KU Medical Image Analysis

Assignment 2

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1 Introduction

The aim of this assignment sheet was to extract vessel structures from a 3D CT volume by using the binary lung mask from Assignment sheet 1. The lung mask indicates where one wants to enhance the vascular structures in the lung. This is done by using an offset medialness-based tubular structure enhancement algorithm.

2 Methodology

In order to enhance the vascular structures in the lung their geometry property of being cylindrical is considered. Mathematically speaking a *tubularity assumption* is used. This means that a cross-section of a vessel approximates a disk. The main idea is that the intensity varies very little along a vascular structure. Across or normal to a vascular structure the intensity profile is similar to a Gaussian intensity profile. To get the shape of the vessel three orthonormal vectors $\boldsymbol{v}_1$, $\boldsymbol{v}_2$, $\boldsymbol{v}_3$ of the tube model are computed by using the Hessian matrix $\mathcal{H}(\boldsymbol{x})$ for each point in an image.

$$\mathcal{H}(\boldsymbol{x}) = \sigma_{\mathcal{H}}^{2\gamma} \nabla^2 I^{(\sigma_{\mathcal{H}})}(\boldsymbol{x})$$

The eigenvalues and their corresponding eigenvectors of the hessian matrix are then used to get the principal directions of the three orthonormal vectors. With λ_1 being close to 0 (since the intensity should not vary along the tube) and λ_2 and λ_3 being either bigger or smaller than 0.

The plane spanned by the eigenvectors $\boldsymbol{v}_2$ and $\boldsymbol{v}_3$ is used to calculate the offset medialness of the tube at radius r. For this, the pre-computed intensity of the boundariness scale space B is sampled (tri-linear interpolation is used to get a better approximation).

$$\boldsymbol{B}(\boldsymbol{x}) = \sigma_{\boldsymbol{B}}^{\gamma} \nabla I^{(\sigma_{\boldsymbol{B}})}(\boldsymbol{x})$$

$$b(\boldsymbol{x}) = ||\boldsymbol{B}(\boldsymbol{x})||_2$$

$$\boldsymbol{g}(\boldsymbol{x}) = \frac{\boldsymbol{B}(\boldsymbol{x})}{b(\boldsymbol{x})}$$

To sample the boundariness scale space a new vector $\boldsymbol{v}_\alpha$ is constructed as follows

$$\boldsymbol{v}_\alpha = \cos(\alpha)\boldsymbol{v}_2 + \sin(\alpha)\boldsymbol{v}_3$$

This results in a rotating phasor with length 1 around the center point. The space $N = [0, 2\pi]$ is segmented into $\lfloor 2\pi\sigma_{\mathcal{H}} + 1 \rfloor$ equidistant segments, and for each segment a phasor $\boldsymbol{v}_{\alpha_i}$ is calculated. Another measure, the circularity c_i, is needed. The circularity measures the effect of the boundariness scale space in the direction of the phasor.

$$c_i = \max\{\langle -\boldsymbol{g}(\boldsymbol{x} + r\boldsymbol{v}_{\alpha_i}), \boldsymbol{v}_{\alpha_i}\rangle, 0\}$$

With this the intensity is sampled and averaged, which results in the initial medialness

$$R_0^+(\boldsymbol{x}, r) = \frac{1}{N}\sum_{i=0}^{N-1} b(\boldsymbol{x} + r\boldsymbol{v}_{\alpha_i})c_i^2 = \frac{1}{N}\sum_{i=0}^{N-1} b_i = \bar{b}$$

To further improve the vessel extraction the symmetry property is exploited. For this, the variance of the boundariness samples is needed

$$s^2(\boldsymbol{x}, r) = \frac{1}{N-1}\sum_{i=0}^{N-1}(b_i - \bar{b})^2$$

$$\mathcal{S}(\boldsymbol{x}, r) = 1 - \frac{s^2(\boldsymbol{x}, r)}{\bar{b}^2}$$

Symmetric structures will have a small variance. $s^2 \ll \bar{b}$ will be true for circular symmetric structures, therefore $\mathcal{S}$ will be close to 1. Otherwise $\mathcal{S}$ will be small. By multiplying the initial medialness with the symmetric constraint the symmetric constraint medialness is calculated

$$R^+(\boldsymbol{x}, r) = R_0^+(\boldsymbol{x}, r)\mathcal{S}(\boldsymbol{x}, r)$$

Last but not least an adaptive threshold is applied. For this the medialness of the center-line is calculated

$$R^-(\boldsymbol{x}, r) = ||\sigma_{\mathcal{H}}^{\gamma}\nabla I^{(\sigma_{\mathcal{H}})}(\boldsymbol{x})||_2$$

The final medialness is obtained as follows

$$R(\boldsymbol{x}, r) = \max\{R^+(\boldsymbol{x}, r) - R^-(\boldsymbol{x}, r), 0\}$$

The medialness was only calculated for the relevant voxel within the lung mask. $\sigma_{\mathcal{H}}$ is set to the radius for all calculations.

