BSAN775 Introduction to Business Analytics

Assignment2: Linear Regression Analysis

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Submitted by:
Arun Rimal - N264S646
Asmita Regmi - F959F423
Buddha Thapa Magar - E923Q669
Prerana Giri - G355R842

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```
[1]: import pandas as pd
     import seaborn as sns
     import matplotlib.pyplot as plt
     import statsmodels.api as sm
     from sklearn.metrics import mean_squared_error, r2_score
```

1 National Employment and GDP

Initially, We did some EDA on the provided data. Following are the codes and findings.

```
[2]: employment_gdp_dataframe = pd.read_excel("./HW2_Datasets.xlsx")
     employment_gdp_dataframe.head()
```

```
[2]:    Quarter  Employment (millions)  GDP (billions)
     0  Q1 2020                    155           21500
     1  Q2 2020                    152           21200
     2  Q3 2020                    150           21700
     3  Q4 2020                    153           22000
     4  Q1 2021                    154           22200
```

```
[3]: employment_gdp_dataframe.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 20 entries, 0 to 19
Data columns (total 3 columns):
 #   Column                 Non-Null Count  Dtype
---  ------                 --------------  -----
 0   Quarter                20 non-null     object
 1   Employment (millions)  20 non-null     int64
 2   GDP (billions)         20 non-null     int64
dtypes: int64(2), object(1)
memory usage: 612.0+ bytes
```

```
[4]: employment_gdp_dataframe.describe()
```

```
[4]:        Employment (millions)  GDP (billions)
     count              20.000000       20.000000
     mean              164.150000    23437.500000
     std                 9.669676     1385.818683
     min               150.000000    21200.000000
     25%               155.750000    22350.000000
     50%               163.000000    23400.000000
     75%               171.500000    24550.000000
     max               181.000000    25750.000000
```

```
[5]: # Splitting Quarter field into years, quarter number.
     employment_gdp_dataframe["Year"] = employment_gdp_dataframe["Quarter"].str[-4:].
      ↪astype(int)
```

```
employment_gdp_dataframe['Quarter_num'] = employment_gdp_dataframe['Quarter'].
↪str[1].astype(int)
employment_gdp_dataframe["Lagged Employment"] =␣
↪employment_gdp_dataframe["Employment (millions)"].shift(1)
employment_gdp_dataframe = employment_gdp_dataframe.dropna()
employment_gdp_dataframe = employment_gdp_dataframe.sort_values(by= ["Year",␣
↪"Quarter_num"]).reset_index().drop(columns= "index")
employment_gdp_dataframe = employment_gdp_dataframe.reset_index().
↪rename(columns={'index': 'Coded Time'})
```

[6]:

```
#OnehotEncoding
from sklearn.preprocessing import OneHotEncoder
encoder = OneHotEncoder(sparse_output= False, drop= "first")

encoded_quarters = encoder.
↪fit_transform(employment_gdp_dataframe[["Quarter_num"]]).astype(int)
df_encoded_quarters = pd.DataFrame(data= encoded_quarters, columns= encoder.
↪get_feature_names_out())
df_encoded = pd.concat([employment_gdp_dataframe, df_encoded_quarters], axis= 1)
df_encoded.head()
```

[6]:

```
   Coded Time  Quarter  Employment (millions)  GDP (billions)  Year  \
0           0  Q2 2020                    152           21200  2020
1           1  Q3 2020                    150           21700  2020
2           2  Q4 2020                    153           22000  2020
3           3  Q1 2021                    154           22200  2021
4           4  Q2 2021                    156           22400  2021

   Quarter_num  Lagged Employment  Quarter_num_2  Quarter_num_3  Quarter_num_4
0            2              155.0              1              0              0
1            3              152.0              0              1              0
2            4              150.0              0              0              1
3            1              153.0              0              0              0
4            2              154.0              1              0              0
```

[7]:

