(Q_1) \end{bmatrix} \right)^{-1} \begin{bmatrix} J_H(Q_1) \\ J_q(Q_1) \end{bmatrix}^T W \begin{bmatrix} \Delta H_1^0 \\ \Delta H_2^0 \\ \Delta q_1^0 \end{bmatrix}$$

$$= 6.9 \text{ L/s}$$

When the leakage of Pipe 1 is  $\Delta Q_1$ , the variation of monitoring values is calculated based on the hydraulic model

$$\begin{bmatrix} \Delta H_1 \\ \Delta H_2 \\ \Delta q_1 \end{bmatrix} \approx \Delta Q_1 \begin{bmatrix} J_H(Q_1) \\ J_q(Q_1) \end{bmatrix} = \begin{bmatrix} -0.5745 \\ -0.1145 \\ 3.3005 \end{bmatrix}$$

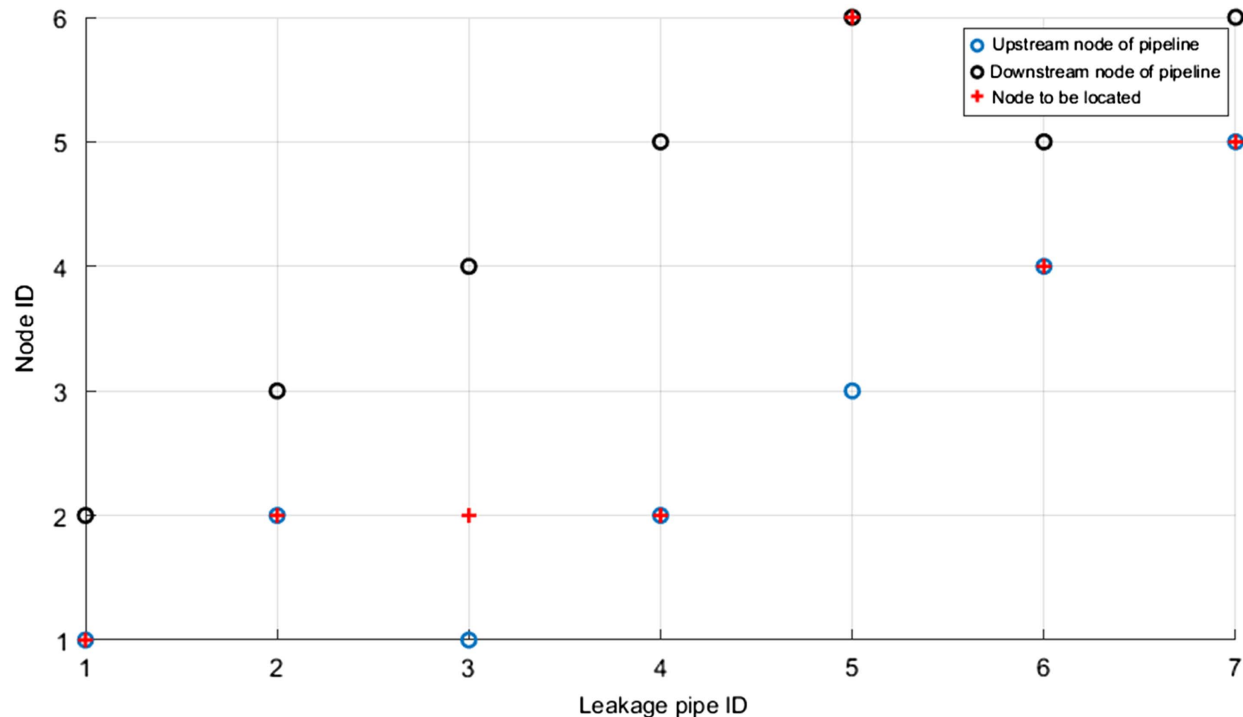
The fitting residuals of the Pipe 1 are calculated

$$\begin{bmatrix} \Delta H_{r1} \\ \Delta H_{r2} \\ \Delta q_{r1} \end{bmatrix} = \begin{bmatrix} \Delta H_1^0 \\ \Delta H_2^0 \\ \Delta q_1^0 \end{bmatrix} - \begin{bmatrix} \Delta H_1 \\ \Delta H_2 \\ \Delta q_1 \end{bmatrix} = \begin{bmatrix} -0.0055 \\ 0.0145 \\ -0.0005 \end{bmatrix}$$

According to the same method, the residual of each pipeline is calculated as indicated in Table 4. It can be seen from Table 4 that when the leakage pipeline from Pipe 1 is the smallest residual which was marked with bold font, Pipeline 1 is determined as the leaking pipeline.