Vessels have different radii and thus the above stated calculations need to be executed for different values for r. In addition to speed up calculations for bigger radii an image pyramid was constructed. The pyramid consists of 3 layers and a sampling factor of 2. Which results in the scales $(256, 256, 256)$, $(128, 128, 128)$, and $(64, 64, 64)$. For each of those scales the above steps were calculated for $r \in \{1, 1.5, 2\}$. The upper levels of the image pyramid were upsampled with linear interpolation.

To get the final multiscale medialness the maximum of each voxel from the resulting images (for each scale and radius) is taken.

3 Results

3.1 Input image

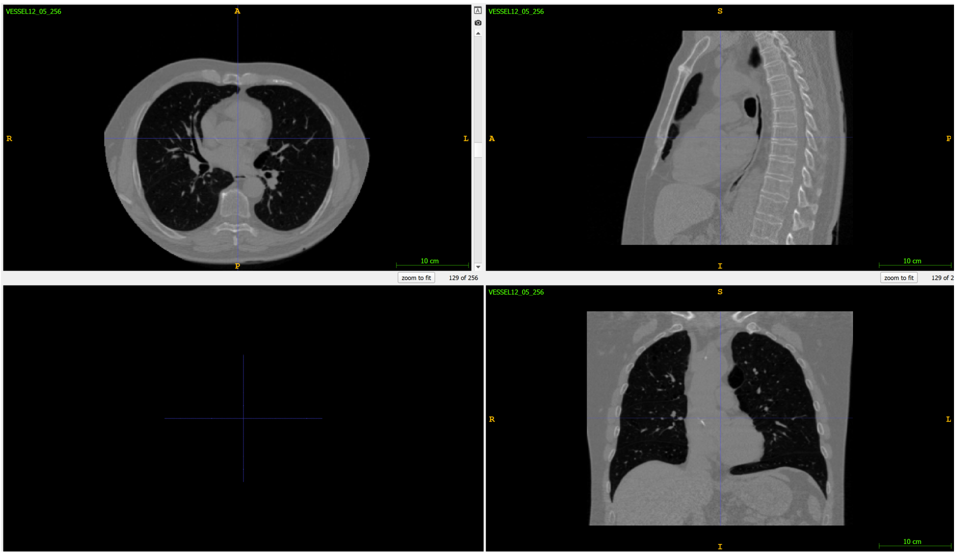

Figure 1: Input image: 3D CT volume of a thorax.

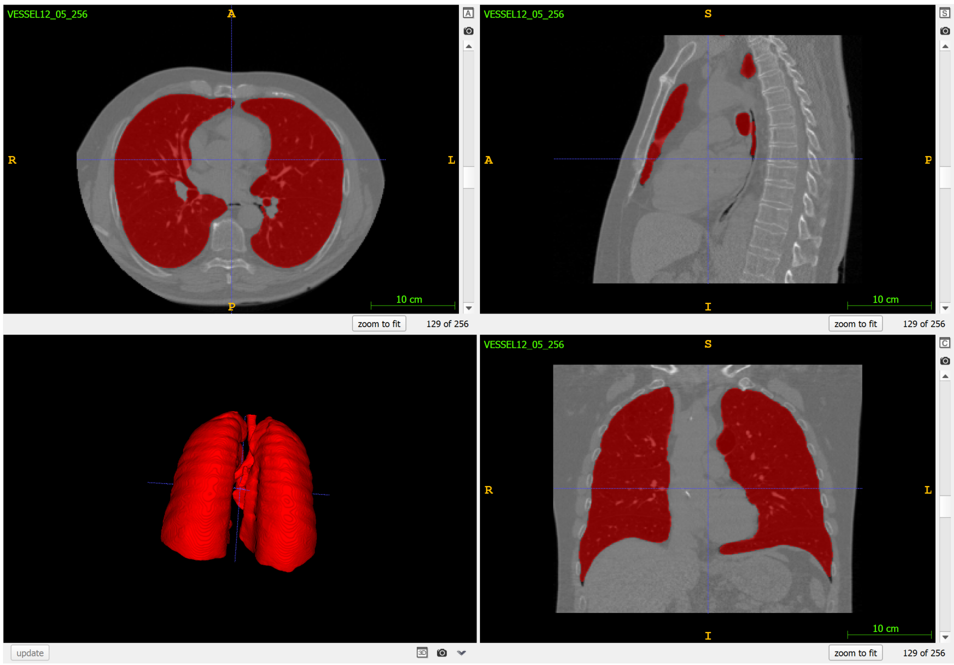

Figure 2: Result of lung segmentation for $\lambda = 5$ with region growing (confidence connected) from Assignment 1.