```
df_encoded.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 19 entries, 0 to 18
Data columns (total 10 columns):
 #   Column                 Non-Null Count  Dtype
---  ------                 --------------  -----
 0   Coded Time             19 non-null     int64
 1   Quarter                19 non-null     object
 2   Employment (millions)  19 non-null     int64
 3   GDP (billions)         19 non-null     int64
 4   Year                   19 non-null     int64
```

```
 5   Quarter_num           19 non-null     int64
 6   Lagged Employment     19 non-null     float64
 7   Quarter_num_2         19 non-null     int64
 8   Quarter_num_3         19 non-null     int64
 9   Quarter_num_4         19 non-null     int64
dtypes: float64(1), int64(8), object(1)
memory usage: 1.6+ KB
```

[8]:
```
# Multivariate linear regression model
X_train = df_encoded.drop(columns= ["Quarter", "GDP (billions)", "Employment␣
 ↪(millions)", "Quarter_num", "Coded Time"])
y_train = df_encoded[["GDP (billions)"]]
X_train_with_const = sm.add_constant(X_train)

multivariate_ols_model = sm.OLS(y_train, X_train_with_const).fit()
```

[9]:
```
y_predicted_multivariiate = multivariate_ols_model.predict(X_train_with_const)
print(f"Mean_squared error for multivariate linear regression:␣
 ↪{mean_squared_error(y_true= y_train, y_pred= y_predicted_multivariiate)}")
```

```
Mean_squared error for multivariate linear regression: 2961.873985624827
```

[10]:
```
#Univariate linear regression model
X_train_univariate = df_encoded[["Lagged Employment"]]

X_train_univariate_with_const = sm.add_constant(X_train_univariate)

univariate_ols_model = sm.OLS(y_train, X_train_univariate_with_const).fit()
```

[11]:
```
y_predicted_univariiate = univariate_ols_model.
 ↪predict(X_train_univariate_with_const)
print(f"Mean_squared error for univariate linear regression:␣
 ↪{mean_squared_error(y_true= y_train, y_pred= y_predicted_univariiate)}")
```

```
Mean_squared error for univariate linear regression: 88189.44444027875
```

1.0.1 Residual Scatterplot: Create a scatterplot of the residuals against the lagged employment.

=> We created the residual scatter plots using multivariate linear regression and univariate linear regression. In multivariate linear regression, we used Year, Lagged Employment and one hot encoded quarter numbers. For Univariate linear regression, We only used Lagged Employment as the predictor and "GDP (Billions)" as dependent variable.

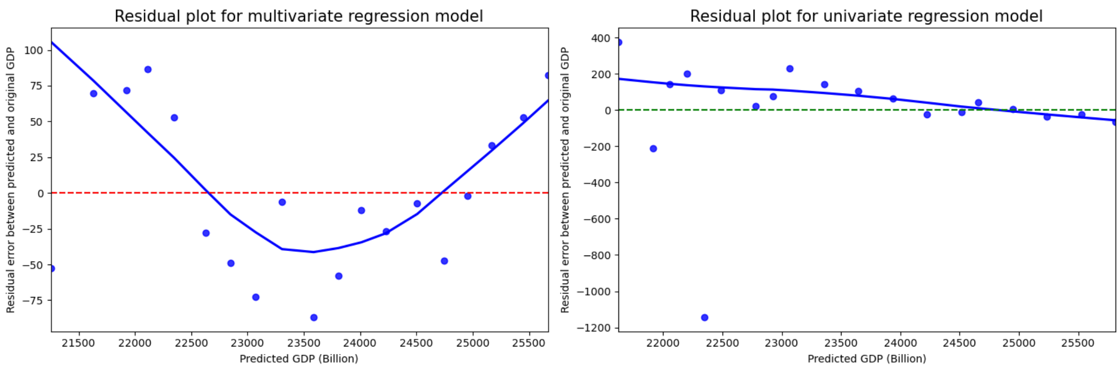
[12]:
```
fig, axs = plt.subplots(1, 2, figsize= (15, 5))

