Budgeted Reinforcement Learning in Continuous State Space

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Introduction — Use-case

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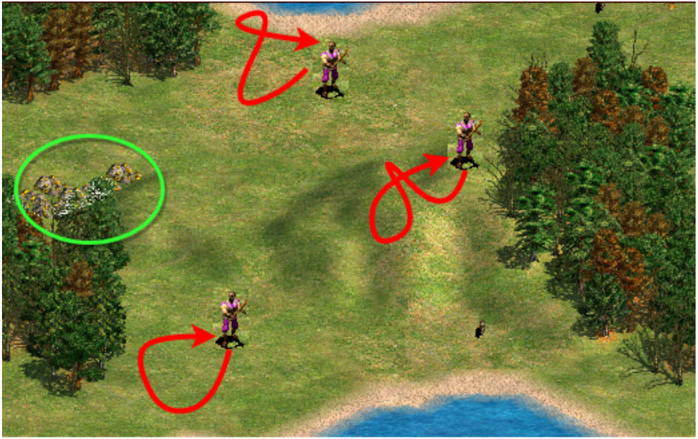

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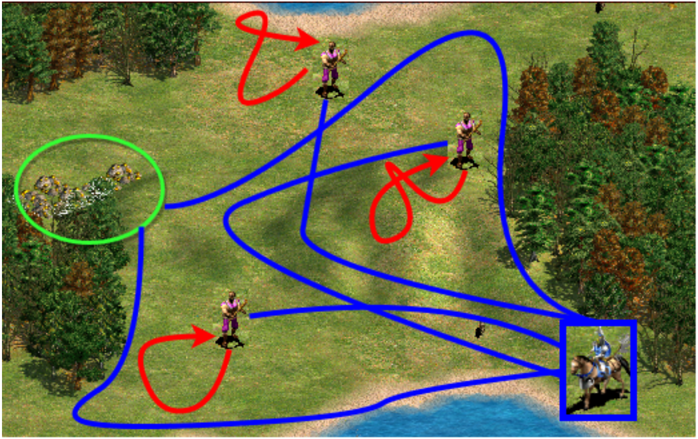

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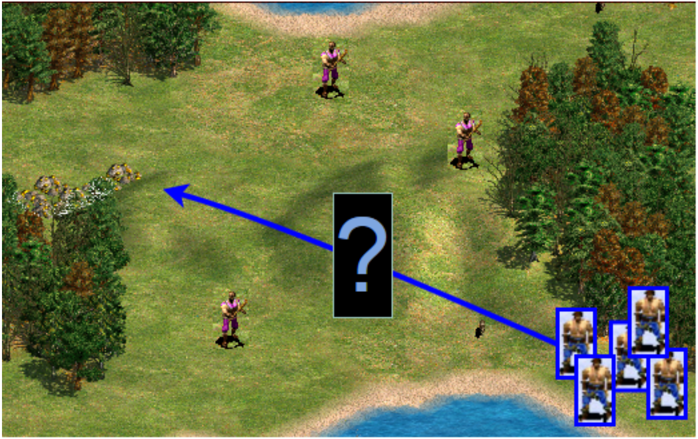

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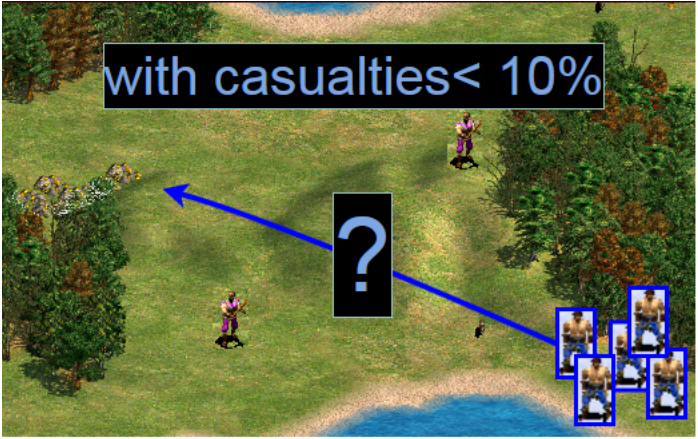

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- gather gold as much as possible;
- limit the villager casualties under some budget;
- being able to change the budget in real time.

