Multi-Task Learning of Component Strengths in Non-Conjugate Admixture Models

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Research

Three Components

1. Multi-Task Learning
2. Admixture Modelling ("Topic Models")
3. Local Variational Bounds

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Multi-Task Learning

Situation: Making many predictions from the same data

1. Predict exam scores in L subjects for several children[12]
2. Predict customers affinity to L observed aspects of a product[1]
3. Propose image captions by predicting $p(word|image)$ from image features for L words (in this case $L > 10,000$)[2]

Multi-Task Learning

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Problem: Can we improve performance by transferring knowledge between tasks[15]

1. Learn correlations between tasks.
2. Learn a low-rank projection of the tasks themselves
3. Learn structure of features by how we use them - via regularization

Regularization

Learn L vectors w_l. How to *transfer* knowledge from inferring $w_1, \ldots, w_{l-1}$ to the task of inferring w_l

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$$y_{nl}|w_l \sim \mathcal{N}\left(w_l^\top x_n, \sigma^2 I\right) \qquad w_l \sim \mathcal{N}\left(\mathbf{0}, \alpha^2 I\right)$$

Regularization

Learn L vectors w_l. How to *transfer* knowledge from inferring $w_1, \ldots, w_{l-1}$ to the task of inferring w_l

Bayesian approach - learn the prior[1]

$$y_{nl}|w_l \sim \mathcal{N}\left(w_l^\top x_n, \sigma^2 I\right) \qquad w_l \sim \mathcal{N}(m, \Sigma)$$

$$m|\Sigma \sim \mathcal{N}\left(m_0, \frac{1}{\lambda}\Sigma\right) \qquad \Sigma \sim \mathcal{W}^{-1}(\Sigma_0, \nu)$$

Regularization

Learn L vectors w_l. How to *transfer* knowledge from inferring $w_1, \ldots, w_{l-1}$ to the task of inferring w_l

Low-Rank Projections of the Feature Space

$$y_{nl} | w_l \sim \mathcal{N}\left(w_l^\top x_n, \sigma^2 I\right)$$

$$w_l | z_l \sim \mathcal{N}\left(U z_l + m, \alpha^2 I\right)$$

$$z_l \sim \mathcal{N}\left(\mathbf{0}, I\right)$$

$$\implies w_l \sim \mathcal{N}\left(m, \alpha^2 I + U U^\top\right)$$

Regularization

Learn L vectors w_l. How to *transfer* knowledge from inferring $w_1, \ldots, w_{l-1}$ to the task of inferring w_l

Exploit sparsity using ARD priors

$$y_{nl} | w_l \sim \mathcal{N}\left(w_l^\top x_n, \sigma^2 I\right) \qquad w_l \sim \mathcal{N}\left(\mathbf{0}, \mathrm{diag}\left(\boldsymbol{\alpha}\right)\right)$$

$$\alpha_f \sim \mathcal{G}\left(a, b\right)$$

Regularization

Learn L vectors w_l. How to *transfer* knowledge from inferring $w_1, \ldots, w_{l-1}$ to the task of inferring w_l

Exploit sparsity using Clustered ARD priors[6]

$$y_{nl} | w_l \sim \mathcal{N}\left(w_l^\top x_n, \sigma^2 I\right) \qquad w_l | z_l \sim \mathcal{N}\left(\mathbf{0}, \mathrm{diag}\left(\boldsymbol{\alpha}_{z_l}\right)\right)$$

$$z_l \sim \mathcal{M}\left(\boldsymbol{\theta}, 1\right) \qquad \alpha_{fk} \sim \mathcal{G}\left(a, b\right)$$

$$\boldsymbol{\theta} \sim \mathcal{D}\left(\beta\right)$$

Regularization

Learn L vectors w_l. How to *transfer* knowledge from inferring $w_1, \ldots, w_{l-1}$ to the task of inferring w_l

This is all just hierarchical Bayeisan modelling

However analogous methods exist in error-optimisation approaches to machine learning which learn regularization functions instead of priors.