**Table 5.** Leakage-detection results based on the sensitivity matrix of pipe flow

Pipe number	Pipe 1	Pipe 2	Pipe 3	Pipe 4	Pipe 5	Pipe 6	Pipe 7
1	<b>0.0205</b>	0.1312	0.1208	0.1773	3.8904	0.2862	1.1974
2	0.0481	<b>0.0061</b>	0.0601	0.0493	1.7004	0.1012	0.4749
3	0.1035	0.1409	<b>0.0455</b>	0.1066	4.2800	0.2236	1.2141
4	0.1329	0.1162	0.0314	<b>0.0201</b>	3.6132	0.1187	0.9550
5	0.4973	0.4911	0.4844	0.4843	<b>0.0014</b>	0.4925	0.4612
6	0.2659	0.2558	0.1560	0.1335	4.0841	<b>0.0183</b>	1.1086
7	0.5204	0.4419	0.4441	0.4081	2.4425	0.3769	<b>0.3024</b>

**Fig. 5.** Leakage-detection results based on the traditional leakage-detection method.

EPANET was used to simulate a leak from each pipe. According to the aforementioned process of leakage detection, the detection results are given in Table 5. It can be seen from Table 5 that when a leaky pipe is used for leakage fitting, the smallest residual is used to locate the leak in each pipe. The smallest residual was marked with bold font in Table 5. For example, when a leak occurs in Pipe 3, Pipe 3 is used for the leakage fitting and the fitting residual is 0.0455, which is the smallest, so Pipe 3 is identified as a leaky pipe.

The experimental results based on the traditional leakage-detection method are shown in Fig. 5. When a leak occurs in Pipe 3, the leakage is located at Node 2 (leakage should be located at Node 1 or Node 4). Therefore, the result based on the traditional leakage-detection method is wrong. The comparison result shows that the proposed method has a great advantage in locating a pipeline leakage.

### Leakage Detection in a Complex Pipe Network with Monitoring Error

In this section, a complex network is simulated. There are two pumping stations and one water tower supplying water to the WDS.

In the WDS, a total of nine pressure monitoring points and two flow monitoring points are arranged. The locations are shown in Fig. 6.

EPANET was used to simulate each pipeline leaks, and the simulated pipeline leakage was 20 L/s, which can be detected for most pipes. The variation of pressure monitoring nodes is different for leaks from different pipes, as shown in Fig. 7.

In order to simulate the actual work of the pipe network, a random error was generated by Monte Carlo simulation (Cong and Oosterlee 2016) to evaluate the influence of the monitoring error on the leakage location. The variance in nodal water pressure monitoring error is  $\sigma_H = 0.1$  m. EPANET was used to simulate Pipe 1's leakage (leakage = 20 L/s).

The proposed method was used to perform 50 leak localization experiments. The statistical results are shown in Fig. 8. In the 50 leak localization experiments, Pipe 1 was detected 33 times, Pipe 15 was detected 16 times, and Pipe 22 was detected one time. When the result of detection was Pipe 1, the calculated average leakage was 19.5153 L/s, which is very close to the true value (20 L/s).