Solution

- This problem can be cast as a Budgeted Markov Decision Process.

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- Find π^* s.t $\forall (s, \beta) \in \mathcal{S} \times \mathcal{B}$:

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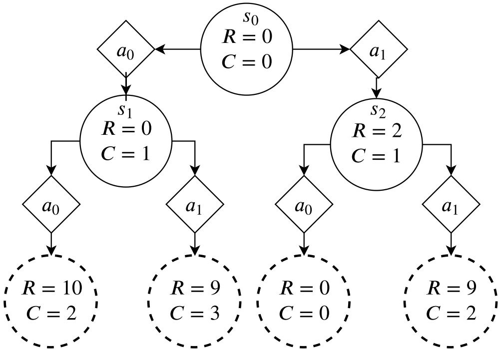
The Dynamic Programming solution: intuition



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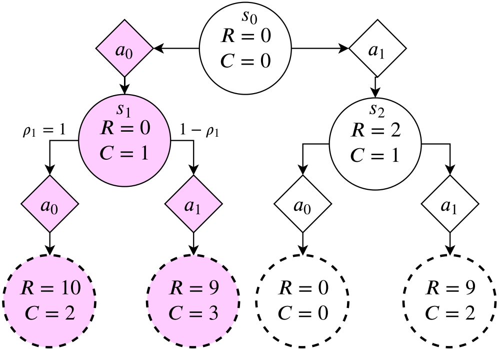

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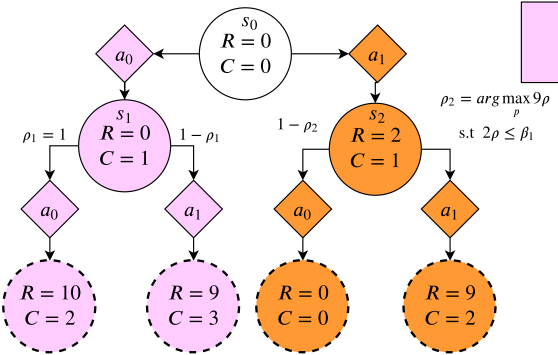

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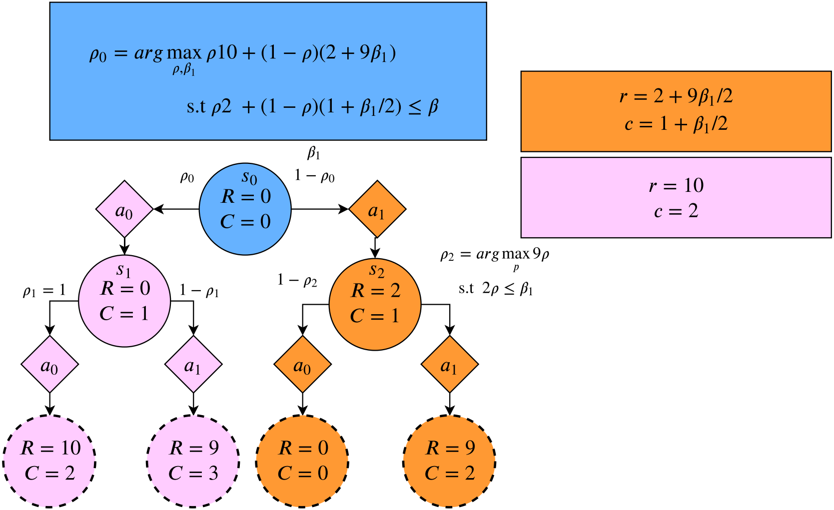

