Linear_Regression_Example1

October 4, 2018

1 Linear Regression Example 1

This notebook is to demonstrate linear regression on a really simple example without the use of any ML libraries

```
In [399]: import numpy as np
          import matplotlib.pyplot as plt
          np.random.seed(12345)
```

2 Get Training Data

We'll create some random sample data from a Normal distribution with `np.random.randn(dim0, dim1)` and add some "gaussian noise" in the form of another random vector from Normal distribution to our response variable `Y = theta0*x0 + theta1*x1 + noise` so that we don't just get a straight line of data, because that's too easy

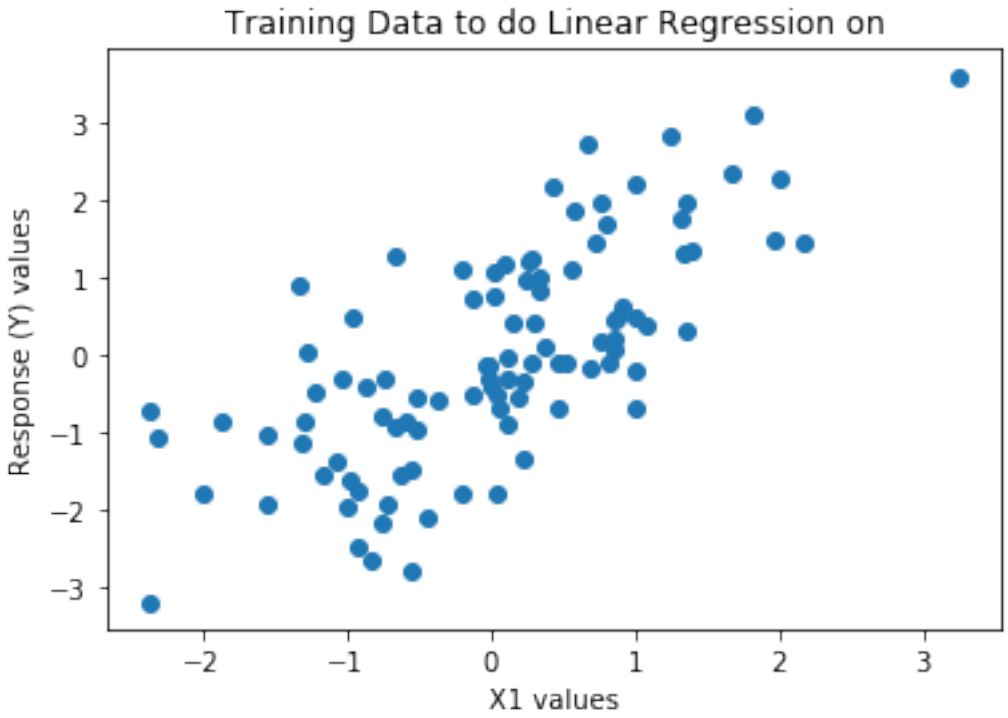
```
In [400]: N = 100 # Number of data points
          x0 = np.ones([N, 1]) # Vector of ones to match up with y-intercept for matrix multip
          x1 = np.random.randn(N, 1) # Feature taken from normal distribution
          X = np.hstack((x0, x1)) # 100x2 Matrix of column1 = 1's and column2 = feature1
          noise = np.random.randn(N, 1) # Random noise vector to simulate real world data

          theta0 = 0.0 # True value of y-intercept
          theta1 = 1.0 # True value of slope / coefficient for x1
          true_thetas = np.array([[theta0, theta1]]) # Put theta coefficients into a vector
          Y = theta0*x0 + theta1*x1 + noise # Linear equation for true Y
          #Y = np.matmul(true_thetas, X.T) # Matrix equation for true Y
          print(Y.shape)
```

```
(100, 1)
```

3 Plot Training Data

```
In [401]: plt.scatter(X[:, 1], Y)
          plt.xlabel('X1 values')
```

```
plt.ylabel('Response (Y) values')
plt.title('Training Data to do Linear Regression on')
plt.show()
```



4 Cost Function

Mean Squared Error (MSE) Cost function defined in class:

$$Cost = \frac{1}{2N}\sum_{i=1}^{N}(y_{pred_i} - y_{true_i})^2$$

Essentially just computing how far each prediction is from the true value

```
In [402]: def computeCost(X, y, theta):
              inner = np.power(((X @ theta.T) - y), 2) # @ means matrix multiplication of arra
              return np.sum(inner) / (2 * len(X))
```

5 Function to Optimize Cost/Loss (Gradient Descent)

Repeat for K iterations or until convergence:

$$\theta = \theta - \frac{\partial}{\partial\theta}\left[\frac{\alpha}{2N}\sum_{i=1}^{N}(y_{pred_i} - y_{true_i})^2\right]$$

$$\theta = \theta - \frac{\alpha}{N} \sum_{i=1}^{N} (y_{pred_i} - y_{true_i}) * X$$

```
In [408]: def gradientDescent(X, y, theta=None, alpha=0.001, iters=100):
              if theta is None:
                  # Set the seed for reproducibility
                  np.random.seed(12345)
                  theta = np.random.randn(1, 2)

              for i in range(iters):
                  theta = theta - (alpha/len(X)) * np.sum((X @ theta.T - y) * X, axis=0)
                  cost = computeCost(X, y, theta)

                  if i % 100 == 0: # just look at cost every ten loops for debugging
                      print('Iteration {}:    Cost = {}'.format(i, cost))

              return (theta, cost)
```

