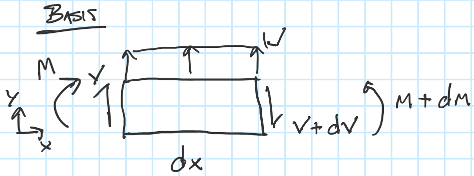
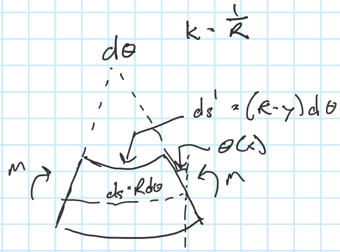
Timoshenko Point Load

Thursday, August 31, 2023 1:00 PM

BASIS

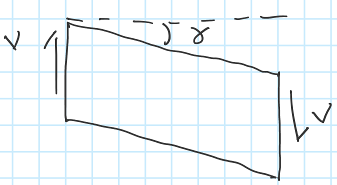




- POSITIVE LOADS POINT IN POSITIVE Y
- POSITIVE MOMENT = COUNTER CLOCKWISE
- POSITIVE INTERNAL SHEAR IN NEGATIVE Y

THIS MEANS POSITIVE NORMAL ROTATION OF CROSS SECTION, θ, IS ALSO COUNTER CLOCKWISE

PURE SHEAR DEFORMATION



$V = GkA\gamma$, WHERE kA = SHEAR AREA

OR

$$\gamma = \frac{V}{GkA}$$

- ASSUME NO LONG. STRAIN FROM SHEAR DEFORMATION

$$\therefore \frac{dv}{dx} = \theta - \gamma$$

NOTE "−" SIGN.

PUTTING IT TOGETHER:

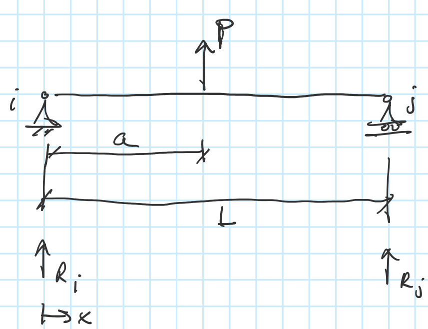
WHERE v = DISPLACEMENT IN Y, POSITIVE UPWARD.

$$dv/dx = \theta(x) - \frac{V(x)}{GkA} \Rightarrow V(x) = GkA\left(\theta(x) - \frac{dv}{dx}\right)$$

$$d\theta/dx = \frac{M(x)}{EI}$$

$$M(x) = EI\,\frac{d\theta}{dx}$$

POINT LOAD



STATICS

$+\uparrow \Sigma F_y = 0 \Rightarrow R_i + P + R_j = 0$

$$R_i = -R_j - P = \frac{P(a-L)}{L} \text{ or } \frac{-P(L-a)}{L}$$

$\circlearrowleft + \Sigma M_i = 0 \Rightarrow Pa + R_j L = 0$ $\qquad R_i + P = -R_j$

$$R_j = \frac{-Pa}{L}$$

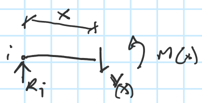
PIECEWISE FUNCTIONS:

$0 \le x \le a$

$a \le x \le L$

PIECEWISE FUNCTIONS:

$0 \le x \le a$



$+\uparrow \Sigma F_y = 0 = R_i - V(x) = 0$

$V(x) = R_i$

$\circlearrowleft \Sigma M_i = 0 = -V(x)\,x + M(x) = 0$

$M(x) = V(x)\,x = R_i\,x$

$V(x) = R_i$

$M(x) = R_i\,x$

APPLICATION OF TIMOSHENKO RELATIONSHIPS

① $R_i = kAG\left(\theta(x) - \frac{du}{dx}\right)$

② $R_i\,x = EI\frac{d\theta}{dx}$

From ②

$$\int \frac{R_i x}{EI}\,dx = \int d\theta$$

$$\frac{R_i x^2}{2EI} + C_1 = \theta \quad ⑤$$

From ① & ⑤

$$R_i = kAG\left(\frac{R_i x^2}{2EI} + C_1 - \frac{du}{dx}\right)$$

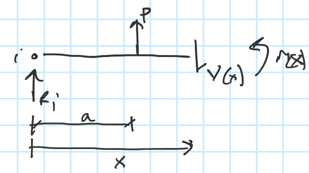
$$\frac{R_i}{kAG} = \frac{R_i x^2}{2EI} + C_1 - \frac{du}{dx}$$

