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MEDICAL IMAGE ENHANCEMENT USING HISTOGRAM PROCESSING TECHNIQUES FOLLOWED BY MEDIAN FILTER

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Abstract: Contrast enhancement plays a key role in analyzing medical images taken through different diagnosing methods (like CT scan, X-ray and MRI) especially in biomedical applications and secures digital image transmission. The principle objective of image enhancement technique is to improve the characteristics or quality of an input image. To improve the image contrast, numerous enhancement techniques have been proposed in literature and the conventional methods include Global Histogram Equalization (GHE), Local Histogram Equalization (LHE) techniques etc. However, the main limitations of these techniques are unpleasant visual artifacts such as over enhancement, level saturation and raised noise level. To overcome these, researchers suggested techniques like Brightness Preserving Bi-Histogram Equalization (BBHE) and Dualistic Sub-Image Histogram Equalization (DSIHE) which failed to eliminate the impulse noise. In this paper, the low contrast noisy medical images are preprocessed for image enhancement followed by noise reduction phase through filtering to reconstruct the original image. Various well-known contrast enhancement techniques like GHE, LHE, BBHE and DSIHE followed by median filter are implemented and their comparative performance analyzed using subjective and objective image quality parameters.

Keywords: Histogram equalization, medical image, median filter, Mean Opinion Score(MOS), Absolute Mean Brightness Error(AMBE), Maximum Difference(MD), Mean Square Error(MSE), Normalized Cross Correlation(NK).

1. INTRODUCTION

Many real world images are acquired with low contrast and are unsuitable for human eyes to read such as medical images. Medical imaging techniques like Computed Tomography (CT) scan, Magnetic Resonance Imaging (MRI), X-ray and Ultrasound have introduced a formidably powerful tool in medicine. One of the most important stages in computerized medical image analysis is stone recognition in any part of body or bone fracture detection. This analysis is heavily affected by image quality. However, the acquired medical images are usually characterized by low Signal-to-Noise-Ratio (SNR) and low Contrast-to-Noise Ratio (CNR) along with multiple and discontinuous edges. So, various image enhancement methods have been suggested in literature to improve the appearance of images for better visual interpretation, understanding and image analysis [6,10,14]. Medical image enhancement technologies have attracted much attention since advanced medical equipments were put into use in the medical field [3]. Enhanced medical images are desired by a surgeon to assist diagnosis and interpretation as the medical image qualities are often deteriorated by noise and other data acquisition devices, illumination conditions, etc. Medical image enhancement also targets the problems of low contrast and high level noise in a medical image. An enhancement algorithm achieves better quality image by either suppressing the noise or increasing the image contrast.

1.1 Medical Images

Medical images are a special kind of images that can be used for the diagnostics of diseases in the patients [3]. A number of modalities like CT- scan, MRI and X-ray imaging exist for obtaining these images. This work focuses on the images obtained through MRI, X-ray and CT scan imaging. These types of imaging methods generally results in poor contrast medical images thereby impeding the physician’s observation and may lead to a wrong diagnosis. Use of image enhancement techniques [15], thus becomes vital for medical imaging.

1.2 Image Enhancement

Image enhancement is a process that focuses on processing an image in such a way that the processed image is more suitable than the original one for the specific application. The word 'specific' has significance. It gives a clue that the results of such an operation are highly application dependent. In other words, an image enhancement technique that works well for X-ray topographic images may not work well for MR images.

Image enhancement techniques work in frequency and spatial domains. In the frequency domain methods, Fourier Transforms of the image [6,7] are considered while spatial domain techniques considers whole number of pixels in the image and directly operates on them. The process can be expressed as:

$$g(x, y) = T[f(x, y)]$$

Where $f(x, y)$ is the input image, $g(x, y)$ is the processed image and T is an operator on f defined over some neighborhood of (x, y). A number of enhancement techniques exist in the spatial domain [6,7,10,11]. Among these are histogram processing, enhancement using arithmetic and logical operations and filters.

1.2.1 Histogram Equalization

The Histogram Equalization (HE) spreads out intensity values along the total range of values in order to achieve higher contrast.

For example, the result of applying histogram equalization to the image in Figure 1 is presented in Figure 2.

