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Gamma Distribution Probability Model for Asian Summer Monsoon Monthly Rainfall

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ABSTRACT—Using data from 39 well-distributed and long-record stations over the area, we found gamma distribution to be the most suitable probability model from among the Pearsonian models that show good fit to monthly rainfall in the Asian summer monsoon. We show that the monthly rainfall distribution is not Gaussian and the simple square-root, cube-root, and logarithmic transformations are of limited utility for normalizing the rainfall distribution.

A Craig type chart indicates that the rainfall distribution is a Type I distribution or a special or limiting case of this distribution; these distributions are fitted to monthly rainfall, and the goodness-of-fit is tested by the chi-square test. The gamma distribution (Pearson's Type III), which is a limiting case of Type I distribution and next to the Gaussian distribution in simplicity, gives a good fit to monthly rainfall at all the stations in each of the summer monsoon months; the Kolmogorov-Smirnov test and the variance ratio test confirm this good fit. The Type I distribution shows good fit to June rainfall at 26 stations, July rainfall at 31 stations, August rainfall at 24 stations, and September rainfall at 23 stations. Type IX, a special case of Type I, shows good fit to June rainfall at four stations, July rainfall at two stations, August rainfall at four stations, and September rainfall at three stations.

In cases where the gamma and other Pearsonian distributions show good fit, the gamma distribution is found to be the most suitable. The spatial distribution of the scale and shape parameters of the gamma distribution applied to monthly rainfall over the area is examined and the chief features of the distribution are indicated and explained. Deciles of the mixed gamma distribution applied to monthly rainfall are tabulated; these can be used to obtain the monthly rainfall probabilities required by any user.

1. INTRODUCTION

Because of the importance of rainfall distribution to agriculture and efficient utilization of the water resources, considerable effort has been made to graduate the rainfall of different time scales by fitting appropriate frequency functions. Sankaranarayanan (1933) tested for normality the frequency distribution of the southwest monsoon season rainfall at 68 representative stations in India, Pakistan, Burma, and Ceylon; he found that at the 5-percent level the moment coefficients of skewness, g_1, and kurtosis, g_2, were significantly different from zero for 34 and 15 stations, respectively. Pramanik and Jagannathan (1953) examined the annual rainfall series at 30 well-distributed stations over India and Pakistan and found significant departures from the Gaussian distribution at 13 stations. On the basis of data for 11 representative stations over India, Mooley and Crutcher (1968) showed that the monthly rainfall during the southwest monsoon season is gamma-distributed.

Barger and Thom (1949) found that the gamma distribution provides good fit to precipitation series in the United States. Thom (1951) considered precipitation and no-precipitation situations produced by different physical systems. He found that the actual rainfall distribution, which consists of precipitation and no precipitation, is a mixed distribution.

Momiyama and Mitsudera (1952) showed good fit of the gamma distribution to the monthly rainfall over Japan. Suzuki (1964, 1967) showed that the hyper gamma distribution gives a good fit to the monthly and annual rainfall at Tokyo and Niigata, Japan.

The purpose of this study is to determine whether or not a suitable unified probability model exists for the distribution of monthly rainfall associated with the Asian summer monsoon.

2. DATA

The stations used in this investigation were selected from the area, Equator to 35°N, and 70° to 140°E, since little monsoon influence is felt outside this area. All stations within this area and the periods of data available for them were carefully examined, and a fairly good representative network of 39 rain gage stations, each with a period of data exceeding 50 yr, was selected. This network and the period of data available are shown in figure 1. The rainfall data for these stations for the summer monsoon months, June, July, August, and September, were collected from the World Weather Records (Smithsonian Institution 1927, 1934, 1947, U.S. Department of Commerce 1959, 1967) to and including 1960. For Singapore, data to 1967 were used to get a rainfall record exceeding 50 yr. Singapore data for 1951–67 and Sandakan, Kutaradja, Menado, and Manila data for 1951–60 were obtained from the concerned meteorological services.

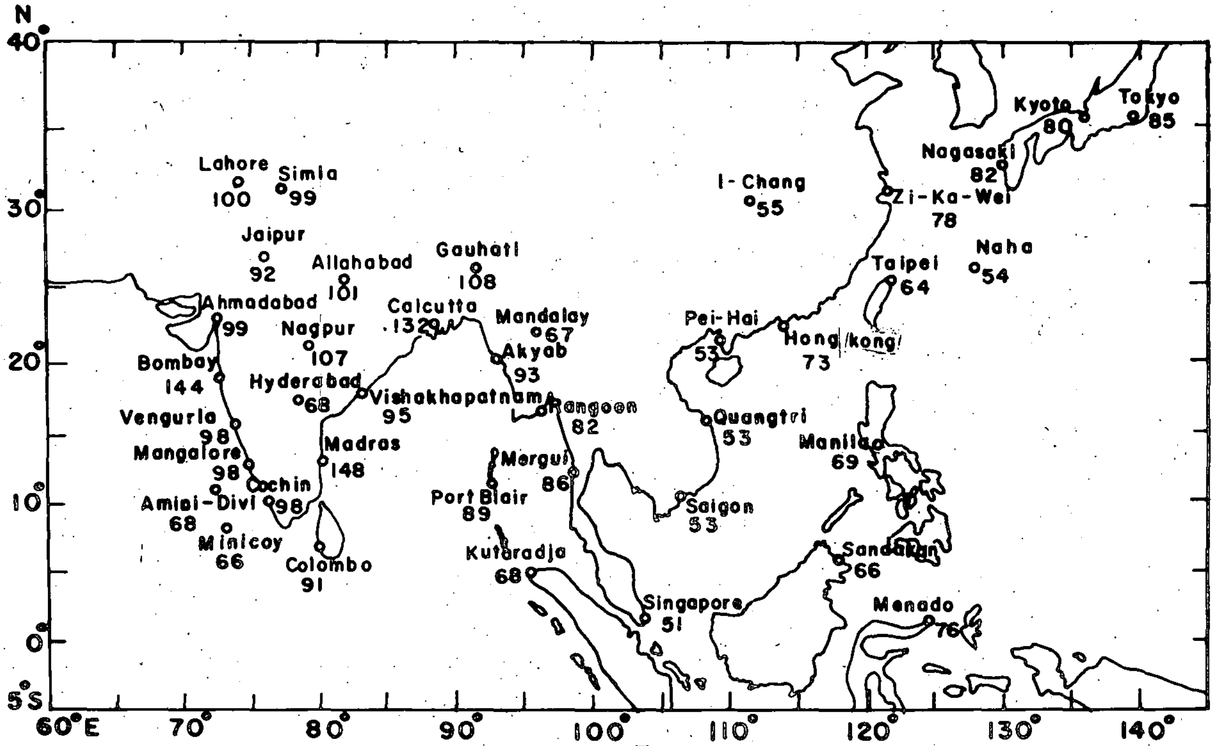

FIGURE 1.—Map showing network of rain gage stations. N, number of years of rainfall data, is given below station name.

TABLE 1.—*Details of noise-producing data sources*

Station	Data
Ahmadabad	Aug. 1868, June 1893, July 1927
Akyab	June 1863, Sept. 1916
Allahabad	June 1916, Aug. 1953
Amini Divi	June 1924, Sept. 1933
Bombay	Sept. 1930, Sept. 1949, Aug. 1958
Calcutta	Sept. 1900
Colombo	July 1878, Sept. 1889
Fort Cochin	July 1924, Aug. 1931
Gauhati	June 1860
I-Chang	July 1935
Jaipur	June 1873, Aug. 1892, Sept. 1924
Lahore	Sept. 1954, Sept. 1958
Madras	June 1870
Mandalay	July 1928, Aug. 1939
Mangalore	June 1868
Manila	Sept. 1914, Aug. 1919
Mergui	June 1888, Aug. 1925
Nagasaki	Sept. 1922, July 1957
Pei-Hai	Aug. 1918, July 1923
Quangtri	July 1919, June 1953
Simla	Aug. 1906
Taipei	July 1930
Tokyo	June 1938, July 1941, Sept. 1958

3. PRELIMINARY CONSIDERATIONS FOR THE CHOICE OF THE FREQUENCY FUNCTION

Pearson (1902*a*) stated that half the difficulty of curve-fitting lies in the choice of suitable function. In selecting a suitable function, Pearson (1902*b*) cautioned against multiplying constants to improve the fit since it is theoretically undesirable and does not necessarily lead to the required result. Elderton and Johnson (1969) also stressed these points; they considered in detail the question of fitting frequency functions to various types of data and stated that Pearsonian curves, which cover a wide range of skewness, have been fitted in various circumstances and agreements are satisfactory. Godske (1968) suggested that the Pearson model may be used for meteorological elements for which distributions are not Gaussian.

Inspection of the rainfall over Southeast Asia reveals that the monthly rainfall covers a wide range of skewness. Hence, in view of what has been stated above, we propose to examine the applicability of the Pearson model to monthly summer monsoon rainfall over Southeast Asia and to obtain the most suitable distribution if two or more Pearsonian distributions show good fit.

4. PEARSON MODEL

Elderton and Johnson (1969) discussed at length the various frequency distributions that come under the Pearson model, their criteria, and the computation of the values of their parameters. They also clearly showed how the differential equation of the Pearsonian system,

$$\frac{dy}{dx}=\frac{y(a+x)}{b_0+b_1x+b_2x^2}, \qquad (1)$$

can be obtained from the elementary propositions of the theory of probability.

5. CHOICE OF A SUITABLE DISTRIBUTION FROM THE PEARSON MODEL

Pearson (1916) gave the criteria for the different frequency distributions of his system in terms of $\beta_1(=\mu_3^2/\mu_2^3)$ and $\beta_2(=\mu_4/\mu_2^2)$ and put these in the form of a diagram called a *Rhind* diagram. The same diagram is also given by Pearson and Hartley (1962). Craig (1936) simplified this diagram by using $\delta=(2\beta_2-3\beta_1-6)/(\beta_2+3)$ instead of β_2. His diagram will hereafter be referred to as the Craig type chart, and this chart will be used for estimating a suitable frequency distribution for monthly rainfall at 39 representative stations in the Asian summer monsoon region. Initially, β_1 and β_2 were computed in each case. The basic purpose of the study is to find a suitable frequency distribution that generally fits the whole body of data. We should not place undue emphasis on fitting a distribution at the extreme ends; if we do that, the fit over the rest of the distribution would suffer. For rainfall distribution, this problem arises in the extreme upper end; that is, in the highest values. Since β_1 and β_2 involve third and fourth powers, respectively, they are subject to high random sampling fluctuations. In nature, events of different probabilities occur in time continuum. When we want to study these events, we take a sample over a period of time separated by two epochs. If, in this period, an event of very low probability has occurred, then such an event will be in the nature of noise over the data sampled for studying the properties of the distribution of the phenomenon. There does not appear to be any suitable way of treating such events of very low probability except to delete them. The number of rainfall observations at stations over the area is generally 70–100, and an event of one in 200 or more (i.e., with a probability ≤ 0.005) is considered in this study as an event of very low probability.

Table 42 of Pearson and Hartley (1962) gives, for the Pearsonian system of curves, percentage points expressed in standard units of the variate for a given β_1 and β_2. Using the computed values of β_1 and β_2 for the whole data in each case and the concerned table in Pearson and Hartley (1962), we examined the values at the upper end of monthly rainfall distribution, and the values having an occurrence probability of 0.005 or less were noted, These values, which are in the nature of noise over data. were deleted. The details of noise-producing data are listed in table 1.

Although these data are noise-producing for the purpose of the present paper, they might well be studied for other specific purposes (e.g., extreme value problem). For the sake of simplicity in data handling, the data for the years mentioned in this table were deleted, and β_1' and β_2', the revised value of β_1 and β_2, respectively, were calculated. From these revised values, δ values were calculated. These β_1' and δ values were entered in the Craig type charts. Figure 2 gives these charts for June, July, August, and September. The bulk of the points lie within the type I (black) region; only a few points appear to have strayed into other regions of the chart. It is possible, therefore, to infer as a first approximation that the monthly rainfall has type I distribution (bell-shaped and J-shaped curves only).