sns.residplot(x = y_predicted_multivariiate, y = multivariate_ols_model.resid,␣
 ↪lowess=True, color='blue', ax= axs[0])
axs[0].axhline(0, color='red', linestyle='--')
```

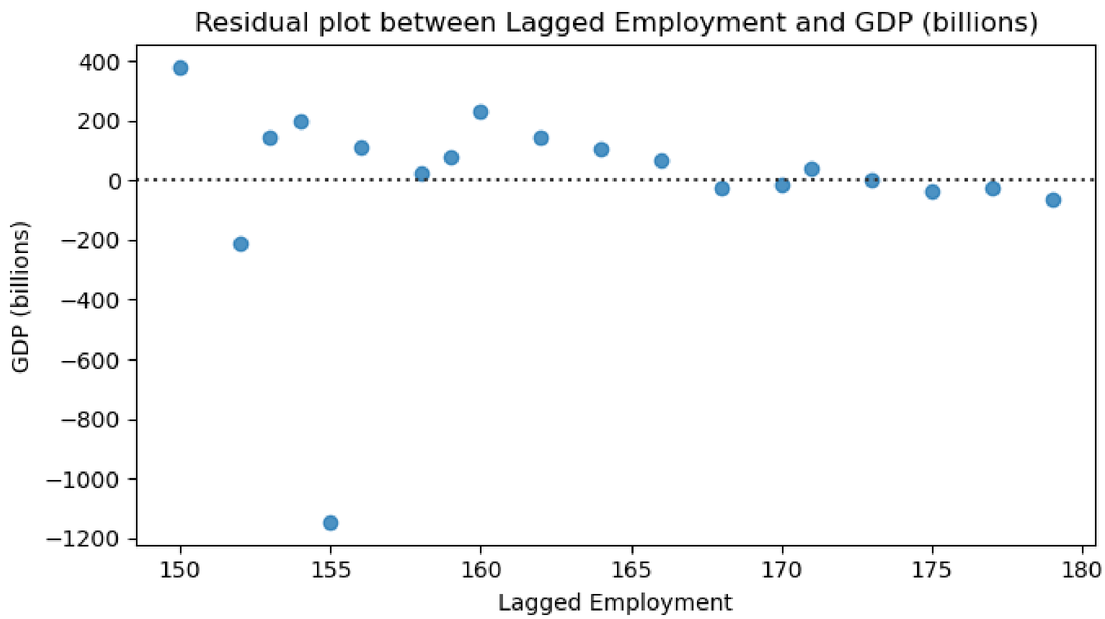
```
axs[0].set_xlabel("Predicted GDP (Billion)")
axs[0].set_ylabel("Residual error between predicted and original GDP")
axs[0].set_title("Residual plot for multivariate regression model", fontsize =␣
↪15)

sns.residplot(x = y_predicted_univariiate, y = univariate_ols_model.resid,␣
↪lowess=True, color='blue', ax= axs[1])
axs[1].axhline(0, color='green', linestyle='--')
axs[1].set_xlabel("Predicted GDP (Billion)")
axs[1].set_ylabel("Residual error between predicted and original GDP")
axs[1].set_title("Residual plot for univariate regression model", fontsize = 15)