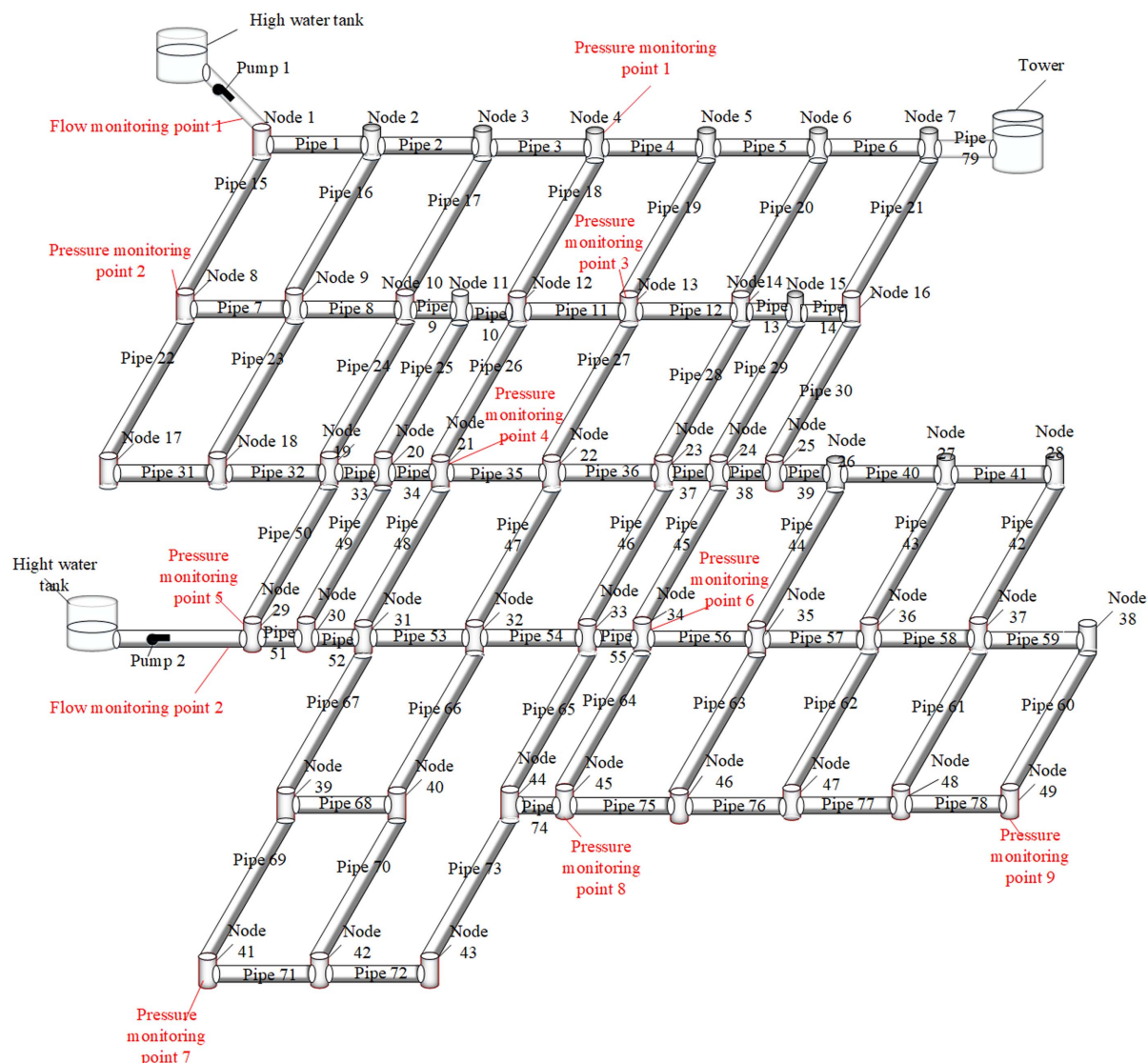


Fig. 6. Complex pipe network structure.

In the 50 experiments, due to the influence of monitoring errors, 33 experimental results were accurate, and the other inaccurate results were also in the vicinity of the real leaky pipe. The pipes that need to be further examined are highlighted in gray in Fig. 9.

The proposed method was used to simulate the leakage of each pipeline. The statistical results of the partial leakage detection are given in Table 6, along with the pipe number and occurrence times. It can be seen from Table 6 that the detection is accurate. The inaccurate location of the pipeline is also in the vicinity of the real leaky pipe.

The traditional leakage-detection method was applied to this complex pipe network, and it can locate the node upstream or downstream of the leaky pipeline. In the case of the accurate localization, a node connects at least two pipes, and the scope of leakage detection has been great. If the location is wrong, the scope of the leakage detection will increase exponentially.

When a leak occurs in Pipe 1, the leakage location results as detected by the traditional method are shown in Fig. 10. The light gray nodes are the detected nodes. The leakage may occur in the dark gray pipe and needs further investigating to confirm the leak-age point. According to the comparison of Figs. 9 and 10, it can be

seen that the proposed method has a smaller leakage investigation scope.

The leakage simulation of different pipelines was carried out, and then the leakage-detection experiment was carried out. The experimental results were statistically analyzed, and the parts of results are provided in Table 7.

It can be seen from Table 7 that the leakage locations have similar properties, which can roughly locate the scope of the leakage, but compared with the proposed method, the scope of investigation is relatively large. Therefore, the advantages of the proposed method can be illustrated, and the leakage investigation range is reduced. Meanwhile, the smaller leak-detection scope can reduce the time needed to find the leaky point and reduce the economic losses. Furthermore, the leakage pipe network can be quickly repaired to ensure the stability of the water supply and improve the utilization rate of water resources.

## Discussion

First, the leakage location-detection method based on the sensitivity matrix of the pipe flow has been proposed to detect leaks in a