Figure 3: Original image size, $r = 1$.

Figure 4: Original image size, $r = 1.5$.

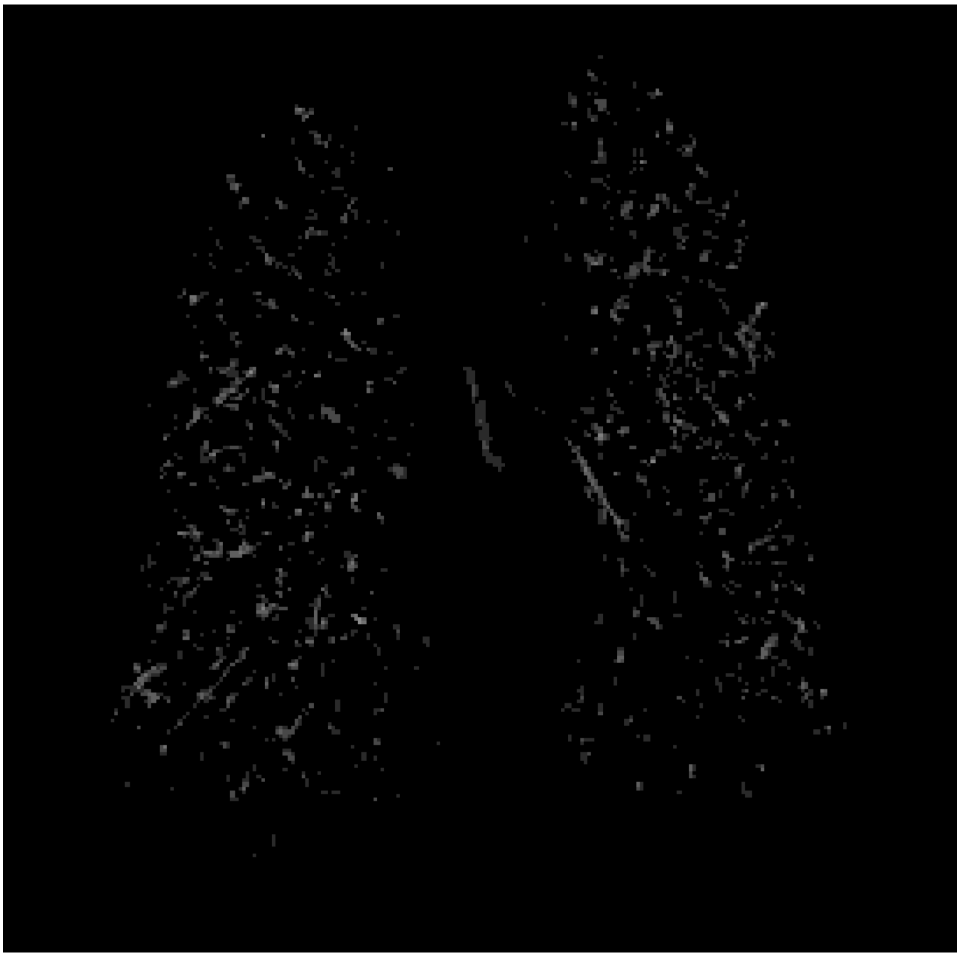

Figure 5: Original image size, $r = 2$.

