$+$ $\times$

$-$ $\div$

TITLE: CONCRETE – P v M SURFACE

SCALE:

DATE: 4/1/23

GENERAL STRAIN PLANE:

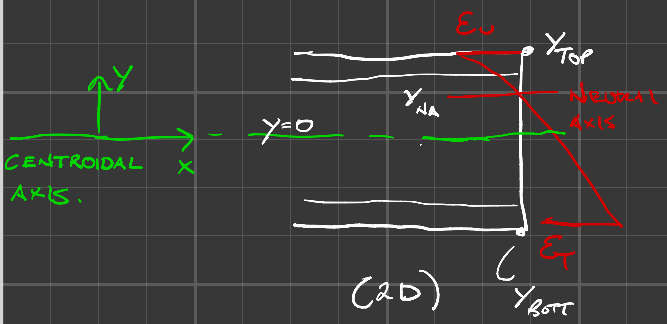


FIG 7

PLANE SECTIONS REMAIN PLANE

$\therefore$ STRAIN = ε IS LINEAR

2D CASE (TEXTBOOK BEAM PROBLEM)

$$\varepsilon(y) = C_1 y + C_3$$

FOR CONCRETE STRAIN AT EXTREME COMPRESSION FIBER AT THE ULTIMATE CONDITION 0.003,

$\varepsilon_U = 0.003$

EXTREME COMPRESSION FIBER IS THE TOP EDGE OF THE SECTION IN FIG. 7 = Y_{TOP}

STRAIN IS 0 AT THE NEUTRAL AXIS.

$\therefore \varepsilon(Y_{TOP}) = \varepsilon_U$; $\varepsilon(Y_{NA}) = 0$

2 EQ
2 UNKNOWNS

$$\left[\begin{array}{l} \varepsilon_U = C_1 Y_{TOP} + C_3 \\ 0 = C_1 Y_{NA} + C_3 \Rightarrow C_3 = -C_1 Y_{NA} \end{array}\right.$$

$$\varepsilon_U = C_1 Y_{TOP} - C_1 Y_{NA}$$

$$C_1 = \frac{\varepsilon_U}{Y_{TOP} - Y_{NA}} \qquad C_3 = \frac{-\varepsilon_U Y_{NA}}{Y_{TOP} - Y_{NA}}$$

DEFINE $C = Y_{TOP} - Y_{NA}$ = NEUTRAL AXIS DEPTH.

ALSO $Y_{NA} = Y_{TOP} - C$

$$\varepsilon(y) = \frac{\varepsilon_U}{c} y + \frac{-\varepsilon_U (Y_{TOP} - c)}{c}$$

$$\boxed{\varepsilon(y) = \frac{\varepsilon_U}{c}\left(y - Y_{TOP} + c\right)}$$

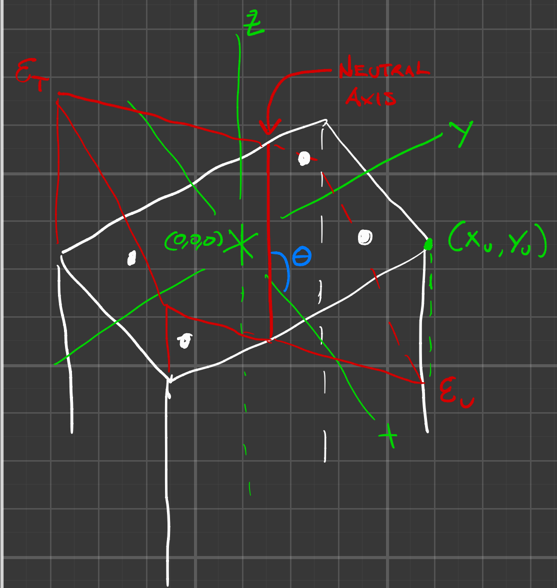
* NOTE COORD. SYSTEM DEFINITION WHERE y=0 AT THE CENTROIDAL AXIS.