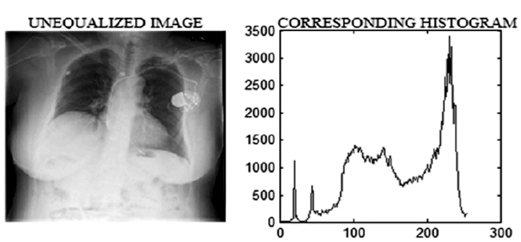


Figure 1: An Image and Its Histogram Before Equalization

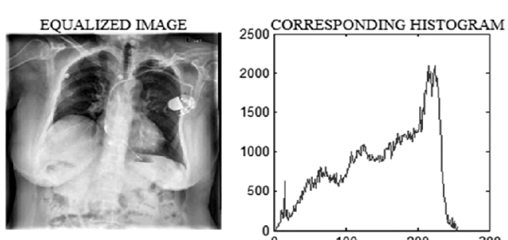


Figure 2: An Image and Its Histogram After Equalization

This method is especially useful when an image is represented by close contrast values, such as images in which both the background and foreground are bright at the same time, or else both are dark at the same time.

The HE techniques can be classified into various categories like Global Histogram Equalization (GHE) [6,10], Local Histogram Equalization (LHE) [10,16], Brightness preserving Bi-Histogram Equalization (BBHE) [1,2,12] and Dualistic Sub-Image Histogram Equalization (DSIHE) [1,2,8,14]. GHE uses the histogram information of entire input image for its transformation function i.e. the histogram of whole input image is first obtained, then the Cumulative Distribution Function (CDF) is calculated and gray transfer function is derived from the CDF [12]. Though this global approach is suitable for overall enhancement, it fails to preserve the local brightness features of the input image. When there exits some gray levels in the image with very high frequencies, they are usually dominating the other gray levels with lower frequencies. In such situation, GHE remaps the gray levels in such a way that the contrast stretching becomes limited in some dominating gray levels having larger image components and causes significant contrast loss for other smaller ones.

LHE tries to eliminate such problem. It uses a small window that slides through every pixel of the image sequentially and only the block of pixels that fall in this window are taken into account for HE and then gray level mapping for enhancement is done only for the center pixel of that window. Thus, it makes use of the local information in a better manner. However, this technique has the problem of enhancing the noises in input image along with the image features.

BBHE and DSIHE are the variants of HE based contrast enhancement. BBHE divides the input image's histogram into two parts based on mean of the input image and then each part is equalized independently [2, 12]. This method tries to overcome the problem of brightness preservation. DSIHE method is similar to BBHE except that it separates the histogram based on median [1]. Though these methods can perform good contrast enhancement, they also cause more annoying side effects depending on the variation of gray level distribution in the histogram.

1.2.2 *Filtering Techniques*

The fundamental requirements of noise filtering methods for medical images are: (*i*) safeguarding important information of the object boundaries and detailed structures, (*ii*) ability to efficiently remove noise in homogeneous regions, and (*iii*) the ability to enhance morphological definitions by sharpening discontinuities. Filtering techniques are broadly categorized as i.e. linear and non-linear filtering. The median filter is a non-linear filtering technique [4], frequently used to remove noise from images. It is also called as sliding window filter, which replace the center value in the window with the median of all pixel values in the window [6]. It is mostly used to reduce speckle noise and salt and pepper noise. Its edge-preserving nature makes it practical in cases where edge blurring is undesirable.

In this paper, a comparative study is carried out to enhance the quality of medical images using various well known histogram processing techniques like GHE, LHE, BBHE and DSIHE followed by median filtering noise removal technique. This paper presents the performance analysis of comparison using objective and subjective image quality measures.

The rest of the paper is organized as follows: Section 2 provides implementation of various histogram processing techniques followed by filtering techniques. Section 3 describes subjective and objective image quality measures used in the work. Experimental results are presented in Section 4 and finally conclusions are drawn in Section 5.