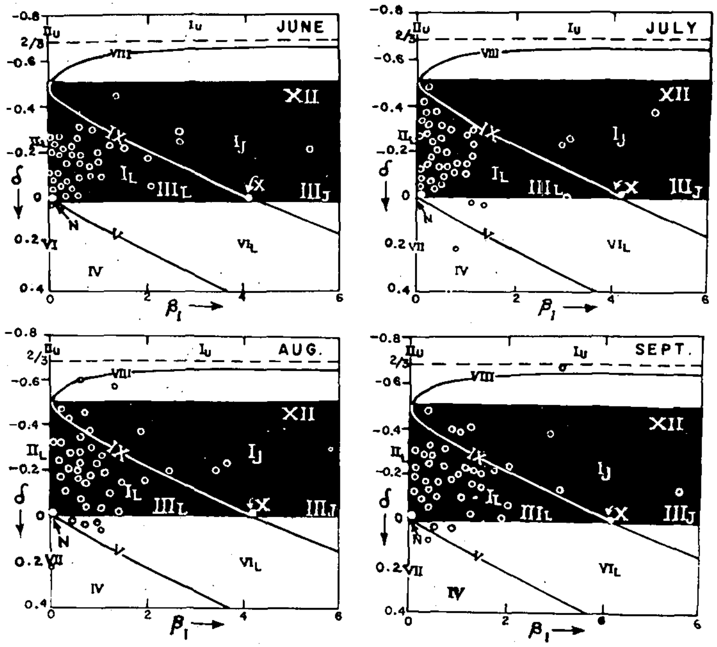

FIGURE 2.—Craig type chart for monthly rainfall distribution (subscripts L and J refer to bell- and J-shaped curves). The black area covers bell- and J-shaped Type I curves; $\delta=0$ and $\delta=-0.5$ define Type III and Type XII distributions, respectively.

Type I distribution with origin at the mode is given by

$$y=\frac{m_1^{m_1}m_2^{m_2}\Gamma(m_1+m_2+2)\left(1+\dfrac{x}{a_1}\right)^{m_1}\left(1-\dfrac{x}{a_2}\right)^{m_2}}{(a_1+a_2)(m_1+m_2)^{(m_1+m_2)}\Gamma(m_1+1)\Gamma(m_2+1)} \qquad (2)$$

where m_1, m_2 are the shape parameters. The beginning of the distribution is a_1 units before the mode and the end is a_2 units after the mode. The range is thus (a_1+a_2). The shape parameters, m_1 and m_2, are related to a_1 and a_2 by the relation $m_1/a_1=m_2/a_2$. The distribution covers U-, J-, and bell-shaped curves. When the beginning of the curve is referred to as the origin, this distribution transforms into

$$y=\frac{\Gamma(m_1+m_2+2)}{(a_1+a_2)\Gamma(m_1+1)\Gamma(m_2+1)}\left(\frac{x}{a_1+a_2}\right)^{m_1}\left(1-\frac{x}{a_1+a_2}\right)^{m_2} \qquad (3)$$

a form that is convenient for use with J-shaped curves.

If m_1 and m_2 are not small, the distribution tails off at both ends; if m_1 and m_2 are small, it rises abruptly at both ends. The following distributions are special cases of this distribution:

1. Gaussian or normal—limiting case, when $m_1=m_2=m$, $a_1=a_2=a$, a and $m\rightarrow\infty$, and (m/a^2) remains finite.
2. Type II—when $m_1=m_2=m$ and $a_1=a_2=a$.
3. Type III (gamma)—limiting case, when a_2 and $m_2\rightarrow\infty$ and (m_2/a_2) remains finite.
4. Type VIII—when m_1 is negative and $m_2=0$.
5. Type IX—when m_1 is positive and m_2 is zero, or m_2 is positive and m_1 is zero.

6. Type X (exponential)—limiting case, when $m_1=0$, a_2 and $m_2 \to \infty$, and (m_2/a_2) remains finite.

7. Type XII—when m_1 and m_2 are both arithmetically less than unity and are of opposite signs. This has a twisted J-shape.

6. ESTIMATION OF PARAMETERS

Pearson (1902*a*, 1902*b*) showed that the method of moments gives good results and is generally applicable. The least-square method also gives good results, but its applicability is limited to frequency distributions of type $y=a+bx+cx^2+\ .\ .\ .$ or one that can be converted into this type. Fisher (1922) showed that the moment estimates and the maximum likelihood (M.L.) estimates differ little when the distribution is close to normal but that the efficiency of moment estimates falls off rapidly with increasing deviations from normality. Consequently, he has advised the use of efficient M.L. estimates under these circumstances. It has also been shown that the M.L. method gives consistent estimates and that if a sufficient estimate exists, it is the M.L. estimate. In addition, Fisher (1924) showed that, when inconsistent and inefficient estimates of parameters are used, the computed chi square measures not only the deviation of observation from the hypothesis but also the deviation due to error in estimation of parameters. For that reason also, he has advised the use of M.L. estimates. In some cases, the equations from which M.L. estimates are to be obtained cannot be explicitly solved; however, in such cases, solutions of requisite accuracy can be obtained on the computer by iteration method. The method of minimum chi square generally leads to difficulties since the equations cannot be solved except on computer by iteration method. Because of the special advantages of estimation by the M.L. method, this method will be used in this study.

Each distribution has location, scale, and shape parameters. A distribution may have more than one shape parameter. In the case of monthly rainfall over southeast Asia, zero rainfall can be considered as an attainable lower bound, although the probability of attaining it would vary from one rainfall regime to another. In wet regimes, the probability of rainfall attaining the lower bound zero would be vanishingly small, and this is taken care of by the high value of the shape parameters, which leads to contact of very high order at the origin (i.e., at the zero rainfall point). Hence, in this study, the location parameter (i.e., the beginning of the distribution) is zero except in the case of normal distribution.

7. TEST FOR NORMALITY

The normal distribution is a limiting case of Type I distribution. The Craig type charts show that in some cases the monthly rainfall is close to normal distribution. It was therefore decided to test monthly rainfall for normality.

Rao (1952) mentioned that the goodness-of-fit test applied to observed frequency distribution to test nor-

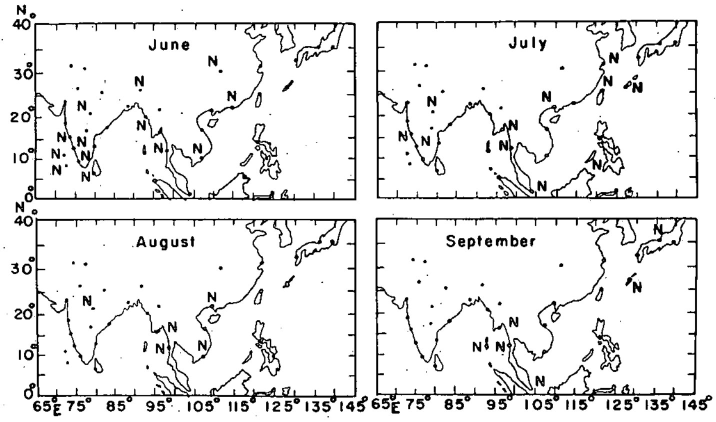

FIGURE 3.—Stations for which rainfall distribution is normal (N).

mality is insensitive in testing for some specific aspects of the distribution such as symmetry and kurtosis. Therefore, $g_1(=\mu_3/\mu_2^{3/2})$ and $g_2(=\mu_4/\mu_2^2)$, Fisher's measures of skewness and kurtosis, respectively, were computed and their departures from zero were tested for significance, in addition to applying the chi-square test. To compute g_1 and g_2, we used expressions from Cramer (1946) for consistent and unbiased estimates of second, third, and fourth moments of the distribution. To test the significance of g_1 and g_2, we used exact expressions for their mean and variance as given by Fisher (1930). The distribution is considered normal if none of the quantities, g_1, g_2, and the chi-square statistic, is significant at the 5-percent level. Figure 3 shows the stations for which the monthly rainfall is normal. Generally, the monthly rainfall distribution is not normal except over the southeastern Arabian sea, the west coast of India south of Bombay, the west coast of Ceylon, and the Burma coast, during June. The test for normality clearly indicates that monthly summer monsoon rainfall over southeast Asia is not Gaussian.

8. NORMALIZING TRANSFORMATIONS

We must next determine if monthly rainfall can be transformed into a Gaussian distribution by any transformation. Bartlett (1947), who considered in some detail the use of transformations, showed that the variance-stabilizing transformation often has the effect of improving the closeness of the distribution to normality and suggested the square root and the logarithmic transformations for variance stabilization. Constancy of variance is one of the important requisites for applicability of the analysis of variance. Freeman and Tukey (1950) suggested the transformation $\sqrt{(x)}+\sqrt{(1+x)}$. This was used by Landsberg et al. (1959).

Stidd (1953) applied the cube-root transformation to rainfall of different time scales in different climatic regimes and found the transformed distribution to be normal in many cases. However, he reported that this transformation was not satisfactory for rainfall at Malden Island (near the Equator), Laurie Island (high southerly latitudes), and some Hawaiian Islands. From this he inferred that the precipitation series of small ocean islands are not normalized by the cube-root transformation.

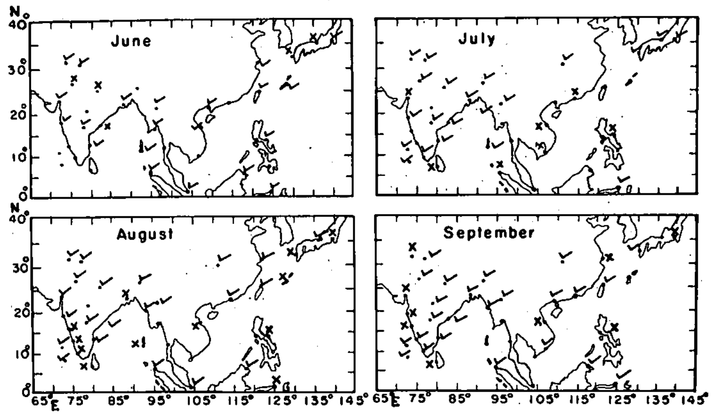

FIGURE 4.—Effect of square-root transformation on monthly rainfall. The tick marks and crosses, respectively, denote normalization and non-normalization on transformation.

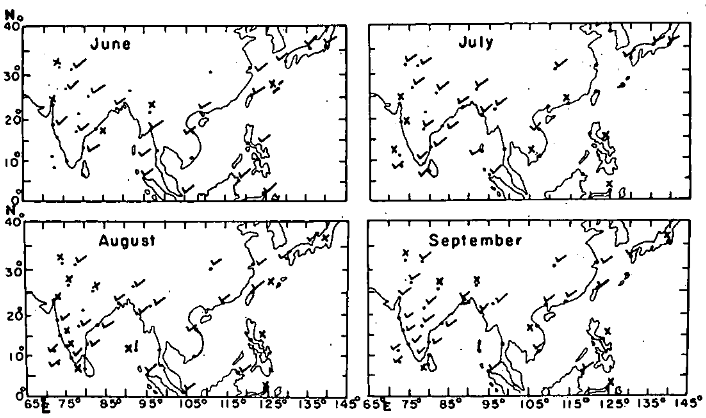

FIGURE 5.—Same as figure 4 for cube-root transformation.

Brooks and Carruthers (1953) mentioned the following forms for normalization:

$$Z=a+b\log(x+c), \tag{4}$$

$$Z=a_0+a_1x+a_2x^2+\ .\ .\ .+a_nx^n, \tag{5}$$

and

$$Z=a+bx^{1/n}. \tag{6}$$

Obviously, the appropriate transformation should be selected for each series of data. In applying a transformation, one must find the constants for each data series. The total number of constants to be evaluated is the number of constants in the normalizing form plus two constants for the transformed normal distribution. Determining so many constants may not offer any specific advantage. We propose to determine if the simple transformations, $\sqrt{x}$, $x^{1/3}$, and $\log(1+x)$ are suitable over the area under consideration.

