Standard notations for Deep Learning

This document has the purpose of discussing a new standard for deep learning mathematical notations.

1 Neural Networks Notations.

General comments:

· superscript (i) will denote the i^{th} training example while superscript [l] will denote the l^{th} layer

Sizes:

$\cdot m$: number of examples in the dataset

$\cdot n_x$: input size

$\cdot n_y$: output size (or number of classes)

$\cdot n_h^{[l]}$: number of hidden units of the l^{th} layer

In a for loop, it is possible to denote $n_x = n_h^{[0]}$ and $n_y = n_h{}^{[\text{number of layers }+1]}$.

$\cdot L$: number of layers in the network.

Objects:

$\cdot X \in \mathbb{R}^{n_x \times m}$ is the input matrix

$\cdot x^{(i)} \in \mathbb{R}^{n_x}$ is the i^{th}example represented as a column vector

$\cdot Y \in \mathbb{R}^{n_y \times m}$ is the label matrix

$\cdot y^{(i)} \in \mathbb{R}^{n_y}$ is the output label for the i^{th} example

$\cdot W^{[l]} \in \mathbb{R}^{\text{number of units in next layer} \times \text{number of units in the previous layer}}$ is the weight matrix,superscript [l] indicates the layer

$\cdot b^{[l]} \in \mathbb{R}^{\text{number of units in next layer}}$ is the bias vector in the l^{th} layer

$\cdot \hat{y} \in \mathbb{R}^{n_y}$ is the predicted output vector. It can also be denoted $a^{[L]}$ where L is the number of layers in the network.

Common forward propagation equation examples:

$a = g^{[l]}(W_x x^{(i)} + b_1) = g^{[l]}(z_1)$ where $g^{[l]}$ denotes the l^{th} layer activation function

$\hat{y}^{(i)} = softmax(W_h h + b_2)$

· General Activation Formula: $a_j^{[l]} = g^{[l]}(\sum_k w_{jk}^{[l]} a_k^{[l-1]} + b_j^{[l]}) = g^{[l]}(z_j^{[l]})$

· $J(x, W, b, y)$ or $J(\hat{y}, y)$ denote the cost function.

Examples of cost function:

· $J_{CE}(\hat{y}, y) = -\sum_{i=0}^{m} y^{(i)} \log \hat{y}^{(i)}$

· $J_1(\hat{y}, y) = \sum_{i=0}^{m} \mid y^{(i)} - \hat{y}^{(i)} \mid$

2 Deep Learning representations

For representations:

· nodes represent inputs, activations or outputs

· edges represent weights or biases

Here are several examples of Standard deep learning representations

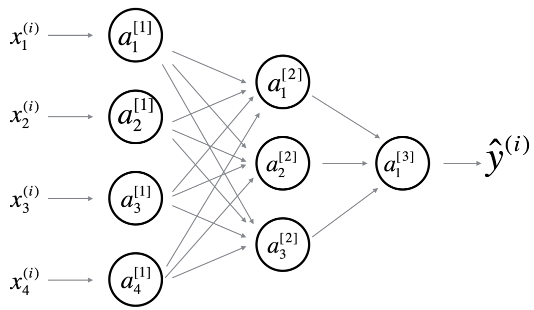


Figure 1: Comprehensive Network: representation commonly used for Neural Networks. For better aesthetic, we omitted the details on the parameters ($w_{ij}^{[l]}$ and $b_i^{[l]}$ etc...) that should appear on the edges

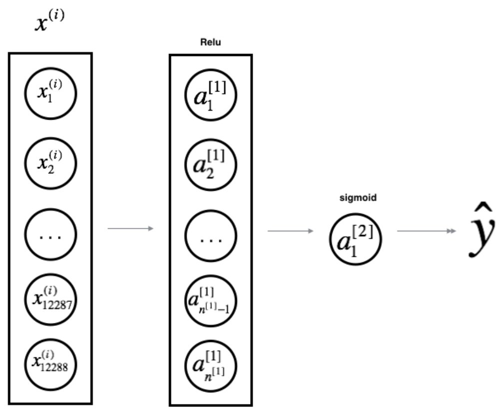


Figure 2: Simplified Network: a simpler representation of a two layer neural network, both are equivalent.

