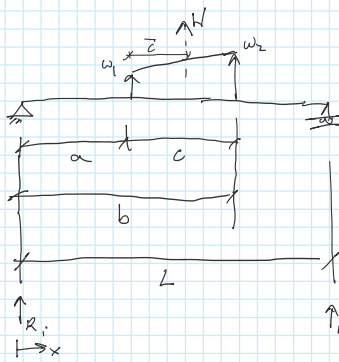


Linear Distributed Load

Sunday, January 15, 2023 3:40 PM



$$c = b - a$$

$$W = \frac{1}{2} c \cdot (w_2 - w_1) + w_1 c$$

$$\frac{1}{2} c w_2 - \frac{1}{2} c w_1 + w_1 c$$

$$\frac{1}{2} c w_2 + \frac{1}{2} c w_1$$

$$W = \frac{1}{2} c (w_2 + w_1)$$

$$\uparrow \sum F_y = 0 = W + R_1 + R_2$$

$$R_1 = -W - R_2$$

$$\uparrow \sum M_i = 0 = R_2 L + W(a + \bar{c})$$

$$R_2 = -\frac{W(a + \bar{c})}{L}$$

$$\bar{c} = \frac{w_1 + 2w_2}{3(w_1 + w_2)} c$$

FIXED END MOMENTS

$$\begin{bmatrix} -C_7 / EI \\ -\frac{1}{EI} \left(\frac{1}{2} C_3 L^2 + C_6 L + C_9 \right) \end{bmatrix} = \begin{bmatrix} \frac{1}{3EI} & -\frac{L}{6EI} \\ -\frac{1}{6EI} & \frac{1}{3EI} \end{bmatrix} \begin{bmatrix} M_i \\ M_j \end{bmatrix}$$

$$\begin{bmatrix} M_i \\ M_j \end{bmatrix} = \begin{bmatrix} -\frac{C_3 L^2 + 2C_6 L + 2C_9 + 4C_7}{L} \\ -\frac{2C_3 L^2 + 4C_6 L + 4C_9 + 2C_7}{L} \end{bmatrix}$$

LOAD FUNCTION ($w = \frac{dV}{dx}$)

$$w_1 + t(w_2 - w_1), \quad 0 \leq t \leq 1$$

$$t = (x - a) / c$$

$$\underline{0 \leq x \leq a}$$

$$w = 0$$

$$\underline{a \leq x \leq b}$$

$$w = w_1 + \frac{x-a}{c} (w_2 - w_1)$$

$$\underline{b \leq x \leq L}$$

$$w = 0$$

SHEAR ($\int w dx = \int dV \Rightarrow \int w dx = V$)

$$\underline{0 \leq x \leq a}$$

$$V = C_1$$

$$\underline{a \leq x \leq b}$$

$$V = \frac{w_2}{2c} x^2 - \frac{w_1}{2c} x^2 - \frac{aw_2}{c} x + \frac{aw_1}{c} x + w_1 x + C_2$$

$$\underline{b \leq x \leq L}$$

$$V = C_3$$

MOMENT ($\int V dx = \int dM \Rightarrow \int V dx = M$)

$$\underline{0 \leq x \leq a}$$

$$M = C_1 x + C_4$$

$$\underline{a \leq x \leq b}$$

$$M = \frac{w_2}{6c} x^3 - \frac{w_1}{6c} x^3 - \frac{aw_2}{2c} x^2 + \frac{aw_1}{2c} x^2 + \frac{w_1}{2} x^2 + C_2 x + C_5$$

$$\underline{b \leq x \leq L}$$

$$M = C_3 x + C_6$$

SLOPE ($\frac{dy^2}{dx^2} = \frac{M}{EI} \Rightarrow \frac{dy}{dx} = \int \frac{M}{EI} dx$)

$$\underline{0 \leq x \leq a}$$

$$EI \theta = \frac{1}{2} C_1 x^2 + C_4 x + C_7$$

$$\underline{a \leq x \leq b}$$

$$EI \theta = \frac{w_2}{24c} x^4 - \frac{w_1}{24c} x^4 - \frac{aw_2}{6c} x^3 + \frac{aw_1}{6c} x^3 + \frac{w_1}{6} x^3 + \frac{C_2}{2} x^2 + C_5 x + C_8$$

$$\underline{b \leq x \leq L}$$

$$EI \theta = \frac{1}{2} C_3 x^2 + C_6 x + C_9$$

DEFLECTION ($\frac{dy}{dx} = \int \theta dx = \int \int \frac{M}{EI} dx dx \rightarrow EI \text{ constant} \Rightarrow EI y = \int \int M dx dx$)