The non-normal monthly rainfall has been transformed using these transformations, and the transformed distributions have been tested for normality in the manner indicated in section 7. The results are presented in figures 4–6 and summarized in table 2. In about 70 percent of the cases, each of the two transformations (the simple square root and the cube root) leads to normalization. The performance of the logarithmic transformation, however, is poor.

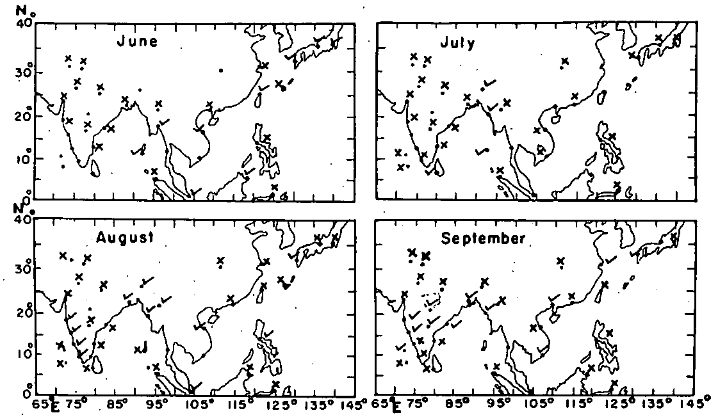

FIGURE 6.—Same as figure 4 for logarithmic transformation.

TABLE 2.—*Effect of normalizing transformations on the non-normal Asian summer monsoon monthly rainfall*

Month	No. of non-normal rainfall distributions	No. of distributions normalized by		
		$\sqrt{X}$	$X^{1/3}$	$\log(1+X)$
June	26	21	21	9
July	28	21	21	4
Aug.	34	22	22	12
Sept.	33	24	25	12

Note: The number refers to number of stations.

As stated previously, Stidd (1953) found that rainfall of some island stations are not normalized on cube-root transformation. Figure 5 shows that, in the present study, such stations are not confined to islands. In the 30 percent of the cases where the square-root or the cube-root transformation did not lead to normalization, there are a few cases for which none of the three transformations leads to normalization. The number of such stations is, one for June, four for July, and five each for August and September. These transformations, therefore, have limited utility from the viewpoint of the normalization of the frequency function for monthly rainfall over southeast Asia during the summer monsoon.

9. GAMMA DISTRIBUTION

The gamma distribution (i.e., Type III from the Pearsonian system) is a limiting case of Type I distribution. It is next to the normal distribution in simplicity, and, at the same time, it covers a wide range of skewness. We therefore decided to test the fit of monthly rainfall to gamma distribution for which the probability density function is given by

$$\left.\begin{aligned} f(x)&=\frac{x^{\gamma-1}e^{-x/\beta}}{\beta^\gamma\Gamma(\gamma)} && \text{for } x>0;\ \gamma,\ \beta>0 \\ \text{and}\quad f(x)&=0 && \text{for } x\leq 0 \end{aligned}\right\} \tag{7}$$

where β and γ are scale and shape parameters, respectively. The exponential distribution is a particular case when $\gamma=1$.

Thom (1958) reviewed important properties of the gamma distribution, computation of M.L. estimates of parameters, efficiency of moment estimates, and variance-covariance matrix of parameters. The skewness and the kurtosis coefficients, g_1 and g_2, respectively, and the coefficient of variation are simple functions of the shape parameter only. If ξ' and ξ'' are two independent gamma variates having shape parameters γ' and γ'', respectively, and a common scale parameter β, then $\xi(=\xi'+\xi'')$ is a gamma variate with scale parameter β and shape parameter $\gamma(=\gamma'+\gamma'')$. This is referred to as the additive or reproductive property of the gamma distribution. If the two scale parameters, β' and β'', are not identical, the additive property can still be used, provided β' and β'' are not significantly different. In this case, β can be taken as $(\beta'+\beta'')/2$. In situations of the common scale parameter, β, Weatherburn (1961) has shown that $\xi'/(\xi'+\xi'')$ or $\xi''/(\xi'+\xi'')$ is a beta variate of the first kind with parameters γ' and γ'' or γ'' and γ', respectively. This property can be used to get the distribution of a ratio like, (July rain)/(rainfall for July and August).

M. L. Estimates of the Parameters

Thom (1958) showed that $\hat{g}$, the M.L. estimate of γ, can be obtained approximately by solving a quadratic equation; he has given a table of corrections to be applied to this value of $\hat{g}$. In this study, $\hat{g}$ was obtained by solving the following equation by Newton's method on the computer:

$$\xi(\hat{g})\equiv\psi(\hat{g})-\ln(\hat{g})-\ln(G_m/A_m)=0. \tag{8}$$

Here, ln is the natural logarithm, G_m and A_m are the geometric and the arithmetic means, respectively, of the rainfall amounts $x_1, x_2, x_3, \cdots, x_n$, and

$$\psi(\hat{g})=\frac{\partial \ln \Gamma(\hat{g})}{\partial \hat{g}}=\text{di-gamma function.}$$

To solve the equation, let $g^{(0)}$ be the first guess. Then put $g^{(1)}=g^{(0)}+h$ for $\hat{g}$ in eq (8), expand to the first power of h, solve for h, and get

$$h=\frac{-\psi(g^{(0)})+\ln(g^{(0)})+\ln\left(\dfrac{G_m}{A_m}\right)}{\psi'(g^{(0)})-\left(\dfrac{1}{g^{(0)}}\right)} \tag{9}$$

where $\psi'=\partial\psi/\partial\hat{g}=$ tri-gamma function.

Next, put the second approximation, $g^{(2)}=g^{(1)}+h$, for $\hat{g}$ in eq (8) and continue the iterations until $g^{(n)}\approx\hat{g}$ is obtained to the desired accuracy. The process is discontinued after n iterations when

$$|\xi(\hat{g})-\xi(g^{(n)})|\leq 0.0001. \tag{10}$$

Convergene was rapid. After obtaining $\hat{g}$, we obtained $\hat{b}$ from the relation

$$\hat{b}\hat{g}=\frac{1}{n}\sum_{i=1}^{n} x_i.$$

Variance-Covariance Matrix

As shown by Thom (1958), the variance-covariance matrix of the parameters of the gamma distribution is given by

$$\begin{bmatrix} \operatorname{var}(\hat{b}) & \operatorname{cov}(\hat{b},\hat{g}) \\ \operatorname{cov}(\hat{b},\hat{g}) & \operatorname{var}(\hat{g}) \end{bmatrix} = \begin{bmatrix} \dfrac{\hat{b}^2\psi'(\hat{g})}{n(\hat{g}\psi'(\hat{g})-1)} & \dfrac{-\hat{b}}{n(\hat{g}\psi'(\hat{g})-1)} \\ \dfrac{-\hat{b}}{n(\hat{g}\psi'(\hat{g})-1)} & \dfrac{\hat{g}}{n(\hat{g}\psi'(\hat{g})-1)} \end{bmatrix}. \tag{11}$$

These matrices were computed for M.L. estimates of the parameters of the gamma distribution fitted to monthly rainfall. These give large sample variances. Fisher (1922) demonstrated that M.L. estimates are normally distributed in large samples. Since data in excess of 50 yr have been used in this study, these can be considered as large samples. Therefore, the variances provided by these matrices could be used to obtain the confidence limits for $\hat{b}$ and $\hat{g}$. These confidence limits could be utilized to test the significance of the difference between the parameters for 2 mo.

The gamma function is required in the computation of the theoretical probabilities based on the gamma model, and the di-gamma and the tri-gamma functions are required in the computation of the M.L. estimates, $\hat{b}$ and $\hat{g}$, of the parameters of the gamma model by Newton's method and in the computation of the variances of $\hat{b}$ and $\hat{g}$. Tables for the gamma, the di-gamma, and the tri-gamma functions are given by Davis (1933) and Abramowitz and Stegun (1964). These tabulated values cannot be used, however, when the iteration procedure has to be followed or the number of computations is large. In this study, these functions were evaluated by computer. For computing $\ln\Gamma(\gamma)$, the logarithmic form of Sterling's series (Davis 1933, Vol. I, p. 181) truncated to include only terms containing the 15th and smaller powers of $(1/\gamma)$ has been used; for $\gamma\geq 4$, the function is accurate to 11 decimal places. The value of $\psi(\gamma)$ is found to an accuracy of eight decimal places for $\gamma\geq 50$ by using the series obtained by differentiating the truncated series for $\ln\Gamma(\gamma)$. The value of $\psi'(\gamma)$ is computed to an accuracy of nine decimal places for $\gamma\geq 6$ by using the series obtained by differentiating the truncated series for $\ln\Gamma(\gamma)$ twice. For γ smaller than these limits, the functions can be computed without loss of accuracy by using the concerned reduction formulas.

Tests for Gamma Distribution

Chi-square test. The chi-square test of goodness-of-fit was applied to monthly rainfall. The number of intervals over which the test was applied varied from seven to 12, and each interval had a frequency of not less than five. The number of degrees of freedom is three less than the number of intervals. The chi-square statistic, χ_0^2, was calculated in each case. Table 3 gives the frequency for different ranges of $P(\chi^2\geq\chi_0^2)$. In no case is the chi-square statistic significant at the 5-percent level. The null

TABLE 3.—*Chi-square test for the null hypothesis that monthly rainfall is gamma-distributed*

Month	Frequency for different intervals of $P(\chi^2 \geq \chi_0^2)$							
	<0.01	≥0.01 but <0.05	≥0.05 but <0.10	≥0.10 but <0.25	≥0.25 but <0.50	≥0.50 but <0.75	≥0.75	Total
June	0	0	2	11	5	9	12	39
July	0	0	2	6	14	9	8	39
Aug.	0	0	1	13	10	10	5	39
Sept.	0	0	3	4	14	11	7	39
Total	0	0	8	34	43	39	32	156

hypothesis is therefore not contradicted, and the monthly rainfall can be taken to be gamma-distributed.

Because of the good fit of the gamma distribution to monthly rainfall at all the stations over the vast monsoon area, we decided to apply additional tests to confirm the exceptionally good fit. Accordingly, the Kolmogorov-Smirnov test (K-S test) and the variance ratio test were also applied.

K-S test. Massey (1951) showed that this test is more powerful than the chi-square test. The K-S test is applied to determine if there is agreement between an assumed theoretical distribution function and the empirical distribution function. Keeping (1962) mentioned that the test can be applied in situations where the theoretical distribution function is continuous. In the present case, the theoretical distribution is continuous since $\hat{b}$ and $\hat{g}$ are positive and x can assume all values. The test statistic used is

$$D_n = \max|S_n(x) - F(x)| \tag{12}$$

where $S_n(x)$ and $F(x)$ are empirical and theoretical distribution functions, respectively. The distribution of D_n is independent of $F(x)$. The theoretical distribution function has to be completely specified, however. In this study, the theoretical distributions have been computed by utilizing $\hat{b}$ and $\hat{g}$ computed from the samples. In such situations, Massey (1951) indicates that:

1. When the K-S test strongly implies rejection of the null hypothesis, rejection is the correct decision.
2. When rejection of the null hypothesis is not implied and D_n is near the critical level, there is some uncertainty as to the decision not to reject the null hypothesis.
3. When rejection of the null hypothesis is not implied and the D_n value is not near the critical level, then the nonrejection decision is correct.

D_n, the test statistic, was calculated in all cases. In addition, $D_n^{(0.05)}$ and $D_n^{(0.20)}$, the values of the test statistic at the 0.05 and 0.20 levels, respectively, were computed by using the asymptotic formulas, $1.36/\sqrt{n}$ and $1.07/\sqrt{n}$, respectively, as given by Massey (1951). D_n was less than $D_n^{(0.20)}$ in all cases. In 94 percent of the cases, the ratio $D_n/D_n^{(0.20)}$ was ≤ 0.80. Therefore, in accordance with what has been indicated by Massey (1951, para. iii), the null hypothesis is not rejected at the 5-percent level.