plt.tight_layout()
plt.show()
```



[13]:
```
plt.figure(figsize=(7, 4))
sns.residplot(y = "GDP (billions)", x = "Lagged Employment", data= df_encoded)
plt.title("Residual plot between Lagged Employment and GDP (billions)")
plt.tight_layout()
plt.show()
```



1.0.2 Does the plot suggest that the residuals are homoscedastic?

As we can see the residual plots for both univariate and multivariate linear regressions, residual plot does not lie within the specific width around the x-axis. Residuals are randomly scattered around the horizontal axis with no clear pattern. Hence, the residuals are not homoscedastic for both univariate and multivariate linear regressions.

1.0.3 Residual Histogram: Create a histogram of the residuals and compare it to the normal distribution

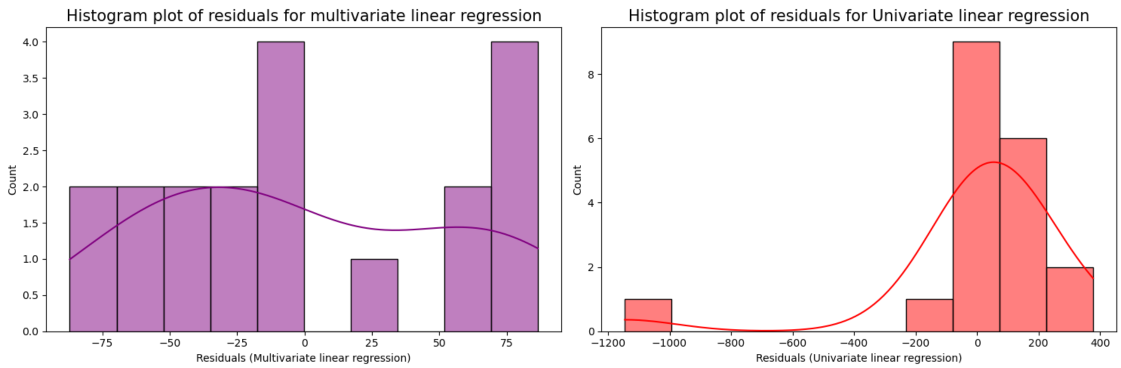
=> We plotted the histogram for the residuals from both univariate linear regression and multivariate linear regression. Below is the code and plot generated.

[14]:
```
fig, axs= plt.subplots(1, 2, figsize= (15, 5))

sns.histplot(data= multivariate_ols_model.resid,  kde= True,bins=10,␣
↪color='purple', ax= axs[0])
axs[0].set_xlabel("Residuals (Multivariate linear regression)", fontsize = 10)
axs[0].set_title("Histogram plot of residuals for multivariate linear␣
↪regression", fontsize = 15)

sns.histplot(data= univariate_ols_model.resid,  kde= True,bins=10, color='red',␣
↪ax= axs[1])
axs[1].set_xlabel("Residuals (Univariate linear regression)", fontsize = 10)
axs[1].set_title("Histogram plot of residuals for Univariate linear regression",␣
↪fontsize = 15)
```

```
plt.tight_layout()
plt.show()
```



1.0.4 Are the residuals approximately normally distributed? Why or why not?

=> No, the residuals from both univariate linear regression and multivariate linear regression are not normally distributed because we we can see in figures plotted above, there is shape of both histogram plots are not resembled with the normal distribution plot.

[15]:
```
multivariate_ols_model.summary()
```

```
/home/buddha-thapa-magar/anaconda3/lib/python3.12/site-
packages/scipy/stats/_axis_nan_policy.py:531: UserWarning: kurtosistest only
valid for n>=20 ... continuing anyway, n=19
  res = hypotest_fun_out(*samples, **kwds)
```

[15]:

Dep. Variable:	GDP (billions)	**R-squared:**	0.998
Model:	OLS	**Adj. R-squared:**	0.998
Method:	Least Squares	**F-statistic:**	1501.
Date:	Sun, 10 Nov 2024	**Prob (F-statistic):**	1.80e-17
Time:	15:19:39	**Log-Likelihood:**	-102.90
No. Observations:	19	**AIC:**	217.8
Df Residuals:	13	**BIC:**	223.5
Df Model:	5		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P> |t|**	**[0.025**	**0.975]**
const	-2.148e+06	1.18e+05	-18.185	0.000	-2.4e+06	-1.89e+06
Year	1075.1900	59.122	18.186	0.000	947.465	1202.915
Lagged Employment	-19.3557	9.134	-2.119	0.054	-39.089	0.377
Quarter_num_2	253.3205	51.444	4.924	0.000	142.183	364.458
Quarter_num_3	572.6762	56.585	10.121	0.000	450.432	694.921
Quarter_num_4	832.0319	62.643	13.282	0.000	696.700	967.364

Omnibus:	3.870	**Durbin-Watson:**	0.790
Prob(Omnibus):	0.144	**Jarque-Bera (JB):**	1.443
Skew:	0.194	**Prob(JB):**	0.486
Kurtosis:	1.707	**Cond. No.**	1.59e+07

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 1.59e+07. This might indicate that there are
strong multicollinearity or other numerical problems.

[16]:
```
univariate_ols_model.summary()
```

