Timoshenko Linear Distributed Load

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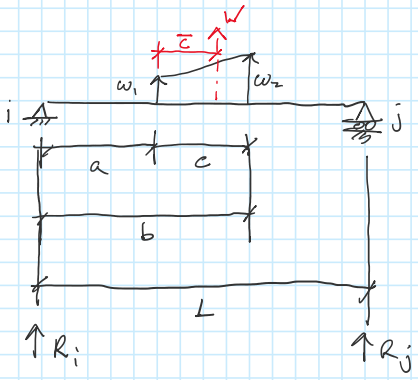

$c = b - a$

$W = c \cdot \omega_1 + \frac{1}{2} c (\omega_2 - \omega_1)$

$= \frac{1}{2} c (\omega_2 + \omega_1)$

$\bar{c} = \frac{\omega_1 + 2\omega_2}{3(\omega_2 + \omega_1)} c$

$+\uparrow \Sigma F_y = 0 = W + R_i + R_j$

$R_i = -W - R_j$

$\circlearrowleft \Sigma M_i = 0 = R_j L + W(a + \bar{c})$

$R_j = -\frac{W(a+\bar{c})}{L}$

LOAD FUNCTION:

$\omega(x) = 0 \qquad 0 \le x \le a$

$\omega(x) = \omega_1 + \left(\frac{x-a}{c}\right)(\omega_2 - \omega_1) \qquad a \le x \le b$ $\Rightarrow$ EXPANDED & SIMPLIFIED $\Rightarrow$ $\underset{\downarrow}{A} x - \underset{\downarrow}{B}$, $A = \frac{(\omega_2 - \omega_1)}{c}$, $B = \frac{a\omega_2 - b\omega_1}{c}$

$\omega(x) = 0 \qquad b \le x \le L$

SHEAR:

$V(x) = C_1 \qquad 0 \le x \le a$

$V(x) = \frac{1}{2} A x^2 - B x + C_2 \qquad a \le x \le b$

$V(x) = C_3 \qquad b \le x \le L$

MOMENT:

$M(x) = C_1 x + C_4 \qquad 0 \le x \le a$

$M(x) = \frac{1}{6} A x^3 - \frac{1}{2} B x^2 + C_2 x + C_5 \qquad a \le x \le b$

$M(x) = C_3 x + C_6 \qquad b \le x \le L$

APPLICATION OF TIMOSHENKO RELATIONSHIP

① $V(x) = kAG(\theta(x) - dv/dx)$

② $M(x) = EI \frac{d\theta}{dx}$

APPLICATION OF (2)

$$C_1 x + C_4 = EI \frac{d\theta}{dx}$$

$$\int \frac{C_1}{EI} x + \frac{C_4}{EI}\, dx = \int d\theta$$

$$\theta(x) = \frac{C_1}{2EI} x^2 + \frac{C_4}{EI} x + C_7 \qquad 0 \le x \le a$$

$$\tfrac{1}{6} A x^3 - B x^2 + C_2 x + C_5 = EI \frac{d\theta}{dx}$$

$$\int \frac{A}{6EI} x^3 - \frac{B}{2EI} x^2 + \frac{C_2}{EI} x + \frac{C_5}{EI}\, dx = \int d\theta$$

$$\theta(x) = \frac{A}{24EI} x^4 - \frac{B}{6EI} x^3 + \frac{C_2}{2EI} x^2 + \frac{C_5}{EI} x + C_9 \qquad a \le x \le b$$

$$C_3 x + C_6 = EI \frac{d\theta}{dx}$$

$$\int \frac{C_3}{EI} x + \frac{C_6}{EI}\, dx = \int d\theta$$

$$\theta(x) = \frac{C_3}{2EI} x^2 + \frac{C_6}{EI} x + C_9 \qquad b \le x \le L$$

APPLICATION OF (1)

$$C_1 = kAG\left[\left(\frac{C_1}{2EI} x^2 + \frac{C_4}{EI} x + C_7\right) - \frac{dv}{dx}\right]$$

$$\frac{C_1}{kAG} - \frac{C_1}{2EI} x^2 - \frac{C_4}{EI} x - C_7 = -\frac{dv}{dx}$$

$$\int \frac{-C_1}{kAG} + \frac{C_1}{2EI} x^2 + \frac{C_4}{EI} x + C_7\, dx = \int dv$$

$$v(x) = \frac{-C_1}{kAG} x + \frac{C_1}{6EI} x^3 + \frac{C_4}{2EI} x^2 + C_7 x + C_{10} \qquad 0 \le x \le a$$

$$\tfrac{1}{2} A x^2 - B x + C_2 = kAG\left[\left(\frac{A}{24EI} x^4 - \frac{B}{6EI} x^3 + \frac{C_2}{2EI} x^2 + \frac{C_5}{EI} x + C_8\right) - \frac{dv}{dx}\right]$$