TABLE 4.—*Analysis of $d_n/d_n^{(0.20)}$*

Month	Frequency for different intervals of the ratio $d_n/d_n^{(0.20)}$						
	≤0.20	>0.20 but ≤0.40	>0.40 but ≤0.60	>0.60 but ≤0.80	>0.80 but ≤1.0	>1.0	Total
June	0	7	15	10	3	4	39
July	0	6	15	11	4	3	39
Aug.	0	7	13	7	10	2	39
Sept.	0	8	12	12	7	0	39
Total	0	28	55	40	24	9	156

Note: $d_n^{(0.20)}$ is obtained by using the asymptotic formula, $0.8/\sqrt{(n)}$, which is midway between those for the normal and the exponential distributions as given by Lilliefors (1967, 1969)

Lilliefors (1967, 1969) showed application of the K-S test to sampling from a normal distribution and from an exponential distribution when the parameters of the distribution are estimated from the sample. The test statistic is $d_n = \max|S_n(x) - F^*(x)|$, where $F^*(x)$ is the theoretical distribution function whose parameters are estimated from the sample. The asymptotic formulas given for the normal and the exponential distributions by Lilliefors (1967, 1969), and generally used for $n > 30$, are respectively, $0.736/\sqrt{n}$ and $0.86/\sqrt{n}$ for the 0.20 level and $0.886/\sqrt{n}$ and $1.06/\sqrt{n}$ for the 0.05 level. The gamma distribution reduces to the exponential distribution when the shape parameter equals unity and approaches the normal distribution when the shape parameter tends to infinity. Hence, the expressions $0.736/\sqrt{n}$ and $0.86/\sqrt{n}$ for the 0.20 level and $0.886/\sqrt{n}$ and $1.06/\sqrt{n}$ for the 0.05 level bracket the asymptotic expressions for the corresponding levels for the gamma distribution with the shape parameter equal to and greater than unity.

In only one individual case in the present study (viz, September rainfall at Lahore, Pakistan) is the shape parameter smaller than unity at the 5-percent level of significance; in all other cases, the shape parameter lies between 1.0 and 17.0. It is therefore reasonable to adopt, in the present case, the asymptotic formulas of $0.80/\sqrt{n}$ for the 0.20 level and $0.973/\sqrt{n}$ for the 0.05 level, which are midway between those for the normal and the exponential distributions. These formulas were used to compute $d_n^{(0.20)}$ and $d_n^{(0.05)}$, the values of d_n for 0.20 and 0.05 levels, respectively, in each case. An analysis of the ratio $d_n/d_n^{(0.20)}$ is given in table 4. In only nine cases (about 6 percent of the total) was $d_n > d_n^{(0.20)}$. In six of these nine cases, $d_n \leq d_n^{(0.15)}$; in one case, $d_n = d_n^{(0.10)}$. In only the two remaining cases, June rainfall at Nagpur, India, and July rainfall of Zi-Ka-Wei, China, was $d_n = d_n^{(0.05)}$. The null hypothesis is, therefore, not rejected at the 5-percent level, and the monthly rainfall is assumed to be gamma-distributed.

It may be noted that the asymptotic formula of $0.973/\sqrt{n}$ used for 0.05 level of d_n may correspond to that for a level of D_n, somewhere between 0.25 and 0.30 of the nonparametric tables, if the asymptotic formulas were extrapolated. But the fact that in 94 percent of the cases the ratio $D_n/D_n^{(0.20)}$ ≤ 0.80 indicates that in all these 94 percent of the cases $D_n \leq D_n^{(0.30)}$. Thus, the K-S test applied in the way suggested by Massey (1951) leads to the same result in the present study.

Variance ratio test. Cochran (1954) suggested this test for Poisson and binomial distributions. It can be used for all distributions for which theoretical variance can be computed independently from parameters estimated by a method other than the method of moments. This has been used here as a test for gamma distribution. Theoretical variance is $\hat{b}^2\hat{g}$ where $\hat{b}$ and $\hat{g}$ are, respectively, M.L. estimates of β and γ, the parameters of the gamma distribution. The test statistic is

$$\chi_\nu^2 = \frac{\left[\sum_{i=1}^{n} (x_i - \bar{x})^2\right]}{\hat{b}^2\hat{g}}. \tag{13}$$

This is to be referred to the chi-square table with $n-1$ degrees of freedom. Fisher and Yates (1957) pointed out that for $n-1$ degrees of freedom, where $n-1$ is greater than about 30, $\sqrt{(2\chi^2)}$ is approximately normally distributed with mean $\sqrt{(2n-3)}$ and standard deviation unity. Hence, to test the significance of χ_ν^2 when degrees of freedom exceed 30, one must calculate the expression $\sqrt{(2\chi_\nu^2)} - \sqrt{(2n-3)}$, which is a normal variate with zero mean and unit standard deviation. We calculated this expression for all cases and found the value to be significant in only three cases. These are September rainfall of Allahabad, India, and July rainfall of Zi-Ka-Wei, China, both significant at the 5-percent level, and June rainfall of Nagpur, significant at the 1-percent level. The number of cases in which χ_ν^2 is significant is not different from that which would be expected by chance. The null hypothesis is not contradicted, therefore, and the monthly rainfall can be taken to be gamma-distributed.

All three independent tests support the conclusion that the gamma distribution gives a good fit to monthly rainfall over southeast Asia during the summer monsoon season.

10. TYPE I DISTRIBUTION

This frequency distribution with origin at the start of the distribution is given by

$$f(x) = \frac{\Gamma(m_1+m_2+2)}{A\Gamma(m_1+1)\Gamma(m_2+1)} \left(\frac{x}{A}\right)^{m_1} \left(1-\frac{x}{A}\right)^{m_2} \quad \text{for } 0 \leq x \leq A$$

and

$$f(x) = 0 \qquad \text{for } x < 0 \tag{14}$$

where A is the scale parameter and m_1 and m_2 are the shape parameters. This is a limited distribution.

Equations for obtaining the M.L. estimates of the parameters of this distribution could not be explicitly solved. It was also not possible to eliminate two parameters from the three equations and obtain an equation in one parameter. An attempt was made to solve the three transcendental simultaneous equations on the computer by iteration method. Difficulties arose in the matter of convergence because the parameter A is involved as $\log(A-x_i)$. We decided, therefore, to estimate A from the data and then obtain the equations for M.L. estimates of m_1 and m_2. If x_M is the highest value of the observations considered, then the scale parameter, A, was estimated as $1.1x_M$. Putting $x/A=Z$, the equation for Type I distribution becomes

$$y = \frac{\Gamma(m_1+m_2+2)}{\Gamma(m_1+1)\Gamma(m_2+1)} Z^{m_1}(1-Z)^{m_2}. \tag{15}$$

This is known as the beta distribution of the first kind.

M.L. Estimates of the Parameters

The equations to be solved for getting M.L. estimates are

$$\psi(\hat{m}_1+\hat{m}_2+2) - \psi(\hat{m}_1+1) + \frac{1}{n}\sum_{i=1}^{n} \ln (Z_i) = 0 \tag{16}$$

and

$$\psi(\hat{m}_1+\hat{m}_2+2) - \psi(\hat{m}_2+1) + \frac{1}{n}\sum_{i=1}^{n} \ln (1-Z_i) = 0. \tag{17}$$

These have been solved on the computer by Newton's method and $\hat{m}_1$ and $\hat{m}_2$ have been obtained.

Variance-Covariance Matrix

Fisher (1922) showed that the variance-covariance matrix giving large sample variances of M.L. estimates is σ_{ij}/n where σ_{ij} is the inverse of σ^{ij} and

$$[\sigma^{ij}] = -E\left\{\frac{\partial^2 \ln [f(x)]}{\partial\theta_i \partial\theta_j}\right\}. \tag{18}$$

Here, θ_i are the parameters of the distribution and E, the expected value operator. Evaluating the elements of the matrix σ^{ij} and inverting, we obtain the variance-covariance matrix

$$\begin{bmatrix} \text{Var}(\hat{m}_1) & \text{Cov}(\hat{m}_1, \hat{m}_2) \\ \text{Cov}(\hat{m}_1, \hat{m}_2) & \text{Var}(m_2) \end{bmatrix} = \begin{bmatrix} \dfrac{-\psi'(\hat{m}_1+\hat{m}_2+2)+\psi'(\hat{m}_2+1)}{n\Delta} & \dfrac{\psi'(\hat{m}_1+\hat{m}_2+2)}{n\Delta} \\ \dfrac{\psi'(\hat{m}_1+\hat{m}_2+2)}{n\Delta} & \dfrac{-\psi'(\hat{m}_1+\hat{m}_2+2)+\psi'(\hat{m}_1+1)}{n\Delta} \end{bmatrix}. \tag{19}$$

where Δ, the determinant of $[\sigma^{ij}]$, is given by

$$\Delta = -\psi'(\hat{m}_1+\hat{m}_2+2)[\psi'(m_1+1)+\psi'(m_2+1)] + \psi'(m_1+1)\psi'(m_2+1). \tag{20}$$

The variance-covariance matrix was computed in each case.

Test of Goodness-of-Fit

The chi-square test was applied to test the goodness-of-fit of the monthly rainfall data to Type I distribution. The number of intervals over which the test was carried out varied generally from six to nine. The number of degrees of freedom varied generally from two to five, since four degrees of freedom were lost through restrictions. The fit was good for 26 stations in June, 31 stations in July, 24 stations in August, and 23 stations in September.

11. TYPE IX DISTRIBUTION

This frequency distribution is given by

$$y=\frac{m+1}{A}\left(1-\frac{x}{A}\right)^m \qquad \text{for } 0\leq x\leq A$$

and

$$y=0 \qquad \text{for } x<0. \tag{21}$$

This is a special case of Type I when $m_1 \approx 0$. It is a J-shaped distribution, cutting the ordinate at $y=(m+1)/A$ and the abscissa at $x=A$. An indication of the cases where Type IX can be tried was provided by the value of m_1 obtained in fitting Type I.

M.L. Estimates of Parameters

The equations to be solved for obtaining the M.L. estimates of parameters are

$$\hat{m}\sum_{i=1}^{n}\frac{1}{(\hat{A}-x_i)}-\frac{n(\hat{m}+1)}{\hat{A}}=0 \tag{22}$$

and

$$\sum_{i=1}^{n}\ln(\hat{A}-x_i)-n\ln(\hat{A})+\frac{n}{(\hat{m}+1)}=0. \tag{23}$$

Eliminating $\hat{m}$ between eq (22) and (23), we get

$$\frac{1}{n}\sum_{i=1}^{n}\ln(\hat{A}-x_i)-\ln(\hat{A})-\frac{1}{\left[\frac{1}{n}\sum_{i=1}^{n}\frac{x_i}{(\hat{A}-x_i)}\right]+1}+1=0. \tag{24}$$

Equation (24) was solved by Newton's method on the computer to obtain $\hat{A}$. Thereafter, $\hat{m}$ was obtained from eq (22).

Variance-Covariance Matrix

The following variance-covariance matrix was obtained by following the method given in the preceding section:

$$\begin{bmatrix}\text{Var}(\hat{A}) & \text{Cov}(\hat{A},\hat{m})\\ \text{Cov}(\hat{A},\hat{m}) & \text{Var}(\hat{m})\end{bmatrix}=\begin{bmatrix}\frac{\hat{A}^2\hat{m}^2(\hat{m}-1)}{n(\hat{m}+1)} & \frac{\hat{A}\hat{m}(\hat{m}^2-1)}{n}\\ \frac{\hat{A}\hat{m}(\hat{m}^2-1)}{n} & \frac{\hat{m}^2(\hat{m}+1)^2}{n}\end{bmatrix}. \tag{25}$$

It can be seen that the M.L. estimates are acceptable only when $m>1$, since, for $m<1$, var (A) becomes negative. This matrix was obtained in each case.

