Macroeconomics I PhD
Problem Set 4

2022/23

Please hand in at the latest December 14, 2022
https://uni-bonn.sciebo.de/s/3FBSLgntT2BXhaT

1 Asset Pricing

The following questions intend to give you an understanding of the interaction of households in financial markets. The first questions introduce you to the workings of complete markets and the limitations for consumption insurance. Thereafter the exercises highlight the interaction of financial markets with heterogeneity and inequality, as well as their propagation.

Consider the following two-period model. There is a unit mass of ex-ante identical households with a subjective discount factor β, and each household $i \in [0,1]$ maximizes the ex-ante utility of consumption

$$u(c_0^i) + \beta \sum_{s \in S} \pi(s) u(c_1^i(s)), \tag{1}$$

where $s \in S$ is a state of the economy occuring with the discrete probability $\pi(s)$. The felicity function $u(\cdot)$ is identical across households, satisfies $u'(\cdot) > 0$, $u''(\cdot) < 0$, and the Inada conditions.

Household i chooses asset portfolio $q^{i,j}$ subject to the natural borrowing limit, where $j \in J$ indexes assets. Assets are traded in period zero only. The price of an asset j is p_0^j. Let $x_1^j(s)$ denote the dividends from the asset j in period $t = 1$ if state $s \in S$ has realized. Gross returns are defined as $R_1^j(s) = \frac{x_1^j(s)}{p_0^j}$. The set of available assets J only contains Arrow securities that cover all aggregate uncertainty, i.e. for all $s \in S$ there exists an asset with dividend $x_1^s(s') = 1$ if $s = s'$ and $x_1^s(s') = 0$ if $s \neq s'$. Let $q_0^{i,s}$ be the number of pure securities that household i buys for state s.

Aggregate risk: Household i receives income y_0^i in period zero and stochastic income $y_1^i(s)$ in period one. Assume for now that all households have the same first period income $y_0^i = y_0^j = y_0 \ \forall i,j \in [0,1]$ and face the same risk, such that $y_1^i(s) = y_1^j(s) = y_1(s) \ \forall i,j \in [0,1]$. Endowments are not storable and there are no other endowments than y_0^i and $y_1^i(s)$. The budget constraint in period $t = 1$ for household i reads

$$c_1^i(s) = y_1^i(s) + q_0^{i,s} \; \forall s \in S. \tag{2}$$

All households enter period t=0 with zero asset holdings (of any category).

(a) State the budget constraint of household i in period $t = 0$.

(b) Set up the household's problem and solve for the first-order condition(s). Derive an expression for p_0^s, the price of the Arrow security $s \in S$.

(c) How many markets are there? State the market clearing conditions.

(d) Derive the demand $q_0^{i,s}$ for each pure security $s \in S$ and the corresponding consumption allocation $c_1^i(s)$ for each household $i \in [0,1]$.

(e) Given the setup with aggregate risk, how good are households insured against the aggregate risk? Would households want to change their portfolio? Can they?

(f) What does your answer from (e) tell you about the effectiveness of financial markets for insurance against risk?

Idiosyncratic risk: So far, we have assumed that all households face the same endowment in $t = 0$, and that they face the same income risk going forward. Hence, we abstracted from heterogeneinty between households. In the next exercises, we will introduce heterogeneinty in endowments and in risk. To simplify, assume that there are only two types of households in the economy, hence $i \in \{0,1\}$. Second, we assume $\pi(s) = \pi(s') = \pi \; \forall s, s' \in S$, as well as $\sum_{s \in S} \pi = 1$. That is, each state of the economy $s \in S$ is equally likely.

(g) Maintain the assumption $y_0^0 = y_0^1 = y > 0$, but now assume that $cov(y_1^0(s), y_1^1(s)) = -1 \; \forall s \in S$ with $\frac{1}{2}\sum_i y_1^i(s) = y$ and $\sum_{s \in S} y_1^0(s) = \sum_{s \in S} y_1^1(s)$. Hence, households now face heterogeneous income risk across states. What is asset demand $q^{i,j} \; \forall j \in J$ and the consumption allocation $\{c_1^0(s), c_1^1(s)\}_{s \in S}$?

(h) Maintain the assumptions made in (g), but now add that households have different, yet positive first period endowments $y_0^i \neq y_1^i$. Describe the allocation of consumption in this case, without determining the exact values.

(i) If a central planner would exist that could determine allocations in the economy such as to maximize utility for both households, how would the allocation differ from the allocation in (h)?

(j) Assume that there exists a central planner that wants to maximize the utility of a household that does not yet know whether he will be household 0 or 1. How would the allocation look like now?

(k) In your own words, what do complete markets achieve and what do they fail to do?

(l) How does the notion of a representative household relate to the discussion?

2 Mechanism behind asset pricing

We now want to analyze the mechanisms that drive asset prices in the complete market case. The following questions shall help you understand how market clearing is an integral feature of this.

In class you derived a pricing equation for any asset a of the form

$$p_t^a = \frac{E_t\{x_{t+1}^a\}}{R_t^f} + \beta \frac{\text{cov}_t(u'(c_{t+1}), x_{t+1}^a)}{u'(c_t)}. \tag{3}$$

(a) In your own words, what is the economic intuition behind equation (3)? Think about the interaction of market clearing and the desire of households to insure against consumption fluctuations.

(b) Based on (3), how does the desire of households to insure themselves against consumption risk influence the equity risk premium?

(c) How does this insight from the pricing of assets relate to your answers from (e), (f) and (k)?