$$\frac{A}{2kAG} x^2 - \frac{B}{kAG} x + \frac{C_2}{kAG} - \frac{A}{24EI} x^4 + \frac{B}{6EI} x^3 - \frac{C_2}{2EI} x^2 - \frac{C_5}{EI} x - C_8 = -\frac{dv}{dx}$$

$$\int \frac{-A}{2kAG} x^2 + \frac{B}{kAG} x - \frac{C_2}{kAG} + \frac{A}{24EI} x^4 - \frac{B}{6EI} x^3 + \frac{C_2}{2EI} x^2 + \frac{C_5}{EI} x + C_8\, dx = \int dv$$

$$v(x) = \frac{-A}{6kAG} x^3 + \frac{B}{2kAG} x^2 - \frac{C_2}{kAG} x + \frac{A}{120EI} x^5 - \frac{B}{24EI} x^4 + \frac{C_2}{6EI} x^3 + \frac{C_5}{2EI} x^2 + C_9 x + C_{11} \qquad a \le x \le b$$

$$C_3 = kAG\left[\left(\frac{C_3}{2EI} x^2 + \frac{C_6}{EI} x + C_9\right) - \frac{dv}{dx}\right]$$

$$\frac{C_3}{kAG} - \frac{C_3}{2EI} x^2 - \frac{C_6}{EI} x - C_9 = -\frac{dv}{dx}$$

$$\int -\frac{C_3}{kAG} + \frac{C_3}{2EI} x^2 + \frac{C_6}{EI} x + C_9\, dx = \int dv$$

$$v(x) = \frac{-C_3}{kAG} x + \frac{C_3}{6EI} x^3 + \frac{C_6}{2EI} x^2 + C_9 x + C_{12} \qquad b \le x \le L$$

12 UNKNOWNS

EQ.

① $x=0$ $V=R_i$

② $x=L$ $V=-R_j$

③ $x=0$ $M=0$

④ $x=L$ $M=0$

⑤ $x=0$ $U=0$

⑥ $x=L$ $U=0$

⑦ $x=a$ M COMPAT.

⑧ $x=b$ M COMPAT.

⑨ $x=a$ ⑤ = COMPAT

⑩ $x=b$ ⑥ = COMPAT.

⑪ $x=a$ U = COMPAT

⑫ $x=b$ U = COMPAT.

① $C_1 = R_i$

② $C_3 = -R_j$

③ $C_4 = 0$

④ $C_3 L + C_6 = 0$

⑤ $C_{10} = 0$

⑥ $-\frac{C_3}{kAG} L + \frac{C_3}{6EI} L^3 + \frac{C_6}{2EI} L^2 + C_9 L + C_{12} = 0$

$\left(\frac{-L}{kAG} + \frac{L^3}{6EI}\right) C_3 + \frac{L^2}{2EI} C_6 + L C_9 + C_{12} = 0$

⑦ $C_1 a + C_4 = \frac{1}{6} A a^3 - \frac{1}{2} B a^2 + C_2 a + C_5$

$a C_1 - a C_2 + C_4 - C_5 = \frac{1}{6} A a^3 - \frac{1}{2} B a^2$

⑧ $\frac{1}{6} A b^3 - \frac{1}{2} B b^2 + C_2 b + C_8 = C_3 b + C_6$

$b C_2 - b C_3 + C_5 - C_6 = -\frac{1}{6} A b^3 + \frac{1}{2} B b^2$

⑨ $\frac{a^2}{2EI} C_1 + \frac{a}{EI} C_4 + C_7 = \frac{A a^4}{24 EI} - \frac{B a^3}{6EI} + \frac{a^2}{2EI} C_2 + \frac{a}{EI} C_5 + C_8$

$\frac{a^2}{2EI} C_1 - \frac{a^2}{2EI} C_2 + \frac{a}{EI} C_4 - \frac{a}{EI} C_5 + C_7 - C_9 = \frac{A a^4}{24 EI} - \frac{B a^3}{6EI}$

⑩ $\frac{A b^4}{24 EI} - \frac{B b^3}{6EI} + \frac{b^2}{2EI} C_2 + \frac{b}{EI} C_5 + C_8 = \frac{b^2}{2EI} C_3 + \frac{b}{EI} C_6 + C_9$

$\frac{b^2}{2EI} C_2 - \frac{b^2}{2EI} C_3 + \frac{b}{EI} C_5 - \frac{b}{EI} C_6 + C_8 - C_9 = \frac{-A b^4}{24 EI} + \frac{B b^3}{6EI}$