Test of Goodness-of-Fit

The chi-square test was applied. The number of degrees of freedom varied from three to six. The fit is found to be good for June rainfall at Madras and Ahmadabad, India, Lahore, Pakistan, and Quangtri, South Vietnam; for July rainfall at Mandalay, Burma, and Nagasaki, Japan; for August rainfall at Ahmadabad, Lahore, Nagasaki, and Quangtri; and for September rainfall at Colombo, Ceylon, I-Chang, China, and Jaipur, India.

Table 5.—Pearsonian distributions that are good fits to monthly rainfall

Station	Pearsonian distributions for monthly rainfall for			
	June	July	Aug.	Sept.
Ahmadabad	III/IX	III/I	III/IX/I	III
Akyab	N/III/I	III/I	III/I	III/I
Allahabad	III	III/I	III/I	III/I
Amini Divi	N/III/I	III	III/I	III/I
Bombay	III/I	III/I	III	III
Calcutta	III	III/I	III	III
Colombo	N/III/I	III	III	III/IX
Fort Cochin	N/III/I	III/I	III	III
Gauhati	N/III/I	III/I	III	III/I
Hong Kong	N/III/I	III/I	III/I	III/I
Hyderabad	III/I	III/I	III/I	III/I
I-Chang	N/III/I	III/I	III	III/IX/I
Jaipur	III	III/I	III/I	III/IX
Kutaradja	III/I	III	III/I	III
Kyoto	III	III	III/I	N/III/I
Lahore	III/IX	III/I	III/IX/I	III
Madras	III/IX	III/I	III/I	III/I
Mandalay	III	III/I/IX	III/I	III/I
Mangalore	N/III/I	N/III/I	III	III/I
Manila	III/I	III/I	III	III/I
Menado	III/I	III/I	III	III/I
Mergui	N/III	N/III/I	N/III	N/III/I
Minicoy	N/III/I	III/I	III/I	III/I
Nagasaki	III	III/IX/I	III/IX/I	III
Nagpur	N/III/I	N/III/I	N/III/I	III/I
Naha	III/I	N/III/I	III	N/III/I
Pei-Hai	III	N/III/I	N/III/I	III/I
Port Blair	III/I	III/I	III	N/III/I
Quangtri	III/IX	III	III/IX	III
Rangoon	III/I	N/III/I	N/III/I	N/III
Saigon	N/III/I	III	N/III/I	III
Sandakan	III/I	N/III/I	III/I	III/I
Simla	III/I	III/I	III/I	III/I
Singapore	III/I	N/III/I	III/I	N/III
Taipei	III/I	N/III/I	III/I	III
Tokyo	III/I	III/I	III	III
Vengurla	N/III/I	N/III/I	III	III/I
Vishakhapatnam	III	III	III/I	III/I
Zi-Ka-Wei	III/I	N/III	III/I	III

12. REMAINING SPECIAL CASES OF TYPE I DISTRIBUTION

Types II, VIII, X, and XII cover the remaining special cases of Type I distribution.

Type II is a symmetrical distribution. An indication of the cases in which Type I approximates this distribution can be had by examining the values of $\hat{m}_1$ and $\hat{m}_2$, shape parameters of the Type I distribution fitted to monthly rainfall. Variances of $\hat{m}_1$ and $\hat{m}_2$ have been computed in all cases wherein Type I shows good fit. Utilizing these, one can test whether $\hat{m}_1$ and $\hat{m}_2$ are approximately equal. Let

$\sigma_{\hat{m}_1}$ and $\sigma_{\hat{m}_2}$ be standard errors of $\hat{m}_1$ and $\hat{m}_2$. If $\hat{m}_2$ lies within the limits $\hat{m}_1 \pm 0.5\sigma_{\hat{m}_1}$ and $\hat{m}_1$ lies within the limits $\hat{m}_2 \pm 0.5\sigma_{\hat{m}_2}$, then $\hat{m}_1$ and $\hat{m}_2$ can be taken to be approximately equal and the distribution can be taken to be Type II. The parameters $\hat{m}_1$ and $\hat{m}_2$ were tested for equality in all cases of good fit of Type I. We found that Type I reduces to Type II for June rainfall at Minicoy and Vengurla, India, July rainfall at Akyab, Burma, and Mangalore, India, August rainfall at Saigon, South Vietnam, and September rainfall at Port Blair, India.

Type III reduces to Type X, the exponential, when γ is unity. γ is taken as unity if unity lies between the limits $\hat{g} \pm 0.5\sigma_{\hat{g}}$. These limits can be obtained in each case by using the computed values of $\hat{g}$ and variance of $\hat{g}$. On applying these limits, we found that γ can be taken as unity for June rainfall of Allahabad and Jaipur.

There was no case of Type I where the parameters $\hat{m}_1$ and $\hat{m}_2$ suggested that Type VIII would give a good fit and only one case (i.e., September rainfall at Lahore) where the parameters $\hat{m}_1$ and $\hat{m}_2$ of Type I suggested that Type XII may be a good fit. It was not considered necessary to fit Type II and Type X distributions and obtain the parameters of these distributions in the forementioned cases since using Type II in place of Type I and Type X instead of Type III would lead to only small differences.

13. COMPARATIVE FIT OF DISTRIBUTIONS

The Pearsonian distributions that show good fit to monthly rainfall are listed in table 5. This table, however, does not include the few cases of Type II and Type X mentioned in the preceding section since these have been covered under Type I and Type III, respectively. In some cases, three distributions show good fit. When two or more distributions show good fit to monthly rainfall, we must decide which distribution is the most suitable. Parrat (1961) presented a criterion for deciding upon the most suitable distribution in such situations. If ϕ, θ, and ψ are three distributions that show good fit to data $Y_{\phi i}$, $Y_{\theta i}$, and $Y_{\psi i}$, where $i=1$ to n, are frequencies for n identical intervals of the variate on the basis of these distributions, Y_i are empirical frequencies for the same intervals of the variate, and C_ϕ, C_θ, and C_ψ are the number of constants that have to be determined in fitting data to these three distributions, then, according to Parrat (1961), the values of the following quantities should be evaluated to determine the most suitable distribution function:

$$\Omega_\phi = \frac{\sum_{i=1}^{n} (Y_i - Y_{\phi i})^2}{n - C_\phi},$$

$$\Omega_\theta = \frac{\sum_{i=1}^{n} (Y_i - Y_{\theta i})^2}{n - C_\theta},$$

and

$$\Omega_\psi = \frac{\sum_{i=1}^{n} (Y_i - Y_{\psi i})^2}{n - C_\psi}.$$

The most suitable distribution function is decided upon by the lowest Ω value. If all three values are close to each other, then the decision as to the most suitable function is not possible on the basis of this criterion. The quantity evaluated in this study is $\sqrt{\Omega}$, which may be designated as the root-mean-square discrepancy (rmsd) between the actual frequency and the theoretical frequency.

The relative variation of the parameters may also be taken into account. If (a_ϕ, b_ϕ), (a_θ, b_θ), and (a_ψ, b_ψ) are the two parameters of the distribution functions ϕ, θ, and ψ, respectively, then the relative variations of the parameters are

$$\sqrt{\mathrm{Var}(a_\phi)}/a_\phi,\ \sqrt{\mathrm{Var}(b_\phi)}/b_\phi;$$

$$\sqrt{\mathrm{Var}(a_\theta)}/a_\theta,\ \sqrt{\mathrm{Var}(b_\theta)}/b_\theta;$$

and

$$\sqrt{\mathrm{Var}(a_\psi)}/a_\psi,\ \sqrt{\mathrm{Var}(b_\psi)}/b_\psi,$$

the variances being large-sample variances. The distribution with the smallest relative variation of the parameters should be preferred. The criteria of the rmsd and the relative variation of parameters would be used to decide upon the most suitable distribution when two or more distributions show good fit to monthly rainfall.

The gamma distribution has good fit in all the cases of monthly rainfall. Comparisons will, therefore, be made between gamma and Type I, gamma and normal, and gamma and Type IX distributions over their respective common areas.

Figure 7A gives a plot of the values of the criterion rmsd for gamma (i.e., Type III) and Type I distributions. For a large majority of the points, rmsd is smaller for the gamma distribution than for Type I distribution. In all 104 cases where Type III and Type I distributions show good fit to monthly rainfall, the relative variation of each of the parameters of the gamma distributions is smaller than the relative variation of each of the parameters of Type I distribution. Thus, on the basis of each criteria, gamma distribution is preferable to Type I distribution.

From figure 7B, a plot of the rmsd for the gamma and the normal distributions, one can see that in a majority of the cases the value of this criterion is smaller for the gamma distribution. In each case, relative variation of each of the parameters of the normal distribution is smaller than that of each of the parameters of the gamma distribution. In this case, a clear-cut decision is difficult—on the basis of the rmsd, the gamma distribution is preferable; on the basis of the relative variation of the parameters, the normal distribution is preferable. In this situation, both of the distributions are equally suitable and any one of them may be applied.

Figure 7C gives a plot of the rmsd for the gamma and Type IX distributions. In each case, the rmsd is smaller for the gamma distribution. In a large majority of the cases, the relative variation of each of the parameters

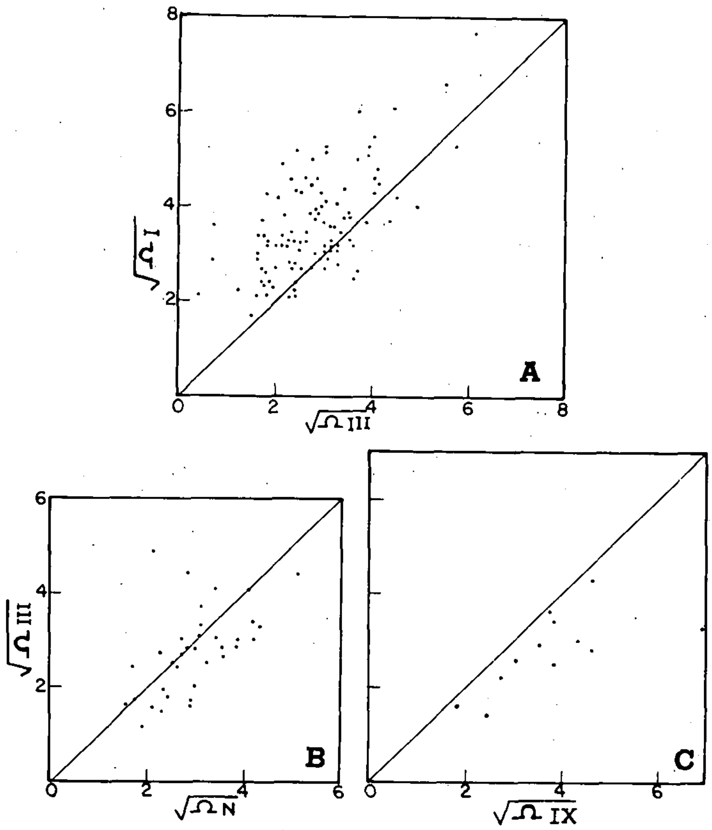

FIGURE 7.—Comparative fit of monthly rainfall for (A) Type I vs. Type III, (B) Type III vs. normal, and (C) Type III vs. Type IX distributions.

of the gamma distribution is smaller than that for each of the parameters of the Type IX distribution. On the basis of each of the criteria, the gamma distribution is preferable to Type IX distribution.

The preceding analysis indicates the general superiority of the fit of gamma distribution to monthly rainfall over that of other Pearsonian distributions during the Asian summer monsoon. The M.L. estimates of the parameters of the gamma distribution applied to monthly rainfall and their variances are given in table 6.