- Low-Rank projections in [3]
- Heterogeneous sparsity in [4]

Research

Three Components

1. Multi-Task Learning
2. **Admixture Modelling ("Topic Models")**
3. Local Variational Bounds

Mixture Models versus Admixture Models

Classical Mixture Model of Text - One topic per document

$$\boldsymbol{\theta} \sim \mathcal{D}(\alpha) \qquad z_d \sim \mathcal{M}(\boldsymbol{\theta}, 1) \qquad w_{dn} \sim \mathcal{M}(\phi_{z_d}, 1)$$

Where each of the component vocabularies is drawn $\phi_k \sim \mathcal{D}(\beta)$.

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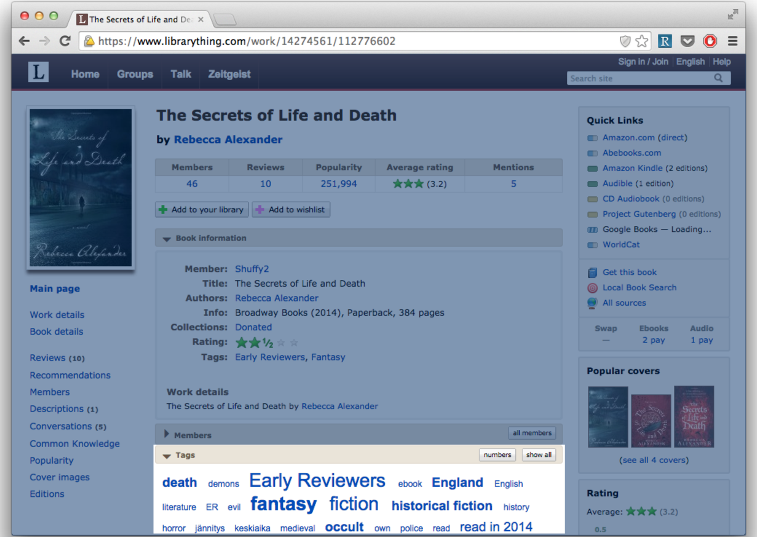
Mixture models struggle to generalise.

- Fewer clusters mean coarser estimates of cluster centroids
- But more clusters mean fewer datapoints per cluster, and thus sparser estimates of cluster centroids (for text), due to the assumption of one cluster per document

Mixture Models versus Admixture Models



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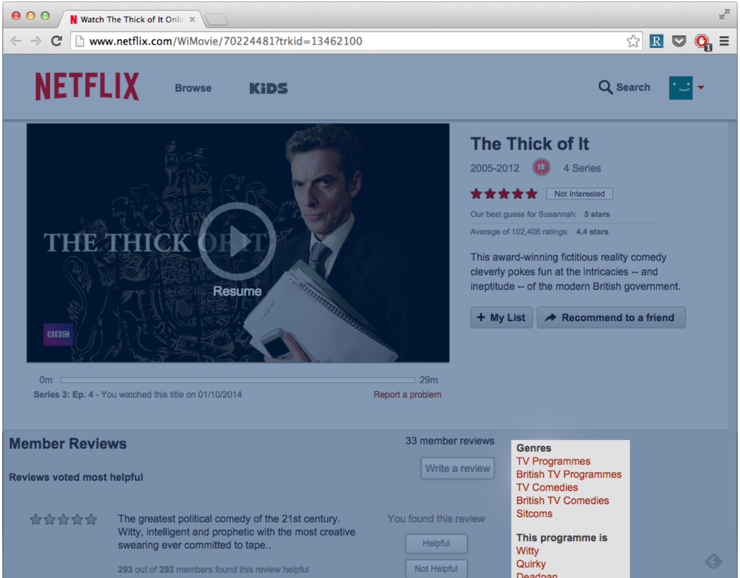