6 Fitting our Model

```
In [409]: # Hyperparameters
          iterations = 2000 # Number of times to repeat gradient descent
          alpha = 0.001 # Learning Rate
```

```
In [410]: # Set random seed for reproducibility
          np.random.seed(12345)
          # Get random values for theta0 and theta1 to start with
          starting_thetas = np.random.randn(1, 2)
          # Get optimized theta0 and theta1 values by minimizing Cost Function via Gradient De
          fitted_thetas, cost = gradientDescent(X, Y, starting_thetas, alpha, iterations)
```

```
Iteration 0:    Cost = 0.6088747389735317
Iteration 100:    Cost = 0.5809574482751166
Iteration 200:    Cost = 0.5584913579998116
Iteration 300:    Cost = 0.5404120453199738
Iteration 400:    Cost = 0.5258629321773909
Iteration 500:    Cost = 0.5141547001785066
Iteration 600:    Cost = 0.5047326304423173
Iteration 700:    Cost = 0.49715032090714145
Iteration 800:    Cost = 0.4910485357807227
Iteration 900:    Cost = 0.486138184989939
Iteration 1000:    Cost = 0.48218662717402405
Iteration 1100:    Cost = 0.47900664724093667
Iteration 1200:    Cost = 0.47644758623199623
Iteration 1300:    Cost = 0.47438820321976516
Iteration 1400:    Cost = 0.47273093103048747
Iteration 1500:    Cost = 0.4713972536236943
```

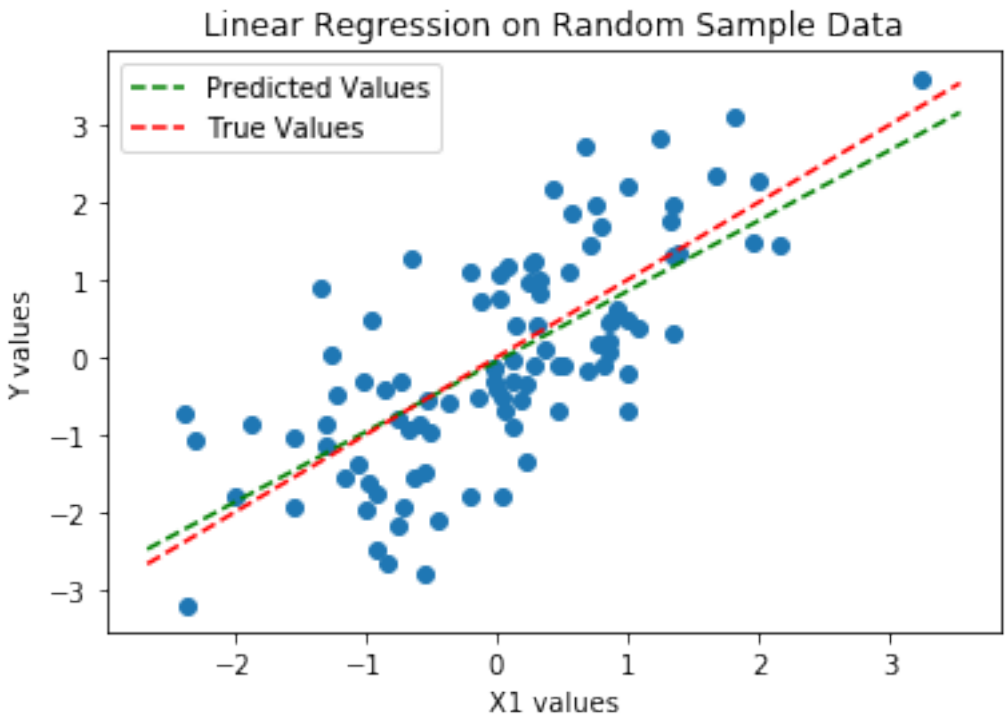
```
Iteration 1600:    Cost = 0.47032398610708925
Iteration 1700:    Cost = 0.4694602811326636
Iteration 1800:    Cost = 0.46876521983662867
Iteration 1900:    Cost = 0.46820587318191387
```

```
In [411]: print('Fitted_Thetas:', fitted_thetas)
          print('True Thetas:', true_thetas)
```

```
Fitted_Thetas: [[-0.053334    0.90830155]]
True Thetas: [[0. 1.]]
```

7 Compare Fitted Model to True Data

```
In [412]: plt.scatter(X[:, 1], Y) # True data
          axes = plt.gca()
          x_vals = np.array(axes.get_xlim()) # Get range of x-axis
          y_pred = fitted_thetas[0][0] + fitted_thetas[0][1]*x_vals
          y_true = theta[0][0] + theta[0][1]*x_vals
          # Plot properties
          plt.plot(x_vals, y_pred, 'g--') # Predicted line of best fit from model
          plt.plot(x_vals, y_true, 'r--') # True line of best fit
          plt.legend(['Predicted Values', 'True Values'])
          plt.title('Linear Regression on Random Sample Data')
          plt.ylabel('Y values')
          plt.xlabel('X1 values')
          plt.show()
```



8 Your Turn

We successfully fitted a model to this training data

Now try seeing how this model performs on different testing data...

```
In [398]: # Create test data (Similar to training set up top)

          # Predict test data y values using the fitted model
          # (no gradient descent or anything needed, we already have the fitted thetas)

          # Compare the predicted y_test values to the true y_test values (Plot it or whatever
```