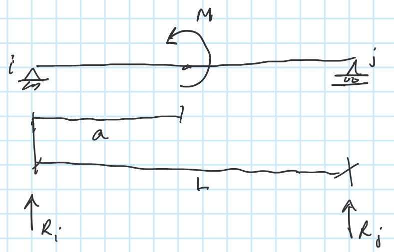
Timoshenko Concentrated Moment

Thursday, August 31, 2023 2:51 PM

(Refer to Timoshenko Point Load for General Basis)



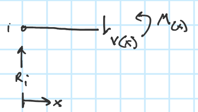
$+\uparrow \Sigma F_y = 0 \Rightarrow R_i + R_j = 0$

$R_i = -R_j = M/L$

$\circlearrowleft + \Sigma M_i = 0 \Rightarrow M + R_j L = 0$

$R_j = -M/L$

$\underline{0 \le x \le a}$



$+\uparrow \Sigma F_y = 0 \Rightarrow R_i - V(x) = 0$

$V(x) = R_i$

$\circlearrowleft + \Sigma M_i = 0 \Rightarrow -V(x)x + M(x) = 0$

$M(x) = V(x)\,x$

$= R_i\, x$

APPLICATION OF TIMOSHENKO RELATIONSHIP

$V(x) = kAG\left(\theta(x) - du/dx\right)$

$M(x) = EI \dfrac{d\theta}{dx}$

① $R_i = kAG\left(\theta(x) - \dfrac{du}{dx}\right)$

② $R_i x = EI\, d\theta/dx$

$\underline{\text{From ②}}$

$\displaystyle\int \frac{R_i x}{EI}\,dx = \int d\theta$

$\dfrac{R_i x^2}{2EI} + C_1 = \theta$ ③

$\underline{\text{From ① ; ③}}$

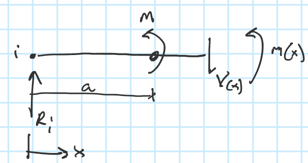
$R_i = kAG\left(\dfrac{R_i x^2}{2EI} + C_1 - \dfrac{du}{dx}\right)$

$\dfrac{R_i}{kAG} - \dfrac{R_i x^2}{2EI} - C_1 = -\dfrac{du}{dx}$

$\displaystyle\int \frac{-R_i}{kAG} + \frac{R_i x^2}{2EI} + C_1\,dx = \int du$

$-\dfrac{R_i x}{kAG} + \dfrac{R_i x^3}{6EI} + C_1 x + C_2 = u$ ④

$\underline{a \le x \le L}$



$+\uparrow \Sigma F_y = 0 \Rightarrow R_i - V(x) = 0$

$V(x) = R_i$

$\circlearrowleft + \Sigma M_i = 0 \Rightarrow M - V(x)x + M(x) = 0$

$M(x) = V(x)x - M$

$= R_i x - M$

APPLICATION OF TIMOSHENKO RELATIONSHIPS

⑤ $R_i = kAG\left(\theta(x) - \dfrac{du}{dx}\right)$

⑥ $R_i x - M = EI\, d\theta/dx$

$\underline{\text{From ⑥}}$

$\displaystyle\int \frac{R_i x}{EI} - \frac{M}{EI}\,dx = \int d\theta$

$\dfrac{R_i x^2}{2EI} - \dfrac{Mx}{EI} + C_3 = \theta$ ⑦

$\underline{\text{From ⑤ ; ⑦}}$

$R_i = kAG\left(\dfrac{R_i x^2}{2EI} - \dfrac{Mx}{EI} + C_3 - \dfrac{du}{dx}\right)$

$\dfrac{R_i}{kAG} - \dfrac{R_i x^2}{2EI} + \dfrac{Mx}{EI} - C_3 = -\dfrac{du}{dx}$

$\displaystyle\int \frac{-R_i}{kAG} + \frac{R_i x^2}{2EI} - \frac{Mx}{EI} + C_3\,dx = \int du$

$-\dfrac{R_i x}{kAG} + \dfrac{R_i x^3}{6EI} - \dfrac{Mx^2}{2EI} + C_3 x + C_4 = u$ ⑧

$\underline{\text{BOUNDARY ; COMPATIBILITY CONDITIONS}}$

$u = 0$	$x = 0$	BC1
$u = 0$	$x = L$	BC2
θ = CONSTANT	$x = a$	BC3
u = CONSTANT	$x = a$	BC4