Suzuki (1964) gave the M.L. estimates of the hyper gamma distribution fitted to monthly rainfall of Toyko and Niigata, Japan, and their variances. These indicate that, for the monthly rainfall at these places, the parameter, α, of the hyper gamma distribution is not significantly different from unity at the 5-percent level except for February, July, and September rainfall at Niigata for which α is just significant at this level. When $\alpha=1$, the hyper gamma distribution reduces to the gamma distribution. Thus Tokyo and Niigata monthly rainfall distributions do not, in general, appear to be significantly different from the gamma distribution.

14. SPATIAL DISTRIBUTION OF THE PARAMETERS OF THE GAMMA MODEL APPLIED TO MONTHLY RAINFALL

In the preceding section, we showed that the gamma probability model is the most suitable among the Pear-

TABLE 6.—*Parameters of gamma distribution fitted to monthly rainfall and their variances*

Station	June $\hat{b}$* / Var($\hat{b}$)	June $\hat{g}$ / Var($\hat{g}$)	July $\hat{b}$ / Var($\hat{b}$)	July $\hat{g}$ / Var($\hat{g}$)	Aug. $\hat{b}$ / Var($\hat{b}$)	Aug. $\hat{g}$ / Var($\hat{g}$)	Sept. $\hat{b}$ / Var($\hat{b}$)	Sept. $\hat{g}$ / Var($\hat{g}$)
Ahmadabad	74.1	1.191	120.1	2.538	129.3	1.582	148.4	0.900
	144	0.024	326	0.119	400	0.044	630	0.013
Akyab	80.2	14.315	95.3	14.154	102.3	10.682	61.3	9.972
	143	4.404	202	4.302	234	2.432	84	2.115
Allahabad	97.1	0.990	73.3	4.166	69.3	4.239	97.0	1.724
	248	0.016	113	0.325	101	0.337	218	0.051
Amini Divi	40.2	9.076	89.4	3.321	84.2	2.221	57.6	2.443
	50	2.408	257	0.304	236	0.131	109	0.160
Bombay	111.7	4.530	126.4	4.985	116.9	3.117	112.1	2.338
	186	0.273	237	0.333	208	0.126	196	0.069
Calcutta	69.8	4,168	37.5	8.631	39.1	8.530	42.8	5.929
	78	0.246	22	1.095	24	1.069	29	0.508
Colombo	59.3	3.332	69.9	1.711	74.5	1.208	90.2	1.493
	84	0.227	125	0.056	151	0.026	213	0.041
Fort Cochin	68.8	10.875	84.1	6.867	61.5	5.420	87.9	2.729
	100	2.391	151	0.937	81	0.577	173	0.139
Gauhati	38.1	8.000	43.2	6.993	42.4	6.315	59.9	3.006
	28	1.160	36	0.881	35	0.715	72	0.154
Hong Kong	109.6	3.792	87.1	4.324	91.8	4.039	111.7	2.446
	347	0.350	217	0.476	242	0.413	371	0.145
Hyderabad	40.6	2.511	32.2	5.337	34.9	4.194	54.5	3.074
	53	0.164	32	0.789	37	0.480	93	0.251
I-Chang	48.4	3.269	61.3	3.437	57.2	3.228	73.6	1.491
	92	0.360	147	0.400	129	0.349	238	0.069
Jaipur	56.2	1.011	68.1	2.905	131.5	1.559	75.4	1.090
	91	0.018	111	0.171	459	0.047	165	0.022
Kutaradja	38.6	2.335	64.4	1.555	43.9	2.477	78.5	2.071
	48	0.141	141	0.059	61	0.160	201	0.107
Kyoto	54.0	4.317	73.2	2.767	60.2	2.447	53.4	3.757
	76	0.433	144	0.171	99	0.132	75	0.521
Lahore	33.2	1.265	73.2	1.942	81.7	1.616	87.5	0.756
	29	0.028	122	0.066	160	0.046	262	0.010
Madras	29.2	1.697	33.2	2.803	35.2	3.319	48.3	2.480
	13	0.033	16	0.096	18	0.136	35	0.076
Mandalay	60.2	2.089	42.1	1.806	25.9	4.129	41.6	3.569
	123	0.116	62	0.085	22	0.486	56	0.354
Mangalore	61.1	15.710	112.1	8.982	90.0	6.601	80.4	3.509
	78	4.983	264	1.604	172	0.855	141	0.235
Manila	88.4	2.883	105.6	4.012	132.9	3.235	58.0	5.904
	250	0.223	349	0.444	560	0.284	106	0.980
Menado	41.4	4.194	73.5	1.982	56.2	2.126	61.1	1.794
	47	0.429	165	0.093	97	0.109	117	0.076
Mergui	64.4	11.847	78.3	10.243	48.2	15.588	58.9	10.863
	100	3.250	149	2.420	56	5.665	84	2.726
Minicoy	37.3	7.839	55.4	4.093	82.0	2.344	75.0	2.147
	43	1.787	97	0.470	223	0.146	188	0.121
Nagasaki	97.1	3.367	116.6	2.192	89.2	2.102	89.6	2.712
	250	0.258	374	0.105	220	0.096	216	0.164
Nagpur	81.6	2.586	60.0	6.037	60.1	4.472	62.6	3.079
	135	0.111	69	0.646	71	0.348	78	0.160
Naha	80.6	3.579	50.7	3.688	115.4	2.320	52.3	3.410
	254	0.434	100	0.462	540	0.175	107	0.393
Pei-Hai	81.2	3.369	110.8	4.216	135.1	3.229	105.8	2.373
	274	0.406	503	0.646	760	0.371	479	0.194
Port Blair	53.3	9.460	60.9	6.452	60.2	6.521	62.4	7.475
	66	1.965	87	0.900	85	0.920	91	1.216
Quangtri	49.2	1.447	53.3	1.332	80.8	1.314	106.1	3.423
	111	0.068	135	0.058	305	0.055	477	0.428
Rangoon	42.1	11.655	41.1	13.715	37.7	13.797	24.7	15.973
	44	3.221	42	4.479	35	4.534	15	6.095
Saigon	23.3	13.490	29.5	9.836	32.4	8.271	28.4	11.803
	21	6.701	33	3.532	40	2.482	31	5.113
Sandakan	31.3	6.221	31.1	5.866	40.7	5.002	51.8	4.666
	31	1.113	30	0.987	52	0.711	85	0.616
Simla	67.9	2.474	58.5	7.520	58.1	7.318	74.7	2.333
	102	0.110	72	1.105	71	1.045	124	0.098
Singapore	29.0	5.866	42.2	3.779	43.9	3.886	25.9	6.131
	34	1.277	74	0.515	79	0.546	27	1.398
Taipei	67.0	4.548	69.1	3.437	128.6	2.245	90.6	2.466
	149	0.612	161	0.342	576	0.140	283	0.171
Tokyo	32.0	5.266	54.9	2.376	68.6	2.163	49.7	4.500
	26	0.636	80	0.121	126	0.099	63	0.460
Vengurla	104.0	8.190	163.4	5.751	101.5	4.984	93.5	2.845
	225	1.316	562	0.638	220	0.475	192	0.148
Vishakhapatnam	43.8	2.377	36.1	3.256	49.0	2.638	58.9	3.078
	44	0.105	29	0.203	55	0.130	78	0.180
Zi-Ka-Wei	54.9	3.294	78.0	1.890	50.1	2.787	51.2	2.561
	82	0.253	175	0.078	69	0.178	73	0.149

*$\hat{b}$ is in mm and Var($\hat{b}$), in mm².

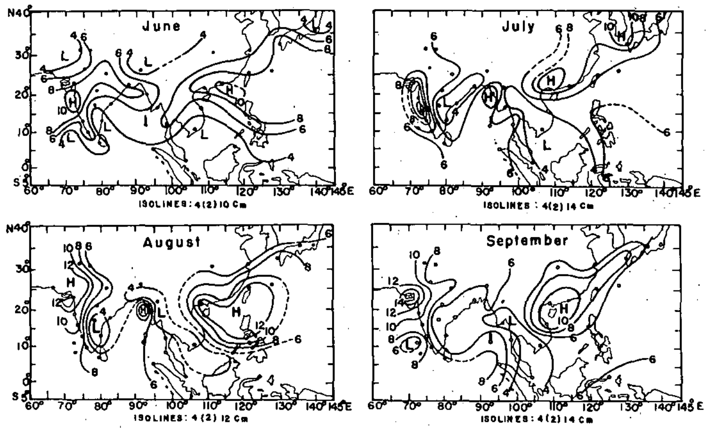

FIGURE 8.—Spatial distribution of scale parameter, $\hat{b}$, of the gamma model fitted to monthly rainfall.

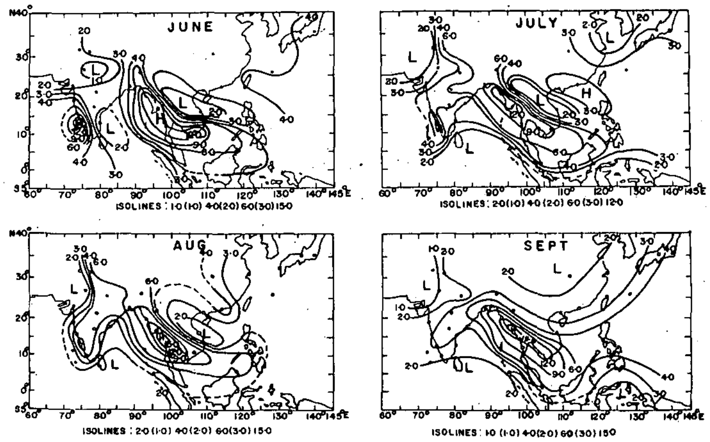

FIGURE 9.—Spatial distribution of shape parameter, $\hat{g}$, of the gamma model fitted to monthly rainfall.

sonian distributions applicable to monthly rainfall over southeast Asia. M. L. estimates, $\hat{b}$ and $\hat{g}$, of the parameters of the gamma distribution vary from one locality to another. Since $\hat{b}$, the scale parameter, equals $\mathrm{Var}(x)/\bar{x}$, any phenomenon that has the effect of increasing the variance more than the mean of the distribution will lead to higher values of the scale parameter. Similarly, since $\hat{g}$, the shape parameter, $=4/\beta_1=6/(\beta_2-3)$, any mechanism that increases the skewness and/or kurtosis coefficient of the distribution would decrease the value of the shape parameter.

Figure 8 shows the spatial distribution of $\hat{b}$. The coding, "Isolines: d_1 (d_2) d_3," has been used to denote that, beginning with the isoline for d_1, isolines are drawn at intervals of d_2, the last one being the isoline for d_3. The chief features of this distribution are the high values (exceeding 10 cm) between 15° and 25°N over western India and between 15° and 30°N east of 105°E. The latter region is influenced by typhoon rainfall, which has the effect of increasing the variance much more than the mean of the rainfall distribution; this leads to higher values of the scale parameter over this region. Westward-moving monsoon depressions located between 75° and 80°E generally intensify as a result of the increased influx of moisture from the Arabian Sea and give high rainfall over the portion of western India between 15° and 25°N. The high rainfall associated with these intensified depressions creates an effect similar to that of the storm rainfall in the eastern parts of Southeast Asia. For this reason, the values of the scale parameter are high between 15° and 25°N over western India.

Figure 9 shows the spatial distribution of the shape parameter, $\hat{g}$, of the gamma model applied to monthly rainfall. The values are high (exceeding nine) over the belt extending from South Vietnam to the Bangladesh coast in all the monsoon months. High values are also found over the central part of the west coast of India during June. Over the southern part of the west coast of India and the adjoining parts of the southeast Arabian sea, the shape parameter decreases markedly from June to July. Low values (less than 2) are observed over northwestern India, Pakistan and the southernmost

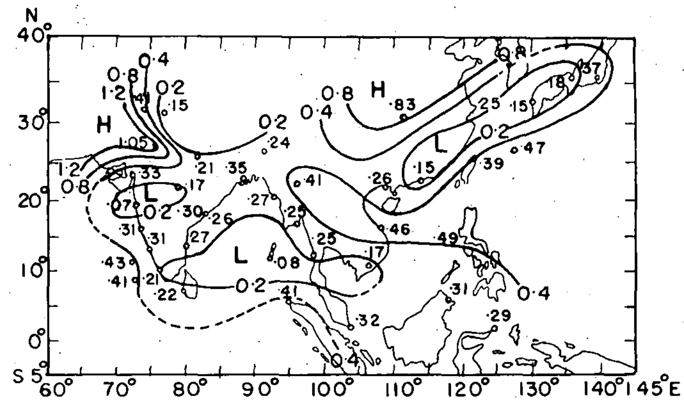

FIGURE 10.—Measure of variation of $\hat{b}$ within season.

