# Asset Pricing - C-CAPM


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Question 1

a. For each $\gamma$ compute the value of $m$ in each year. Use then the entire time series of $m$ and $R_M$ to compute $-\mathrm{Cov}(R_M, m)$ and multiply it by the mean of $1 + r$ . Plot the resulting values against all values of gamma Which of those values comes closest to the model's theoretical prediction (mean value of RM-r)?

For each gamma;

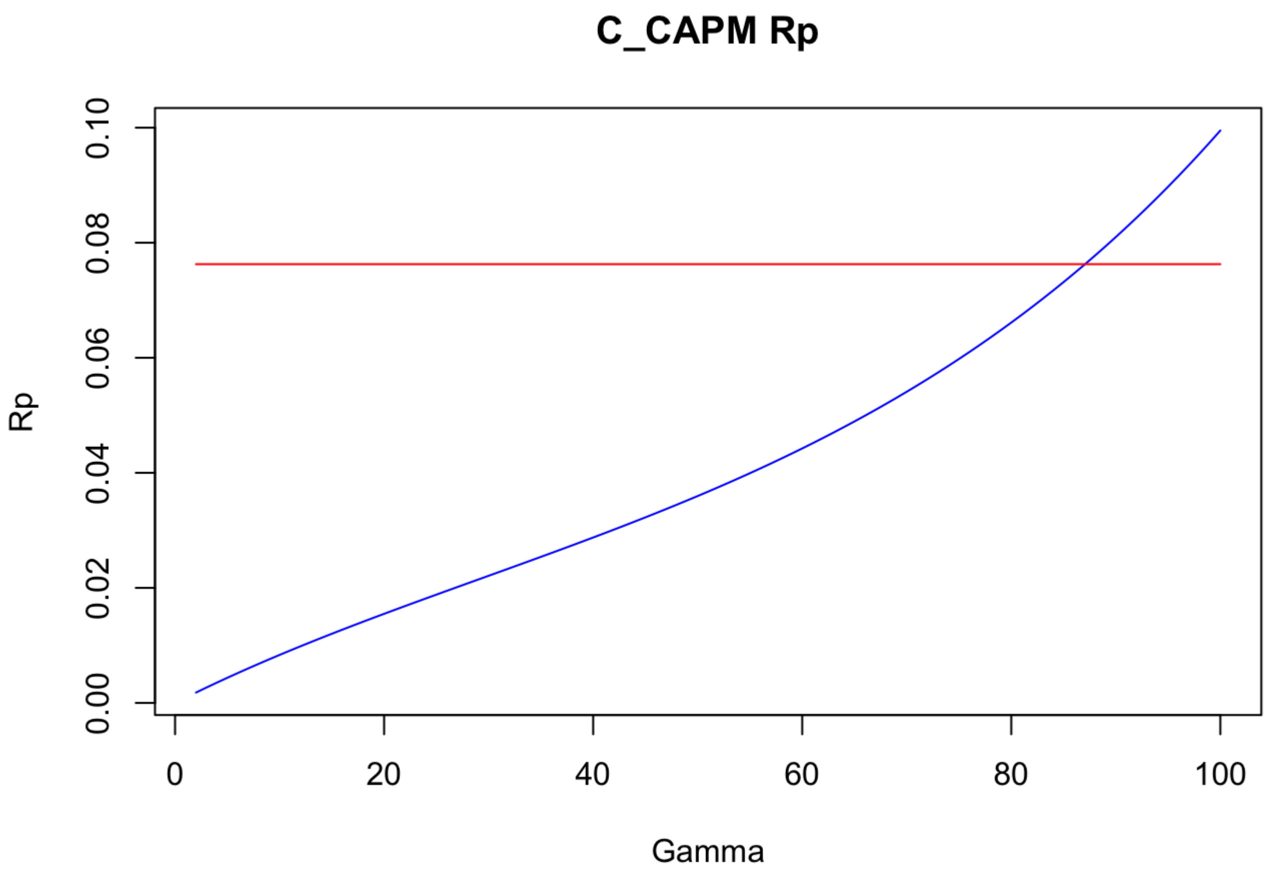
$$m = \frac{U'(c_1)}{U'(c_0)} = \left(\frac{c_0}{c_1}\right)^{\gamma}$$