$$\underline{0 \leq x \leq a}$$

$$EI y = \frac{1}{6} C_1 x^3 + \frac{1}{2} C_4 x^2 + C_7 x + C_{10}$$

$$\underline{a \leq x \leq b}$$

$$EI y = \frac{w_2}{120c} x^5 - \frac{w_1}{120c} x^5 - \frac{aw_2}{24c} x^4 + \frac{aw_1}{24c} x^4 + \frac{w_1}{24} x^4 + \frac{1}{6} C_2 x^3 + \frac{1}{2} C_5 x^2 + C_8 x + C_{11}$$

$$a \leq x \leq b$$

$$E(y) = \frac{\omega_2}{120c} x^5 - \frac{\omega_1}{120c} x^5 - \frac{a\omega_2}{24c} x^4 + \frac{a\omega_1}{24c} x^4 + \frac{\omega_1}{24} x^4 + \frac{1}{6} c_2 x^3 + \frac{1}{2} c_5 x^2 + c_8 x + c_{11}$$

$$b \leq x \leq L$$

$$E(y) = \frac{1}{6} c_3 x^3 + \frac{1}{2} c_6 x^2 + c_9 x + c_{12}$$

unknowns = 12

EG.

- | | |
|------------------|------------------------------------|
| ① $x=0$ $V=R_i$ | ⑦ $x=a$ $M=\text{CONST.}$ |
| ② $x=L$ $V=-R_j$ | ⑧ $x=b$ $M=\text{CONST.}$ |
| ③ $x=0$ $M=0$ | ⑨ $x=a$ $y'=\ominus=\text{CONST.}$ |
| ④ $x=L$ $M=0$ | ⑩ $x=b$ $y'=\text{CONST.}$ |
| ⑤ $x=0$ $y=0$ | ⑪ $x=a$ $y=\text{CONST.}$ |
| ⑥ $x=L$ $y=0$ | ⑫ $x=b$ $y=\text{CONST.}$ |

$$\textcircled{1} \quad c_1 = R_i$$

$$\textcircled{2} \quad c_3 = -R_j$$

$$\textcircled{3} \quad c_4 = 0$$

$$\textcircled{4} \quad c_5 L + c_6 = 0$$

$$\textcircled{5} \quad c_{10} = 0$$

$$\textcircled{6} \quad \frac{1}{6} b^3 c_3 + \frac{1}{2} L^2 c_6 + L c_4 + c_{12} = 0$$

$$\textcircled{7} \quad a c_1 + c_4 = \frac{-a^3 \omega_2}{3c} + \frac{a^3 \omega_1}{3c} + \frac{a^2 \omega_1}{2} + a c_2 + c_5$$

$$a c_1 - a c_2 + c_4 - c_5 = \frac{-a^3 \omega_2}{3c} + \frac{a^3 \omega_1}{3c} + \frac{a^2 \omega_1}{2}$$

$$\textcircled{8} \quad \frac{b^3 \omega_2}{6c} - \frac{a b^2 \omega_2}{2c} - \frac{b^3 \omega_1}{6c} + \frac{a b^2 \omega_1}{2c} + \frac{b^2 \omega_1}{2} + b c_2 + c_5 = b c_3 + c_6$$

$$\frac{b^3 \omega_2}{6c} - \frac{a b^2 \omega_2}{2c} - \frac{b^3 \omega_1}{6c} + \frac{a b^2 \omega_1}{2c} + \frac{b^2 \omega_1}{2} = -b c_2 + b c_3 - c_5 + c_6$$

$$\textcircled{9} \quad \frac{1}{2} a^2 c_1 + a c_4 + c_7 = \frac{-a^4 \omega_2}{8c} + \frac{a^4 \omega_1}{8c} + \frac{a^3 \omega_1}{6} + \frac{1}{2} a^2 c_2 + a c_5 + c_8$$

$$\frac{1}{2} a^2 c_1 - \frac{1}{2} a^2 c_2 + a c_4 - a c_5 + c_7 - c_8 = \frac{-a^4 \omega_2}{8c} + \frac{a^4 \omega_1}{8c} + \frac{a^3 \omega_1}{6}$$

$$\textcircled{10} \quad \frac{b^4 \omega_2}{24c} - \frac{a b^3 \omega_2}{6c} - \frac{b^4 \omega_1}{24c} + \frac{a b^3 \omega_1}{6c} + \frac{b^3 \omega_1}{6} + \frac{1}{2} b^2 c_2 + b c_5 + c_8 = \frac{1}{2} b^2 c_3 + b c_6 + c_9$$