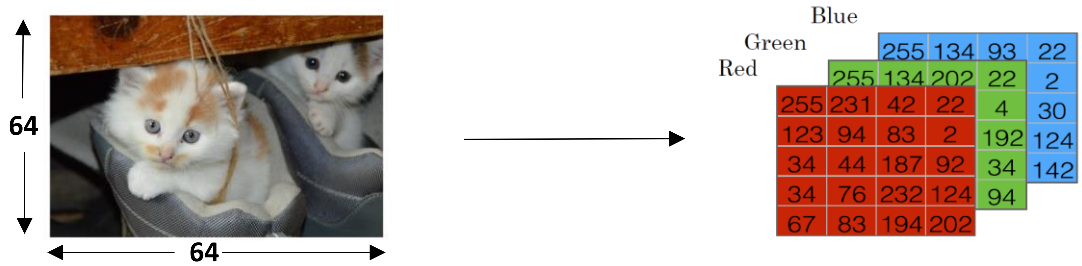
Binary Classification

In a binary classification problem, the result is a discrete value output.

For example - account hacked (1) or compromised (0)
- a tumor malign (1) or benign (0)

Example: Cat vs Non-Cat

The goal is to train a classifier that the input is an image represented by a feature vector, x, and predicts whether the corresponding label y is 1 or 0. In this case, whether this is a cat image (1) or a non-cat image (0).



An image is store in the computer in three separate matrices corresponding to the Red, Green, and Blue color channels of the image. The three matrices have the same size as the image, for example, the resolution of the cat image is 64 pixels X 64 pixels, the three matrices (RGB) are 64 X 64 each.

The value in a cell represents the pixel intensity which will be used to create a feature vector of n-dimension. In pattern recognition and machine learning, a feature vector represents an object, in this case, a cat or no cat.

To create a feature vector, x, the pixel intensity values will be “unroll” or “reshape” for each color. The dimension of the input feature vector x is $n_x = 64\ x\ 64\ x\ 3 = 12\,288$.

$$x = \begin{bmatrix} 255 \\ 231 \\ 42 \\ \vdots \\ 255 \\ 134 \\ 202 \\ \vdots \\ 255 \\ 134 \\ 93 \\ \vdots \end{bmatrix} \begin{array}{l} \left.\begin{matrix} \\ \\ \\ \\ \end{matrix}\right\} \text{red} \\ \left.\begin{matrix} \\ \\ \\ \\ \end{matrix}\right\} \text{green} \\ \left.\begin{matrix} \\ \\ \\ \\ \end{matrix}\right\} \text{blue} \end{array}$$

Logistic Regression

Logistic regression is a learning algorithm used in a supervised learning problem when the output y are all either zero or one. The goal of logistic regression is to minimize the error between its predictions and training data.

