计算物理作业 4

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班会旧语新知："物理学系就业面广。"

1　题目 1：牛顿插值法

1.1　题目描述

Newton interpolation of:

(1) 10 equal spacing points of $\cos(x)$ within $[0, \pi]$;

(2) 10 equal spacing points $\frac{1}{1+25x^2}$ within $[-1, 1]$.

Compare the results with the cubic spline interpolation.

1.2　程序描述

1.3　伪代码

1.4　结果示例

2　题目 2：金属棒温度数据拟合

2.1　题目描述

The table below gives the temperature T along a metal rod whose ends are kept at fixed constant temperatures. The temperature is a function of the distance x along the rod.

(1) Compute a least-squares, straight-line fit to these data using $T(x) = a + bx$.

(2) Compute a least-squares, parabolic-line fit to these data using $T(x) = a + bx + cx^2$.

表 1: Temperature data along the metal rod

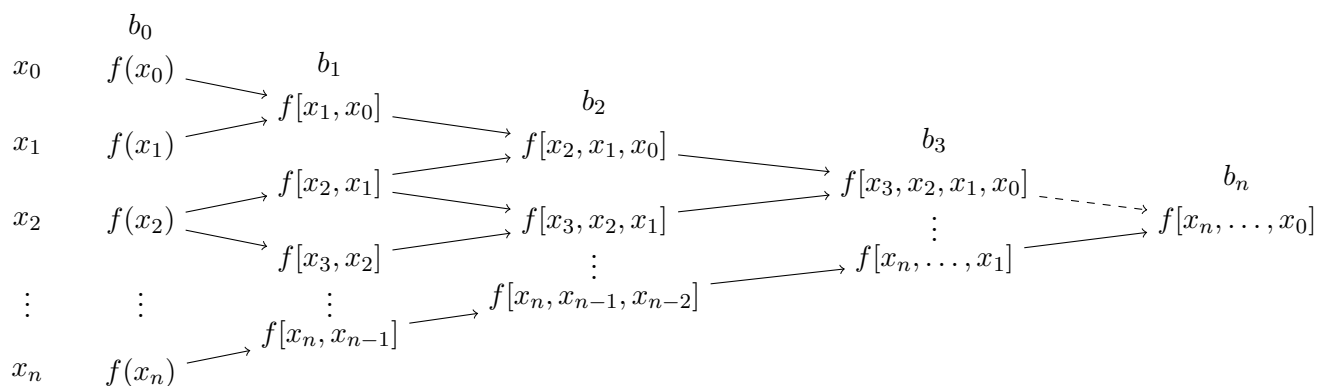| x_i (cm)	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
T_i (℃)	14.6	18.5	36.6	30.8	59.2	60.1	62.2	79.4	99.9

3　补充题：牛顿插值法唯一性探讨

3.1　题目描述

Given $n+1$ points $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$, the n-th order interpolation polynomial using Newton's method is:

$$P_n(x) = f[x_0] + f[x_1, x_0](x - x_0) + f[x_2, x_1, x_0](x - x_0)(x - x_1) + \cdots + f[x_n, x_{n-1}, \ldots, x_0](x - x_0)(x - x_1) \ldots (x - x_{n-1})$$

where $f[x_i, x_{i-1}, \ldots, x_0]$ represents the divided differences. Taking the coefficients from the lower edge of the difference table (i.e., $f[x_n], f[x_n, x_{n-1}], \ldots, f[x_n, \ldots, x_0]$), will this provide higher accuracy for values of x near x_n?

3.2　解答与证明