TITLE:

SCALE:

DATE: 4/1/23

3D CASE (BIAXIAL BENDING PROBLEM)



$$\varepsilon(x,y) = C_1 x + C_2 y + C_3$$

$\varepsilon = 0$ ON NEUTRAL AXIS:

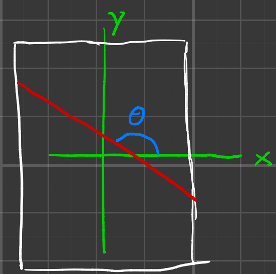
$$0 = C_1 x + C_2 y + C_3$$

$$-C_2 y = C_1 x + C_3$$

$$\boxed{y = -\frac{C_1}{C_2} x + \frac{-C_3}{C_2}}$$

SLOPE-INTERCEPT FORM OF EQ. FOR THE NEUTRAL AXIS.

$$\frac{dy}{dx} = -C_1/C_2 = \text{SLOPE OF NEUTRAL AXIS.}$$



$$\text{TAN}\,\alpha = \frac{\text{RISE}}{\text{RUN}} = \frac{C_1}{C_2}$$

$$C_1 = C_2 \,\text{TAN}\,\alpha \quad (1)$$

$\varepsilon = \varepsilon_U$ AT THE COMPRESSION FIBER FURTHEST FROM THE NEUTRAL AXIS. CALL THIS POINT (x_U, y_U).

$$\varepsilon(x_U, y_U) = \varepsilon_U = C_1 x_U + C_2 y_U + C_3$$

FROM (1) $\Rightarrow$ $\varepsilon_U = C_2 \,\text{TAN}\,\alpha\, x_U + C_2 y_U + C_3$

$$C_3 = \varepsilon_U - C_2\left(x_U \,\text{TAN}\,\alpha + y_U\right) \quad (2)$$

SUBSTITUTION OF (1) AND (2) INTO $\varepsilon(x,y) = x C_2 \,\text{TAN}\,\alpha + C_2 y + \varepsilon_U - C_2(x_U \,\text{TAN}\,\alpha + y_U)$

AFTER SIMPLIFICATION $\boxed{\varepsilon(x,y) = \varepsilon_U + C_2\left[(x - x_U)\,\text{TAN}\,\alpha + (y - y_U)\right]}$

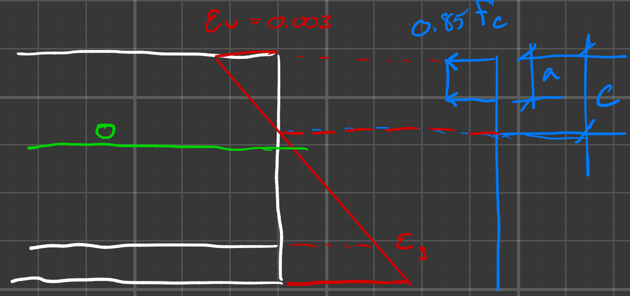
* NOTE $\varepsilon(x,y)$ IS FULLY DEFINED WITH C_2 AND α. ASSUMING CONCRETE SECTION IS DEFINED BY LINEAR BOUNDARIES (x_U, y_U) WILL BE ONE OF THE VERTICES.

TITLE: CONCRETE STRESS - STRAIN RELATIONSHIPS

SCALE:

DATE: 4/1/23

WHITNEY BLOCK



THIS IS THE "k_3" FACTOR ACCOUNTS FOR SIZE, SHAPE, & CASTING POSITION

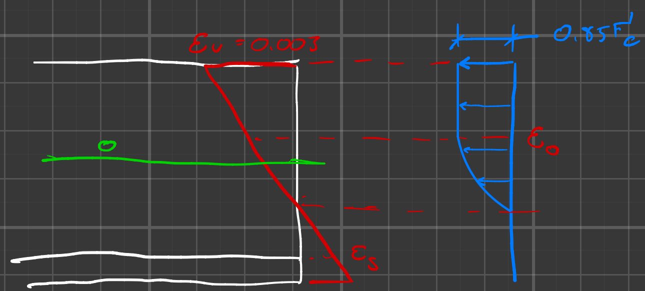
$$\sigma_c = 0.85 f'_c$$

$$a = \beta_1 c$$

STRAIN IS LINEAR SO

$$\varepsilon_a = \varepsilon_u - \beta_1 \varepsilon_u$$

PCA PARABOLIC - CONSTANT



$$\varepsilon_0 = \frac{2(0.85 f'_c)}{E_c}$$

$0 < \varepsilon_c < \varepsilon_0$

$$\sigma_c = 0.85 f'_c \left[2\left(\frac{\varepsilon_c}{\varepsilon_0}\right) - \left(\frac{\varepsilon_c}{\varepsilon_0}\right)^2 \right]$$

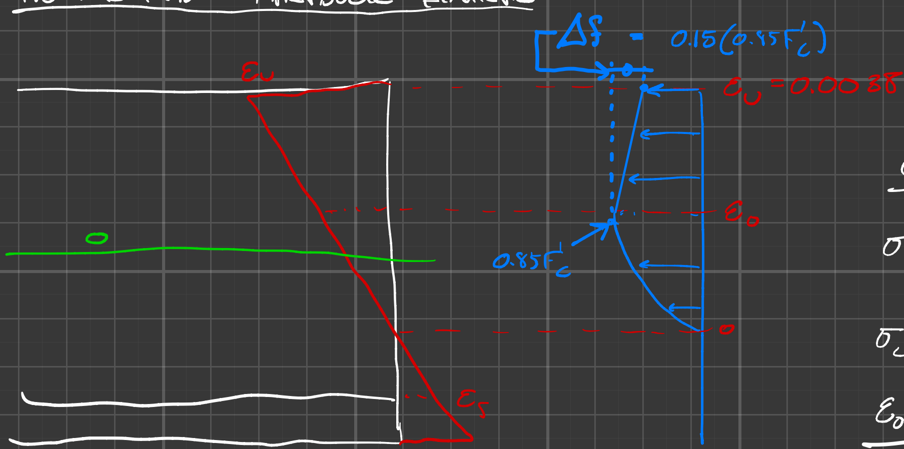
OR

$$\sigma_c = E_c \varepsilon_c - \frac{5 E_c^2}{17 f'_c} \varepsilon_c^2$$

$\varepsilon_c \geq \varepsilon_0$

$$\sigma_c = 0.85 f'_c$$

HOGNESTAD - PARABOLIC - LINEAR



$$\varepsilon_0 = \frac{2(0.85 f'_c)}{E_c}$$

$0 < \varepsilon_c < \varepsilon_0$

$$\sigma_c = 0.85 f'_c \left[2\left(\frac{\varepsilon_c}{\varepsilon_0}\right) - \left(\frac{\varepsilon_c}{\varepsilon_0}\right)^2 \right]$$

OR

$$\sigma_c = E_c \varepsilon_c - \frac{5 E_c^2}{17 f'_c} \varepsilon_c^2$$

$\varepsilon_0 \leq \varepsilon_c \leq \varepsilon_u$

$$\sigma_c = C_1 + C_2 \varepsilon$$

$$C_1 = \frac{17 f'_c (200 E_c \varepsilon_u - 289 f'_c)}{400 (10 E_c \varepsilon_u - 17 f'_c)}$$

$$C_2 = -51 E_c f'_c / 40(10 E_c \varepsilon_u - 17 f'_c)$$

$$\sigma(\varepsilon) = C_1 + C_2 \varepsilon$$

$$\sigma(\varepsilon_0) = C_1 + C_2 \varepsilon_0 = 0.85 f'_c$$

$$\sigma(\varepsilon_u) = C_1 + C_2 \varepsilon_u = 0.85 f'_c - 0.15(0.85 f'_c)$$