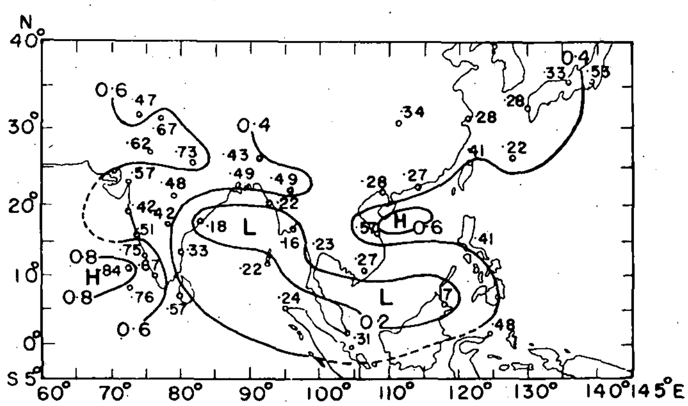

FIGURE 11.—Measure of variation of $\hat{g}$ within season.

parts of the area west of 100°E during all the monsoon months. The high values of $\hat{g}$ are due to a low skewness coefficient, and the low values are due to a high skewness coefficient.

Figures 10 and 11 show, respectively, the variation of $\hat{b}$ and $\hat{g}$ within the monsoon season. The measure adopted is

$$\frac{\sum_{i=1,j=2}^{i=3,j=4}|\hat{b}_i-\hat{b}_j|}{\dfrac{6\left(\sum_{i=1}^{4}\hat{b}_i\right)}{4}} \qquad \text{for } \hat{b},$$

and

$$\frac{\sum_{i=1,j=2}^{i=3,j=4} |\hat{g}_i-\hat{g}_j|}{6\left(\frac{\sum_{i=1}^{4}\hat{g}_i}{4}\right)} \quad \text{for } \hat{g}.$$

subject to the condition that $i<j$. Here, $\hat{b}_i$ and $\hat{g}_i$ (for $i=1$ to 4) are the values of the M.L. estimates of the parameters of the gamma distribution applied to rainfall of the four monsoon months, June through September. Variation of $\hat{b}$ within the monsoon season is small over and near the northern part of the Indian west coast, southern parts of the Bay of Bengal and of South Vietnam, and over the belt from southeastern China to the western part of southern Japan. The variation is high over Pakistan and adjoining parts of northwestern India. The chief features of the variation of $\hat{g}$ within the monsoon season are a narrow belt of small variation from northern Borneo to the northern parts of the east coast of India, large variation over the southern part of the west coast of India and the adjoining southeastern Arabian sea, and larger variation over most parts of India than over the rest of Southeast Asia.

15. DIFFERENCES IN THE SCALE PARAMETERS OF THE GAMMA MODEL APPLIED TO MONTHLY RAINFALL

Let $\hat{b}_1$, $\hat{b}_2$, $\hat{b}_3$, and $\hat{b}_4$ be scale parameters for rainfall at a station for the 4 monsoon mo, June, July, August, and September and $\sigma_{\hat{b}_1}$, $\sigma_{\hat{b}_2}$, $\sigma_{\hat{b}_3}$, and $\sigma_{\hat{b}_4}$, the corresponding standard errors. Fisher (1922) showed that large sample M.L. estimates are normally distributed. If each of the scale parameters lies within each of the intervals, $\hat{b}_1 \pm \sigma_{\hat{b}_1}$, $\hat{b}_2 \pm \sigma_{\hat{b}_2}$, $\hat{b}_3 \pm \sigma_{\hat{b}_3}$, and $\hat{b}_4 \pm \sigma_{\hat{b}_4}$, which constitute 68 percent confidence intervals for the scale parameters, then the four parameters can be assumed to be not different. If three satisfy this criterion, then these three will be taken as not different. In case only two satisfy this criterion, then they are considered to be not different. Table 7 gives months for which the scale parameters are not different. Mooley (1971) demonstrated that monthly rainfall over Southeast Asia during the summer monsoon season is pairwise independent. Using the additive property of the gamma distribution, we can, therefore, obtain the shape and the scale parameters for the rainfall distribution at a station for a 2-mo period from the shape and the scale parameters for any 2 mo mentioned in table 7 for that station. From these parameters, probabilities for the 2-mo rainfall can be obtained by using the tables of the gamma distribution (Salvosa 1930, Pearson 1957, Wilk et al. 1962, Thom 1968).

16. RAINFALL PROBABILITIES

Rainfall probabilities are required by a wide variety of clientele and the requirements vary widely. The gamma probability model has been found to be the most suitable for application to monthly rainfall for the Asian summer monsoon. This model can be used to obtain the rainfall probabilities. Yao (1971) computed precipitation probabilities for eastern Asia. Since the requirements of the users vary widely, it would not serve much useful purpose to compute and tabulate the probabilities of rainfall not exceeding/exceeding specified rainfall amounts. Computation and tabulation of the deciles of the mixed gamma distribution applied to monthly rainfall would, however, be very useful since any user can easily obtain the rainfall probabilities he needs from these values. These deciles should be computed for each of the monsoon months for each of the stations.

TABLE 7.—*Months for which scale parameters are not different*

Station	Months	Station	Months
Ahmadabad	July, Aug.	Mergui	June, Sept.
Akyab	July, Aug.	Minicoy	Aug., Sept.
Allahabad	July, Aug.	Nagasaki	June, Aug., Sept.
Do.	June, Sept.		
Amini Divi	July, Aug.	Nagpur	July, Aug., Sept.
Bombay	June, July, Aug., Sept.	Naha	July, Sept.
Calcutta	July, Aug., Sept.	Pei-Hai	July, Sept.
		Port Blair	June, July, Aug.
Colombo	July, Aug.		
Cochin	June, Aug.	Do.	July, Aug., Sept.
Do.	July, Sept.		
Gauhati	June, July, Aug.	Quangtri	June, July
		Rangoon	June, July Aug.
Hong Kong	July, Aug.		
Do.	June, Sept.	Saigon	July, Aug., Sept.
Hyderabad	July, Aug.		
I-Chang	July, Aug.	Sandakan	June, July
Jaipur	July, Sept.	Simla	July, Aug.
Kutaradja	June, Aug.	Do.	June, Sept.
Kyoto	June, Aug., Sept.	Singapore	July, Aug.
		Do.	June, Sept.
Lahore	July, Aug.	Taipei	June, July
Do.	Aug., Sept.	Tokyo	July, Sept.
Madras	July, Aug.	Vengurla	June, Aug., Sept.
Do.	June, July.		
Mandalay	July, Sept.	Vishakhapatnam	June, Aug.
Mangalore	Aug., Sept.	Zi-Ka-Wei	June, Aug., Sept.
Menado	Aug., Sept.		

Let the deciles be denoted by x_d where $d=1$ to 9. To obtain these, we must solve the following equation for x_d:

$$\frac{P_{x_d}-P}{1-P}=\int_0^{x_d} \frac{x^{\hat{g}-1}e^{-x/\hat{b}}}{(\hat{b}^{\hat{g}})\Gamma(\hat{g})}\,dx. \tag{26}$$

In eq (26), P_{x_d} stands for the probability of rainfall not exceeding x_d, and P stands for the empirical probability of no rain. For different deciles, $P_{x_d}=0.1, 0.2, \ldots,$ 0.9. All quantities in eq (26) except x_d are known. The values for $\hat{b}$ and $\hat{g}$ can be obtained from table 6. P is obtained from the rainfall data. If P is 0.1 or more, the first decile is indeterminate. In solving eq (26) for x_d, we followed a procedure similar to that used by Wilk et al. (1962).

The first step consists in obtaining the lower limits of the deciles, $x_{1L}, x_{2L}, \ldots, x_{9L}$, of the mixed gamma distribution.

Putting $x = x_d Z$ in eq (26), we get

$$\frac{P_{x_d} - P}{1-P} = x_d^{\hat{g}} \int_0^1 \frac{Z^{\hat{g}-1} e^{-x_d Z/\hat{b}}}{(\hat{b}^{\hat{g}})\Gamma(\hat{g})}\, dZ$$

and (27)

$$\frac{P_{x_d} - P}{1-P} \leq x_d^{\hat{g}} \int_0^1 \frac{Z^{\hat{g}-1}}{(\hat{b}^{\hat{g}})\Gamma(\hat{g})}\, dZ$$

since the maximum value of the exponential function is unity at $Z=0$. The lower limit of the decile is given, therefore, by

$$x_{dL} = \hat{b}\left[\Gamma(\hat{g}+1)\left(\frac{P_{x_d} - P}{1-P}\right)\right]^{1/\hat{g}}. \qquad (28)$$

The lower limit has been calculated for each decile ($d=1$ to 9).

The second step consists of obtaining two limits, one lower (Ll) and the other upper (Lu), between which the ninth decile, x_9, lies. This is done as shown in figure 12A by successively putting $x_d = x_{9L}, 2x_{9L}, 3x_{9L}, \ldots$ in eq (26) and noting the stage when the right side of eq (26) exceeds the left side.

The third step is the process of halving between Ll and Lu and continuing the process as shown in figure 12B until x_9 is obtained with desired accuracy. Let the first halving point be 1. We determine if this point is to the left or right of x_9 by evaluating the integral on the right side of eq (26) and comparing it with left side. As long as the halving point continues to remain to the left of x_9, halving is continued between the most recent halving point and Lu. Once the halving point goes to the right of x_9, halving is done between this point and the immediately preceding halving point. With every subsequent halving process, we must determine if the halving point is to the left or right of x_9. If it is to the right, then halving is done between this point and the immediately preceding halving point on the left of x_9. If it is to the left, then halving is done between this point and the immediately preceding halving point on the right of x_9. Halving is continued following this principle. The process of halving is stopped after n repetitions when $|P(x \leq x_{\text{mid}}^{(n)}) - P(x \leq x_9)| < \epsilon$ for a predetermined small value of ϵ. The term $x_{\text{mid}}^{(n)}$ is the value attained after n repetitions of the halving process and $P(x \leq x_9)$ is 0.9. In this study, $\epsilon = 0.0001$. At this stage, $x_{\text{mid}}^{(n)}$ is assumed equal to x_9. In a similar manner, we commence the process of halving between x_{8L} and x_9 and continue until we arrive at x_8. Following the same procedure, $x_7, x_6, \ldots, x_2$ and x_1, the remaining deciles, are obtained. Table 8 gives the deciles, in millimeters, of the mixed gamma distribution applied to monthly rainfall during the summer monsoon season at all the stations in Southeast Asia. Probabilities of rainfall less than any specified amount can be obtained by linear interpolation from this table with an accuracy that is sufficient for most purposes. If higher accuracy is desired, however, a smooth graph between the deciles and the probability may be prepared and the requisite probability interpolated from this smooth graph.

