Problem 2

11 January 2022 09:59 AM

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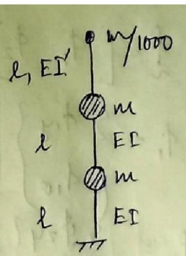
Q(2)



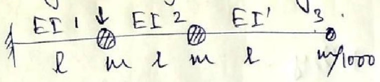
$m = 85t$
$= 85000 \text{ kg}$.
$EI = 2.8 \times 10^5 \text{ KNm}^2$
$EI' = 31.7 \text{ KNm}^2$
$l = 3m$.
$\xi = 0.05$

Mass matrix –

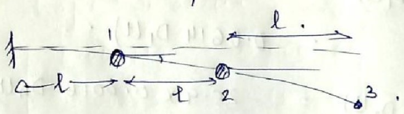
$$\underset{\sim}{m} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m/1000 \end{bmatrix}.$$

$$\underset{\sim}{m} = 10^3 \begin{bmatrix} 85 & 0 & 0 \\ 0 & 85 & 0 \\ 0 & 0 & 0.085 \end{bmatrix} \text{kg}$$

for finding the flexibility matrix –



apply unit load at ① and measure disp. at other points –



$$f_{11} = \frac{l^3}{3EI}$$

$$f_{21} = \frac{l^3}{3EI} + \left(\frac{l^2}{2EI}\right) \times l. \quad \text{Slope @ ①}$$

$$= \frac{5l^3}{6EI}.$$

$$f_{31} = \frac{l^3}{3EI} + 2l\left(\frac{l^2}{2EI}\right) = \frac{4}{3}\frac{l^3}{EI}$$

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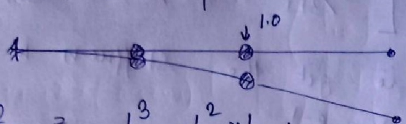
Similarly applying unit load @ 2 and measuring the disp. at all other points –



$$f_{12} = \frac{l^3}{3EI} + \frac{l^2}{2EI} \times L.$$

$$= \frac{5L^3}{6EI}.$$

$$f_{22} = \frac{(2l)^3}{} = \frac{8L^3}{}$$

$$\frac{}{3EI} \qquad \frac{}{3EI}.$$

$$f_{32} = \frac{8l^3}{3EI} + \frac{4l^2}{2EI} \times l.$$

$$= \frac{14 l^3}{3EI}.$$

and for 3rd column of the flexibility matrix —

apply unit load @ ③ and finding disp. at all points

$$f_{13} = f_{31} = \frac{4l^3}{3EI} \quad \text{(due to symmetry)}$$

$$f_{23} = f_{32} = \frac{14 l^3}{3EI} \quad (\;"\quad "\quad "\;).$$

$$f_{33} = \frac{l^3}{3EI'} + \underbrace{\left[\frac{(2l)^3}{3EI} + \frac{(l)(2l)^2}{2EI}\right]}_{\Delta\text{ part}} + \underbrace{\left\{\frac{2l^2}{2EI} + \frac{2l}{EI}\right\}l}_{\theta\text{ part}}$$

$$= \frac{l^3}{3EI'} + \frac{14 l^3}{3EI} + \frac{4 \cdot l^3}{EI}.$$

$$= \frac{l^3}{3EI'} + \frac{26 l^3}{3EI}$$

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$$\therefore f = \begin{bmatrix} l^3/3EI & 5l^3/6EI & 4l^3/3EI \\ 5l^3/6EI & 8l^3/3EI & 14l^3/3EI \\ 4l^3/3EI & 14l^3/3EI & \frac{26l^3}{3EI} + \frac{l^3}{3EI'} \end{bmatrix}.$$

Now to get the stiffness matrix,

$$K = f^{-1}$$

$$\therefore K = 10^8 \begin{bmatrix} 1.422 & -0.444 & 0.0000603 \\ -0.444 & 0.1779 & -0.0000805 \\ 0.0000603 & (-0.0000805) & 0.0000352 \end{bmatrix}$$