2. IMPLEMENTATION

The GHE, LHE, BBHE and DSIHE techniques were implemented using MATLAB platform. The details are outlined below:

2.1 Global HE Method

The Histogram of digital image $I = \{I(i, j)\}$, with L discrete intensity levels denoted by $\{I_0, I_1, \ldots \ldots \ldots, I_{L-1}\}$, is defined as:

$$h(I_k) = n_k, \quad \text{for } k = 0, 1, \ldots, L-1$$

Where I_k is the kth intensity value and n_k is the number of pixels in the image with intensity r_k. For an $M \times N$ image, a normalized histogram known as Probability Density Function (PDF) is defined by:

$$p(I_k) = \frac{n_k}{MN}, \text{ for } k = 0, 1, \ldots, L-1$$

Where $p(I_k)$ gives an estimate of the probability of occurrence of gray level I_k in an image. Based on the PDF, the Cumulative Density Function (CDF) is defined as:

$$C(I_k) = \sum_{j=0}^{k} p(I_j), \quad \text{for } k = 0, 1, \ldots, L-1$$

GHE enhances $I = \{I(i, j)\}$, by using CDF as its transformation function. This transformation function $f(I_k)$ is defined as:

$$f(I_k) = I_0 + (I_{L-1} - I_0)c(I_k)$$

Then the output image produced by GHE, $Y = \{Y(i, j)\}$ can be expressed as [13]:

$$Y = f(I)$$

$$Y = f\{(I(i, j) \mid \forall I(i, j) \in I\}$$

Although GHE successfully increases the contrast in the image, this method does not put any constrain in preserving the mean brightness.

2.2 Local HE Method

The GHE method of histogram equalization takes the global information of image into account and cannot adapt to local light condition [10]. So, local enhancement is used to perform block-overlapped histogram equalization in LHE method. Algorithm for the same can be expressed as:

Copy the image matrix into another location and pad it with zeros on all sides. Then define square or rectangular neighborhood (mask or window) of size say $M \times N$ and move the center from pixel to pixel. For each neighborhood, calculate histogram of the points in the neighborhood. Then obtain histogram equalization/specification function and map gray level of pixel centered in neighborhood. New pixel values and previous histogram should be used to calculate next histogram. Finally locally enhanced image is obtained. By changing the window matrix size i.e. the values of M and N; the histogram equalization can be enhanced. However, LHE demands high computational cost [16] and sometimes causes over-enhancement in some portion of the image. Moreover, this technique has the problem of enhancing the noises in the input image along with the image features.

2.3 Brightness Preserving Bi-HE Method

BBHE first decomposes an input image into two sub-images based on the mean of the input image [13,15]. One of the sub image is set of samples less than or equal to the mean whereas the second one is the set of samples greater than the mean. Then the BBHE equalizes the sub images independently based on their respective histograms with the constraint that the samples in the former set are mapped into the range from the minimum gray level to the input mean and the samples in the latter set are mapped into the range from the mean to the maximum gray level. It means first sub image is equalized over the range up to the mean and the other sub image is equalized over the range as shown in Figure 3.

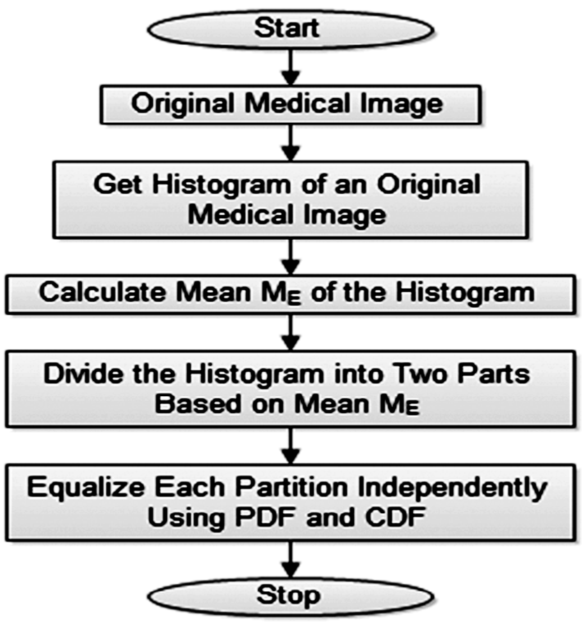


Figure 3: Flow Chart for BBHE

Thus, the resulting equalized sub images are bounded by each other around the input mean, which has an effect of preserving mean brightness.