```
/home/buddha-thapa-magar/anaconda3/lib/python3.12/site-
packages/scipy/stats/_axis_nan_policy.py:531: UserWarning: kurtosistest only
valid for n>=20 ... continuing anyway, n=19
  res = hypotest_fun_out(*samples, **kwds)
```

[16]:

Dep. Variable:	GDP (billions)	**R-squared:**	0.949
Model:	OLS	**Adj. R-squared:**	0.945
Method:	Least Squares	**F-statistic:**	313.1
Date:	Sun, 10 Nov 2024	**Prob (F-statistic):**	2.19e-12
Time:	15:19:39	**Log-Likelihood:**	-135.14
No. Observations:	19	**AIC:**	274.3
Df Residuals:	17	**BIC:**	276.2
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P> |t|**	**[0.025**	**0.975]**
const	-55.0648	1335.336	-0.041	0.968	-2872.378	2762.249
Lagged Employment	144.5184	8.167	17.695	0.000	127.287	161.750

Omnibus:	35.615	**Durbin-Watson:**	0.803
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	87.870
Skew:	-2.871	**Prob(JB):**	8.30e-20
Kurtosis:	11.833	**Cond. No.**	3.03e+03

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 3.03e+03. This might indicate that there are
strong multicollinearity or other numerical problems.

1.0.5 Significance Test: At the .05 significance level, does employment in the previous quarter significantly influence GDP? Support your answer using the regression output.

- Univariate linear regression:
  => If we look into the p-value of the "Lagged Employment", it has the value of 0 which is less than 0.05. It signifies that "Lagged Employment" has significance in determining the "GDP (Billions)".
- Multivariate linear regression:

=> If we look into the p-value of the "Lagged Employment", it has the value of 0.054 which is greater than 0.05. It means that "Lagged Employment" has less significance in determining the "GDP(Billions)" when comparing with other predictors.

1.0.6 Variation Explained: What percentage of the variation in GDP is explained by employment in the previous quarter?

- Univariate Linear Regression:
  => While looking into the R-squared value, it has value of 0.949. This means Employment in the previous quarter can explain the 94.9% of variation in GDP.
- Multivariate Linear Regression:
  => It has R-square value of 0.998. This means 99.8 % of the variation in GDP is explained by predictors. Predictors are "Year", "Lagged Employment", "One hot encoded quarters". But, "Lagged Employment" has the higher p-value among all predictors.

1.0.7 Impact of Employment Increase: Estimate the change in quarterly GDP for a 1 million increase in employment.

- Univariate linear regression:
  => We only have "Lagged Employment" as the predictors. Lagged Employment is in Millions and GDP is in billion. It has the coefficient of around 144.5. This means for 1 million increase in employment, there will be 144.5 billion increase in GDP.
- Multivariate linear regression:
  => We have "Year", "Lagged Employment (Millions)", and One hot encoded quarters as predictors. GDP is in billions. "Lagged Employment" has a coefficient of around -19.35. This means they have inverse associations. 1 Million increase in employment leads to 19.35 billion decrease in GDP.

2 Smartwatch_Pricing

Initially, We perform EDA and prepared multivariate linear regression model.

[17]:
```
smartwatch_price_df = pd.read_excel("./HW2_Datasets.xlsx", sheet_name=␣
↪"Smartwatch_Pricing")
smartwatch_price_df.head()
```

[17]:
```
   Price (USD)  Battery Life (hours)  Display Quality (1-5)  \
0          250                    24                      4
1          200                    18                      3
2          300                    30                      5
3          150                    16                      3
4          400                    36                      5

   Water Resistance (meters)
0                         50
1                         30
2                        100
```