Figure 6: First stage down-sampled image, $r = 1$

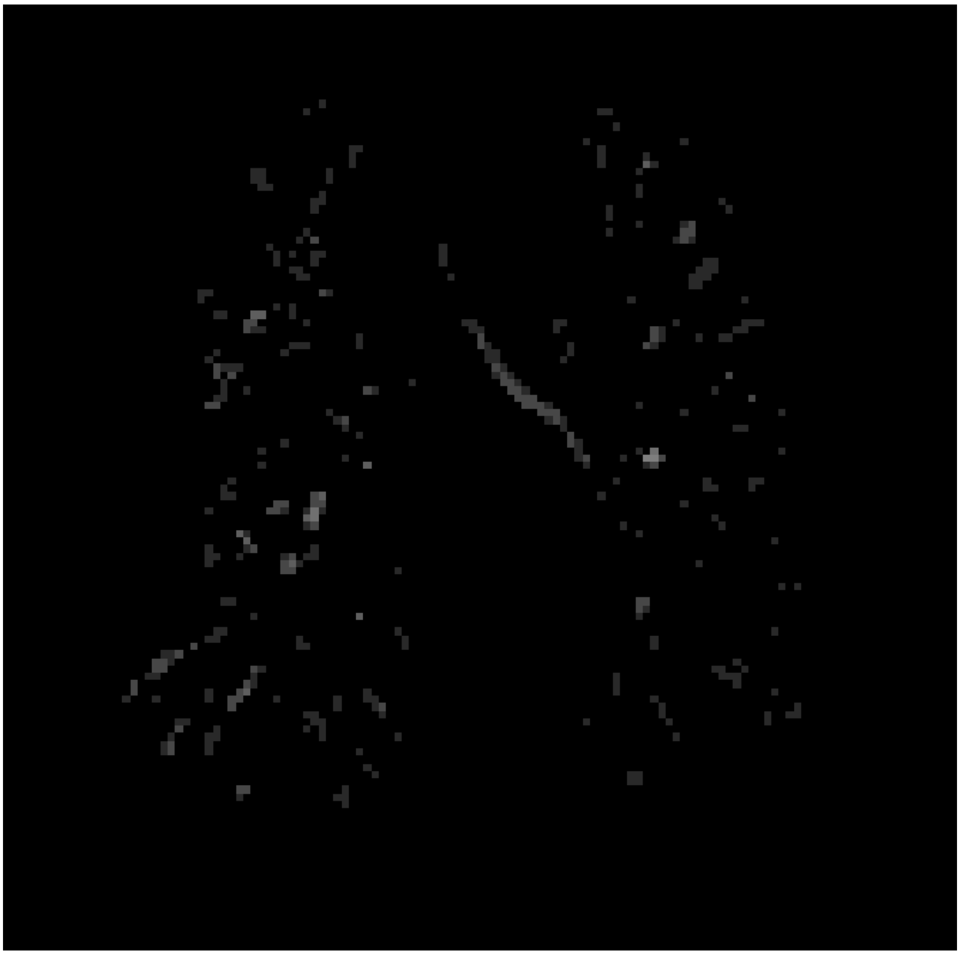

Figure 7: First stage down-sampled image, $r = 1.5$

