Question 4: Solution

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Question #4 [20]	CE629A	C. Kolay, IITK

$$m = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.5 \end{bmatrix} \times 10^4 \text{ kg} \qquad \Phi = \begin{bmatrix} 0.44 & -0.81 & 0.80 \\ 0.83 & -0.17 & -1.0 \\ 1.0 & 1.0 & 0.95 \end{bmatrix}$$

Part (a): [12]

$$M_n = \underline{\phi}_n^T \underline{m}\, \underline{\phi}_n = \sum_{j=1}^{3} m_j \phi_{j,n}^2 \Rightarrow M_1 = 1.3825 \times 10^4 \text{ kg}$$
$$M_2 = 1.185 \times 10^4 \text{ kg}$$
$$M_3 = 2.891 \times 10^4 \text{ kg}$$

+2

$$L_n^h = \underline{\phi}_n^T \underline{m}\, \underline{1} = \sum_{j=1}^{3} m_j \phi_{j,n} \Rightarrow L_1^h = 1.77 \times 10^4 \text{ kg}$$
$$L_2^h = -0.48 \times 10^4 \text{ kg}$$
$$L_3^h = 0.275 \times 10^4 \text{ kg}$$

+2

$$L_n^\theta = \sum_{j=1}^{3} h_j m_j \phi_{j,n} \Rightarrow L_1^\theta = 14.4 \times 10^4 \text{ kg-m}$$
$$L_2^\theta = 1.4 \times 10^4 \text{ kg-m}$$
$$L_3^\theta = 0.9 \times 10^4 \text{ kg-m}$$

+2

$$\Gamma_n = \frac{L_n^h}{M_n} \Rightarrow \Gamma_1 = 1.28$$
$$\Gamma_2 = -0.41$$
$$\Gamma_3 = 0.13$$

+1

Effective modal masses $M_n^* = V_{bn}^{st} = \Gamma_n L_n^h$

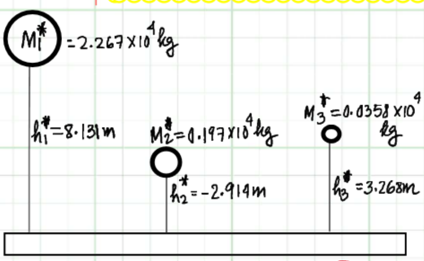
$$\Rightarrow M_1^* = V_{b1}^{st} = 2.267 \times 10^4 \text{ kg}$$
$$M_2^* = V_{b2}^{st} = 0.197 \times 10^4 \text{ kg}$$
$$M_3^* = V_{b3}^{st} = 0.0358 \times 10^4 \text{ kg}$$

+1

Effective modal heights $h_n^* = \frac{M_{bn}^{st}}{V_{bn}^{st}} = \frac{L_n^\theta}{L_n^h}$

$$\Rightarrow h_1^* = 8.131 \text{ m}$$
$$h_2^* = -2.914 \text{ m}$$
$$h_3^* = 3.268 \text{ m}$$

+1



+3

Derivation of necessary formulas:

$$\underline{m}\ddot{\underline{u}} + \underline{c}\dot{\underline{u}} + \underline{k}\underline{u} = -\underline{m}\,\underline{1}\,\ddot{u}_g(t)$$

$$\Rightarrow \underline{\Phi}^T \underline{m} \underline{\Phi} \ddot{\underline{q}}(t) + \underline{\Phi}^T \underline{c} \underline{\Phi} \dot{\underline{q}}(t) + \underline{\Phi}^T \underline{k} \underline{\Phi} \underline{q}(t) = -\underline{\Phi}^T \underline{m}\,\underline{1}\,\ddot{u}_g(t)$$