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$$\rho_0 = arg\max_{\rho,\beta_1} \rho 10 + (1-\rho)(2+9\beta_1)$$

$$\text{s.t } \rho 2 + (1-\rho)(1+\beta_1/2) \leq \beta$$

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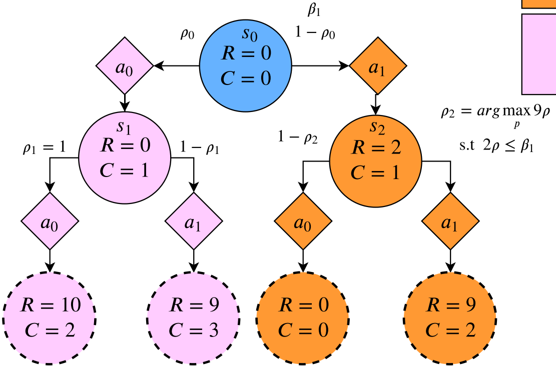

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Policy Evaluation

The Bellman Expectation equationsare preserved, and the Bellman Expectation Operator $\mathcal{T}^\pi$ is a γ-contraction.

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(ii) Maximise the rewards:

$$V_r^*(\overline{s}) \stackrel{\text{def}}{=} \max_{\pi \in \Pi_a(\overline{s})} V_r^\pi(\overline{s}) \qquad \Pi_r(\overline{s}) \stackrel{\text{def}}{=} \arg\max_{\pi \in \Pi_a(\overline{s})} V_r^\pi(\overline{s})$$

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(iii) Minimise the costs:

$$V_c^*(\bar{s}) \stackrel{\text{def}}{=} \min_{\pi \in \Pi_r(\bar{s})} V_c^\pi(\bar{s}), \qquad \Pi^*(\bar{s}) \stackrel{\text{def}}{=} \arg\min_{\pi \in \Pi_r(\bar{s})} V_c^\pi(\bar{s})$$

We define the budgeted action-value function Q^* similarly

Budgeted Bellman Optimality Equation

Theorem (Budgeted Bellman Optimality Equation)

Q^* verifies the following equation:

$$Q^*(\overline{s}, \overline{a}) = \mathcal{T} Q^*(\overline{s}, \overline{a})$$
$$\stackrel{\text{def}}{=} R(\overline{s}, \overline{a}) + \gamma \sum_{\overline{s'} \in \overline{\mathcal{S}}} \overline{P}(\overline{s'} | \overline{s}, \overline{a}) \sum_{\overline{a'} \in \overline{\mathcal{A}}} \pi_{greedy}(\overline{a'} | \overline{s'}; Q^*) Q^*(\overline{s'}, \overline{a'}),$$

where the greedy policy π_{greedy} is defined by:

$$\pi_{greedy}(\overline{a} | \overline{s}; Q) \in \boxed{\arg\min}_{\rho \in \Pi_r^Q} \mathop{\mathbb{E}}_{\overline{a} \sim \rho} Q_c(\overline{s}, \overline{a}),$$

$$\text{where} \quad \Pi_r^Q \stackrel{\text{def}}{=} \boxed{\arg\max}_{\rho \in \mathcal{M}(\overline{\mathcal{A}})} \mathop{\mathbb{E}}_{\overline{a} \sim \rho} Q_r(\overline{s}, \overline{a})$$
$$\text{s.t.} \quad \mathop{\mathbb{E}}_{\overline{a} \sim \rho} Q_c(\overline{s}, \overline{a}) \boxed{\leq \beta}$$

Optimality of the policy

Proposition (Optimality of the policy)

$\pi_{greedy}(\cdot\ ; Q^*)$ is simultaneously optimal in all states $\bar{s} \in \overline{\mathcal{S}}$:

$$\pi_{greedy}(\cdot\ ; Q^*) \in \Pi^*(\bar{s})$$

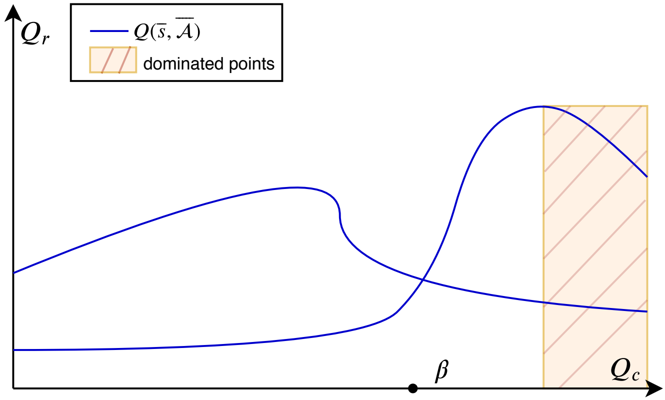
In particular, $V^{\pi_{greedy}(\cdot\,;Q^*)} = V^*$ and $Q^{\pi_{greedy}(\cdot\,;Q^*)} = Q^*$.