$$E(R_m - R_f) = -\mathrm{Cov}(m, R_m)(1 + R_f)$$

```
library("readxl")
library("data.table")
library("tinytex")
df <- read_excel("/Users/guneykan/Desktop/PS1Data.xlsx")
df<-data.table(df)
names(df)<-c("year", "dc", "r_p", "r_f")
```

```
hist_Rp<-df[, mean(r_p)]
hist_Rf<-df[, mean(r_f)]
r_m<-df[, r_p] + df[, r_f] - 1
df[, r_m:=list(r_m)]
df[, m:=list(1/df[, dc])]
gamma<-c(2:100)
m_gamma<-vector("list", 99)
cov_r_m<-vector("list", 99)
c_capm_Rp<-vector("list", 99)
for(i in c(1:99)){
  m_gamma[[i]]<-(df[, m])^gamma[i]
  cov_r_m[[i]]<-cov(m_gamma[[i]], df[, r_m])
  c_capm_Rp[[i]]<-cov_r_m[[i]]*hist_Rf*-1
}
```

```
plot(gamma, unlist(c_capm_Rp),
main="C_CAPM Rp",
ylab="Rp",
xlab = "Gamma",
type="l",
col="blue")
lines(gamma, rep(hist_Rp, 99), col="red")
```



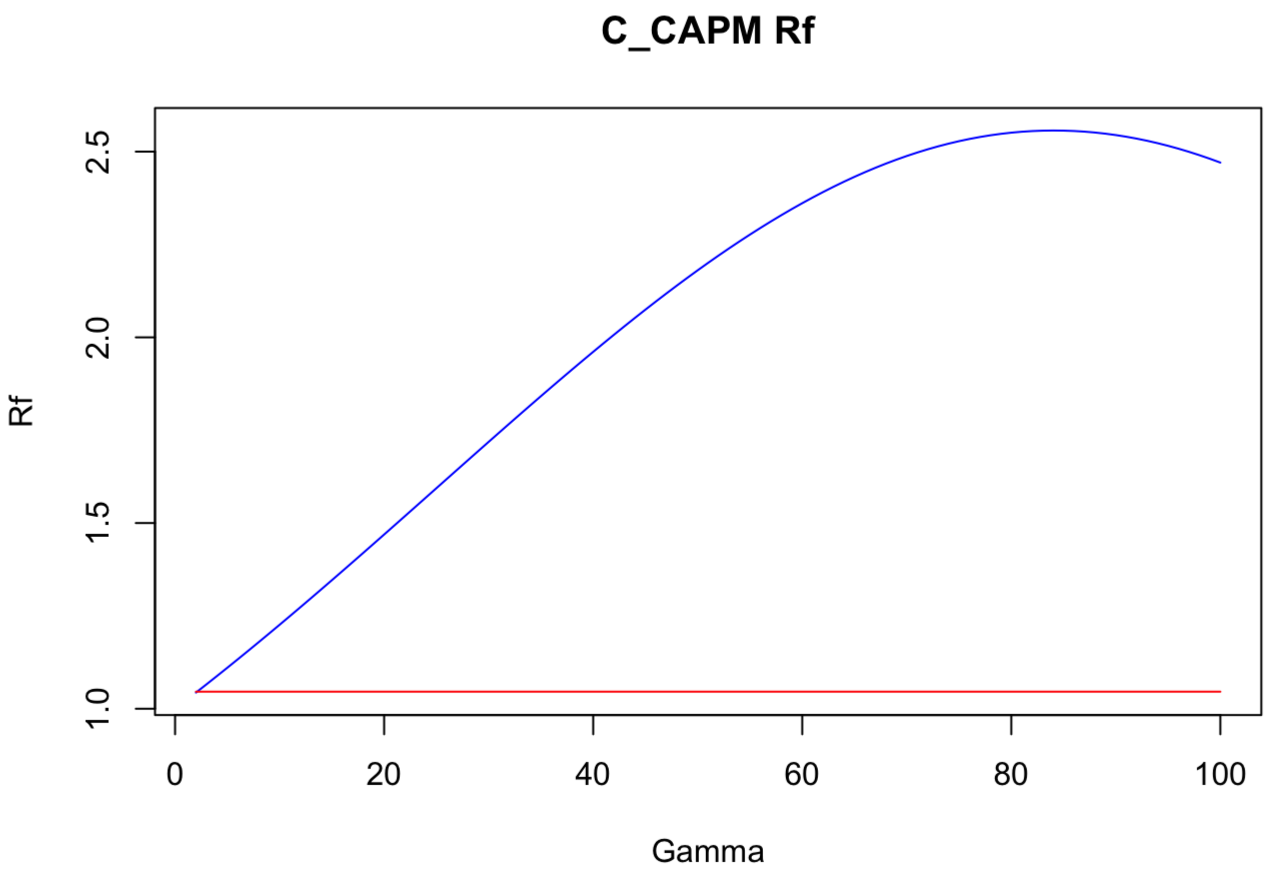

```
index<-which.min(abs(unlist(c_capm_Rp)-hist_Rp))
cat("closest CAPM estimation:", unlist(c_capm_Rp)[index]," ",
    "mean Rp:", hist_Rp," ",
    "respective gamma:", gamma[index])
```

```
## closest CAPM estimation: 0.07618853   mean Rp: 0.0762747   respective gamma: 87
```