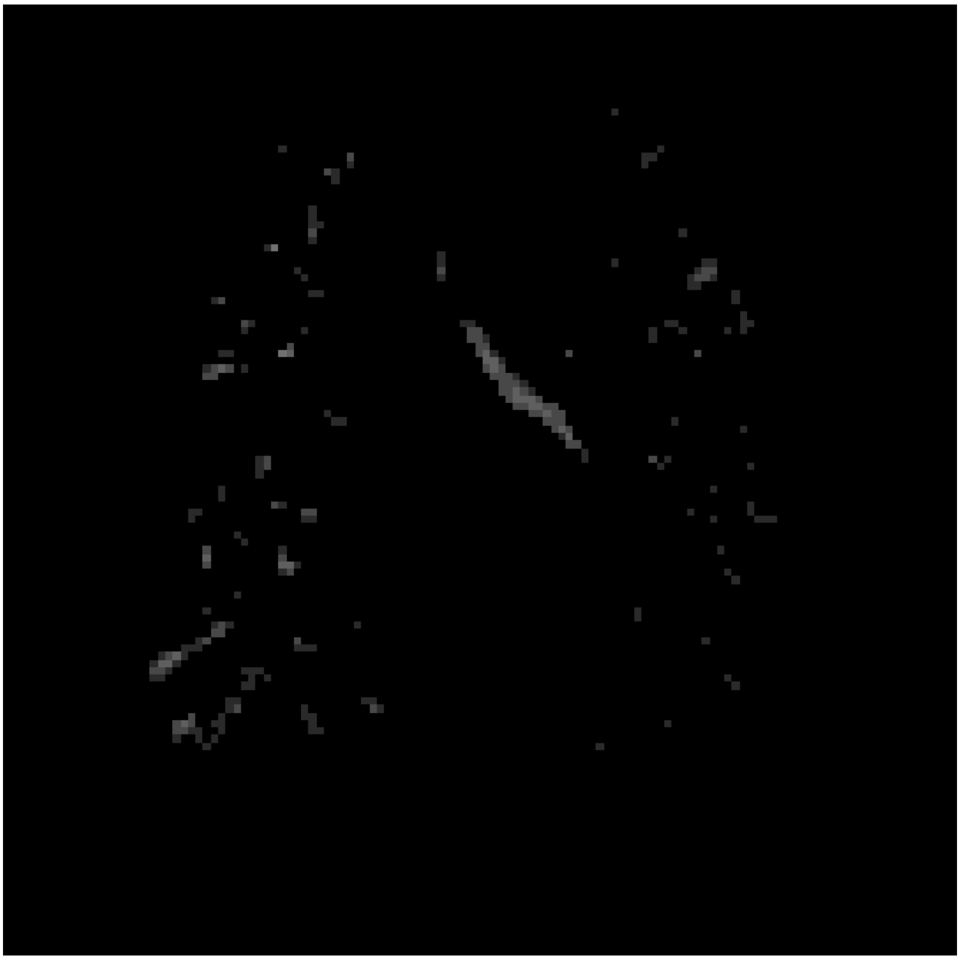

Figure 8: First stage down-sampled image, $r = 2$

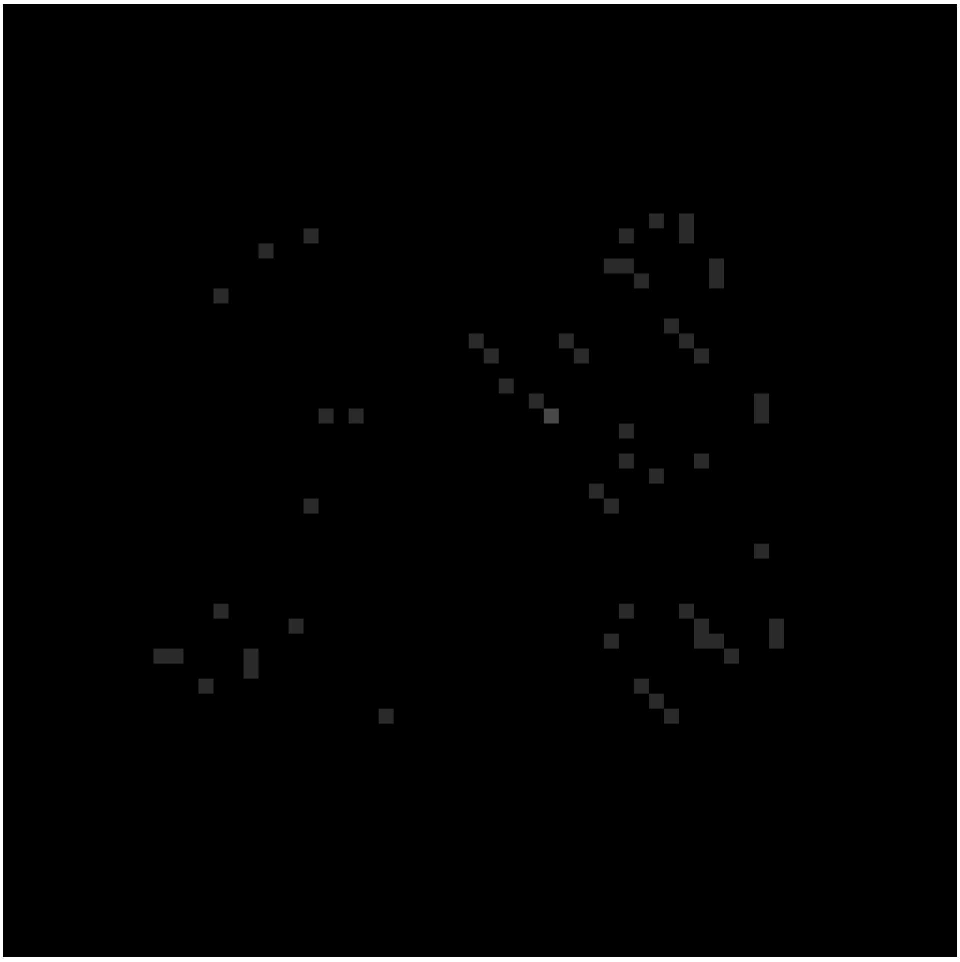

Figure 9: Second stage down-sampled image, $r = 1$