2.4 Dualistic Sub-Image HE Method

DSIHE also separates the input histogram into two sub-sections. Both BBHE and DSIHE are similar except that DSIHE chooses to separate the histogram based on median instead of mean as in BBHE.

Let an image $I(i,j)$ is segmented by a section with gray level of $I = I_m$ and the two sub-images are I_L and I_U, so we have:

$$I = I_L \cup I_U$$

Here $\quad I_L = (I(i,j) \mid I(i,j) < I_m \forall\, I(i,j) \in I)$

and $\quad I_U = (I(i,j) \mid I(i,j) < I_m \forall\, I(i,j) \in I)$

It is obvious that sub-image I_L is composed by gray level of $\{I_0, I_1, \ldots, I_{m-1}\}$, while sub-image I_U is composed of $\{I_m, I_{m+1}, \ldots \ldots \ldots, I_{L-1}\}$. The aggregation of original image's gray level distribution probability is decomposed into $\{p_0, p_1, \ldots \ldots \ldots, p_{m-1}\}$ and $\{p_m, p_{m+1} \ldots \ldots \ldots, p_{L-1}\}$ correspondingly. The corresponding CDF will be:

$$C_L(I_k) = \frac{1}{p}\sum\nolimits_{i=0}^{k} p_i\,,\;\; k = 0, 1, \ldots m-1$$

$$C_U(I_k) = \frac{1}{p-1}\sum\nolimits_{i=m}^{L-1} p_i\,,\;\; k = m, m+1, \ldots L-1$$

Based upon CDF, transform functions of two sub-image's histogram are equalized below:

$$F_L(I_k) = I_0 + (I_{m-1} - I_0)C_L(I_k),\, k = 0, 1, \ldots, m-1$$

$$F_U(I_k) = I_m + (I_{L-1} - I_m)C_U(I_k),\, k = m, m+1 \ldots, L-1$$

At last result of dualistic sub-image histogram is obtained after the two equalized sub-images are composed into one image. Suppose $Y(i, j)$ denotes the processed image then:

$$Y = \{Y(i,j)\} = F_L(I_L) \cup F_U(I_U)$$

or $$Y(i,j) = \begin{cases} I_0 + (I_m - I_0)\, C_L(I_k) \\ I_{m+1} + (I_{L-1} - I_{m+1}) C_U(I_k) \end{cases}$$

Flow chart of various algorithm steps for DSIHE [8, 13] is shown in Figure 4.

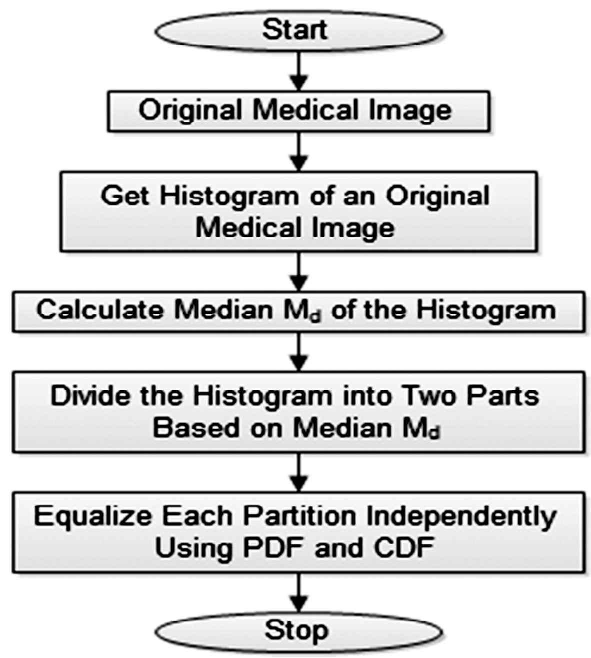


Figure 4: Flow Chart for DSIHE

2.5 Median Filtration Method

A median filter finds the median of the number of pixels at its input [5,9]. The median of a group, containing an odd number of elements, is defined as the middle element, when the elements of the group are sorted. In the standard median filtering applications, a window of size W (where W is odd) is moved along the sampled values of the signal or the image. For each position of the window, the median of the elements within the windows is computed and then written at the output pixel located at the same position as the central element of the window. The median computed at this operation is called the running or the moving median. Since the size of the window is constant, the number of incoming elements is equal to the number of outgoing elements. The dimensions of the filter mask must be odd. Mask sizes are 3 × 3, 5 × 5 or 7 × 7. Minimum mask size is preferred in many cases.

