A 4.1

(a) Jupyter Notebook

(b)

$$f_A(x) = 3^x$$
$$f_B(x) = \sqrt{x} + \log_2 x$$
$$f_C(x) = x^x$$
$$f_D(x) = x \log_2 x$$
$$f_E(x) = 2^{\sqrt{\log_2 x}}$$
$$f_F(x) = x^3 + 12x^2 + 200x + 999$$

Hypothese: $f_E(x) < f_B(x) < f_D(x) < f_F(x) < f_A(x) < f_C(x)$

Beweis:

(1.) $f_E(x) < f_B(x) \Longleftrightarrow 2^{\sqrt{\log_2 x}} \in O(\sqrt{x} + \log_2(x))$

Setze $x_0 = 2,\ c = 1 \Rightarrow f_E(x_0) = 2^1 < 1\sqrt{2} + 1 = c f_B(x_0)$

IS: Sei $f_E(x) \leq c \cdot f_B(x)$

$$f_E(x+1) = 2^{\sqrt{\log_2(x+1)}} \leq c \cdot f_B(x+1) = c \cdot (\sqrt{x+1} + \log_2(x+1))$$

(2.) $f_B(x) < f_D(x) \Longleftrightarrow \sqrt{x} + \log_2(x) \in O(x \log_2(x))$

$$\lim_{x \to \infty} \frac{\sqrt{x} + \log_2(x)}{x \log_2 x} \overset{\text{L'Hosp.}}{=} \lim_{x \to \infty} \frac{(\sqrt{x} + \log_2(x))'}{(x \log_2 x)'} = \frac{\underbrace{\frac{\sqrt{x}\log 2 + 2}{x \log 4}}_{\to 0}}{\underbrace{\frac{\log x + 1}{\log 2}}_{\to \infty}} \Rightarrow 0$$

(3.) $f_D(x) < f_F(x) \Longleftrightarrow x \log_2(x) \in O(x^3 + 12x^2 + 200x + 999)$

$$\Rightarrow \lim_{x \to \infty} \frac{x \log_2(x)}{x^3 + 12x^2 + 200x + 999} \overset{\text{L'Hosp.}}{=} \lim_{x \to \infty} \frac{(x \log_2(x))'}{(x^3 + 12x^2 + 200x + 999)'}$$

$$= \lim_{x \to \infty} \frac{(\log x + 1) \cdot \frac{1}{\log 2}}{3x^2 + 24x + 200} \overset{\text{L'Hosp.}}{=} \lim_{x \to \infty} \frac{(\log x \cdot \frac{1}{\log 2} + \frac{1}{\log 2})'}{(3x^2 + 24x + 200)'}$$

$$= \lim_{x \to \infty} \frac{\overbrace{\frac{1}{x \log 2}}^{\to 0}}{\underbrace{6x + 24}_{\to \infty}} \to 0$$

(4.) $f_F(x) < f_A(x) \Longleftrightarrow x^3+12x^2+200x+999 \in O(3^x)$

$$\lim_{x\to\infty} \frac{x^3+12x^2+200x+999}{3^x} \overset{\text{L'Hospital} \times 3}{=} \lim_{x\to\infty} \frac{(x^3+12x^2+200x+999)'''}{(3^x)'''}$$

$$= \lim_{x\to\infty} \left.\frac{6}{3^x \log^3(3)_{\to\infty}}\right\} \to 0$$

(5.) $f_A(x) \leq f_C(x) \Longleftrightarrow 3^x \in O(x^x)$

Setze $c=3,\ x_0=4$

<u>IA</u>: $\Rightarrow f_A(x) = 3^4 \leq 3\cdot 4^4 = f_C(x)$

<u>IS</u>: Sei $f_A(x) \leq f_C(x) \Rightarrow 3^x \leq x^x$

$\Rightarrow f_A(x+1) = 3^{x+1} = 3\cdot f_A(x)$

$f_C(x+1) = (x+1)^{x+1} = \underbrace{(x+1)\underbrace{(x+1)^x}_{\geq x^x}}_{\geq x^x}$

$\overset{IV}{\Longrightarrow} f_A(x+1) = 3\cdot f_A(x) \overset{c=3}{\leq} 3 f_C(x) \leq 3(x+1)^x \leq 3\cdot(x+1)(x+1)^x = 3 f_C(x+1)$

A 4.2

(a) Siehe Jupyter Notebook

(b)

```
def sieve1(N):
    primes = list(range(N+1))
    primes[1] = 0
    stop = N
    k = 2
    while k <= stop:
        j = 2*k
        while j <= N:
            primes[j] = 0
            j += k
        k += 1
    return [k for k in primes if k != 0]
```

$\in O(N)$

$\frac{j}{k}$ mal $\Rightarrow \in O(\ln N)$

$\in O(N \ln N)$

(c)

```
def sieve2(N):
    primes = list(range(N+1))
    primes[1] = 0
    stop = N
    k = 2
    while k <= stop:
        if primes[k] != 0:
            j = 2*k
            while j <= N:
                primes[j] = 0
                j += k
        k += 1
    return [k for k in primes if k != 0]
```

return [k for k in primes if k != 0]

$N-2$ mal $\Rightarrow \in O(N)$

$\frac{j}{p}$ mal $\Rightarrow \in O(\ln(\ln N))$

$\in O(N \ln(\ln N))$

(d) Siehe Jupyter Notebook