(a) for the natural freq. and mode shapes, we solve the eigen value problem (in MATLAB)

$$|\underset{\sim}{K} - \omega^2 \underset{\sim}{m}| = 0$$

we get $\omega^2 = \{39.39 \quad 43.73 \quad 1840.99\}.$

$$\omega = \{6.276 \quad 6.6132 \quad 42.906\}.$$

$$T = 2\pi/\omega = \{1.001,\ 0.95,\ 0.146\}.$$

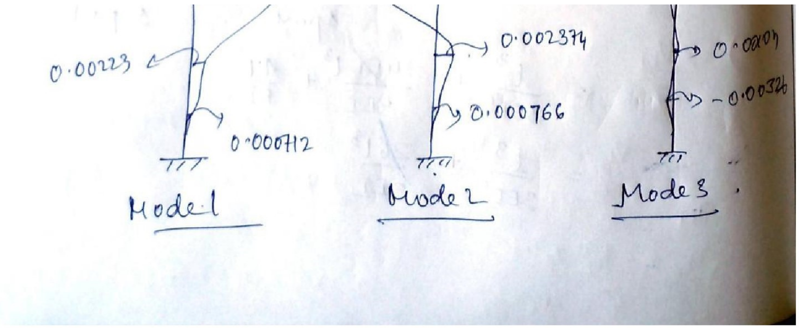
Mode shape matrix $- \underset{\sim}{\Phi}$

$$\underset{\sim}{\Phi} = \begin{bmatrix} 0.000712 & 0.000766 & -0.00326 \\ 0.00223 & 0.002374 & 0.00104 \\ 0.07905 & -0.0742 & -0.00018 \end{bmatrix}$$

(Mass normalised)

0.07905   −0.0742   −0.0098

Corr



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(b) $M_{\cdot} = \underset{\sim}{\Phi}^T \underset{\sim}{m} \underset{\sim}{\Phi} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Which is in accordance as Φ's were all mass normalised.

$\underline{S} = \underset{\sim}{m} \underline{i}$; $i = \{1 \; 1 \; 1\}^T$.

Also, $\underline{S} = \sum_{n=1}^{N} S_n = \sum_{n=1}^{N} \Gamma_n \underset{\sim}{m} \Phi_n$

$\Gamma_n = L_n^h / M_n$ $\quad L_n^h = \underline{\Phi}_n^T \underset{\sim}{m} \underline{i}$.

$L_1^h = \underline{\Phi}_1^T \underset{\sim}{m} \underline{i}$.

$= \{0.000712 \;\; 0.00223 \;\; 0.07905\} * \underset{\sim}{m} * \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$.

$= 257.49$

$L_2^h = \underline{\Phi}_2^T \underset{\sim}{m} \underline{i}$; $\quad L_3^h = \underline{\Phi}_3^T \underset{\sim}{m} \underline{i}$

$= 261.13.$ $\qquad = -188.64.$

$\Gamma_n = \{257.49 \;\; 261.13 \;\; -188.64\}$.

as $M_n = 1.0$.

$\therefore S_1 = \Gamma_1 \underset{\sim}{m} \Phi_1$

$= 257.49 * \underset{\sim}{m} * \begin{Bmatrix} 0.000712 \\ 0.00223 \\ 0.07905 \end{Bmatrix}$.

$= \begin{Bmatrix} 15583.29 \\ 48807.22 \\ 1730.13 \end{Bmatrix}$.

$S_2 = \Gamma_2 \underset{\sim}{m} \Phi_2 = \begin{Bmatrix} 17002.17 \\ 52804.40 \\ -1646.94 \end{Bmatrix}$.

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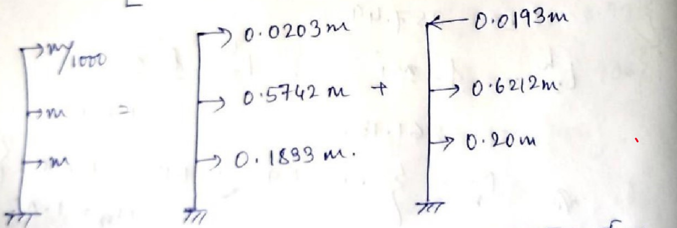
$S_3 = \Gamma_3 \underset{\sim}{m} \Phi_3 = \begin{pmatrix} 52272.14 \\ -16675.77 \\ 2.88 \end{pmatrix}$.