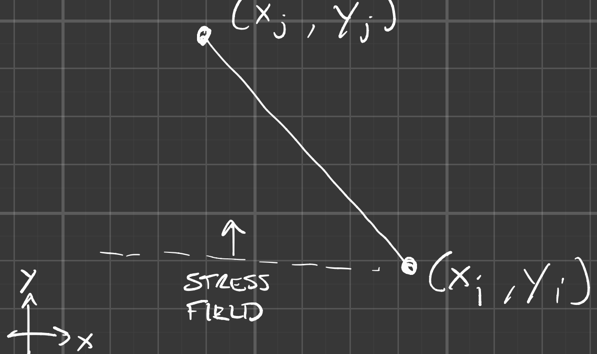
TITLE: GENERAL INTEGRALS FOR CONSTANT STRESS　　SCALE:

DATE: 4/1/23

- SOLVE INTEGRALS FOR THE 2D CASE. 3D CASE CAN BE LIMITED TO 2D BY COORD. ROTATION OF $-\alpha$ SO THAT STRAIN ONLY VARIES IN Y.

- ASSUME CROSS SECTION IS DEFINED BY PIECEWISE LINEAR BOUNDARIES IN COUNTER CLOCKWISE ORDER AND THE SECTION IS CLOSED, FIRST POINT = LAST. THIS ALLOWS FOR SUMMATION OF THE LINE INTEGRALS ON THE BOUNDARY.

- GREENS THEOREM: $\int_A f(x,y)\,dA = \iint \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA = \int P\,dx + Q\,dy$

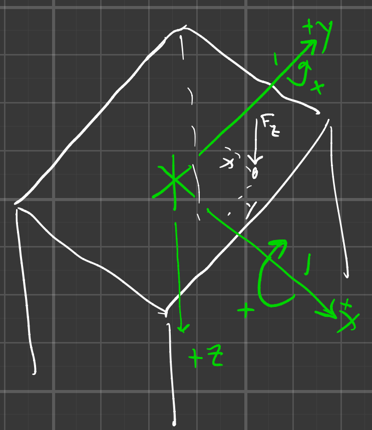


NOTE WILL NEED TO SUBDIVIDE BOUNDARIES THAT CROSS THE NEUTRAL AXIS.

GENERAL BOUNDARY SEGMENT IN THE STRESS FIELD

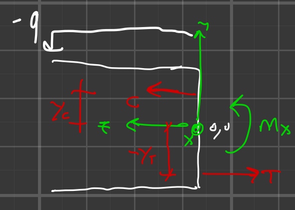
ESTABLISH SIGN CONVENTION:

$\downarrow F_z$ = COMPRESSION (TAKE COMPRESSION AS POSITIVE)



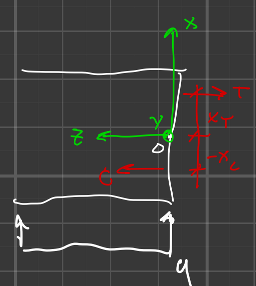
$\Rightarrow$ $F_z \cdot y = M_x$ (+ PUTS TOP OF SECTION IN COMPRESSION)

$F_z \cdot x = -M_y \Rightarrow M_y = F_z \cdot -x$ (+ PUTS LEFT IN COMPRESSION)



$\Sigma M_{x_0} = C \cdot y_c + (-T)(-y_T)$

$= C y_c + T y_T$ (+) ✓



$\Sigma M_{y_0} = C \cdot (-1)(-x_c) + (-T)(-1)(x_T)$

$= C x_c + T x_T$ (+) ✓

TITLE: WHITNEY BLOCK

SCALE:

DATE: 4/1/23

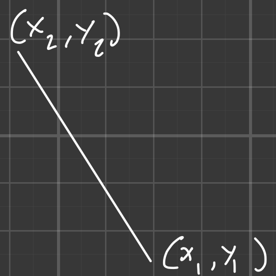
ESTABLISH ε DOMAIN, $\varepsilon(y) = \frac{\varepsilon_u}{c}\left(y - y_{TOP} + c\right)$

$$a = \beta_1 c \Rightarrow y_a = y_{TOP} - \beta_1 c \Rightarrow \varepsilon(a) = \frac{\varepsilon_u}{c}\left(y_{TOP} - \beta_1 c - y_{TOP} + c\right)$$

$$= \frac{\varepsilon_u}{c}\left(-\beta_1 c + c\right) = \varepsilon_u\left(-\beta_1 + 1\right)$$

$$\therefore \quad f(\varepsilon) = \begin{cases} 0.85 f'_c & \varepsilon_u(-\beta_1 + 1) \le \varepsilon \le \varepsilon_u \\ 0 & \text{ELSE} \end{cases}$$

(NOTE $f(\varepsilon)$ IS CONSTANT $\therefore$ CAN BE REMOVED FROM INTEGRALS)



DEFINE PARAMETRIC FUNCTIONS FOR X & Y

$$x(t) = x_1 + t(x_2 - x_1) \qquad dx/dt = x_2 - x_1 \quad \text{OR} \quad dx = (x_j - x_i)dt$$

$$y(t) = y_1 + t(y_2 - y_1) \qquad dy/dt = y_2 - y_1 \quad \text{OR} \quad dy = (y_j - y_i)dt$$

$$\int_A f(\varepsilon)\, dA = \iint f(\varepsilon)\, dx\, dy = \iint \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA = \int P dx + Q dy$$

VOLUME OF STRESS FIELD = AXIAL FORCE

TRY $Q = x f \quad P = 0 \Rightarrow \frac{\partial Q}{\partial x} = f \quad \frac{\partial P}{\partial y} = 0 \Rightarrow \iint f - 0\, dA = \iint f dA$ ✓

$$\int x f\, dy \Rightarrow f \int_0^1 \left[x_1 + t(x_2 - x_1)\right](y_2 - y_1)\, dt$$

$$P = \tfrac{1}{2} f\, (x_2 + x_1)(y_2 - y_1)$$

FIRST MOMENT OF STRESS FIELD = MOMENT

$$M_{yy} = \int_A -x\, f(\varepsilon)\, dA \quad ; \quad M_{xx} = \int_A y\, f(\varepsilon)\, dA$$

CHOOSE Q & P: $\quad Q = 0 \quad P = y x f(\varepsilon) \qquad\qquad Q = x y f(\varepsilon) \quad P = 0$

$$\frac{\partial Q}{\partial x} = 0 \quad \frac{\partial P}{\partial y} = x f(\varepsilon) \Rightarrow 0 - x f(\varepsilon) = -x f(\varepsilon) ✓ \qquad \frac{\partial Q}{\partial x} = y f(\varepsilon) \quad \frac{\partial P}{\partial y} = 0 \Rightarrow y f(\varepsilon) - 0 = y f(\varepsilon) ✓$$

$$\int y x f\, dx \qquad\qquad \int x y f\, dy$$

$$f \int_0^1 \left[y_1 + t(y_2 - y_1)\right]\left[x_1 + t(x_2 - x_1)\right](x_2 - x_1)\, dt \qquad f \int_0^1 \left[x_1 + t(x_2 - x_1)\right]\left[y_1 + t(y_2 - y_1)\right](y_2 - y_1)\, dt$$

$$M_{yy} = \tfrac{1}{6} f\, (x_2 - x_1)(2x_2 y_2 + x_1 y_2 + x_2 y_1 + 2x_1 y_1) \qquad M_{xx} = \tfrac{1}{6} f\, (y_2 - y_1)(2x_2 y_2 + x_1 y_2 + x_2 y_1 + 2x_1 y_1)$$

* WHERE $f = 0.85 f'_c$ = CONSTANT $\therefore$ THE ABOVE WORK FOR ANY CONSTANT STRESS FIELD