$$\frac{b^4 \omega_2}{24c} - \frac{a b^3 \omega_2}{6c} - \frac{b^4 \omega_1}{24c} + \frac{a b^3 \omega_1}{6c} + \frac{b^3 \omega_1}{6} = -\frac{1}{2} b^2 c_2 + \frac{1}{2} b^2 c_3 - b c_5 + b c_6 - c_8 + c_9$$

$$\textcircled{11} \quad \frac{1}{6} a^3 c_1 + \frac{1}{2} a^2 c_4 + a c_7 + c_{10} = \frac{-a^5 \omega_2}{30c} + \frac{a^5 \omega_1}{30c} + \frac{a^4 \omega_1}{24} + \frac{1}{6} a^3 c_2 + \frac{1}{2} a^2 c_5 + a c_8 + c_{11}$$

$$\frac{1}{6} a^3 c_1 - \frac{1}{6} a^3 c_2 + \frac{1}{2} a^2 c_4 - \frac{1}{2} a^2 c_5 + a c_7 - a c_8 + c_{10} - c_{11} = \frac{-a^5 \omega_2}{30c} + \frac{a^5 \omega_1}{30c} + \frac{a^4 \omega_1}{24}$$

$$\textcircled{12} \quad \frac{b^5 \omega_2}{120c} - \frac{a b^4 \omega_2}{24c} - \frac{b^5 \omega_1}{120c} + \frac{a b^4 \omega_1}{24c} + \frac{b^4 \omega_1}{24} + \frac{1}{6} b^3 c_2 + \frac{1}{2} b^2 c_5 + b c_8 + c_{11} = \frac{1}{6} b^3 c_3 + \frac{1}{2} b^2 c_6 + b c_9 + c_{12}$$

$$\frac{b^5 \omega_2}{120c} - \frac{a b^4 \omega_2}{24c} - \frac{b^5 \omega_1}{120c} + \frac{a b^4 \omega_1}{24c} + \frac{b^4 \omega_1}{24} = -\frac{1}{6} b^3 c_2 + \frac{1}{6} b^3 c_3 - \frac{1}{2} b^2 c_5 + \frac{1}{2} b^2 c_6 - b c_8 + b c_9 - c_{11} + c_{12}$$

$$\begin{aligned}
& \frac{\left(2b^2+(-a-3L)b-a^2+3La\right)w_2+\left(b^2+(a-3L)b-2a^2+3La\right)w_1}{6L} \\
& \frac{\left(2b^3+(-3a-3L)b^2+6Lab+a^3\right)w_2+\left(b^3-3Lb^2-3a^2b+2a^3\right)w_1}{6Lb-6La} \\
& \frac{(2b^2-a b-a^2)w_2+(b^2+a b-2a^2)w_1}{6L} \\
& 0 \\
& -\frac{a^3w_2+(2a^3-3a^2b)w_1}{6b-6a} \\
& -\frac{(2b^2-a b-a^2)w_2+(b^2+a b-2a^2)w_1}{6} \\
& \frac{\left(12b^4+(-3a-45L)b^3+(-3a^2+15La+40L^2)b^2+(-3a^3+15La^2-20L^2a)b-3a^4+15La^3-20L^2a^2\right)w_2+\left(3b^4+(3a-15L)b^3+(3a^2-15La+20L^2)b^2+(3a^3-15La^2+20L^2a)b-12a^4+45La^3-40L^2a^2\right)w_1}{360L} \\
& \frac{\left(12b^5+(-15a-45L)b^4+(60La+40L^2)b^3-60L^2ab^2+3a^5+20L^2a^3\right)w_2+\left(3b^5-15Lb^4+20L^2b^3+(-15a^4-60L^2a^2)b+12a^5+40L^2a^2\right)w_1}{360Lb-360La} \\
& \frac{\left(12b^4-3ab^3+(40L^2-3a^2)b^2+(-3a^3-20L^2a)b-3a^4-20L^2a^2\right)w_2+\left(3b^4+3ab^3+(3a^2+20L^2)b^2+(3a^3+20L^2a)b-12a^4-40L^2a^2\right)w_1}{360L} \\
& 0 \\
& -\frac{a^5w_2+(4a^5-5a^4b)w_1}{120b-120a} \\
& -\frac{(4b^4-ab^3-a^2b^2-a^3b-a^4)w_2+(b^4+ab^3+a^2b^2+a^3b-4a^4)w_1}{120}
\end{aligned}$$