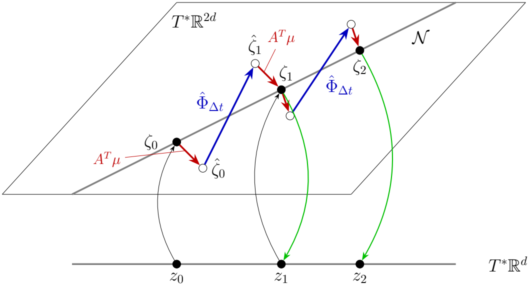
Extended Phase Space Integrators with Symmetric Projection

Given an extended phase space integrator $\hat{\Phi}_{\Delta t}\colon T^*\mathbb{R}^{2d} \to T^*\mathbb{R}^{2d}$ and $z_n = (q_n, p_n) \in T^*\mathbb{R}^d$,

1. $\zeta_n := (q_n, q_n, p_n, p_n) \in \mathcal{N}$
2. Symmetric projection onto $\mathcal{N}$
   1. $\hat{\zeta}_n := \zeta_n + A^T\mu$
   2. $\hat{\zeta}_{n+1} := \hat{\Phi}_{\Delta t}(\hat{\zeta}_n)$
   3. $\zeta_{n+1} := \hat{\zeta}_{n+1} + A^T\mu \in \mathcal{N}$
3. $z_{n+1} := (q_{n+1}, p_{n+1})$



$\mu \in \mathbb{R}^{2d}$ must be determined so that $\zeta_{n+1} \in \mathcal{N}$ where

$$\mathcal{N} := \left\{(q, q, p, p) \in T^*\mathbb{R}^{2d} \mid q, p \in \mathbb{R}^d\right\} = \ker A, \quad A := \begin{bmatrix} I_d & -I_d & 0 & 0 \\ 0 & 0 & I_d & -I_d \end{bmatrix}.$$

semiexplicit: $\hat{\Phi}$ is **explicit** but finding μ is **implicit**.