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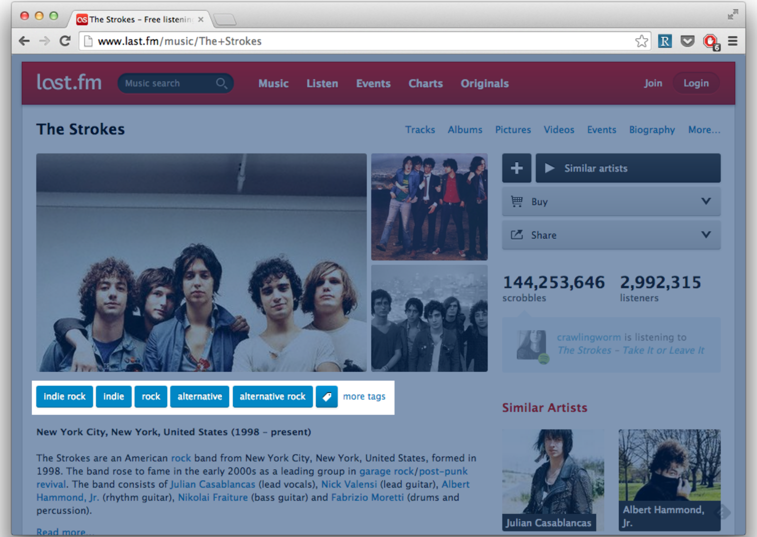

Mixture Models versus Admixture Models

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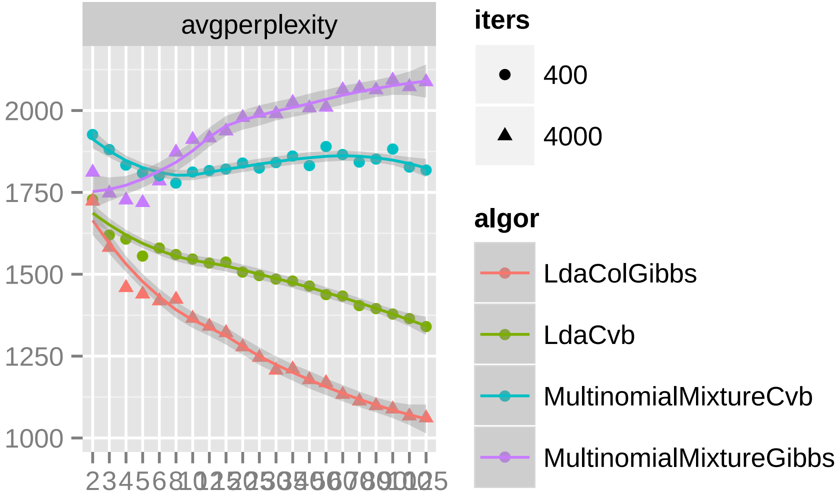
Where each of the component vocabularies is drawn $\phi_k \sim \mathcal{D}(\beta)$.

Admixture models assign a *mixture* of topics to each document, In the case of text the “topic-model” implementation[10] assigns a single topic to each word w_{dn}.

$$\boldsymbol{\theta}_d \sim \mathcal{D}(\alpha) \qquad z_{dn} \sim \mathcal{M}(\boldsymbol{\theta}_d, 1) \qquad w_{dn} \sim \mathcal{M}(\phi_{z_{dn}}, 1)$$

Mixture Models versus Admixture Models



Topic Models: Multinomial Admixtures

Five broad areas of research

- Richer Observation Models
  - Language Models[41][44][26]
  - Alternative distrbutions such as logistic-normal[9][17] or von Mises - Fisher[34]

Topic Models: Multinomial Admixtures

Five broad areas of research

- Richer Observation Models
- Alternatives to the Dirchlet prior
  - The "Logistic Normal" prior for the Correlated Topic Model[7]

$$\boldsymbol{\eta}_d \sim \mathcal{N}(\mu, \Sigma)$$
$$\theta_{dk} = \sigma_k(\boldsymbol{\theta}_d) = \frac{\exp(\eta_{dk})}{\sum_j \exp(\eta_{dj})}$$

  - Same quality of model fit with fewer topics

Topic Models: Multinomial Admixtures

Five broad areas of research

- Richer Observation Models
- Alternatives to the Dirchlet prior
- Bayesian Non-Parametrics to estimate Topic Counts
    - Hierarchical Dirichlet Processes[38]
    - Discrete Infinite Logistic Normal Distribution[32]

Topic Models: Multinomial Admixtures

Five broad areas of research

- Richer Observation Models
- Alternatives to the Dirchlet prior
- Bayesian Non-Parametrics to estimate Topic Counts
- Scalable Implementations:
    - Gibbs Sampling[33] and Collapsed Gibbs Sampling[19]
    - Variational[10] inference and Collapsed Variational[39][21] inference
    - MAP and other approximations[5]
    - Distributed "Big Data" approaches[37][31][16]
    - Optimised online approaches[22][23][29]