b. For each $\gamma$ compute the inverse of the sample mean of $m$ and plot against all values of $\gamma$ Which of those values comes closest to the model's theoretical prediction (mean value of $1 + r$))?

$$1 + R_f = 1/E(m)$$

```
mu_m<-sapply(m_gamma, mean)
c_capm_rf<-1/mu_m
plot(gamma, c_capm_rf,
main="C_CAPM Rf",
ylab="Rf",
xlab = "Gamma",
type="l",
col="blue")
lines(gamma, rep(hist_Rf, 99), col="red")
```




```
index_0<-which.min(abs(c_capm_rf-hist_Rf))
cat("closest CAPM estimation:", c_capm_rf[index_0], " ",
    "mean Rf:", hist_Rf, " ",
    "respective gamma:", gamma[index_0])
```

```
## closest CAPM estimation: 1.043475   mean Rf: 1.04595   respective gamma: 2
```

c. What conclusions do you draw from these results?

From Taylor Expansion we obtain;

$$E(R_m - R_f) = \gamma \mathrm{Cov}\left(\frac{(c_1 - c_0)}{c_0}, R_m\right)(R_f + 1)$$

From Relative Risk Aversion;

$$R(C) = -\frac{U''(c)}{U'(c)}c = \frac{\gamma}{c}c = \gamma$$

From the relative risk aversion function, we see that, as gamma increases, investors become more risk-averse. Thus we can conclude as γ increases, investors become more risk-averse and expected risk premium increases. Relative Risk Aversion depends on the gamma hence relative risk aversion is same for all investors, and we can implement C-CAPM using aggregate consumption

Comparing data with the results of the Consumption CAPM model, γ needs to be around 87 for the C-CAPM model to match the historical Risk Premium, meaning investors need to be extremely risk-averse.

On the other hand, when we look at the estimation of the risk-free rate by the C-CAPM model, gamma needs to be around 2 to match the historical Risk-free rate. These two estimations of the model do not match with each other, or the data itself. While risk premium estimation indicates that investors need to be highly risk-averse to explain empirical risk premium, comparison of the historical risk-free and the model estimation shows gamma needs to be around 2, which is the complete opposite of the result we obtained at 1(a). We see that as the covariance of the stochastic discount factor, and market return goes more negative as the investors become more risk-averse; this is due to covariance reflects the expectation of the investors, thus their risk awareness. As investors become more risk-averse, they require a higher risk premium to compensate for the risk they are taking by holding the asset. In this particular case, the asset is the market portfolio.