2.6 Experimental Set-up

The experimental analysis was carried out using two main steps : (*i*) enhancement of medical images using GHE/LHE/BBHE/DSIHE Histogram Processing Techniques (HPTs) and (*ii*) median filtration to eliminate noise from enhanced medical images, as depicted in the following flow chart (Figure 5).

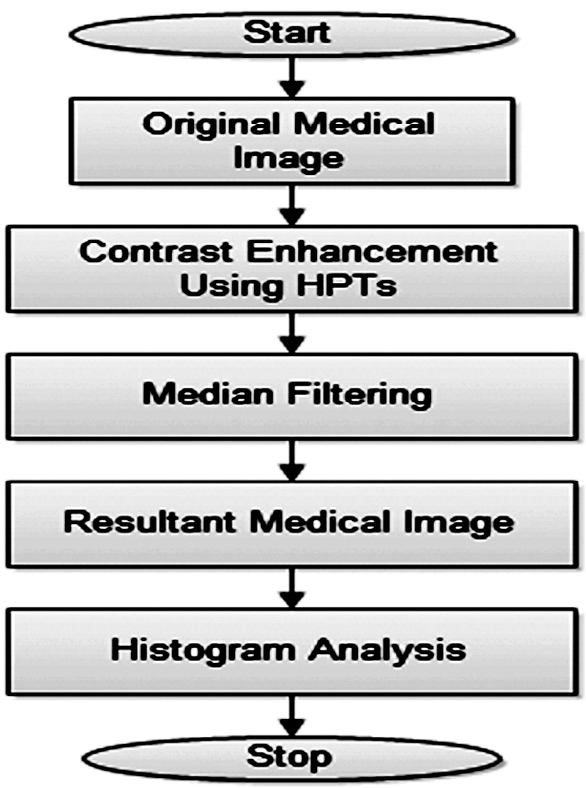


Figure 5: Flow Chart of Experimental Set-up

In this paper, five medical images like MR, *X*-ray of jpg formats (size: 410 × 377, 500 × 392, 500 × 392, 256 × 256 and 512 × 512) were taken as test images as depicted in Figure 6 and median filter of 3 × 3 mask size (dimensions) was used.

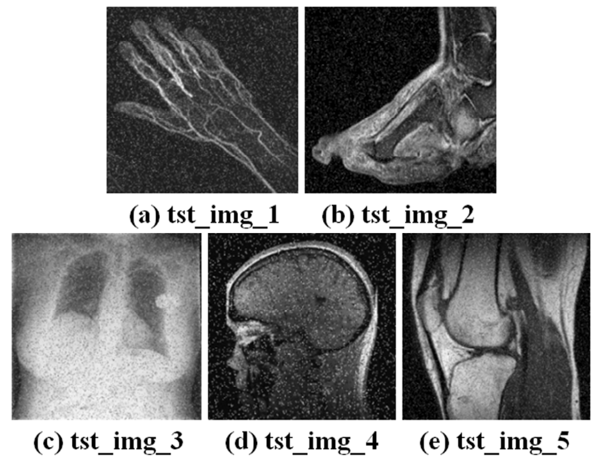


Figure 6: Five Test Images

3. PERFORMANCE METRICS

IQM (Image Quality Measure) is vital in the development of image processing algorithms such as enhancement, deblurring, denoising etc. and are used to evaluate their performances in terms of quality of processed image. There are two basic types of quality measures (subjective and objective) used in literature.

3.1 Subjective Quality Measure

In fact, in image enhancement system, the truly definitive measure of image quality is perceptual quality. The enhanced image quality is specified by Mean Opinion Score (MOS), which is result of perception based subjective evaluation [11]. The meaning of the 5-level grading scales of MOS is 5-pleasant or excellent, 4-good, 3-acceptable, 2-poor quality and 1-unacceptable. MOS is defined as follow:

$$MOS = \frac{1}{S}\sum_{i=0}^{s} iP_i,$$

Where i is an image score, P_i is image score probability and S is number of observer.