Solving the untractable program

Proposition ($\pi_{\text{greedy}} = \pi_{\text{hull}}$)

π_{greedy} can be computed explicitly, as a mixture of two points that lie on the convex hull of Q.

Solving the non-linear programming problem: intuition



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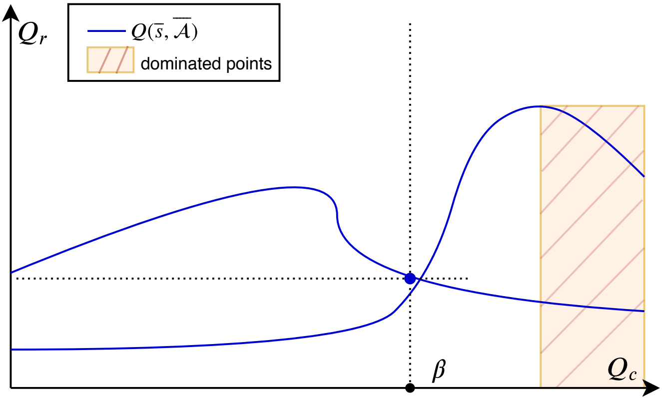

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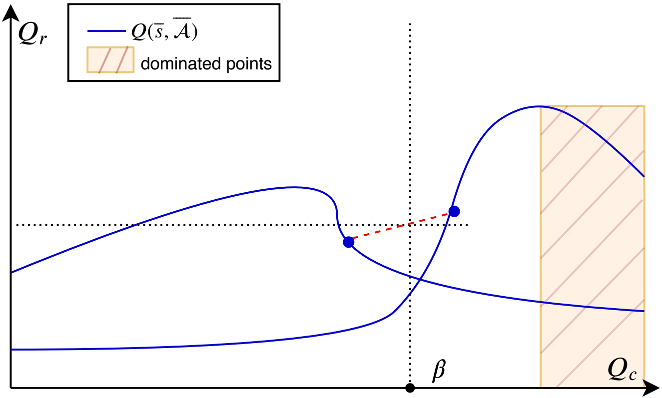

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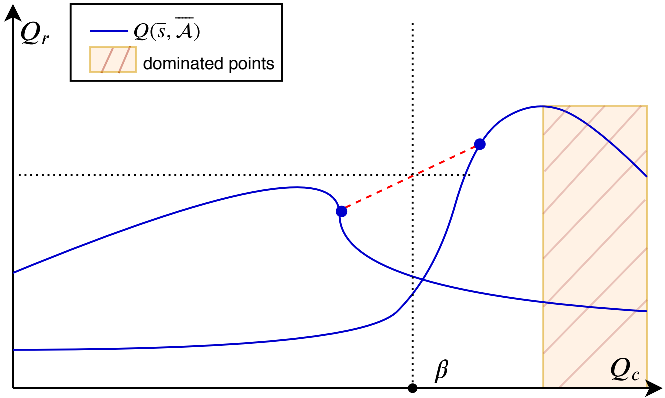

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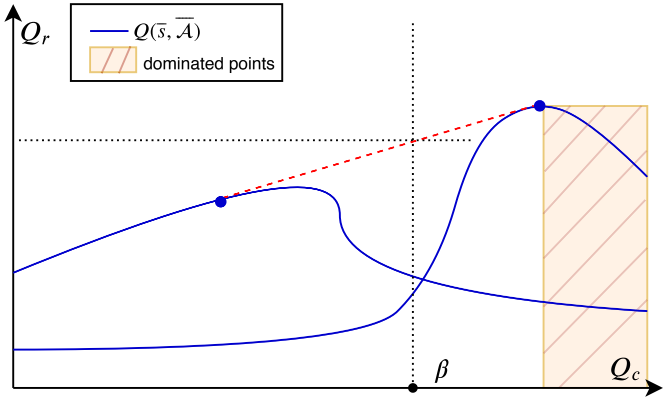