Also, the model estimation of risk-free shows that as the investors become more risk-averse (gamma increases), the risk-free rate also increases. This does not make sense in the sense that if the investors are more risk-averse, they tend to invest more in bonds, which will push the price of bonds up while bringing the risk-free rate down. What we see here is the opposite, but this can be seen from the different view, which is investors are well compensated for the risk they are taking, so the market is in equilibrium.

Question 2

b. Stochastic discount factor

$$U(c_t) = \left(\frac{c_t}{x_t - 1}\right)^{1-\gamma}$$

$$U'(c_t) = \frac{c_t^{-\gamma}}{x_{t-1}^{(1-\gamma)}}$$

$$U'(c_{t-1}) = \frac{c_{t-1}^{-\gamma}}{x_{t-2}^{1-\gamma}}$$

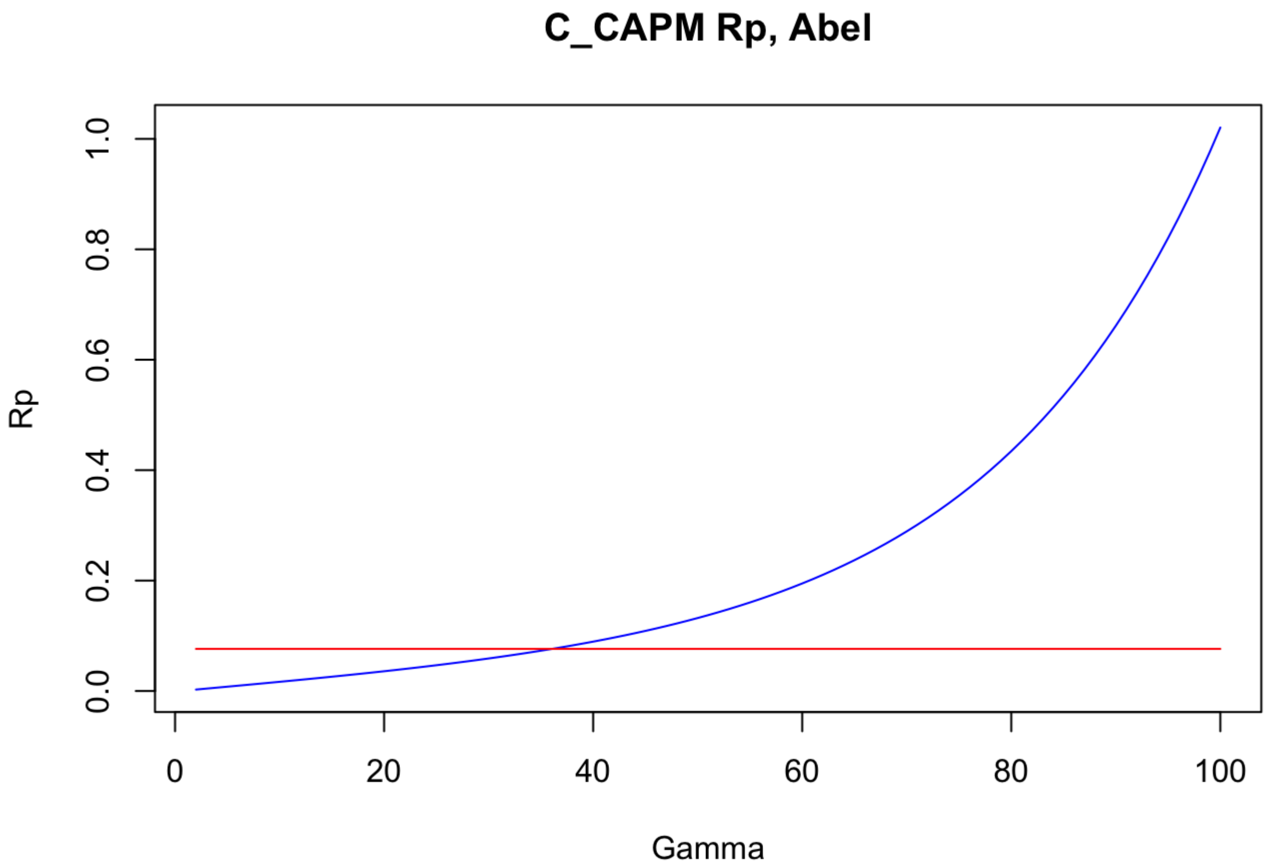
$$m = \frac{U'(c_t)}{U'(c_{t-1})} = \left(\frac{c_t}{c_{t-1}}\right)^{-\gamma}\left(\frac{x_{t-1}}{x_{t-2}}\right)^{\gamma-1}$$

