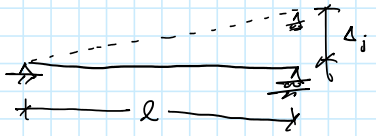
Timoshenko End Delta

Friday, March 29, 2024 5:19 PM



V

$V(x) = C_1$

M

$M(x) = C_1 x + C_2$

APPLICATION OF TIMOSHENKO RELATIONSHIP

① $V(x) = kAG(\theta(x) - dv/dx) \longrightarrow -\frac{V(x)}{kAG} + \theta(x) = \frac{dv}{dx} \rightarrow \int -\frac{V(x)}{kAG} + \theta(x)\,dx = v(x)$

② $M(x) = EI\frac{d\theta}{dx} \longrightarrow \frac{M(x)}{EI} = \frac{d\theta}{dx} \rightarrow \int \frac{M(x)}{EI}\,dx = \theta(x)$

APPLICATION OF ②

$C_1 x + C_2 = EI\frac{d\theta}{dx}$

$\int \frac{C_1}{EI}x + \frac{C_2}{EI}\,dx = \int d\theta$

$\frac{C_1}{2EI}x^2 + \frac{C_2}{EI}x + C_3 = \theta(x)$

APPLICATION OF ①

$C_1 = kAG\left[\left(\frac{C_1}{2EI}x^2 + \frac{C_2}{EI}x + C_3\right) - \frac{dv}{dx}\right]$

$\int -\frac{C_1}{kAG} + \frac{C_1}{2EI}x^2 + \frac{C_2}{EI}x + C_3\,dx = \int dv$

$-\frac{C_1}{kAG}x + \frac{C_1}{6EI}x^3 + \frac{C_2}{2EI}x^2 + C_3 x + C_4 = v(x)$

B.C.S

① $M(0) = 0$

② $M(\ell) = 0$

③ $v(0) = 0$

④ $v(\ell) = v_j$

BC1: $C_2 = 0$

BC2: $C_1 = 0$

BC3: $C_4 = 0$

BC4: $C_3\ell = v_j$

$C_3 = \frac{v_j}{\ell}$

BCH: $C_3 L = u_j$

$$C_3 = \frac{u_j}{L}$$

FIXED END MOMENTS

$$\begin{bmatrix} -\frac{u_i}{L} \\ -\frac{u_j}{L} \end{bmatrix} = \begin{bmatrix} k_{s,ii} & k_{s,ij} \\ k_{s,ji} & k_{s,jj} \end{bmatrix} \begin{bmatrix} m_i \\ m_j \end{bmatrix}$$