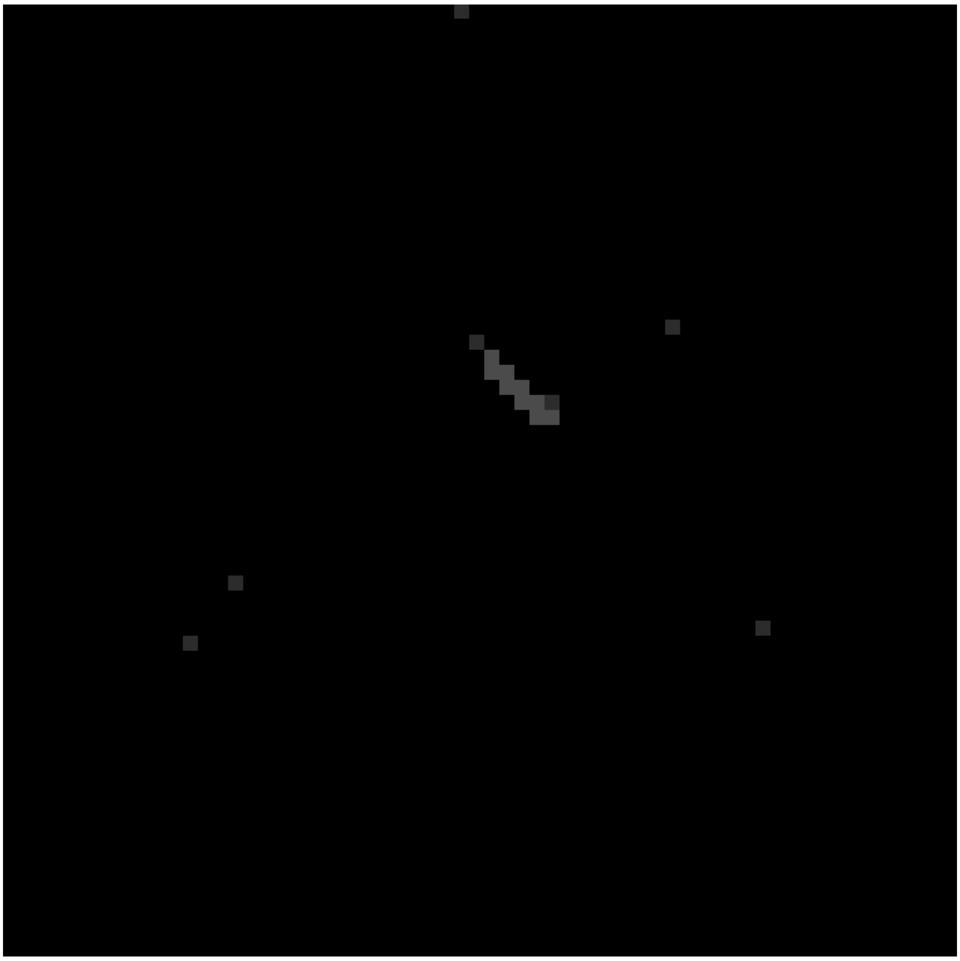

Figure 10: Second stage down-sampled image, $r = 1.5$

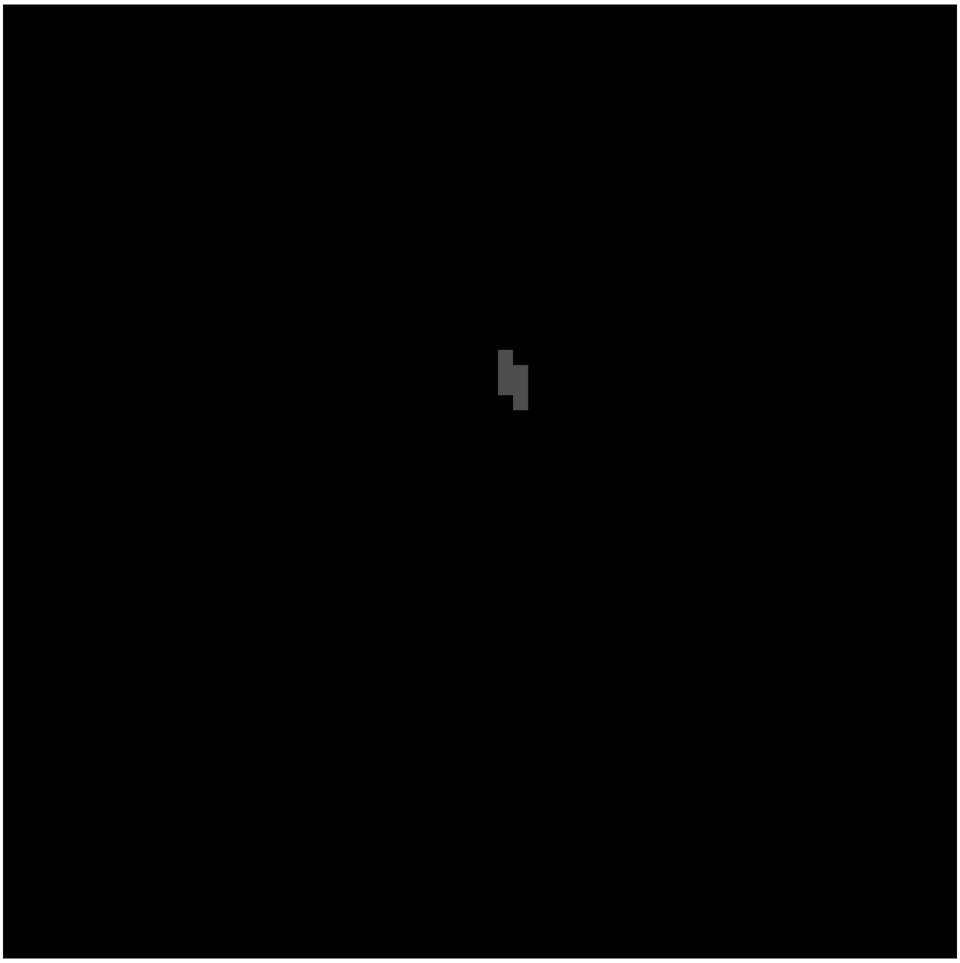