```
3                          20
4                         150
```

[18]:
```
smartwatch_price_df.describe()
```

[18]:
```
       Price (USD)  Battery Life (hours)  Display Quality (1-5)  \
count    20.000000             20.000000              20.000000
mean    298.000000             27.600000               3.850000
std      89.639629              7.315449               0.988087
min     140.000000             15.000000               2.000000
25%     242.500000             21.750000               3.000000
50%     297.500000             28.500000               4.000000
75%     367.500000             33.250000               5.000000
max     430.000000             40.000000               5.000000

       Water Resistance (meters)
count                  20.000000
mean                   78.250000
std                    46.205348
min                    10.000000
25%                    38.750000
50%                    70.000000
75%                   112.500000
max                   160.000000
```

[19]:
```
smartwatch_price_df.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 20 entries, 0 to 19
Data columns (total 4 columns):
 #   Column                     Non-Null Count  Dtype
---  ------                     --------------  -----
 0   Price (USD)                20 non-null     int64
 1   Battery Life (hours)       20 non-null     int64
 2   Display Quality (1-5)      20 non-null     int64
 3   Water Resistance (meters)  20 non-null     int64
dtypes: int64(4)
memory usage: 772.0 bytes
```

[20]:
```
X_train_sw = smartwatch_price_df.drop(columns= ["Price (USD)"])
y_train_sw = smartwatch_price_df[["Price (USD)"]]

X_train_sw_with_const = sm.add_constant(X_train_sw)

sw_ols_model = sm.OLS(y_train_sw, X_train_sw_with_const).fit()
```

[21]:
```
sw_ols_model.summary()
```

[21]:

Dep. Variable:	Price (USD)	**R-squared:**	0.925
Model:	OLS	**Adj. R-squared:**	0.911
Method:	Least Squares	**F-statistic:**	65.52
Date:	Sun, 10 Nov 2024	**Prob (F-statistic):**	3.32e-09
Time:	15:19:39	**Log-Likelihood:**	-91.915
No. Observations:	20	**AIC:**	191.8
Df Residuals:	16	**BIC:**	195.8
Df Model:	3		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P> |t|**	**[0.025**	**0.975]**
const	-80.8143	56.929	-1.420	0.175	-201.499	39.870
Battery Life (hours)	12.9974	3.057	4.252	0.001	6.518	19.477
Display Quality (1-5)	13.2338	8.426	1.571	0.136	-4.629	31.096
Water Resistance (meters)	-0.3944	0.512	-0.771	0.452	-1.479	0.690

Omnibus:	1.532	**Durbin-Watson:**	2.622
Prob(Omnibus):	0.465	**Jarque-Bera (JB):**	1.186
Skew:	0.388	**Prob(JB):**	0.553
Kurtosis:	2.095	**Cond. No.**	901.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

2.0.1 Model Significance: Is the overall model significant at the 0.05 level? Explain your reasoning.

=> F-statistic signifies the overall model significant. For out multivariate linear regression model, It has F-statistics value of $3.32 * 10^{-9}$ which is much smaller than 0.05. This means overall model has significance at the 0.05 level.

2.0.2 Predicted Price: Use the model to predict the price of the new smartwatch

Tech Innovate expects their new product to have 20 hours of battery life, a display quality of 4, and 30 meters of water resistance.
We predict the price of the new product by using the linear regression model we trained.

[22]:
```python
new_smartwatch_with_const = pd.DataFrame({
    "const": [1],
    "Battery Life (hours)": [20],
    "Display Quality (1-5)": [4],
    "Water Resistance (meters)": [30]
})

new_sw_predicted_price = sw_ols_model.predict(new_smartwatch_with_const)

print(f"Predicted price for the new smartwatch: ${new_sw_predicted_price.iloc[0]:
 ↪.2f}")
```