3.2 Objective Quality Measures

3.2.1 *Absolute Mean Brightness Error*

Absolute Mean Brightness Error (AMBE) is defined to rate the performance in preserving the original brightness. Smaller value of AMBE indicates a better preservation of the brightness property. It is difference between original image and enhanced image [12] and is given as:

$$AMBE = |I(i,j) - \hat{I}(i,j)|$$

Where $I(i, j)$ is average intensity of input image and $\hat{I}(i,j)$ is average intensity of enhanced image. Its value range is of [0, ∞].

3.2.2 *Maximum Difference*

The large the value of Maximum Difference (MD) means that image is poor quality. MD measures which takes the maximum of the difference between original and enhanced image and is defined as follows [10]:

$$MD = \max[|I(i,j) - \hat{I}(i,j)|]$$

Where $I(i, j)$ denotes sample of input image and $\hat{I}(i,j)$ denotes the samples of enhanced image. Its value range is of [0, ∞].

3.2.3 *Mean Square Error*

The simplest of image quality measures, is Mean Square Error (MSE). The large value of MSE means that image is poor quality. MSE is defined as follows [9, 10]:

$$MSE = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}|I(i,j) - \hat{I}(i,j)|^2$$

Where $I(i, j)$ denotes sample of input image and $\hat{I}(i,j)$ denotes the samples of enhanced image. M and N are number of pixels in row and column directions, respectively. Its value range is of [0, ∞].

3.2.4 *Normalized Cross Correlation*

It is defined as the ratio of sum of multiplication of input image and enhanced image to square sum of original image [11]. It is denoted by *NK*. The small value of *NK* means that image is poor quality.

$$NK = \frac{\sum_{i=1}^{M}\sum_{j=1}^{N} I(i,j)\,\hat{I}(i,j)}{\sum_{i=1}^{M}\sum_{j=1}^{N} I(i,j)^2}$$

Where $I(i, j)$ denotes sample of input image and $\hat{I}(i,j)$ denotes the samples of enhanced image. Its value range is of [0, 1].

4. PERFORMANCE ANALYSIS

In this paper, various image enhancement techniques based on GHE/LHE/BBHE/DSIHE are taken up for comparison on a single platform using five different medical images and their performances in terms of AMBE/MD/MSE/NK are calculated and presented in Tables 1- 5.

Table 1
Image Quality Measures for tst_img_1

IQMs *HPTs*	*AMBE*	*MD*	*MSE*	*NK*
GHE	68.6624	242	8658.06	1.40399
LHE	13.9469	251	2752.2	0.882138
BBHE	37.2414	235	3840.79	0.295167
DSIHE	32.3546	255	3448.27	0.422895

Table 2
Image Quality Measures for tst_img_2

IQMs *HPTs*	*AMBE*	*MD*	*MSE*	*NK*
GHE	75.7579	240	8637.75	1.26263
LHE	14.0075	251	3182.77	0.895715
BBHE	16.0726	230	3442	0.431795
DSIHE	28.3538	255	3494.38	0.422883

Table 3
Image Quality Measures for tst_img_3

IQMs *HPTs*	*AMBE*	*MD*	*MSE*	*NK*
GHE	31.8054	255	3336.941	0.789601
LHE	9.64372	251	2681.91	0.886626
BBHE	39.8047	255	4341.79	0.691365
DSIHE	105.275	255	13463.7	0.372864

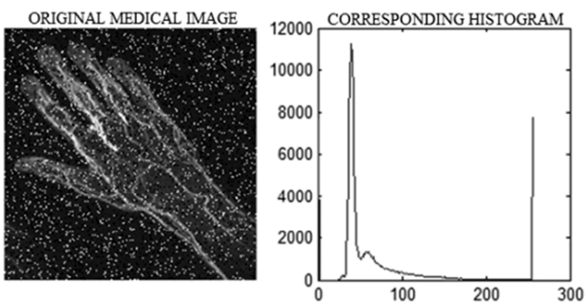
Table 4
Image Quality Measures for tst_img_4

IQMs *HPTs*	*AMBE*	*MD*	*MSE*	*NK*
GHE	72.3432	241	7936.77	1.34804
LHE	18.6457	255	3124.11	0.938378
BBHE	19.4708	255	3112.9	0.474373
DSIHE	347.7412	255	3643.54	0.41441