Applying check –

$$\underset{\sim}{s} = \underset{\sim}{m}\underset{\sim}{i} = \begin{Bmatrix} 85000 \\ 85000 \\ 85 \end{Bmatrix}.$$

$$\Sigma s_n = \begin{Bmatrix} 84999.98 \\ 84989.98 \\ 85.07 \end{Bmatrix} \approx \underset{\sim}{s} \quad (\because \text{ok}).$$

$$\frac{S_n}{m} = \begin{bmatrix} 0.1833 & 0.20 & 0.6149 \\ 0.5742 & 0.6212 & -0.1961 \\ 0.0203 & -0.0193 & 3.388\times10^{3} \end{bmatrix}.$$



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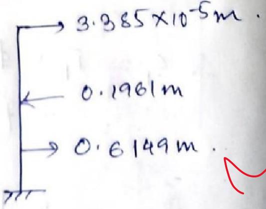

(c) static response

(i) displacement of Appendage mass –

$$u_{3n}^{st} = \frac{\Gamma_n \phi_{3n}}{\omega_n^2}$$

$$u_{3n}^{st} = \{\, 0.516 \quad -0.443 \quad 1.8844\times10^{5} \,\}.$$

(ii) Shear force at base of Appendage

$$V_{3n}^{st} = \sum_{j=3}^{3} S_{3n}.$$

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$$V_{3n}^{st} = \{\, 1730.13 \quad -1646.94 \quad 2.88 \,\}.$$

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(iii) Shear force @ the base of tower

$$V_{bn}^{st} = M_n^* = L_n^h \times \Gamma_n = \sum_{j=1}^{3} S_{jn}$$

$$= \{\, 66301.10 \quad 68188.87 \quad 35585.04 \,\}.$$

(d) We observe that the 1st two periods are close to each other and for this reason the 1st two modes contribute to the appendage displacement (which are high in these two modes). Also ~~base~~ shear at the roof/appendage level is also majorly contributed by the 1st two closely spaced modes.

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(e) Done in MATLAB.

(f) " " " .

(g) " ... the corresponding values are

plots are
given along with the plots.

(h) Seismic coefficients

(i) for appendage $= \dfrac{|\max(V_3)|}{m_3 \times g}$

$= \dfrac{4.132 \times 10^3}{85 \times \cancel{10^3} \times 9.81}$

$= 4.96$

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(ii) for whole tower $= \dfrac{|\max(V_b)|}{(2 \times 85 \times 10^3 + 85) \times g}$

$= \dfrac{7.583 \times 10^5}{(2 \times 85 \times 10^3 + 85) \times 9.81}$

$= 0.455$ 0.45

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EQ_HW4...


```
%Assignment 4 P2a Modal static analysis upto effectie earthquake forces
clc
clearvars
m=10^3*[85    0     0      %mass matrix
         0   85     0
         0    0  0.085];
k=10^8*[1.422  -0.444    0.0000603      %stiffness matrix
       -0.444   0.1779  -0.0000805
        0.0000603  -0.0000805 0.0000352];
[V,D]=eig(k,m); %solving eigen value problem to get natural ferq. and mode shapes
M=V'*m*V; %modal mass matrix
Lnh=V'*m*[1;1;1]; %Lnh vector for all the modes
S1=Lnh(1,1)*m*V(:,1); %1st mode contribution to the effective earthqukae force
S2=Lnh(2,1)*m*V(:,2); %similarly 2nd mode contribution
S3=Lnh(3,1)*m*V(:,3); %3rd mode contribution
S=S1+S2+S3; %Total effectiv earthquake static forces
```