⑪ $\frac{-a}{kAG} C_1 + \frac{a^3}{6EI} C_1 + \frac{a^2}{2EI} C_4 + a C_7 + C_{10} = \frac{-A a^3}{6kAG} + \frac{B a^2}{2kAG} - \frac{a}{kAG} C_2 + \frac{A a^5}{120 EI} - \frac{B a^4}{24 EI} + \frac{a^3}{6EI} C_2 + \frac{a^2}{2EI} C_5 + a C_8 + C_{11}$

$\left(\frac{-a}{kAG} + \frac{a^3}{6EI}\right) C_1 + \left(\frac{a}{kAG} - \frac{a^3}{6EI}\right) C_2 + \frac{a^2}{2EI} C_4 - \frac{a^2}{2EI} C_5 + a C_7 - a C_8 + C_{10} - C_{11} = \frac{-A a^3}{6kAG} + \frac{B a^2}{2kAG} + \frac{A a^5}{120 EI} - \frac{B a^4}{24 EI}$

⑫ $\frac{-A b^3}{6kAG} + \frac{B b^2}{2kAG} - \frac{b}{kAG} C_2 + \frac{A b^5}{120 EI} - \frac{B b^4}{24 EI} + \frac{b^3}{6EI} C_2 + \frac{b^2}{2EI} C_5 + b C_8 + C_{11} = \frac{-b}{kAG} C_3 + \frac{b^3}{6EI} C_3 + \frac{b^2}{2EI} C_6 + b C_9 + C_{12}$

$\left(\frac{-b}{kAG} + \frac{b^3}{6EI}\right) C_2 + \left(\frac{b}{kAG} - \frac{b^3}{6EI}\right) C_3 + \frac{b^2}{2EI} C_5 - \frac{b^2}{2EI} C_6 + b C_8 - b C_9 + C_{11} - C_{12} = \frac{A b^3}{6kAG} - \frac{B b^2}{2kAG} - \frac{A b^5}{120 EI} + \frac{B b^4}{24 EI}$

* SYMBOLIC SOLUTION OF THE INTEGRATION CONSTANTS IS NOT PRACTICAL FOR ENTRY.

BUILD AND SOLVE FOR THE COEFFICIENTS WITH IN THE LOAD CLASS

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & L & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & \frac{L^3}{6EI} - \frac{L}{A_s G} & 0 & 0 & \frac{L^2}{2EI} & 0 & 0 & L & 0 & 0 & 1 \\
a & -a & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & b & -b & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{a^2}{2EI} & -\left(\frac{a^2}{2EI}\right) & 0 & \frac{a}{EI} & -\left(\frac{a}{EI}\right) & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & \frac{b^2}{2EI} & -\left(\frac{b^2}{2EI}\right) & 0 & \frac{b}{EI} & -\left(\frac{b}{EI}\right) & 0 & 1 & -1 & 0 & 0 & 0 \\
\frac{a^3}{6EI} - \frac{a}{A_s G} & \frac{a}{A_s G} - \frac{a^3}{6EI} & 0 & \frac{a^2}{2EI} & -\left(\frac{a^2}{2EI}\right) & 0 & a & -a & 0 & 1 & -1 & 0 \\
0 & \frac{b^3}{6EI} - \frac{b}{A_s G} & \frac{b}{A_s G} - \frac{b^3}{6EI} & 0 & \frac{b^2}{2EI} & -\left(\frac{b^2}{2EI}\right) & 0 & b & -b & 0 & 1 & -1
\end{bmatrix}
=
\begin{bmatrix}
K_i \\ K_j \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{6}Aa^3 - \frac{1}{2}Ba^2 \\ -\frac{1}{6}Ab^3 + \frac{1}{2}Bb^2 \\ \frac{Aa^4}{24EI} - \frac{Ba^3}{6EI} \\ \frac{-Ab^4}{24EI} + \frac{Bb^3}{6EI} \\ \frac{-Aa^3}{6kAG} + \frac{Ba^2}{2kAG} + \frac{Aa^5}{120EI} - \frac{Ba^4}{24EI} \\ \frac{Ab^3}{6kAG} - \frac{Bb^2}{2kAG} - \frac{Ab^5}{120EI} + \frac{Bb^4}{24EI}
\end{bmatrix}
$$

* $A_s = kA$

FIXED END MOMENTS

$$
\begin{bmatrix} -C_7 \\ -\frac{C_3 L^2}{2EI} - \frac{C_6 L}{EI} - C_7 \end{bmatrix}
=
\begin{bmatrix} k_{s,i,i} & k_{s,i,j} \\ k_{s,j,i} & k_{s,j,j} \end{bmatrix}
\begin{bmatrix} M_i \\ M_j \end{bmatrix}
$$