BC1

$$C_2 = 0$$

BC2

$$-\frac{R_i L}{kAG} + \frac{R_i L^2}{6EI} - \frac{ML^2}{2EI} + C_3 L + C_4 = 0$$

$$L C_3 + C_4 = \frac{R_i L}{kAG} - \frac{R_i L^3}{6EI} + \frac{ML^2}{2EI}$$

BC3

$$\frac{R_i a^2}{2EI} + C_1 = \frac{R_i a^2}{2EI} - \frac{Ma}{EI} + C_3$$

$$C_1 - C_3 = -\frac{Ma}{EI}$$

BC4

$$-\frac{R_i a}{kAG} + \frac{R_i a^3}{6EI} + C_1 a + C_2 = -\frac{R_i a}{kAG} + \frac{R_i a^3}{6EI} - \frac{Ma^2}{2EI} + C_3 a + C_4$$

$$aC_1 + C_2 - aC_3 - C_4 = \frac{-Ma^2}{2EI}$$

IN MATRIX FORM:

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & L & 1 \\ 1 & 0 & -1 & 0 \\ a & 1 & -a & -1 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{R_i L}{kAG} - \frac{R_i L^3}{6EI} + \frac{ML^2}{2EI} \\ -\frac{Ma}{EI} \\ -\frac{Ma^2}{2EI} \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} \frac{Ma^2}{2EIL} - \frac{Ma}{EI} + \frac{ML}{3EI} + \frac{M}{kAGL} \\ 0 \\ \frac{Ma^2}{2EIL} + \frac{ML}{3EI} + \frac{M}{kAGL} \\ -\frac{1 \cdot Ma^2}{2EI} \end{bmatrix} = \begin{bmatrix} \left[\frac{3a^2 - 6La + 2L^2}{6EIL} + \frac{1}{kAGL}\right] M \\ 0 \\ \left[\frac{3a^2 + 2L^2}{6EIL} + \frac{1}{kAGL}\right] M \\ -M \cdot \frac{a^2}{2EI} \end{bmatrix}$$

- USE ABOVE TO SOLVE FOR FIXED-END MOMENTS BY SETTING SLOPE AT ENDS = TO THE INVERSE OF THE END SLOPE FROM INTERNAL LOADING.

$$M_i (a=0) \Rightarrow \quad \theta(0) = \left[\frac{L}{3EI} + \frac{1}{kAGL}\right] M_i$$

$$\theta(L) = \frac{R_i L^2}{2EI} - \frac{ML}{EI} + \frac{M_i 2L}{6EI} + \frac{M_i}{kAGL} \Rightarrow \left[-\frac{L}{6EI} + \frac{1}{kAGL}\right] M_i$$

$$M_j (a=L) \Rightarrow \quad \theta(0) = \left[\frac{-L}{6EI} + \frac{1}{kAGL}\right] M_j$$

$$\theta(L) = \frac{R_i L^2}{2EI} - \frac{M_j L}{EI} + \frac{5 L m_j}{6EI} + \frac{M_j}{kAGL} \Rightarrow \left[\frac{L}{3EI} + \frac{1}{kAGL}\right] M_j$$

$$\begin{bmatrix} \theta_i \\ \theta_j \end{bmatrix} = \begin{bmatrix} \left(\frac{L}{3EI} + \frac{1}{kAGL}\right) & \left(\frac{-L}{6EI} + \frac{1}{kAGL}\right) \\ \left(\frac{-L}{6EI} + \frac{1}{kAGL}\right) & \left(\frac{L}{3EI} + \frac{1}{kAGL}\right) \end{bmatrix} \begin{bmatrix} M_i \\ M_j \end{bmatrix}$$

$$\begin{bmatrix} \theta_i \\ \theta_j \end{bmatrix} = \begin{bmatrix} k_{s_{ii}} & k_{s_{ij}} \\ k_{s_{ji}} & k_{s_{jj}} \end{bmatrix} \begin{bmatrix} M_i \\ M_j \end{bmatrix}$$

FIXED END MOMENTS

$$\begin{bmatrix} -C_i \\ \frac{-R_i L^2}{2EI} + \frac{mL}{EI} - C_j \end{bmatrix} = \begin{bmatrix} k_{s_{ii}} & k_{s_{ij}} \\ k_{s_{ji}} & k_{s_{jj}} \end{bmatrix} \begin{bmatrix} M_i \\ M_j \end{bmatrix}$$