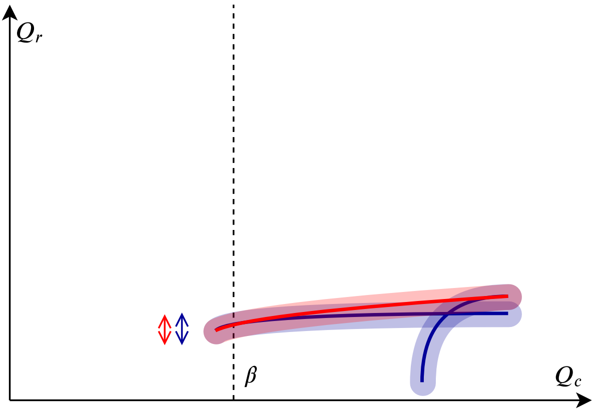
Not a contraction

Theorem (Non-Contractivity)

For any BMDP $(\mathcal{S}, \mathcal{A}, P, R_r, R_c, \gamma)$ with $|\mathcal{A}| \geq 2$, $\mathcal{T}$ is not a contraction.

$$\forall \varepsilon > 0, \exists Q^1, Q^2 \in (\mathbb{R}^2)^{\overline{\mathcal{S}\mathcal{A}}} : \|\mathcal{T} Q^1 - \mathcal{T} Q^2\|_\infty \geq \frac{1}{\varepsilon} \|Q^1 - Q^2\|_\infty$$

Not a contraction: intuition



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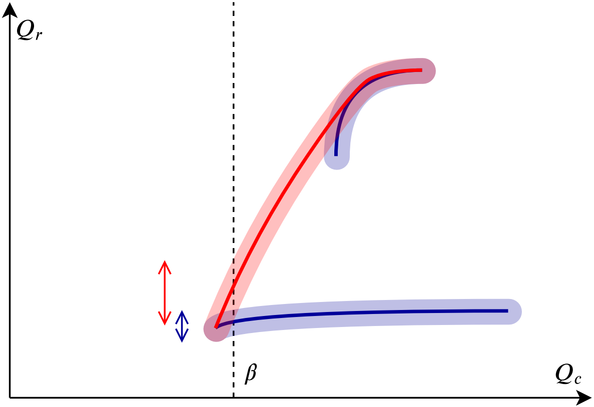
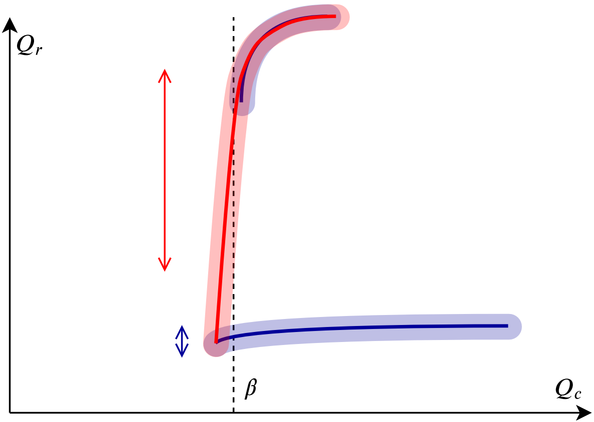


Not a contraction: intuition



Contractivity on smooth Q-functions

Conjecture (Contractivity $\mathcal{L}_\gamma$)

$\mathcal{T}$ is a contraction when restricted to the subset $\mathcal{L}_\gamma$ of Q-functions such that "Q_r is L-Lipschitz with respect to Q_c", with $L < \frac{1}{\gamma} - 1$.