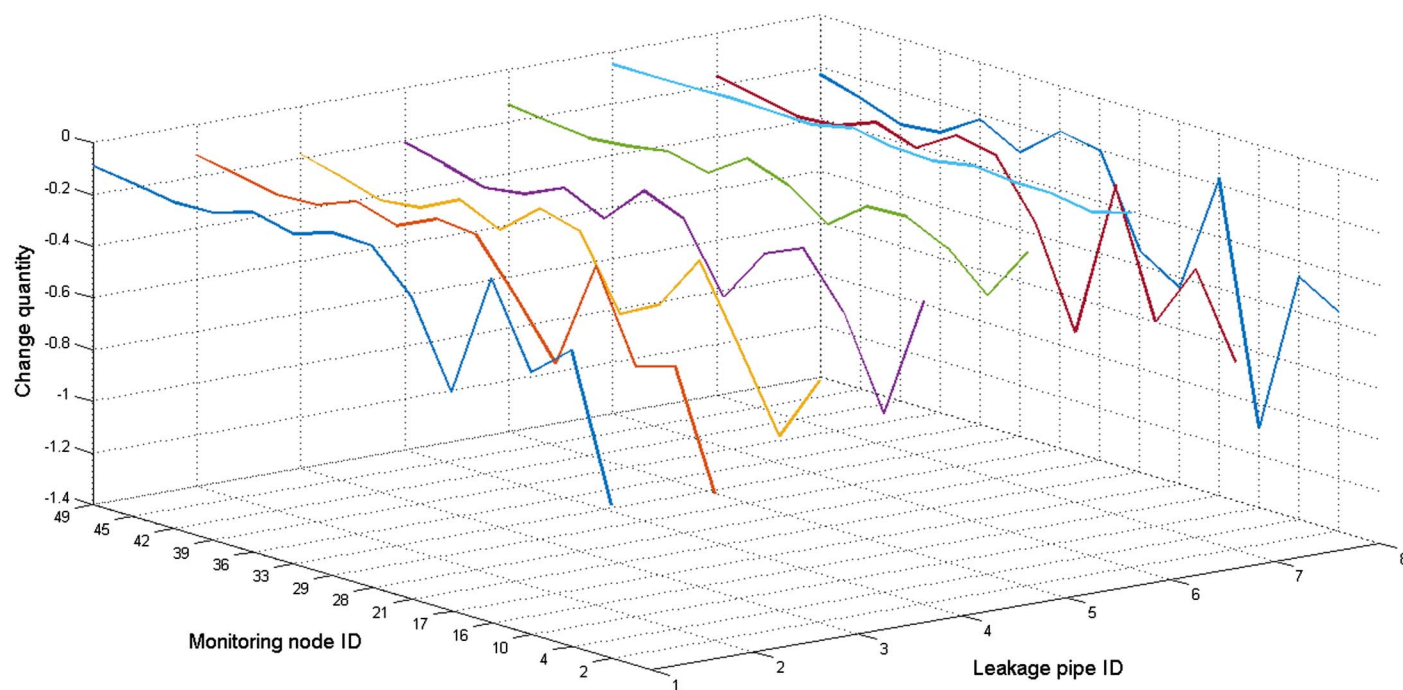


Fig. 7. Changes in pressure monitoring.

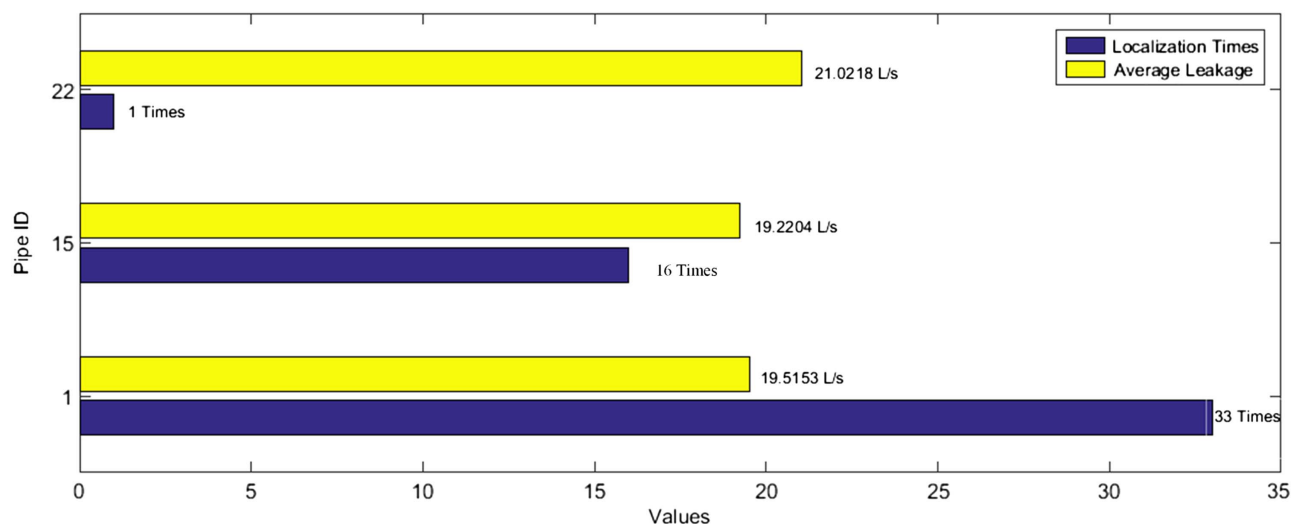


Fig. 8. Leakage location statistics of Pipe 1 leakage.

pipe network. The sensitivity matrices regarding the pipe flow to nodal pressure and pipe flow have been deduced by the sensitivity matrices regarding the nodal flow to nodal pressure and pipe flow. Then, the actual leakage state of the pipe was fitted by the least-square method based on the sensitivity matrices of the pipe flow. Moreover, the leakage pipeline was determined using fitting residuals of each pipeline.

Second, compared with the method based on the sensitivity matrix of nodal flow, the superiority of the proposed method has been illustrated by a simple pipe network. Moreover, in order to illustrate its feasibility in a practical engineering application, Monte Carlo simulation was used to add random error to the monitoring data in a complex pipe network model. The experimental results showed that the proposed method is accurate in the

presence of monitoring errors, and the area of a leakage can be roughly located.