```
Predicted price for the new smartwatch: $220.24
```

2.0.3 Significant Drivers: Based on the model, which factors are significant drivers of price? Are there any variables that seem counterintuitive?

Variable	coef	std err	t	P>|t|
const	-80.8143	56.929	-1.420	0.175
Battery Life (hours)	12.9974	3.057	4.252	0.001
Display Quality (1-5)	13.2338	8.426	1.571	0.136
Water Resistance (meters)	-0.3944	0.512	-0.771	0.452

As we can see in the table above, "Battery Life (hours)" has least p-value which means it is the most significant factor while determining variance in price. After that, "Display Quality" has the moderate significance and "Water Resistance (meters)" has least significance.

While looking into the coefficients of 3 predictors, - "Battery Life (hours)" and "Display Quality (1-5)" has positive coefficient values. This means whenever battery life or display quality increases, it increases the price of the watch. - But, "Water Resistance (meters)" has negative coefficient. It is the counterintuitive variable. whenever, water resistance increases, it tends to decrease the price of watch.

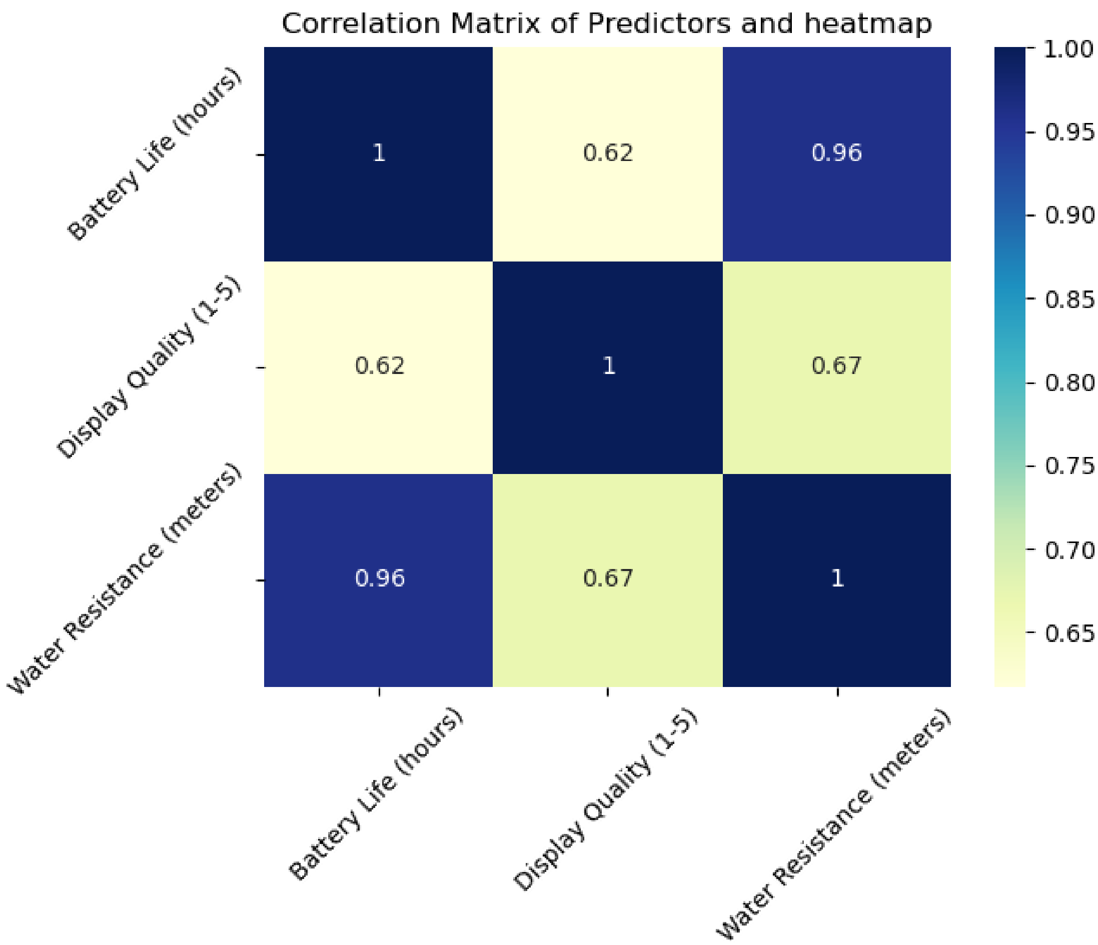
2.0.4 Correlation Analysis: Check for multicollinearity by calculating the correlations between the drivers. Which pairs of variables show a strong correlation?

[23]:
```
# Price field dropped as we are trying to check multicollinearity among␣
↪predictors
sw_correlation_matrix = smartwatch_price_df.drop(columns= ["Price (USD)"]).corr()
sw_correlation_matrix
```

[23]:
```
                           Battery Life (hours)  Display Quality (1-5)  \
Battery Life (hours)                   1.000000               0.617456
Display Quality (1-5)                  0.617456               1.000000
Water Resistance (meters)              0.960879               0.668344

