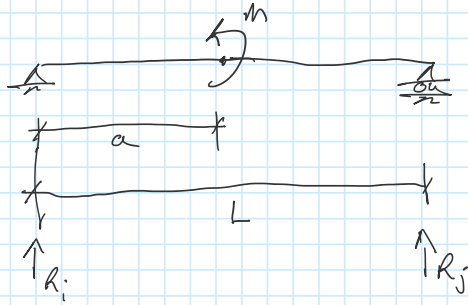# Concentrated Moment

Monday, January 23, 2023   10:22 AM

$$\uparrow_+ \Sigma F_y = 0 \implies R_i + R_j = 0$$
$$R_i = -R_j = \frac{M}{L}$$

$$\circlearrowleft) \Sigma M_i = 0 \implies M + R_j L = 0$$
$$R_j = -M/L$$

$\underline{V}$

$\underline{0 \le x \le a}$

$$V = R_i$$

$\underline{a \le x \le L}$

$$V(x) = R_i$$

$\underline{M}$

$\underline{0 \le x \le a}$

$$M(x) = R_i x$$

$$\circlearrowleft) \Sigma M_x = 0 = M_x - R_i x \, {}^? 0$$
$$M_x = R_i x$$

$\underline{a \le x \le L}$

$$M(x) = R_i x - M$$

$$\circlearrowleft) \Sigma M_x = 0 \implies M_x + M - R_i x = 0$$
$$M_x = R_i x - M$$

$\underline{\theta}$

$\underline{0 \le x \le a}$

$$EI \, \theta(x) = \tfrac{1}{2} R_i x^2 + C_1$$

$\underline{a \le x \le L}$

$$EI \, \theta(x) = \tfrac{1}{2} R_i x^2 - M x + C_2$$

$\underline{\Delta}$

$\underline{0 \le x \le a}$

$$EI \, y(x) = \tfrac{1}{6} R_i x^3 + C_1 x + C_3$$

$\underline{a \le x \le L}$

$$EI\, y(x) = \tfrac{1}{6} R_i x^3 + C_1 x + C_3$$

$\underline{a \le x \le L}$

$$EI\, y(x) = \tfrac{1}{6} R_i x^3 - \tfrac{1}{2} M x^2 + C_2 x + C_4$$

① $\quad y=0 \quad x=0$

② $\quad y=0 \quad x=L$

③ $\quad y=y \quad x=a$

④ $\quad \theta=\theta \quad x=a$

① $\quad C_3 = 0$

② $\quad \tfrac{1}{6} R_i L^3 - \tfrac{1}{2} L^2 M + L C_2 + C_4 = 0 \quad \Rightarrow \quad L C_2 + C_4 = -\tfrac{1}{6} R_i L^3 + \tfrac{1}{2} L^2 M$

③ $\quad \tfrac{1}{6}\cancel{R_i a^3} + a C_1 + C_3 = \tfrac{1}{6}\cancel{R_i a^3} - \tfrac{1}{2} a^2 M + a C_2 + C_4$

$\qquad a C_1 - a C_2 + C_3 - C_4 = -\tfrac{1}{2} a^2 M$

④ $\quad \tfrac{1}{2}\cancel{R_i a^2} + C_1 = \tfrac{1}{2}\cancel{R_i a^2} - M a + C_2$

$\qquad C_1 - C_2 = -M a$

$$C_1 = \frac{3 M a^2 - 6 L M a + 2 L^2 M}{6 L}$$

$$C_2 = \frac{3 M a^2 + 2 L^2 M}{6 L}$$

$$C_3 = 0$$

$$C_4 = -\frac{M a^2}{2}$$

$\underline{\text{FOR END MOMENTS}}$

$M_i \quad (a=0) \qquad \theta(0) = \frac{1}{EI}\left(\frac{L M_i}{3}\right) = \frac{L}{3EI} M_i$

$\qquad\qquad\qquad\qquad \theta(L) = \frac{1}{EI}\left(\frac{-L M_i}{6}\right) = \frac{-L}{6EI} M_i$

$M_j \quad (a=L) \qquad \theta(0) = \frac{1}{EI}\left(\frac{-L M_j}{6}\right) = \frac{-L}{6EI} M_j$

$\qquad\qquad\qquad\qquad \theta(L) = \frac{1}{EI}\left(\frac{L M_j}{3}\right) = \frac{L}{3EI} M_j$

$\qquad\qquad \theta(0) = \frac{L}{3EI} M_i - \frac{L}{6EI} M_j$

$\qquad\qquad \theta(L) = -\frac{L}{6EI} M_i + \frac{L}{3EI} M_j$

$$\begin{bmatrix} \theta_0 \\ \theta_L \end{bmatrix} = \begin{bmatrix} \dfrac{L}{3EI} & -\dfrac{L}{6EI} \\ -\dfrac{L}{6EI} & \dfrac{L}{3EI} \end{bmatrix} \begin{bmatrix} m_i \\ M_j \end{bmatrix} \quad \longleftarrow \quad$$

USE TO SOLVE FOR FIXED END MOMENTS,
SET $\theta = -\theta$ FROM LOAD

$$\begin{bmatrix} -C_1/EI \\ \dfrac{1}{EI}\left(-\tfrac{1}{2}L^2 R_i + ML - C_2\right) \end{bmatrix} =$$

$$\begin{bmatrix} M_j \\ M_i \end{bmatrix} = \begin{bmatrix} \dfrac{-\left(3M a^2 - 4LMa + L^2 M\right)}{L^2} \\ \dfrac{-\left(3M a^2 - 2LMa\right)}{L^2} \end{bmatrix}$$