Third, this proposed method further reduces the scope of leakage detection. However, the distribution of monitoring points in the pipe network will affect the detection outcomes. Therefore, according to the sensitivity matrix, cluster analysis will be used to obtain the optimal distribution of monitoring points.

## Conclusion

This paper proposed a leakage location identification method based on the sensitivity matrix of the pipe flow. This proposed method can locate the specific leaky pipeline and further reduce the

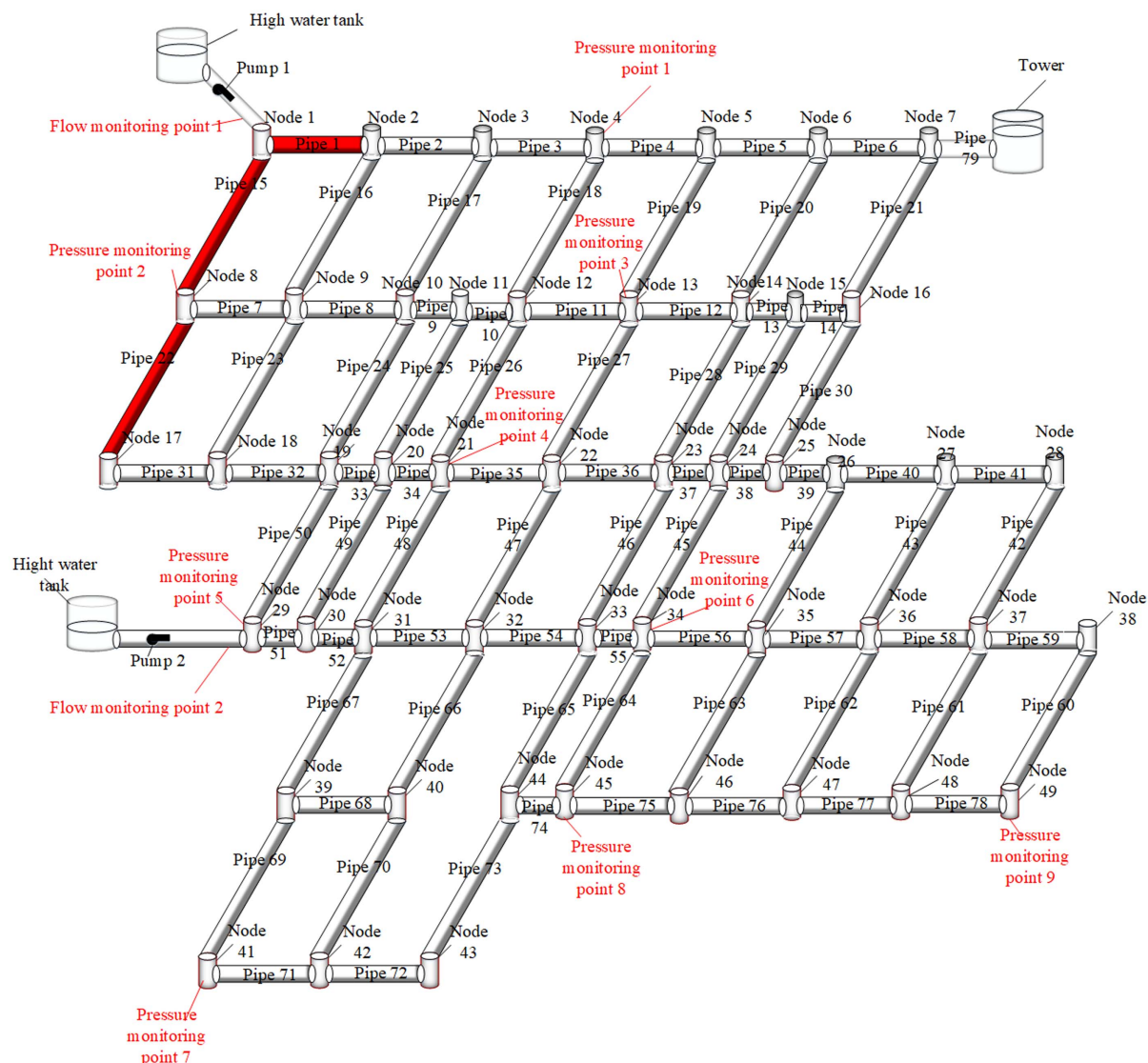


Fig. 9. Detected range based on the proposed method.