                           Water Resistance (meters)
Battery Life (hours)                        0.960879
Display Quality (1-5)                       0.668344
Water Resistance (meters)                   1.000000
```

[24]:
```
sns.heatmap(data= sw_correlation_matrix, annot= True, cmap= "YlGnBu")
plt.xticks(rotation=45)
plt.yticks(rotation=45)
plt.title('Correlation Matrix of Predictors and heatmap')
plt.show()
```



According to the above correlation matrices, it is found that - battery life and water resistance has the maximum correlation coefficient of 0.96. It means there is strong association and collinearity between “Battery Life (hours)” and “Water Resistance (meter)”.

2.0.5 Revised Model: If necessary, remove any highly correlated drivers and rerun the regression. Has the model improved?

=> As we can see, there is high correlation between “Battery Life (hours)” and “Water Resistance (meter)”, we will be removing “Water Resistance (meter)” from predictors and preparing new linear regression model.

[25]:
```
new_X_train_sw = smartwatch_price_df.drop(columns= ["Price (USD)", "Water␣
↪Resistance (meters)"])

new_X_train_sw_with_const = sm.add_constant(new_X_train_sw)

new_sw_ols_model = sm.OLS(y_train_sw, new_X_train_sw_with_const).fit()
```

[26]:
```python
new_sw_ols_model.summary()
```

[26]:

Dep. Variable:	Price (USD)	**R-squared:**	0.922
Model:	OLS	**Adj. R-squared:**	0.913
Method:	Least Squares	**F-statistic:**	100.4
Date:	Sun, 10 Nov 2024	**Prob (F-statistic):**	3.85e-10
Time:	15:19:40	**Log-Likelihood:**	-92.280
No. Observations:	20	**AIC:**	190.6
Df Residuals:	17	**BIC:**	193.5
Df Model:	2		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P> |t|**	**[0.025**	**0.975]**
const	-42.1472	26.583	-1.585	0.131	-98.233	13.939
Battery Life (hours)	10.7902	1.056	10.222	0.000	8.563	13.017
Display Quality (1-5)	10.9969	7.815	1.407	0.177	-5.492	27.486

Omnibus:	1.555	**Durbin-Watson:**	2.494
Prob(Omnibus):	0.460	**Jarque-Bera (JB):**	1.287
Skew:	0.464	**Prob(JB):**	0.525
Kurtosis:	2.172	**Cond. No.**	130.

Yes, Removing the highly correlated predictor (In this case Water Resistance) has improved the model.

The newly trained linear regression model has a performance similar to the original model. - -

- The New model has an R-squared value of 0.922 which mean Battery life and Display quality can explain the 92.2% variance in price. But in the old model, there is a 0.925 R-squared value which means Battery life, Display quality, and Water Resistance (meters) 92.5% variance in price.
- If we look into the adjusted R-squared value of both the old model and the new model, the new model has a higher adjusted R-squared value of 0.913 in comparison with the old model which has adjusted R-squared value of 0.911. This means, that removing the water resistance variable as a predictor improves the predictive ability of the model.