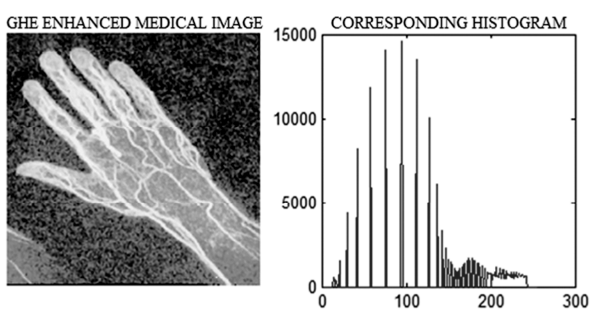
Table 5
Image Quality Measures for tst_img_5

IQMs *HPTs*	*AMBE*	*MD*	*MSE*	*NK*
GHE	35.1185	241	3485.43	1.10712
LHE	15.9064	251	3086.14	0.953166
BBHE	26.5744	210	3914.29	0.5517
DSIHE	64.0486	255	6895.93	0.336004

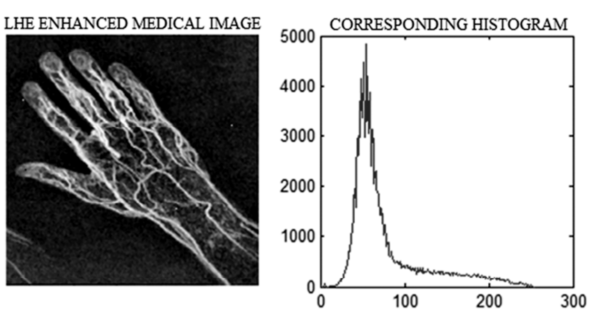
The medical image enhancement techniques are followed by median filter to minimize noise in addition to enhancing the acquired medical images (like MR and X-ray) to achieve high contrast noise free medical images. The visual results and their corresponding histograms are shown only for two test images namely tst_img_1 (Figure 7) and for tst_img_2 (Figure 8).



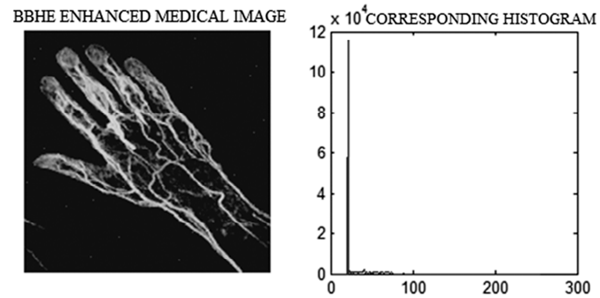
(*a*) Original Image and Its Histogram



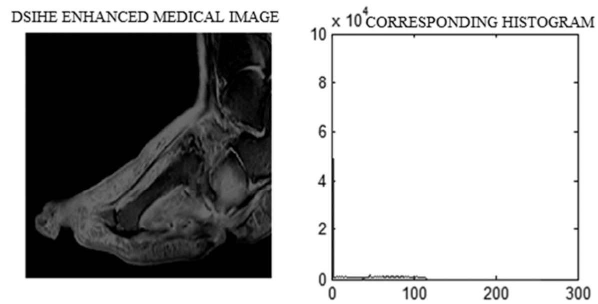
(*b*) GHE Enhanced Image and Its Histogram