**Table 6.** Location statistics of pipe parts based on proposed method

Leaking pipe number	Detected pipeline number	Number of times located
9	9	36
	24	2
	25	12
10	10	36
	18	7
	25	4
11	26	3
	11	24
	19	22
12	26	1
	34	3
	12	30
13	20	12
	27	8
	13	26
	20	4
	28	20

leakage scope of the investigation. Meanwhile, two cases of a water distribution system were given to illustrate the proposed method. Compared with the method based on sensitivity matrix of the nodal flow, the proposed method was more accurate in the absence of monitoring errors. Moreover, the random error based on Monte Carlo simulation was added to the pressure monitoring data in a complex pipe network. The detection results of the proposed method have a smaller scope of leakage investigation when the monitoring error is added. The superiority and applicability of this proposed method has thus been fully demonstrated. Furthermore, the smaller leak-detection scope can reduce the time required to find the leaky point and reduce the economic loss. Meanwhile, the leaking pipe network can be quickly repaired to ensure the stability of the water supply and improve the utilization rate of water resources.

In a future work, the authors will use sensitivity analysis and cluster analysis to optimize the distribution of monitoring points so that a monitoring point can fully respond to the impact of a leakage on the pipe network. Similarly, a different leakage rate should be chosen and more experiments could be done to determine the leakage rate for each pipe. Meanwhile, an actual WDS will be used

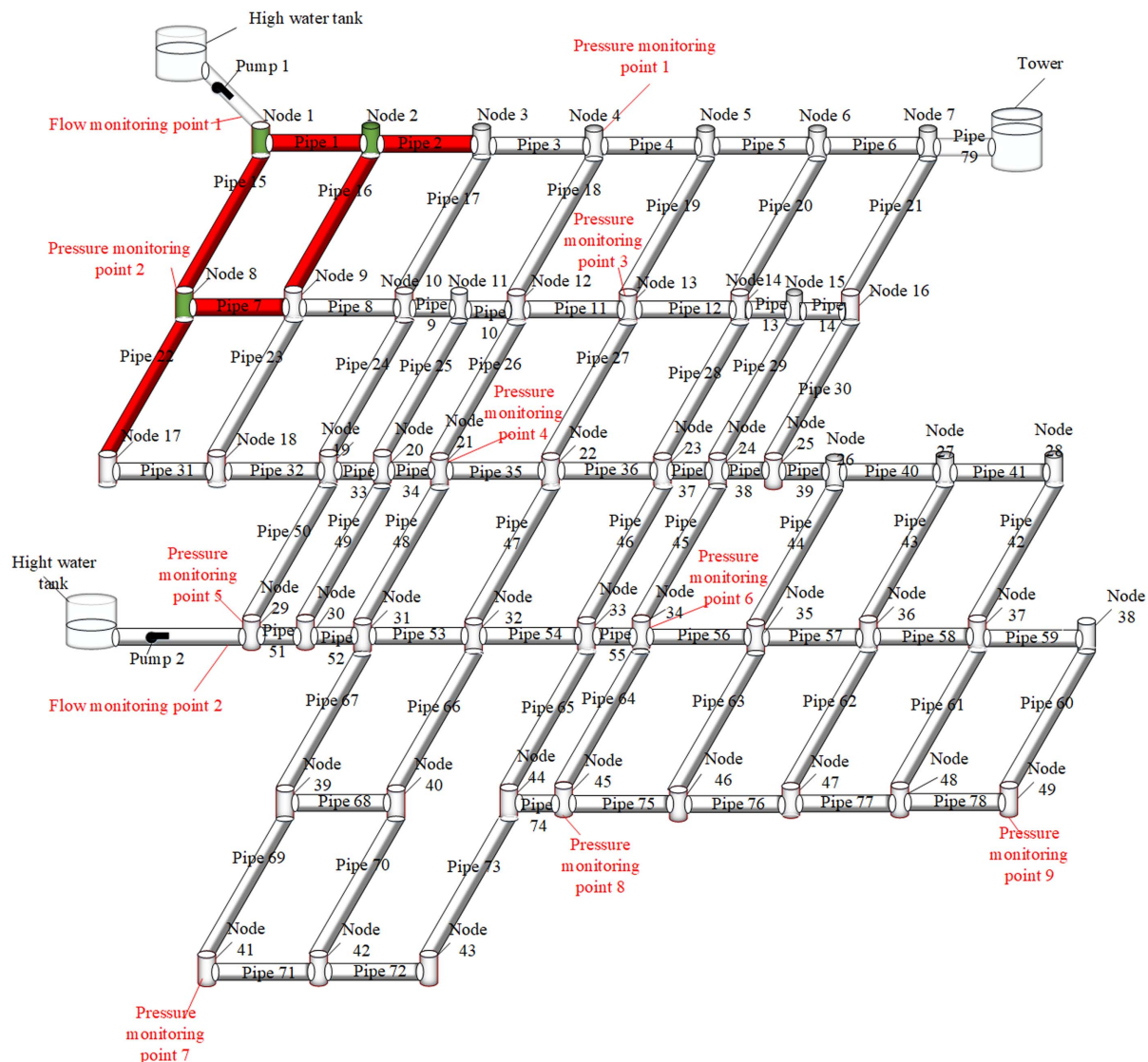


Fig. 10. Detected range based on the traditional method.