$$m = \left(\frac{c_t}{c_{t-1}}\right)^{-\gamma}\left(\frac{c_{t-1}}{c_{t-2}}\right)^{\gamma-1}$$

```
m_gamma_abel<-vector("list", 99)
cov_r_m_abel<-vector("list", 99)
c_capm_Rp_abel<-vector("list", 99)
abel_m<-c(NA, df[2:63, m])
df[, abel_factor:=list(abel_m)]
for(t in c(1:99)){
  for(i in c(2:63)){
    m_gamma_abel[[t]][i-1]<-((df[(i-1), m])^(1-gamma[t]))*(df[i, abel_factor]^(gamma
[t]))
  }
}
for(i in c(1:99)){
  cov_r_m_abel[[i]]<-cov(m_gamma_abel[[i]], df[2:63, r_m])
  c_capm_Rp_abel[[i]]<-cov_r_m_abel[[i]]*hist_Rf*-1
}
```

c. Risk Premium estimation for each gamma

```
plot(gamma, unlist(c_capm_Rp_abel),
main="C_CAPM Rp, Abel",
ylab="Rp",
xlab = "Gamma",
type="l",
col="blue")
lines(gamma, rep(hist_Rp, 99), col="red")
```

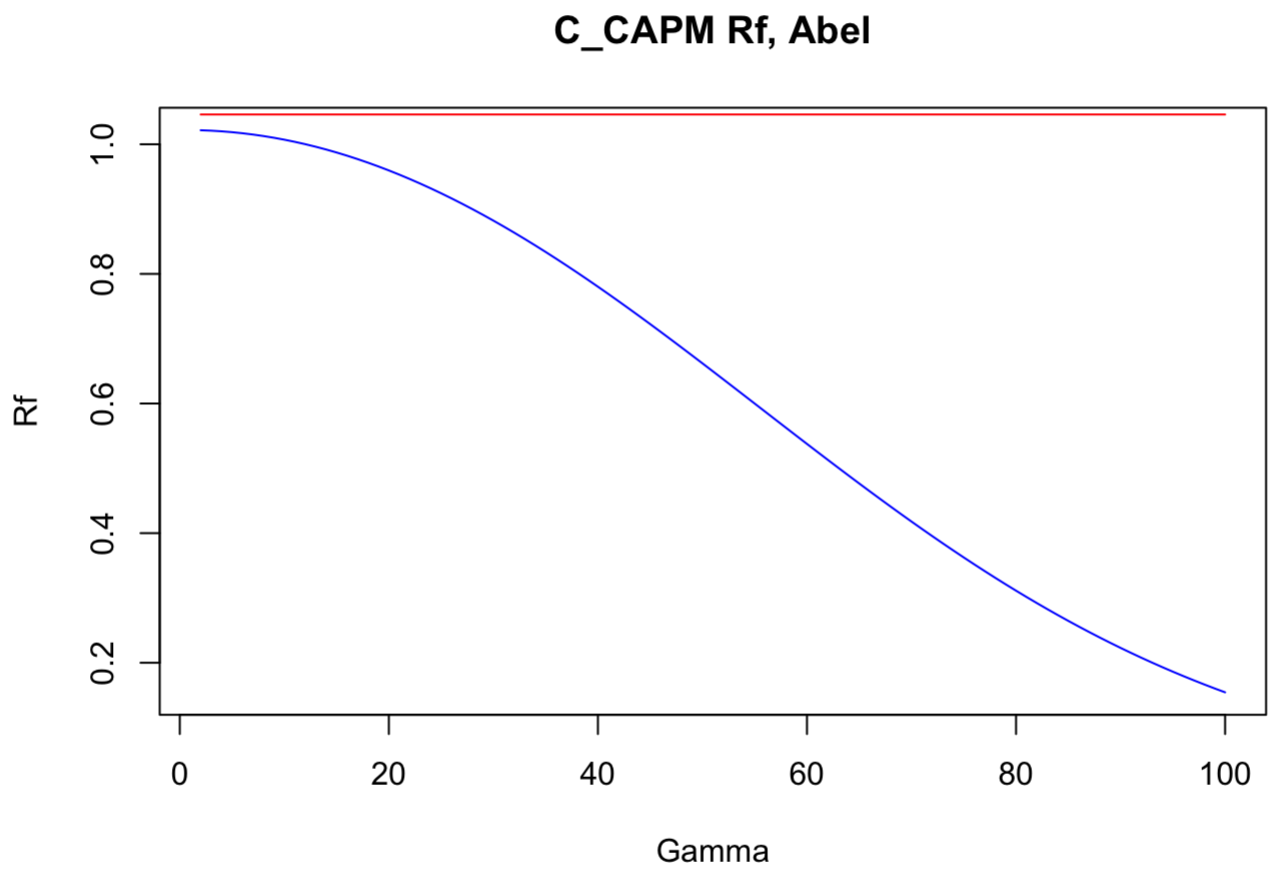



```
index<-which.min(abs(unlist(c_capm_Rp_abel)-hist_Rp))
cat("closest CAPM estimation:", unlist(c_capm_Rp_abel)[index]," ",
    "mean Rp:", hist_Rp," ",
    "respective gamma:", gamma[index])
```

```
## closest CAPM estimation: 0.07606894   mean Rp: 0.0762747   respective gamma: 36
```

d. Risk-Free rate estimation for each gamma

```
mu_m_abel<-sapply(m_gamma_abel, mean)
c_capm_rf_abel<-1/mu_m_abel
plot(gamma, c_capm_rf_abel,
main="C_CAPM Rf, Abel",
ylab="Rf",
xlab = "Gamma",
type="l",
col="blue")
lines(gamma,rep(hist_Rf, 99), col="red")
```




```
index_0<-which.min(abs(c_capm_rf_abel-hist_Rf))
cat("closest CAPM estimation:", c_capm_rf_abel[index_0], " ",
    "mean Rf:", hist_Rf, " ",
    "respective gamma:", gamma[index_0])
```

```
## closest CAPM estimation: 1.021638   mean Rf: 1.04595   respective gamma: 2
```

e. How does the model compare relative to the model with simpler preferences of question 1?

Considering updated utility function from Relative Risk Aversion we obtain;

$$R(C) = -\frac{U''(c)}{U'(c)}c = \gamma$$

Note that Relative Risk Aversion is still constant so we can use aggregate consumption

Compared to question 1(a), now gamma, which matches the historical risk premium is around 36. While this value implies less risk aversion relative to the estimation in question 1(a), there is still a high degree of risk aversion, and it still does not match the gamma estimated through matching the model estimation of risk-free rate to the historical risk-free rate. Due to the fact utility function takes into account previous consumption growth in addition to current consumption growth, it is more realistic compared to C-CAPM. In the pricing model of Abel, we see that as gamma increases (as investors become more risk-averse) risk-free rate decreases which is due to stochastic discount factor taking previous consumption growth into account in addition to current consumption growth. This makes the Abel version of the asset pricing model more realistic, compared to the model given in question 1.