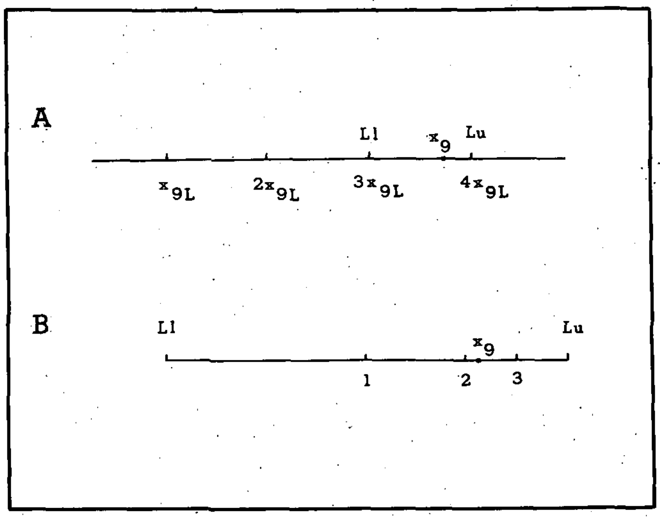

FIGURE 12.—Diagram of the procedure for (A) finding the lower limit, Ll, and upper limit, Lu, between which the ninth decile, X_9, lies and (B) further specifying X_9 to the desired accuracy.

TABLE 8.—*Deciles of mixed gamma distribution applied to monthly rainfall*

Station and period of rainfall	Deciles (mm)								
	1st	2d	3d	4th	5th	6th	7th	8th	9th
Ahmadabad									
June	9	21	33	47	62	81	105	137	192
July	99	144	184	224	266	313	370	444	561
Aug.	42	71	100	130	163	202	250	315	421
Sept.	6	21	38	58	83	113	153	210	309
Akyab									
June	780	888	971	1047	1121	1199	1285	1392	1549
July	915	1042	1141	1230	1318	1409	1512	1638	1825
Aug.	692	863	896	978	1059	1144	1241	1360	1537
Sept.	380	445	497	544	591	640	696	766	869
Allahabad									
June	8	19	32	47	64	86	114	153	220
July	136	178	213	247	281	319	363	419	506
Aug.	132	172	206	238	271	307	349	402	485
Sept.	36	60	84	108	135	165	203	253	336
Amini Divi									
June	221	261	293	323	352	383	418	461	527
July	116	159	195	231	268	308	356	418	515
Aug.	55	82	107	133	160	191	228	277	355
Sept.	45	65	84	102	122	144	171	206	261
Bombay									
June	235	303	360	414	470	530	599	688	825
July	306	389	458	522	588	660	742	847	1008
Aug.	137	190	235	280	326	378	438	517	641
Sept.	80	119	154	189	226	268	319	386	492
Calcutta									
June	130	170	203	235	268	304	346	399	482
July	193	229	258	285	312	340	372	411	471
Aug.	198	236	266	293	321	350	383	424	486
Sept.	133	164	190	215	239	266	296	334	393
Colombo									
June	78	106	130	154	178	205	237	279	343
July	27	44	61	78	97	119	146	182	241
Aug.	13	25	37	51	67	86	109	142	198
Sept.	26	45	64	84	106	132	164	208	281
Fort Cochin									
June	476	554	615	671	726	784	849	930	1050
July	319	389	445	498	550	606	670	751	872
Aug.	168	211	246	279	313	349	391	444	525
Sept.	83	118	149	179	211	247	290	346	435
Gauhati									
June	178	213	241	267	292	320	351	390	449
July	168	204	234	261	288	317	350	392	455
Aug.	144	177	204	229	254	281	312	351	410
Sept.	66	92	115	137	161	186	217	257	319
Hong Kong									
June	172	229	277	324	373	426	488	568	693
July	171	222	265	306	348	394	447	515	619
Aug.	162	213	257	298	340	387	441	511	618
Sept.	86	127	163	199	237	280	331	399	507

TABLE 8.—Continued

Station and period of rainfall	Deciles (mm)								
	1st	2d	3d	4th	5th	6th	7th	8th	9th
Hyderabad									
June	33	48	61	75	89	105	124	149	188
July	86	108	127	144	161	180	202	229	271
Aug.	65	85	102	118	135	153	174	201	242
Sept.	63	87	108	128	150	174	202	238	296
I-Chang									
June	61	84	104	123	142	164	190	223	276
July	84	114	140	165	190	219	252	296	363
Aug.	71	97	120	142	166	191	222	261	322
Sept.	18	34	50	66	85	106	132	168	228
Jaipur									
June	5	12	20	29	39	52	68	91	130
July	71	100	125	150	176	205	239	283	354
Aug.	35	66	95	125	159	198	247	312	420
Sept.	7	17	29	41	56	74	97	129	183
Kutaradja									
June	27	41	53	65	78	92	110	133	169
July	20	35	48	63	80	99	122	154	207
Aug.	35	51	65	79	94	111	132	159	201
Sept.	45	68	91	113	137	165	198	242	314
Kyoto									
June	106	138	164	190	216	244	277	319	384
July	70	100	126	152	179	209	245	292	366
Aug.	47	68	88	107	128	151	179	215	273
Sept.	85	113	137	160	183	209	240	279	340
Lahore									
June	3	9	15	22	29	38	49	64	89
July	37	57	77	97	119	143	173	213	278
Aug.	24	44	63	82	104	129	159	201	268
Sept.		1	7	16	28	43	64	94	149
Madras									
June	10	18	24	32	40	49	60	75	100
July	33	46	58	70	82	96	112	134	168
Aug.	46	62	77	91	105	121	140	165	203
Sept.	38	56	72	87	104	123	145	175	222
Mandalay									
June	35	53	70	88	106	128	153	187	242
July	18	29	40	51	63	76	93	115	152
Aug.	47	62	74	86	98	112	127	147	177
Sept.	61	81	99	117	134	154	177	207	253
Mangalore									
June	666	753	820	881	940	1002	1071	1156	1281
July	607	719	807	889	969	1055	1152	1273	1453
Aug.	323	396	455	509	564	623	690	775	903
Sept.	114	154	188	221	256	293	338	395	484
Manila									
June	91	128	161	193	226	263	308	365	456
July	185	243	293	340	389	442	504	584	707
Aug.	166	227	281	332	387	446	517	608	750
Sept.	181	225	261	294	328	364	406	458	538
Menado									
June	78	101	121	140	160	181	206	238	287
July	29	53	74	95	117	143	174	214	280
Aug.	23	44	61	78	96	117	141	173	225
Sept.	17	35	51	68	85	105	130	162	215
Mergui									
June	496	573	633	688	742	799	862	941	1058
July	502	587	654	716	776	840	913	1002	1136
Aug.	520	588	641	689	735	784	839	905	1003
Sept.	407	473	526	574	621	670	726	795	898
Minicoy									
June	169	203	230	255	280	307	337	375	432
July	100	131	158	183	209	237	270	312	377
Aug.	59	87	113	138	166	197	234	282	360
Sept.	46	69	91	113	137	164	196	239	308
Nagasaki									
June	129	176	216	255	295	340	392	460	566
July	74	111	146	181	218	260	311	379	487
Aug.	52	80	105	131	159	190	228	279	360
Sept.	83	119	150	181	214	251	294	351	441
Nagpur									
June	70	101	128	156	185	217	256	306	387
July	191	236	273	308	342	380	423	477	559
Aug.	124	160	191	220	249	281	319	366	439
Sept.	72	100	124	147	172	200	232	274	340
Naha									
June	118	159	194	227	262	300	345	403	493
July	78	104	126	148	170	195	223	260	317
Aug.	81	121	156	192	230	274	326	394	503
Sept.	71	96	118	139	161	186	214	251	308

TABLE 8.—Concluded

Station and period of rainfall	Deciles (mm)								
	1st	2d	3d	4th	5th	6th	7th	8th	9th
Pei-Hai									
June	108	147	181	213	247	284	328	385	473
July	209	273	327	378	431	488	555	640	772
Aug.	168	230	284	337	392	453	524	617	762
Sept.	78	114	148	181	217	257	305	368	470
Port Blair									
June	309	364	408	447	486	528	576	634	722
July	212	260	300	336	373	412	457	514	600
Aug.	213	261	300	336	373	412	456	513	598
Sept.	265	320	364	405	446	489	539	601	694
Quangtri									
June	13	23	33	44	56	70	87	111	150
July	10	20	30	41	53	67	85	110	151
Aug.	17	32	46	63	81	102	129	167	229
Sept.	134	189	235	279	324	373	431	506	623
Rangoon									
June	318	368	406	442	477	513	555	606	682
July	379	433	407	513	550	589	632	686	765
Aug.	351	406	439	474	508	544	584	634	707
Sept.	275	310	338	363	387	412	440	475	526
Saigon									
June	211	241	265	286	307	329	354	384	429
July	180	211	235	258	280	304	331	364	413
Aug.	157	188	212	235	257	281	308	341	392
Sept.	218	252	278	302	326	351	379	414	465
Sandakan									
June	104	128	147	166	184	204	227	255	299
July	95	118	137	154	172	191	213	241	283
Aug.	99	126	148	169	190	213	240	274	326
Sept.	113	146	173	199	225	253	286	327	391
Simla									
June	54	78	100	121	144	170	200	240	304
July	252	304	345	384	422	463	510	568	656
Aug.	240	290	331	368	406	446	492	549	635
Sept.	53	78	101	125	149	177	210	254	324
Singapore									
June	88	110	127	144	160	178	199	224	264
July	67	90	109	127	146	166	190	221	270
Aug.	73	97	117	136	156	178	203	236	287
Sept.	84	104	120	135	150	167	186	209	245
Taipei									
June	142	183	217	250	283	319	361	414	497
July	95	129	158	186	215	247	285	333	409
Aug.	85	128	166	205	247	294	351	426	547
Sept.	71	104	133	163	194	229	271	326	414
Tokyo									
June	84	106	124	141	158	176	198	225	267
July	40	59	77	94	113	133	158	191	244
Aug.	42	64	84	104	126	151	181	220	283
Sept.	104	134	159	183	207	234	265	304	365
Vengurla									
June	499	596	674	746	817	893	979	1087	1248
July	485	604	702	793	886	985	1099	1244	1464
Aug.	247	313	369	421	474	532	598	683	812
Sept.	94	133	167	200	236	275	321	382	478
Vishakhapatnam									
June	32	47	61	75	90	106	126	153	194
July	46	62	71	91	106	122	141	166	205
Aug.	43	62	79	96	113	133	157	187	236
Sept.	68	94	117	139	162	188	218	258	320
Zi-Ka-Wei									
June	70	96	119	140	163	188	217	255	314
July	37	59	79	100	122	148	180	222	291
Aug.	49	69	87	105	123	144	169	201	252
Sept.	43	62	79	96	115	135	159	191	241

Because of the pairwise independence of monthly rainfall shown by Mooley (1971), joint probability as may be required can be easily computed from the probabilities for the individual months.

Sajnani (1964) showed that the pentad rainfall of the rain gage station at Bombay (Colaba) during the different months of the southwest monsoon is representative of that over the Colaba district. The representative character of monthly station rainfall is expected to be much better. We feel, therefore, that the probabilities obtained for the individual stations could be applied to areas much larger than a district.

17. CONCLUSIONS

1. The monthly rainfall over Southeast Asia is not normally distributed, and the simple square-root, cube-root, and logarithmic transformations are of limited utility for normalizing the rainfall distribution.

2. The chi-square test, the Kolmogorov-Smirnov test, and the variance ratio test all show clearly that monthly rainfall in the Asian summer monsoon is gamma-distributed.

3. In cases where the gamma and other Pearsonian distributions show good fit to monthly rainfall, we find that, on the basis of the root-mean-square discrepancy of the actual frequency from the theoretical frequency and the relative variation of the parameters, the gamma distribution is the most suitable.

4. The values of the scale parameter of the gamma model applied to monthly rainfall are generally high over western India between 15° and 25°N and over parts of Southeast Asia between 15° and 30°N and east of 105°E. Heavy to very heavy rainfall associated with the intensified depressions over the former area and typhoons over the latter area lead to high scale-parameter values.

5. High values of the shape parameter are found over the belt from South Vietnam to the Bangladesh coast. These maxima are due to a small skewness coefficient over this belt. Over the southern part of the west coast of India and adjoining parts of the south-eastern Arabian Sea, the shape parameter decreases markedly from June to July. The variation of the shape parameter within the summer monsoon season is greater over most parts of India than over the rest of Southeast Asia.

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