Budgeted Dynamic Programming

Algorithm 1: Budgeted Value-Iteration

Data: P, R_r, R_c

Result: Q^*

$Q_0 \leftarrow 0$

repeat

  $Q_{k+1} \leftarrow \mathcal{T} Q_k$

until *convergence*

Budgeted Reinforcement Learning

We address several limitations of Budgeted Value-Iteration

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   - Work with a batch of samples $\mathcal{D} = \{(\overline{s}_i, \overline{a}_i, r_i, \overline{s}'_i\}_{i \in [0,N]}$
   - Replace $\mathcal{T}$ with a sampling operator $\hat{\mathcal{T}}$:

$$\hat{\mathcal{T}} Q(\overline{s}_i, \overline{a}_i, r_i, \overline{s}'_i) \stackrel{\text{def}}{=} r_i + \gamma \sum_{\overline{a'_i} \in \mathcal{A}_i} \pi_{\text{greedy}}(\overline{a'_i} | \overline{s'_i}; Q) Q(\overline{s'_i}, \overline{a'_i}).$$

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2. If $\mathcal{S}$ is continuous:
   - Employ function approximation Q_θ, and minimise a regression loss

$$\mathcal{L}(Q_\theta, Q_{\text{target}}; \mathcal{D}) = \sum_{\mathcal{D}} ||Q_\theta(\overline{s}, \overline{a}) - Q_{\text{target}}(\overline{s}, \overline{a}, r, \overline{s}')||_2^2$$

Budgeted Fitted-Q

Algorithm 2: Budgeted Fitted-Q Iteration

Data: $\mathcal{D}$

Result: Q^*

$Q_{\theta_0} \leftarrow 0$

repeat

  $\theta_{k+1} \leftarrow \arg\min_\theta \mathcal{L}(Q_\theta, \hat{\mathcal{T}} Q_{\theta_k}; \mathcal{D})$

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- CPU parallel computing of the targets $\sum_{\overline{a'_i} \in \mathcal{A}_i} \pi_{\text{greedy}}(\overline{a'_i} | \overline{s'_i}; Q) Q(\overline{s'_i}, \overline{a'_i}) \; \forall i$

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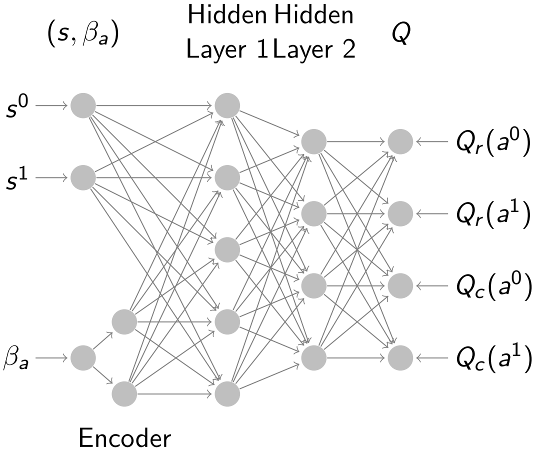
- Same for samples generation.
- Neural Network as function approximator:



Experiments: performances of BFTQ

- Baseline: λ-FTQ, Lagrangian relaxation
  - $R_r(s, a) \leftarrow R_r(s, a) - \lambda R_c(s, a)$ where $\lambda \geq 0$

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- Applications:
  - dialogue systems
  - autonomous driving

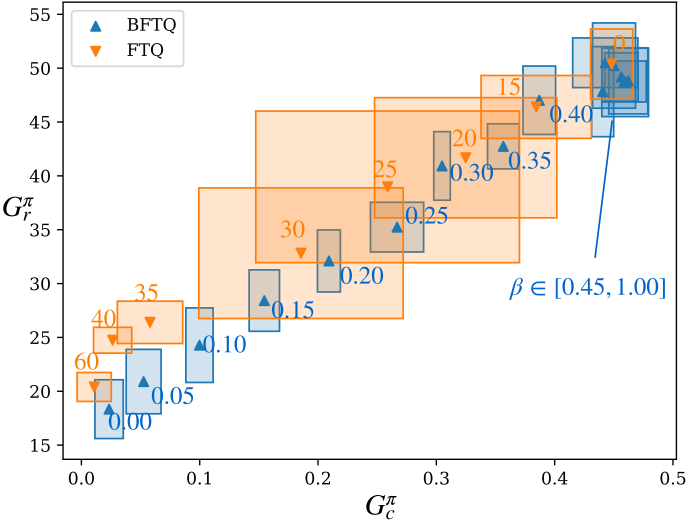
Experiments: dialogue systems

- A slot-filling problem: the agent (the dialogue system) fills a form by asking the user each slot.