Figure 11: Second stage down-sampled image, $r = 2$

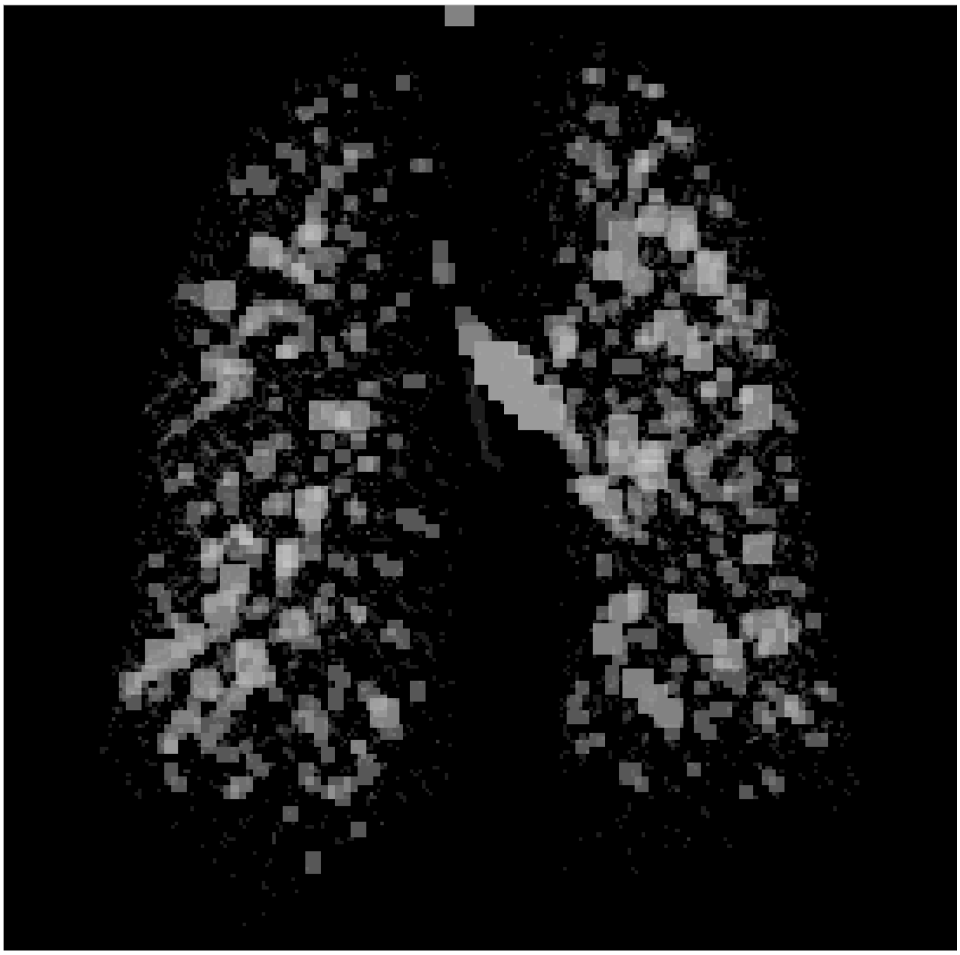

Figure 12: Combined image across all scales and radii.