(*c*) LHE Enhanced Image and Its Histogram

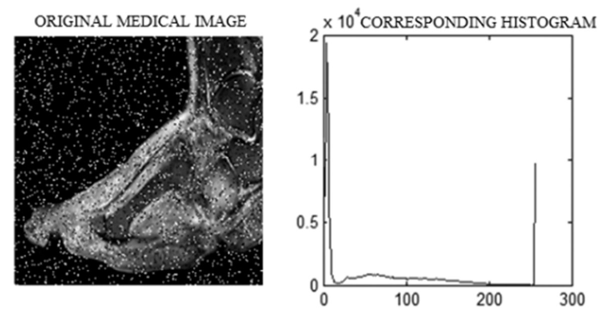


(*d*) BBHE Enhanced Image and Its Histogram

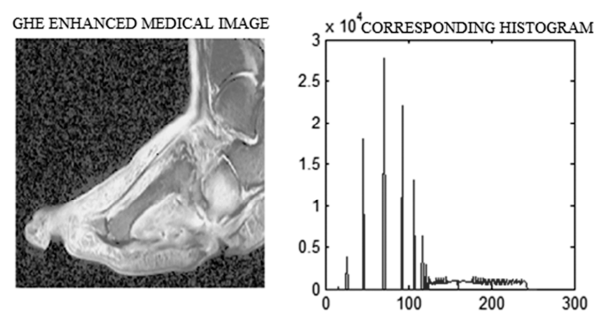


(*e*) DSIHE Enhanced Image and Its Histogram

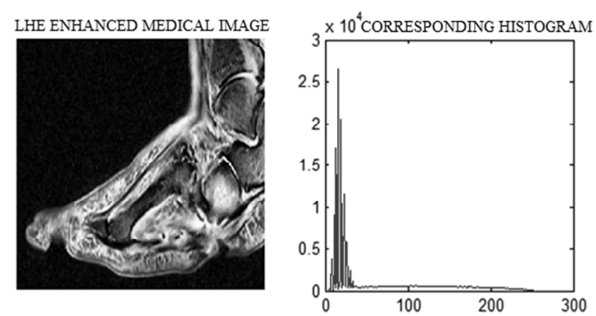
Figure 7: tst_img_1 before and after HE



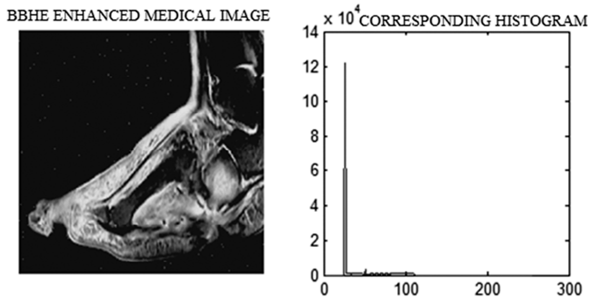
(*a*) Original Image and Its Histogram



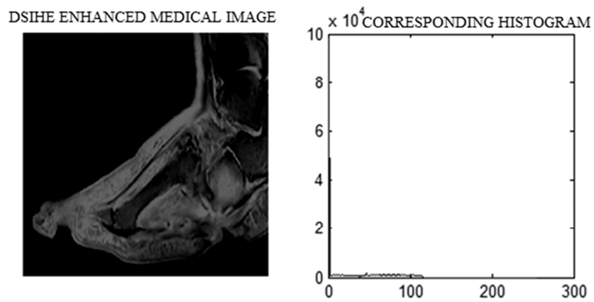
(*b*) GHE Enhanced Image and Its Histogram



(*c*) LHE Enhanced Image and Its Histogram



(*d*) **BBHE Enhanced Image and Its Histogram**



(*e*) **DSIHE Enhanced Image and Its Histogram**

Figure 8: tst_img_2 Before and After HE

For subjective assessment; 10 observers who are senior in information engineers with some background in digital image processing, subjectively evaluate the enhancing quality using MOS. Table 1-5 indicate that LHE has lowest AMBE, lower MSE and NK approaches to 1. Also BBHE and DSIHE have lower values of AMBE for test images, which reflect brightness preserving property of both. As there is lesser variation in MD after applying various medical image enhancement techniques; so, it does not reflect much meaningful information to decide which one technique is better or not.

The reliability of objective image quality measurement can be evaluated by finding the relation between objective and subjective image quality measurements. As reflected from these results, the image quality improves after image enhancement and filtering.

5. CONCLUSION

The performance of various Histogram Processing Techniques (HPTs) like GHE, LHE, BBHE and DSIHE is analyzed on five test images and results are presented. These results reflect that their high quality after applying LHE and DSIHE techniques followed by median filtering. LHE gave better results than GHE. Moreover, the method is simple and computationally effective that makes it easy to implement. Both BBHE and DSIHE methods preserve the brightness of original images after enhancement.

In future, work shall be extended to include distortions like Gaussian noise and blurring.

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