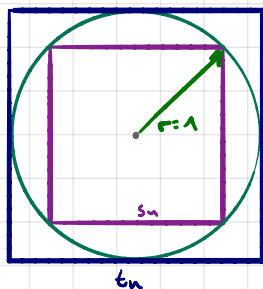


Aufgabe 3

Hoellriegel, Schmitt



$$U(\square) < U(\bigcirc) < U(\square)$$

$$\Rightarrow n s_n < 2\pi < n t_n$$

$$\Rightarrow \frac{n}{2} s_n < 2\pi < \frac{n}{2} t_n$$

$$s_{2n} = \sqrt{2 - \sqrt{4 - s_n^2}}$$

$$t_{2n} = \frac{2}{t_n} (\sqrt{4 + t_n^2} - 2)$$

A3b

$$s_{2n} = \sqrt{2 - \sqrt{4 - s_n^2}} = \sqrt{2 - 2 \cdot \underbrace{\sqrt{1 - \frac{s_n^2}{4}}}_{\substack{\rightarrow 0 \\ \rightarrow -1 \\ \rightarrow 2}}}$$

\Rightarrow Subtraktion zweier fast identischer Zahlen!

\Rightarrow Auslöschung!!! \Rightarrow Ergebnis besteht fast nur noch aus Rundungsfehlern.

A3c

alt: $\sqrt{2 - \sqrt{4 - s^2}}$

, $\frac{2}{t} (\sqrt{4 + t^2} - 2)$

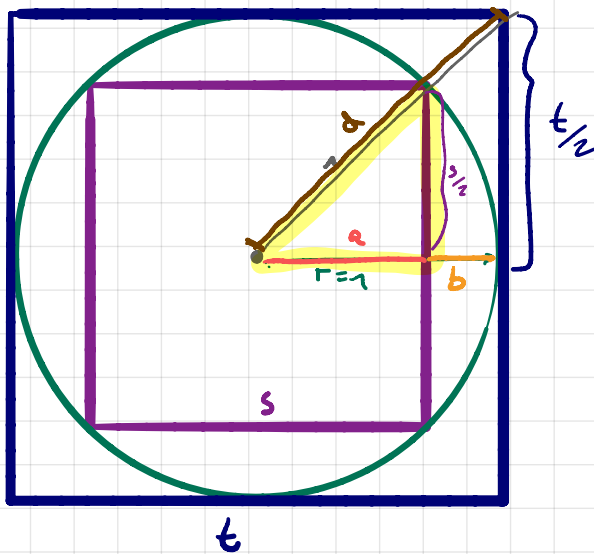
neu: $\frac{s}{\sqrt{2 + \sqrt{4 - s^2}}}$

, $\frac{2t}{\sqrt{4 + t^2} + 2}$

$$\frac{2}{t} \cdot (\sqrt{4 + t^2} - 2)$$

= ... ")