**Table 7.** Location statistics of the pipe parts based on the traditional leakage-detection method

Leaking pipe number	Upstream and downstream node number of leaking pipeline	Detected pipeline number	Number of times located
9	10	4	1
	11	11	6
		19	43
10	11	5	7
	12	12	15
		20	28
11	12	6	20
	13	12	5
		13	5
		21	20
12	13	7	8
	14	13	41
		14	1
13	14	14	43
	15	15	2
		23	5

to prove the effectiveness of the proposed method. Moreover, some more optimization methods, such as nonlinear mechanistic models and particle swarm optimization, among others, will be combined to compare with the results from the current work.

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## References

- Aksela, K., M. Aksela, and R. Vahala. 2009. "Leakage detection in a real distribution network using a SOM." *Urban Water J.* 6 (4): 279–289. <https://doi.org/10.1080/15730620802673079>.
- Andersen, J. H., and R. S. Powell. 2000. "Implicit state-estimation technique for water network monitoring." *Urban Water J.* 2 (2): 123–130. [https://doi.org/10.1016/S1462-0758\(00\)00050-9](https://doi.org/10.1016/S1462-0758(00)00050-9).
- Cong, F., and C. W. Oosterlee. 2016. "Multi-period mean-variance portfolio optimization based on Monte-Carlo simulation." *J. Econ. Dyn. Control* 64: 23–38. <https://doi.org/10.1016/j.jedc.2016.01.001>.