Topic Models: Multinomial Admixtures

Five broad areas of research

- Richer Observation Models
- Alternatives to the Dirchlet prior
- Bayesian Non-Parametrics to estimate Topic Counts
- Scalable Implementations:
- Use of Covariates x_d
  - Ad-hoc: Time[43], author[28], region[17]
  - "Downstream" Models[8][35][40]

$$p(w_d, x_d) = \int_{\boldsymbol{\theta}_d} p(w_d|\boldsymbol{\theta}_d)p(x_d|\boldsymbol{\theta}_d)p(\boldsymbol{\theta}_d) \tag{1}$$

  - "Upstream" Models:[30]

$$p(w_d|x_d) = \int_{\boldsymbol{\theta}_d} p(w_d|\boldsymbol{\theta}_d)p(\boldsymbol{\theta}_d|x_d) \tag{2}$$

.

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LDA also linked to multinomial PCA[14] and Non-Negative Matrix Factorization[18]

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3. **Local Variational Bounds**

Local Variational Bounds

The softmax transformation

$$\eta_d \sim \mathcal{N}(\boldsymbol{\mu}, \Sigma) \qquad z_{dn} \sim \mathcal{M}(\boldsymbol{\sigma}(\boldsymbol{\eta}_d), 1) \tag{1}$$

Problems

- Prior (a Gaussian) is not conjugate to the likelihood (a multinomial mixture)
- No analytic form for the posterior.

Approach is to approximate the likelihood using bounds

- Classic approach is a global bound over the entire likelihood (e.g. the Laplace approximation)
- A better fit, and more flexibility, can be obtained by using *local* bounds over the likelihood of each document.

Local Variational Bounds

- Global Bounds: Laplace, Delta Method[42]
- Quadratic Bounds: Bohning[11], Bouchard[13]
- Other more recent bounds
    - Piecewise Quadratic Bounds[27]
    - Tilted Bound[25]
    - Stick-breaking bound[24]

Research

Use-Cases: Microtexts

- Tweets: Predict text given user features (username, time tweet was posted). Notably predict absent words such as hashtags.
- Image captions: Predict caption given image features.

Incorporate multi-task learning into an upstream topic model

Partial Model

The Model

$$\eta_d \sim \mathcal{N}(Ax_d, \Sigma) \qquad \boldsymbol{\theta}_d = \boldsymbol{\sigma}(\eta_d)$$

$$z_{dn} \sim \mathcal{M}(\boldsymbol{\theta}_d, 1) \qquad w_{dn} \sim \mathcal{M}(\phi_{z_{dn}}, 1)$$

Partial Model

The Model

$$\eta_d \sim \mathcal{N}(Ax_d, \Sigma) \qquad \theta_d = \sigma(\eta_d)$$

$$z_{dn} \sim \mathcal{M}(\theta_d, 1) \qquad w_{dn} \sim \mathcal{M}(\phi_{z_{dn}}, 1)$$

Prior over A?

Matrix-Variate Priors

Matrix-Variate Normal Distribution [20] :

$$A \sim \mathcal{N}(M, \Omega, \Sigma) \implies \mathrm{vec}(A) \sim \mathcal{N}(\mathrm{vec}(M), \Sigma \otimes \Omega)$$

Matrix-Variate Priors

Matrix-Variate Normal Distribution [20] :

$$A \sim \mathcal{N}(M, \Omega, \Sigma) \implies \mathrm{vec}(A) \sim \mathcal{N}(\mathrm{vec}(M), \Sigma \otimes \Omega)$$

$$\ln p(W) = -\frac{FL}{2}\ln 2\pi - \frac{F}{2}\ln|\Omega| - \frac{L}{2}\ln|\Sigma| - \frac{1}{2}\mathrm{etr}\left(\Sigma^{-1}(W-M)\Omega^{-1}(W-M)^\top\right)$$

where $\mathrm{etr}(X) = \exp(\mathrm{tr}(X))$

Matrix-Variate Priors

Matrix-Variate Normal Distribution [20] :

$$A \sim \mathcal{N}(M, \Omega, \Sigma) \implies \mathrm{vec}(A) \sim \mathcal{N}(\mathrm{vec}(M), \Sigma \otimes \Omega)$$

$$V \sim \mathcal{N}(0, I, I) \qquad A|V \sim \mathcal{N}(UZ, I, \Sigma)$$

$$A \sim \mathcal{N}\left(0, I + UU^\top, \Sigma\right)$$

See also [2] for examples of matrix-variate normal priors.