For n^{th} mode,

$$M_n \ddot{q}_n(t) + C_n \dot{q}_n(t) + K_n q_n(t) = -\underline{\phi}_n^T \underline{m}\,\underline{1}\,\ddot{u}_g(t)$$

$$\Rightarrow \ddot{q}_n(t) + 2\zeta_n \omega_n \dot{q}_n(t) + \omega_n^2 q_n(t) = -\frac{\underline{\phi}_n^T \underline{m}\,\underline{1}}{M_n}\ddot{u}_g(t) = -\frac{L_n^h}{M_n}\ddot{u}_g = -\Gamma_n \ddot{u}_g(t)$$

Compare this to SDF eqn of motion:

$$\ddot{D}_n(t) + 2\zeta_n \omega_n \dot{D}_n(t) + \omega_n^2 D_n(t) = -\ddot{u}_g(t)$$

$$\therefore q_n(t) = \Gamma_n D_n(t) \text{ where } \boxed{\Gamma_n = \frac{\underline{\phi}_n^T \underline{m}\,\underline{1}}{M_n} = \frac{L_n^h}{M_n}}$$

$$\therefore \underline{u}_n(t) = \underline{\phi}_n q_n(t) = \Gamma_n \underline{\phi}_n D_n(t) = \frac{\Gamma_n}{\omega_n^2}\phi_n \cdot A_n(t) = u_n^{st} A_n(t)$$

Now, $\underline{s}_n = \underline{k} \cdot \underline{u}_n^{st} = \underline{k} \frac{\Gamma_n}{\omega_n^2}\phi_n = \Gamma_n \underline{m}\, \underline{\phi}_n$

$$V_{bn}^{st} = \sum_{j=1}^{N} s_{jn} = \Gamma_n L_n^h \text{ because } \underline{\phi}_n^T \underline{m}\,\underline{1} = \underline{m}\,\underline{\phi}_n$$
$$= M_n^* = \text{Effective modal mass}$$

$$M_{bn}^{st} = \sum_{j=1}^{N} h_j s_{jn} = \sum_{j=1}^{N} h_j \Gamma_n m_j \phi_{j,n} = \Gamma_n L_n^\theta = V_{bn}^{st} h_n^*$$

$$\therefore \boxed{L_n^\theta = \sum_{j=1}^{N} h_j m_j \phi_{j,n}}$$

$$\boxed{h_n^* = \frac{M_{bn}^{st}}{V_{bn}^{st}} = \frac{\Gamma_n L_n^\theta}{V_{bn}^{st}} = \frac{\Gamma_n L_n^\theta}{M_n^*} = \frac{L_n^\theta}{L_n^h}}$$

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Part (b): [5]

Modal base moment $M_{bn}^{st} = \sum_{j=1}^{3} h_j s_{jn} = \Gamma_n L_n^\theta \Rightarrow M_{b1}^{st} = 18.432 \times 10^4$ kg-m

$M_{b2}^{st} = -0.574 \times 10^4$ kg-m

$M_{b3}^{st} = 0.117 \times 10^4$ kg-m

+3

$\therefore$ Total base shear $V_b(t) = \sum_{n=1}^{3} V_{bn}^{st} A_n(t)$

$= [0.267A_1(t) + 0.197A_2(t) + 0.0358 \times A_3(t)] \times 10^4$ ...

+1

$= [2.264 A_1(t) + 0.174 A_2(t) + 0.0358 \times A_3(t)] \times 10 \; N$ +1

$\therefore$ Total base moment $M_b(t) = \sum_{n=1}^{3} M_{bn}^{st} A_n(t)$

$$= [18.432 A_1(t) - 0.574 A_2(t) + 0.117 A_3(t)] \; N \cdot m$$ +1

Part (c) :

Yes, peak values of modal responses can be determined using the response spectrum for each mode because $r_n(t) = r_n^{st} \cdot A_n(t)$

$$\therefore r_{no} = \max_t |r_n(t)| = r_n^{st} \cdot \max_t |A_n(t)| = r_n^{st} \cdot PSA(\zeta_n, \omega_n)$$

+3