- Du, K., T. Y. Long, J. H. Wang, and J. S. Guo. 2015. "Inversion model of water distribution systems for nodal demand calibration." *J. Water Resour. Plann. Manage.* 141 (9): 04015002. [https://doi.org/10.1061/\(ASCE\)WR.1943-5452.0000506](https://doi.org/10.1061/(ASCE)WR.1943-5452.0000506).
- Farley, B., S. R. Mounce, and J. B. Boxall. 2010. "Field testing of an optimal sensor placement methodology for event detection in an urban water distribution network." *Urban Water J.* 7 (6): 345–356. <https://doi.org/10.1080/1573062X.2010.526230>.
- Ferrante, M., and B. Brunone. 2003. "Pipe system diagnosis and leak detection by unsteady-state tests. 2: Wavelet analysis." *Adv. Water Resour.* 26 (1): 107–116. [https://doi.org/10.1016/S0309-1708\(02\)00102-1](https://doi.org/10.1016/S0309-1708(02)00102-1).
- Kang, D., and K. Lansey. 2015. "Novel approach to detecting pipe bursts in water distribution networks." *J. Water Resour. Plann. Manage.* 140 (1): 121–127. [https://doi.org/10.1061/\(ASCE\)WR.1943-5452.0000264](https://doi.org/10.1061/(ASCE)WR.1943-5452.0000264).
- Lee, P. J., J. P. Vítkovský, M. F. Lambert, A. R. Simpson, and J. A. Liggett. 2005. "Leak location using the pattern of the frequency response diagram in pipelines: A numerical study." *J. Sound Vibr.* 284 (3–5): 1051–1073. <https://doi.org/10.1016/j.jsv.2004.07.023>.
- Liggett, J. A., and L. C. Chen. 1996. "Inverse transient analysis in pipe networks." *J. Hydraul. Eng.* 120 (8): 934–955. [https://doi.org/10.1061/\(ASCE\)0733-9429\(1994\)120:8\(934\)](https://doi.org/10.1061/(ASCE)0733-9429(1994)120:8(934)).
- Liu, N. D., K. Du, J. P. Tu, and W. X. Dong. 2017. "Analytical solution of Jacobian matrices of WDS models." *Proc. Eng.* 186: 388–396. <https://doi.org/10.1016/j.proeng.2017.03.236>.
- Liu, T., Y. Liu, J. Li, and X. Dong. 2013. "Leakage situation and control solution of China water supply pipeline." *J. Fudan Univ. (Natural Science)* 52 (06): 807–810.
- Mamo, T. G., I. Juran, and I. Shahrour. 2014. "Virtual DMA municipal water supply pipeline leak detection and classification using advance pattern recognizer multi-class SVM." *J. Pattern Recognit. Res.* 9 (1): 25–42. <https://doi.org/10.13176/11.548>.
- Ministry of Construction of the People's Republic of China. 2002. "Standard for leakage control and assessment of urban water supply distribution system." CJJ 92-2002 J187-2002. Shanghai: China Architecture & Building Press.
- Mounce, S. R., J. B. Boxall, and J. Machell. 2009. "Development and verification of an online artificial intelligence system for detection of bursts and other abnormal flows." *J. Water Resour. Plann. Manage.* 13 (4): 672–686. [https://doi.org/10.1061/\(ASCE\)WR.1943-5452.0000030](https://doi.org/10.1061/(ASCE)WR.1943-5452.0000030).
- Mounce, S. R., A. Khan, A. S. Wood, A. J. Day, P. D. Widdop, and J. Machell. 2003. "Sensor-fusion of hydraulic data for burst detection and location in a treated water distribution system." *Inf. Fusion* 4 (3): 217–229. [https://doi.org/10.1016/S1566-2535\(03\)00034-4](https://doi.org/10.1016/S1566-2535(03)00034-4).
- Mounce, S. R., R. B. Mounce, and J. B. Boxall. 2011. "Novelty detection for time series data analysis in water distribution systems using support vector machines." *J. Hydroinf.* 13 (4): 672–686. <https://doi.org/10.2166/hydro.2010.144>.
- Mpesha, W., M. Hanif Chaudhry, and S. L. Gassman. 2002. "Leak detection in pipes by frequency response method using a step excitation." *J. Hydraul. Res.* 40 (1): 55–62. <https://doi.org/10.1080/00221680209499873>.
- Palau, C. V., F. J. Arregui, and M. Carlos. 2011. "Burst detection in water networks using principal component analysis." *J. Water Resour. Plann. Manage.* 138 (1): 47–54. [https://doi.org/10.1061/\(ASCE\)WR.1943-5452.0000147](https://doi.org/10.1061/(ASCE)WR.1943-5452.0000147).
- Pérez, R., V. Puig, J. Pascual, A. Peralta, E. Landeros, and L. Jordanas. 2009. "Pressure sensor distribution for leak detection in Barcelona water distribution network." *Water Sci. Technol. Water Supply* 9 (6): 715–721. <https://doi.org/10.2166/ws.2009.372>.
- Pérez, R., V. Puig, J. Pascual, J. Quevedo, E. Landeros, and A. Peralta. 2011. "Methodology for leakage isolation using pressure sensitivity analysis in water distribution networks." *Control Eng. Pract.* 19 (10): 1157–1167. <https://doi.org/10.1016/j.conengprac.2011.06.004>.
- Poulakis, Z., D. Valougeorgis, and C. Papadimitriou. 2003. "Leakage detection in water pipe networks using a Bayesian probabilistic framework." *Probab. Eng. Mech.* 18 (4): 315–327. [https://doi.org/10.1016/S0266-8920\(03\)00045-6](https://doi.org/10.1016/S0266-8920(03)00045-6).
- Puust, R., Z. Kapelan, D. Savic, and T. Koppel. 2008. "Probabilistic leak detection in pipe networks using the SCEM-UA algorithm." In *Proc., Water Distribution Systems Analysis Symp.*, 1–12. Reston: ASCE.
- Romano, M., Z. Kapelan, and D. A. Savić. 2010. "Real-time leak detection in water distribution systems." In *Proc., Water Distribution Systems Analysis*, 1074–1082. Reston: ASCE.
- Schwaller, J., and J. E. Van Zyl. 2014. "Modeling the pressure-leakage response of water distribution systems based on individual leak behavior." *J. Hydraul. Eng.* 141 (5): 04014089. [https://doi.org/10.1061/\(ASCE\)HY.1943-7900.0000984](https://doi.org/10.1061/(ASCE)HY.1943-7900.0000984).
- Steffelbauer, D., M. Neumayer, M. Günther, and D. Fuchs-Hanusch. 2014. "Sensor placement and leakage localization considering demand uncertainties." *Proc. Eng.* 89: 1160–1167. <https://doi.org/10.1016/j.proeng.2014.11.242>.
- Vítkovský, J. P., A. R. Simpson, and M. F. Lambert. 2000. "Leak detection and calibration using transients and genetic algorithms." *J. Water Resour. Plann. Manage.* 126 (4): 262–265. [https://doi.org/10.1061/\(ASCE\)0733-9496\(2000\)126:4\(262\)](https://doi.org/10.1061/(ASCE)0733-9496(2000)126:4(262)).
- Vries, D., B. van den Akker, E. Vonk, W. de Jong, and J. van Summeren. 2016. "Application of machine learning techniques to predict anomalies in water supply networks." *Water Sci. Technol. Water Supply* 16 (6): 1528–1535. <https://doi.org/10.2166/ws.2016.062>.
- Ye, G., and R. A. Fenner. 2010. "Kalman filtering of hydraulic measurements for burst detection in water distribution systems." *J. Pipeline Syst. Eng. Pract.* 2 (1): 14–22. [https://doi.org/10.1061/\(ASCE\)PS.1949-1204.0000070](https://doi.org/10.1061/(ASCE)PS.1949-1204.0000070).
- Yeh, W. W. G. 1986. "Review of parameter identification procedures in groundwater hydrology: The inverse problem." *Water Resour. Res.* 22 (2): 95–108. <https://doi.org/10.1029/WR022i002p00095>.
- Zhang, J. 1997. "Designing a cost-effective and reliable pipeline leak-detection system." *Pipes Pipelines Int.* 42 (1): 20–26. <https://doi.org/10.1016/j.jlp.2008.09.012>.