4 Discussion

The images 3 to 5 show the vessel enhancement in its original image size with different radii. The images 6 to 8 show a down-sampled version of the first images with different radii. The images 9 to 11 show the last layer of the image pyramid with different radii. $\sigma_{\boldsymbol{B}}$ was set to 0.9 for all scales.

The images within a stage of the image processing pyramid show the effect of a different radius assumption quite well. With an increasing radius, the vessels increase in thickness but decrease in number. This is expected behavior because small vessels are more numerous than large ones.

The different stages of the image pyramid have a similar effect as increasing the radius. By decreasing the resolution the radius implicitly increases (e.g. a down-sampled image with sampling factor 2 and radius 1.5 would correspond to the normal image with radius 3) and execution time can be reduced drastically. When down-sampling the number of voxels within the lung mask are reduced, and the amount of sampling/interpolating steps necessary. The sampling and interpolating steps are the most expensive operation in the implementation.

The last stage of the image pyramid is most likely not necessary as nearly no new information can be obtained.

Figure 12 shows the combination across all scales and radii of the whole image processing pipeline. There are interpolation artifacts from up-sampling the higher stages of the image pyramid. When using more sophisticated interpolation approaches (e.g. B-Spline) the artifacts increase in severity. I am not sure if this is a result of a wrongly chosen interpolation approach or an issue with how I visualized the results with ImageJ.

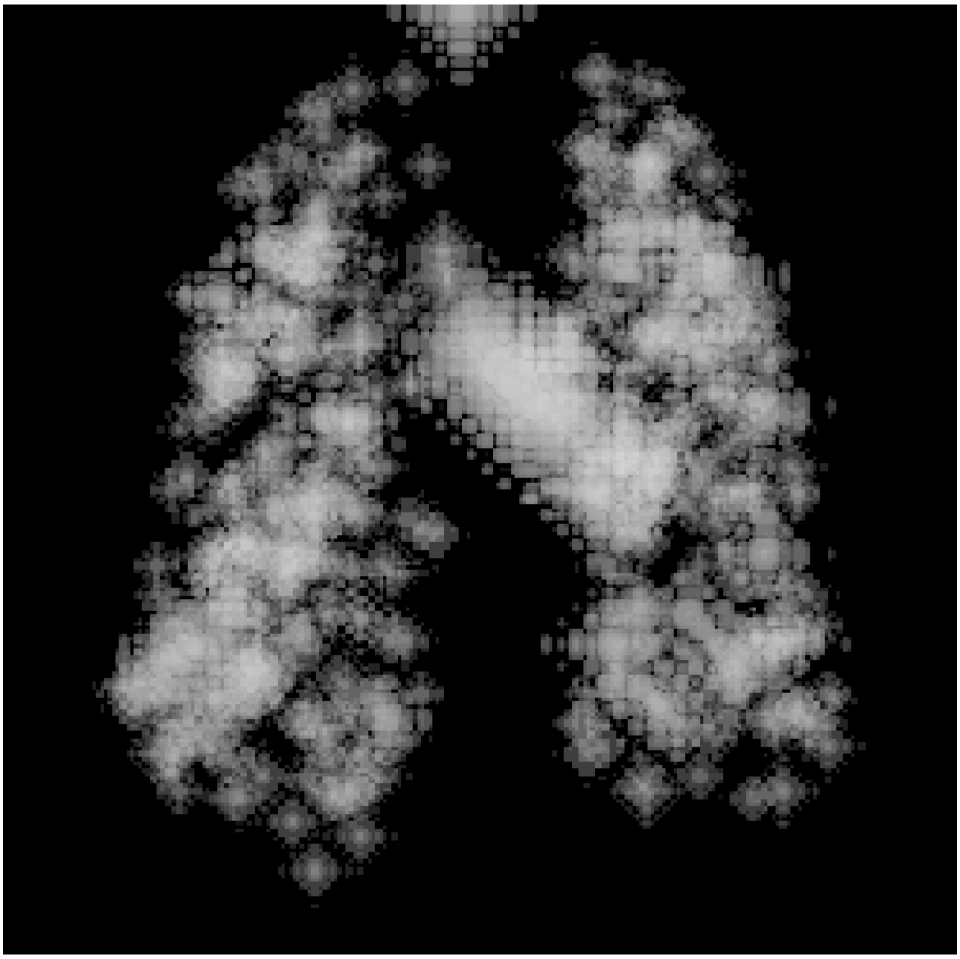

Figure 13: Artifacts after B-Spline interpolation.