Experiments: dialogue systems

- A slot-filling problem: the agent (the dialogue system) fills a form by asking the user each slot.
- Two ways to deal with recognition errors:
  - ask to repeat with voice (safe/slow),
  - Ask to repeat with numeric pad (unsafe/fast).

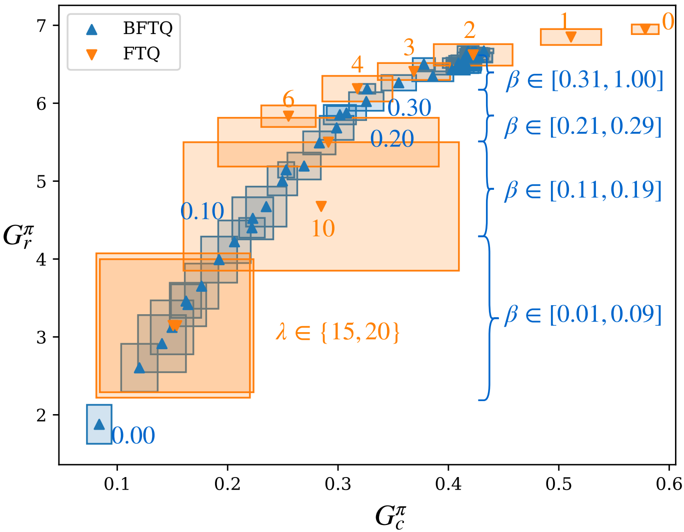
Experiments: dialogue systems



Experiments: autonomous driving

- the agent (the car) is on a two-way road with a car in front of it:
  - it can stay behind (safe/slow),
  - it can overtake (unsafe/fast).

Experiments: autonomous driving



Experiments: autonomous driving

▸ BFTQ on the highway environment

Risk-sensitive exploration

How to collect the batch $\mathcal{D}$?

- We propose an ε-greedy exploration procedure

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$$\overline{a} = (a, \beta_a) \sim \mathcal{U}(\Delta_{\mathcal{AB}})$$
$$\text{where } \Delta \text{is s.t. } \mathbb{P}\,(a, \beta_a | s, \beta) \,\text{verifies}\, \mathbb{E}[\beta_a] \leq \beta$$

Experiments: risk-sensitive exploration

- Validate the risk-sensitive exploration procedure on the corridor environment

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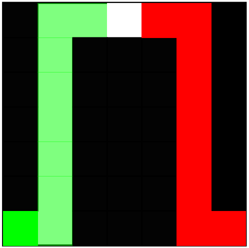
- Validate the risk-sensitive exploration procedure on the corridor environment
- Learn 2 BFTQ policies with respectively:
  - A batch generated by a risk-neutral ε-greedy procedure
  - A batch generated by a risk-sensitive ε-greedy procedure

Experiments: corridors

- 2 corridors:
  - high costs/high rewards around the starting state
  - no costs/low rewards around the starting state
- The outermost cell is the one yielding the most reward



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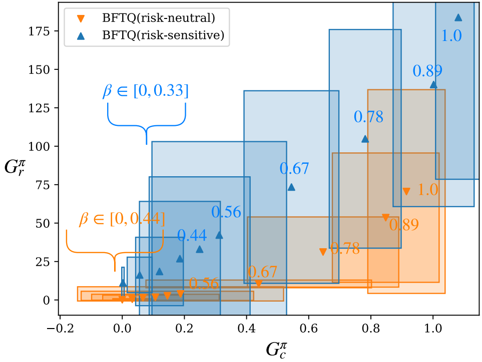

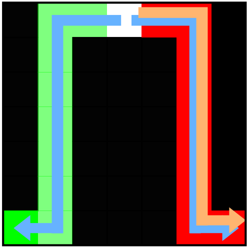

Experiments: corridors

▸ Risk-sensitive vs Risk-Neutral on the corridors environment

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