Full Model

The Model

$$V \sim \mathcal{N}(0, I, I) \qquad A|V \sim \mathcal{N}(UV, I, \Sigma) \tag{2}$$

$$\boldsymbol{\eta}_d \sim \mathcal{N}(Ax_d, \Sigma) \qquad \boldsymbol{\theta}_d = \boldsymbol{\sigma}(\boldsymbol{\eta}_d) \tag{3}$$

$$z_{dn} \sim \mathcal{M}(\boldsymbol{\theta}_d, 1) \qquad w_{dn} \sim \mathcal{M}(\phi_{z_{dn}}, 1) \tag{4}$$

Inference

- Bound the likelihood using quadratic bounds
- Use the variational bound with the mean-field approximation $q(V, A, \Theta, Z, \Phi) = q(V)q(A)q(\Theta)q(Z)q(\Phi)$
- Numerically eliminate $Z \in \mathbb{R}^{D \times N \times K}$ as it consumes huge amounts of computer memory.

Model

Two Datasets NIPS Papers from 1987 to 1999

- 682 Documents. Vocabulary of 12503 words
- Median document length of 1,532 words, total of 1,075,323 words across the corpus
- Features are: authors; citations; the year

Tweets from April to September 2013 (inclusive)

- 735,868 tweets from 572 users. Vocabulary of 82,698 words
- Median tweet length is 10 words, total of 7,272,228 word observations across the corpus
- Features are: authors; time at various granularities (hour, day, week, month)

Images are ongoing

Twitter Hashtags

- Our model predicts words according to $p(w_d|x_d)$
- So given a tweets features, we can generate words ourselves, instead of using the observed words
- We can generate hashtags
  - So given all tweets that do not have a given hashtag (like `#eurozone`)
  - In which tweet is it most likely to occur given the features

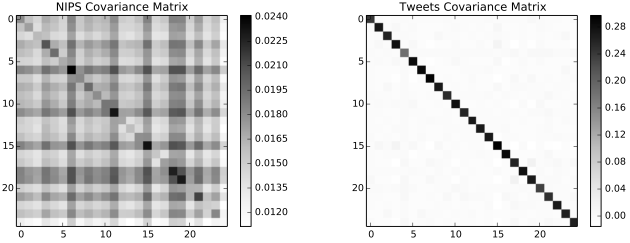
Twitter Hashtags

Hashtag	Tweet Words
#eurozone	#German finmin #Schaeuble argues #Karlsruhe may have no jurisdiction over #ECB measures.1st question by court's judges also focuses on this.
#usopen	Essa foi apenas a 2a vitoria de Gasquet nas oitavas de um Grand Slam. O frances alcanca sua primeira QFs em GS desde Wimbledon 2007 —
#f1	MT @f1paddockpass: ...a big shout out to the marshalls & volunteers here in Singapore. To all of you, heartfelt thanks @F1NightRace

Twitter Hashtags

Hashtag	**Tweet Words**
#tcot	#Delaware Senate rejects bill to keep guns away from unstable people deemed danger to others http://hrld.us/110xnGt #guncontrol #NRA
#beer	No-Li Brewhouse on track for 150% growth, expanding annual capacity to 10,000 barrels ...
#obamacare	@michellemalkin #feded supporters simply believe you won't challenge them. #stopcommoncore @afpne

Covariances



(a) NIPS (b) Tweets

Figure: Covariances over topics inferred using the Bohning implementation of the model for $K = 25$

What Next?

- Look at more complex matrix-variate priors, with low-rank approximations to both row and column covariances (the obvious approach doesn't work well)
- Large Scale inference (to 33 million tweets), potentially via SGD[23]
- Look at alternatives to the low-rank decomposition such as the use of matrix-variate Gaussian Scale models for sparsity[46]
- Look at richer downstream models for covariates such as collective-matrix[36] and tensor factorization[45].

Questions?

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