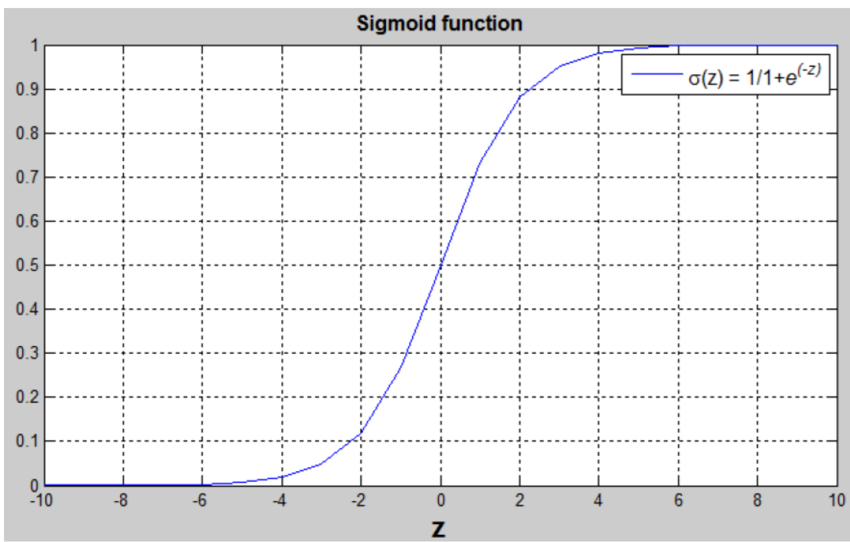
Example: Cat vs No - cat

Given an image represented by a feature vector x, the algorithm will evaluate the probability of a cat being in that image.

$$Given\ x\ ,\ \hat{y} = P(y = 1|x), \text{where } 0\ \leq \hat{y} \leq 1$$

The parameters used in Logistic regression are:

- The input features vector: $x\ \in \mathbb{R}^{n_x}$, where n_x is the number of features
- The training label: $y \in 0,1$
- The weights: $w\ \in \mathbb{R}^{n_x}$, where n_x is the number of features
- The threshold: $b\ \in \mathbb{R}$
- The output: $\hat{y} = \sigma(w^T x + b)$
- Sigmoid function: s = $\sigma(w^T x + b)$ = $\sigma(z)$= $\frac{1}{1+\ e^{-z}}$



$(w^T x + b)$ is a linear function $(ax + b)$, but since we are looking for a probability constraint between [0,1], the sigmoid function is used. The function is bounded between [0,1] as shown in the graph above.

Some observations from the graph:

- If z is a large positive number, then $\sigma(z)$ = 1
- If z is small or large negative number, then $\sigma(z)$ = 0
- If $z = 0$, then $\sigma(z)$ = 0.5

Logistic Regression: Cost Function

To train the parameters w and b, we need to define a cost function.

$x^{(i)}$ the i-th training example

Recap:

$\hat{y}^{(i)} = \sigma(w^T x^{(i)} + b)$, where $\sigma(z^{(i)}) = \frac{1}{1+e^{-z^{(i)}}}$

$Given\ \{(x^{(1)}, y^{(1)}), \cdots, (x^{(m)}, y^{(m)})\},\ we\ want\ \hat{y}^{(i)} \approx y^{(i)}$

Loss (error) function:

The loss function measures the discrepancy between the prediction ($\hat{y}^{(i)}$) and the desired output ($y^{(i)}$). In other words, the loss function computes the error for a single training example.

$$L(\hat{y}^{(i)}, y^{(i)}) = \frac{1}{2}(\hat{y}^{(i)} - y^{(i)})^2$$

$$L(\hat{y}^{(i)}, y^{(i)}) = -(\, y^{(i)} \log(\hat{y}^{(i)}) + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})$$

- If $y^{(i)} = 1$: $L(\hat{y}^{(i)}, y^{(i)}) = -\log(\hat{y}^{(i)})$ where $\log(\hat{y}^{(i)})$ and $\hat{y}^{(i)}$ should be close to 1
- If $y^{(i)} = 0$: $L(\hat{y}^{(i)}, y^{(i)}) = -\log(1 - \hat{y}^{(i)})$ where $\log(1 - \hat{y}^{(i)})$ and $\hat{y}^{(i)}$ should be close to 0

Cost function

The cost function is the average of the loss function of the entire training set. We are going to find the parameters $w\ and\ b$ that minimize the overall cost function.

$$J(w, b) = \frac{1}{m} \sum_{i=1}^{m} L(\hat{y}^{(i)}, y^{(i)}) = -\frac{1}{m} \sum_{i=1}^{m} [(\, y^{(i)} \log(\hat{y}^{(i)}) + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})]$$