$$\frac{R_i}{kAG} - \frac{R_i x^2}{2EI} - C_1 = -\frac{du}{dx}$$

$$\int -\frac{R_i}{kAG} + \frac{R_i x^2}{2EI} + C_1\,dx = \int du$$

$$\frac{-R_i x}{kAG} + \frac{R_i x^3}{6EI} + C_1 x + C_2 = u \quad ⑥$$

$a \le x \le L$



$+\uparrow \Sigma F_y = 0 \Rightarrow \underbrace{R_i + P}_{-R_j} - V(x) = 0$

$V(x) = -R_j$

$\circlearrowleft \Sigma M_i = 0 \Rightarrow Pa - V(x)\,x + M(x) = 0$

$M(x) = V(x)\,x - Pa$

$= -R_j\,x - Pa$

APPLICATION OF TIMOSHENKO RELATIONSHIPS

③ $-R_j = kAG\left(\theta(x) - \frac{du}{dx}\right)$

④ $-R_j\,x - Pa = EI\frac{d\theta}{dx}$

From ④

$$\int \frac{-R_j x}{EI} - \frac{Pa}{EI}\,dx = \int d\theta$$

$$\frac{-R_j x^2}{2EI} - \frac{Pax}{EI} + C_3 = \theta \quad ⑦$$

From ③ & ⑦

$$-R_j = kAG\left(\frac{-R_j x^2}{2EI} - \frac{Pax}{EI} + C_3 - \frac{du}{dx}\right)$$

$$\frac{-R_j}{kAG} = \frac{-R_j x^2}{2EI} - \frac{Pax}{EI} + C_3 - \frac{du}{dx}$$

$$\int \frac{R_j}{kAG} - \frac{R_j x^2}{2EI} - \frac{Pax}{EI} + C_3\,dx = \int du$$

$$\frac{R_j x}{kAG} - \frac{R_j x^3}{6EI} - \frac{Pax^2}{2EI} + C_3 x + C_4 = u \quad ⑧$$

BOUNDARY & COMPATIBILITY CONDITIONS

$u = 0$	$x = 0$	BC1
$u = 0$	$x = L$	BC2
θ = CONSTANT	$x = a$	BC3
u = CONSTANT	$x = a$	BC4

BC1

$C_2 = 0$

BC2

$$\frac{R_j L}{kAG} - \frac{R_j L^3}{6EI} - \frac{PaL^2}{2EI} + C_3 L + C_4 = 0$$

$$L C_3 + C_4 = -\frac{R_j L}{kAG} + \frac{R_j L^3}{6EI} + \frac{PaL^2}{2EI}$$

BC3

$$\frac{R_i a^2}{2EI} + C_1 = \frac{-R_j a^2}{2EI} - \frac{Pa^2}{EI} + C_3$$

$$C_1 - C_3 = \frac{-R_i a^2}{2EI} - \frac{R_j a^2}{2EI} - \frac{Pa^2}{EI}$$

BC4

$$\frac{-R_i a}{kAG} + \frac{R_i a^3}{6EI} + C_1 a + C_2 = \frac{R_j a}{kAG} - \frac{R_j a^3}{6EI} - \frac{Pa^3}{2EI} + C_3 a + C_4$$

$$a C_1 + C_2 - a C_3 - C_4 = \frac{R_i a}{kAG} - \frac{R_i a^3}{6EI} + \frac{R_j a}{kAG} - \frac{R_j a^3}{6EI} - \frac{Pa^3}{2EI}$$

$$\begin{pmatrix} \frac{P a^3}{6 E I L} - \frac{P a^2}{2 E I} + \frac{L P a}{3 E I} \\ 0 \\ \frac{P a^3}{6 E I L} + \frac{L P a}{3 E I} \\ \frac{P a}{A_s G} - \frac{P a^3}{6 E I} \end{pmatrix} \begin{matrix} = C_1 \\ = C_2 \\ = C_3 \\ = C_4 \end{matrix}$$

$A_s = kA$

FIXED END MOMENTS

$$\begin{bmatrix} -C_1 \\ \frac{R_j L^2}{2EI} + \frac{PaL}{EI} - C_3 \end{bmatrix} = \begin{bmatrix} k_{sii} & k_{sij} \\ k_{sji} & k_{sjj} \end{bmatrix} \begin{bmatrix} M_i \\ M_j \end{bmatrix}$$