A3d

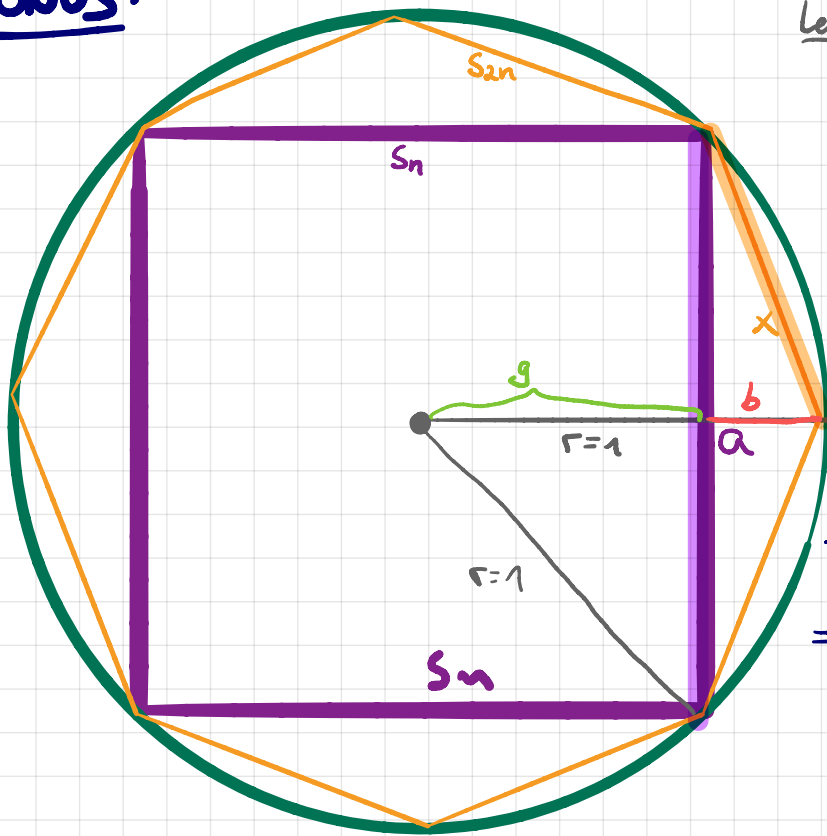


$$\begin{aligned} a^2 + \left(\frac{s}{2}\right)^2 &= 1^2 \\ \Rightarrow a^2 &= 1 - \frac{s^2}{4} \\ \Rightarrow a &= \sqrt{1 - \frac{s^2}{4}} \\ \Rightarrow b &= 1 - a = 1 - \sqrt{1 - \frac{s^2}{4}} \end{aligned}$$

$$\begin{aligned} \text{2. Strahlensatz} \quad \frac{t/2}{s/2} &= \frac{a}{1} \\ \Rightarrow t/2 &= a \cdot \frac{s}{2} \\ \Rightarrow t &= 2a \end{aligned}$$

$$\begin{aligned} \text{1. Strahlensatz: } \frac{d}{1} &= \frac{a+b}{a} \\ \Rightarrow t &= \frac{a+b}{a} \cdot s \\ &= \frac{a+1-a}{a} \cdot s = \frac{1}{a} \cdot s = \frac{1}{\sqrt{1 - \frac{s^2}{4}}} \cdot s \\ &= \frac{s}{\sqrt{1 - \frac{s^2}{4}}} \cdot 2 = \frac{2s}{\sqrt{4 - s^2}} = \frac{2s}{\sqrt{4 \cdot (1 - \frac{s^2}{4})}} = \frac{2s}{\sqrt{4 - s^2}} \end{aligned}$$

Bonus:



Lesbarkeit:

$$\text{sei } x := s_{2n}$$

$$\text{sei } a := s_n$$

$$\frac{a^2}{4} + b^2 = x^2$$

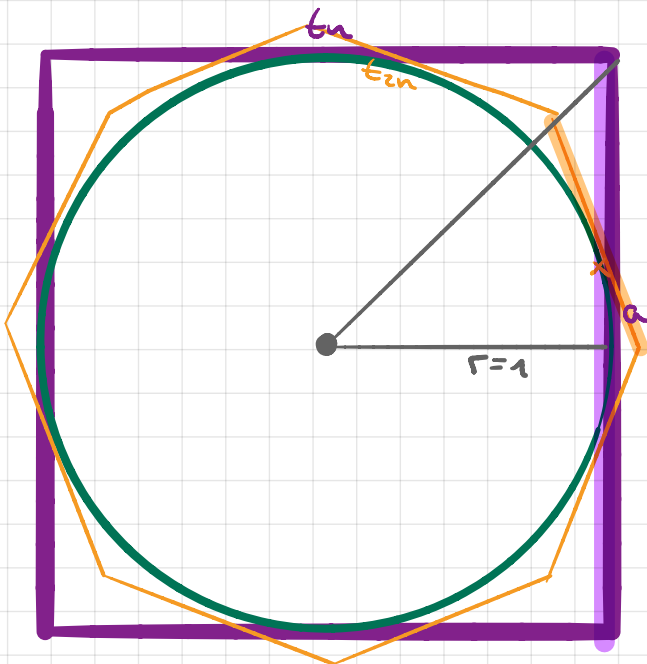
$$b = 1 - g = 1 - \sqrt{1 - \frac{a^2}{4}}$$

$$g^2 = 1 - \frac{a^2}{4} \Rightarrow g = \sqrt{1 - \frac{a^2}{4}}$$

$$\Rightarrow x^2 = \frac{a^2}{4} + (1 - \sqrt{1 - \frac{a^2}{4}})^2$$

$$\begin{aligned} \Rightarrow x^2 &= \frac{a^2}{4} + (1 - \sqrt{1 - \frac{a^2}{4}})^2 \\ &= \frac{a^2}{4} + 1^2 - 2\sqrt{1 - \frac{a^2}{4}} + 1 - \frac{a^2}{4} \\ &= -2\sqrt{1 - \frac{a^2}{4}} + 2 \\ &= 2 - \sqrt{1 - \frac{a^2}{4}} \end{aligned}$$

$$\Rightarrow x = s_{2n} = \sqrt{2 - \sqrt{1 - \frac{a^2}{4}}} = \sqrt{2 - \sqrt{1 - s_n^2}}$$



Lesbarkeit:

$$x := t_{2n}$$

$$a := t_n$$