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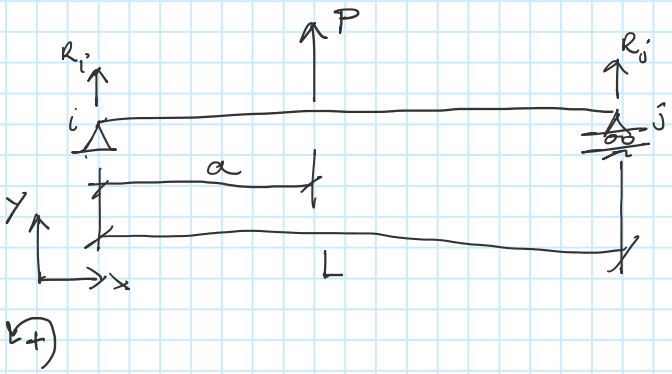
Tuesday, January 3, 2023 3:51 PM



$\uparrow_+ \Sigma F_y = 0 \Rightarrow R_i + P + R_j = 0$

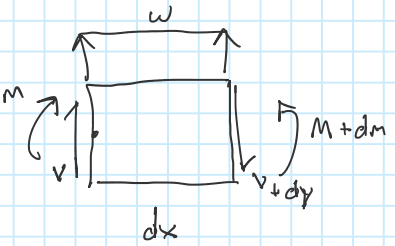
$R_i = -P - R_j$

$\Sigma M_i = 0 \Rightarrow Pa + R_j L = 0$

$R_j = -\frac{Pa}{L}$

$R_i = -P + \frac{Pa}{L}$

$= P\frac{(a-L)}{L}$



$V + w\,dx - (V + dV) = 0$

$V + w\,dx - V - dV = 0$

$w = dV/dx$

$-M + (M + dM) - \frac{1}{2} w\,dx^2 - V\,dx = 0$

$dM = \frac{1}{2} w\,dx^2 + V\,dx$

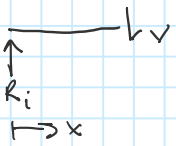
$\frac{dM}{dx} = \frac{1}{2} w\,dx + V$

$\lim dx \to 0$

$\frac{dM}{dx} = V$

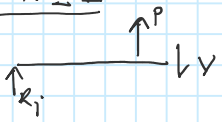
SHEAR (+ INTERNAL SHEAR ↓)

$x \le a$



$V(x) = R_i$

$a < x \le L$



$R_i + P - V = 0$

$V(x) = R_i + P$

MOMENT ($\int V\,dx$)

$x \le a$

$M(x) = R_i x + C_1$

BOUNDARY COND.

$M = 0$ @ $x = 0$ $\therefore C_1 = 0$

$a < x \le L$

$M = 0$ @ $x = L \Rightarrow 0 = R_i L + PL + C_2$

$a < x \leq L$

$M(x) = R_1 x + Px + C_2$

$M = 0$ e $x = L \Rightarrow 0 = R_1 L + PL + C_2$

$C_2 = -R_1 L - PL$

$-P(a-L) - PL$

$-Pa + PL - PL$

$C_2 = -Pa$

EIS $(\int M_{dx})$

$x \leq a$

$EIS(x) = \frac{1}{2} R_1 x^2 + C_1 x + C_3$

$a < x \leq L$

$EIS(x) = \frac{1}{2} R_1 x^2 + \frac{1}{2} P x^2 + C_2 x + C_4$

EId $(\int S \, dx)$

$x \leq a$

$EId(x) = \frac{1}{6} R_1 x^3 + \frac{1}{2} C_1 x^2 + C_3 x + C_5$

$a < x \leq L$

$EId(x) = \frac{1}{6} R_1 x^3 + \frac{1}{6} P x^3 + \frac{1}{2} C_2 x^2 + C_4 x + C_6$

BOUNDARY COND:

$d = 0$ e $x = 0 \Rightarrow C_5 = 0$

$d = 0$ e $x = L \Rightarrow \frac{R_1 L^3}{6} + \frac{PL^3}{6} + \frac{L^2}{2} C_2 + C_4 L + C_6 = 0 \Rightarrow \frac{L^2}{2} C_2 + C_4 L + C_6 = \frac{-R_1 L^3}{6} - \frac{PL^3}{6}$

$s_a = s_a$ e $x = a \Rightarrow \frac{R_1 a^2}{2} + C_1 a + C_3 = \frac{R_1 a^2}{2} + \frac{Pa^2}{2} + C_2 a + C_4$

$C_3 = \frac{Pa^2}{2} + C_2 a + C_4$

$d_a = d_a$ e $x = a$

$\frac{R_1 a^3}{6} + \frac{C_1 a^2}{2} + C_3 a + C_5 = \frac{R_1 a^3}{6} + \frac{Pa^3}{6} + \frac{C_2 a^2}{2} + C_4 a + C_6$

$\frac{Pa^3}{2} + C_2 a^2 + C_4 a = \frac{Pa^3}{6} + \frac{C_2 a^2}{2} + C_4 a + C_6$

$\frac{Pa^3}{3} + \frac{C_2 a^2}{2} = C_6$

Integration Constants:

$$\begin{pmatrix} 0 \\ -P\,a \\ \frac{P\,a\,(a-2\,L)\,(a-L)}{6\,L} \\ \frac{P\,a\,(a^2+2\,L^2)}{6\,L} \\ 0 \\ -\frac{P\,a^3}{6} \end{pmatrix}$$

Fixed End Moments:

$$\begin{pmatrix} -\frac{P\,a\,(a-L)^2}{L^2} \\ -\frac{P\,a^2\,(a-L)}{L^2} \end{pmatrix}$$