EQ_HW4...


```
%Assignment 4 P2b Modal response for different modes used in Modal analysis
%of 3 storey cantilever tower
clc
clearvars
fid = fopen('El Centro Ground Motion data.txt') ;  % open the text file
S = textscan(fid,'%s');    % text scan the data
fclose(fid) ;      % close the file
S = S{1} ;
a_g = cellfun(@(x)str2double(x), S);  % convert the cell array to double
% Remove NaN's which were strings earlier
a_g(isnan(a_g))=[];
col = 2;
count = 0;
temp_arr =[];
temp_row = [];
for i = 1:length(a_g)
    if count == col
        temp_arr = [temp_arr;
                    temp_row];
        count = 0;
        temp_row = [];
    end
    temp_row = [temp_row,a_g(i)];
    count = count +1;
end
temp_arr = [temp_arr;
            temp_row];
```

```
a_g = temp_arr(:,2:end);
a_g=a_g.*9.81;
clear temp_arr temp_row S;
% Creating Time axis
t=zeros(length(a_g),1);
for i=2:length(a_g)
    t(i)=t(i-1)+0.02;
end
t1=0:0.005:31.18;
a_g1=interp1(t,a_g,t1);
del_t=0.005;
%Producing System response data
Tn=[1.001,0.95,0.146]; %Natural Period of the N-no. of modes
Z=0.05; %Damping ratio for each mode
u=zeros(length(a_g1),3); %Initialising displacement response vector of the SDOF system
v=zeros(length(a_g1),3); %Initialising velocity response vector of the SDOF system
a=zeros(length(a_g1),3); %Initialising Acceleration response vector for the SDOF
system
A_t=zeros(length(a_g1),3); %Matrix to store PSA for the diff. modes
for j=1:length(Tn)
Wn=(2*pi)/Tn(j); %Natural Frequency
Wd=Wn*sqrt(1-Z^2); %Damped Natural Frequency
%Defining Parameters required A,B,C,D & A1,B1,C1,D1
A=exp(-Z*Wn*del_t)*((Z/sqrt(1-Z^2))*sin(Wd*del_t)+cos(Wd*del_t));
B=exp(-Z*Wn*del_t)*(sin(Wd*del_t)/Wd);
C=(((2*Z)/(Wn*del_t))+exp(-Z*Wn*del_t)*(((1-2*Z^2)/(Wd*del_t)-(Z/sqrt(1-Z^2)))*sin
(Wd*del_t)-(1+((2*Z)/(Wn*del_t)))*cos(Wd*del_t)))/Wn^2;
D=(1-((2*Z)/(Wn*del_t))+exp(-Z*Wn*del_t)*(((2*Z^2-1)/(Wd*del_t))*sin(Wd*del_t)+((2*Z)/
```



```
(Wn*del_t))*cos(Wd*del_t)))/Wn^2;
A1=-exp(-Z*Wn*del_t)*((Wn/sqrt(1-Z^2))*sin(Wd*del_t));
B1=exp(-Z*Wn*del_t)*(cos(Wd*del_t)-(Z/sqrt(1-Z^2))*sin(Wd*del_t));
C1=((-1/del_t)+exp(-Z*Wn*del_t)*(((Wn/(sqrt(1-Z^2)))+(Z/(del_t*sqrt(1-Z^2))))*sin
(Wd*del_t)+(cos(Wd*del_t)/del_t)))/Wn^2;
D1=(1-exp(-Z*Wn*del_t)*((Z/sqrt(1-Z^2))*sin(Wd*del_t)+cos(Wd*del_t)))/(Wn^2*del_t);
for i=1:length(a_g1)-1
    u(i+1,j)=A*u(i,j)+B*v(i,j)-C*a_g1(i)-D*a_g1(i+1);
    v(i+1,j)=A1*u(i,j)+B1*v(i,j)-C1*a_g1(i)-D1*a_g1(i+1);
    a(i+1,j)=-a_g1(i+1)-2*Z*Wn*v(i+1,j)-Wn^2*u(i+1,j);
    A_t(i+1,j)=Wn^2*u(i+1,j); %Pseudo Spectral Acceleration(PSA)
end
end
figure(1)
plot(t1(1:3000),A_t(1:3000,1),'r','linewidth',2);
hold on
plot(t1(1:3000),A_t(1:3000,2),'b','linewidth',2);
hold on
plot(t1(1:3000),A_t(1:3000,3),'k--','linewidth',1.5);
xlabel('Time(sec)')
ylabel('PSA (m/s^{2})')
legend('Mode 1','Mode 2','Mode 3')
grid on
figure(2)
plot(t1(1:3000),u(1:3000,1),'r','linewidth',2);
hold on
plot(t1(1:3000),u(1:3000,2),'b','linewidth',2);
hold on
plot(t1(1:3000),u(1:3000,3),'k','linewidth',2);
xlabel('Time(sec)')
ylabel('D_{n} (m)')
legend('Mode 1','Mode 2','Mode 3')
grid on
U3n_st=[0.516,-0.443,1.844*10^-5]; %Vector of static roof displacements for the 'n'
modes
V3n_st=[1730.13,-1646.94,2.88]; %vector of static 3rd storey shear for the 'n' modes
Vbn_st=[66301.10,68188.87,35585.04]; %vector of the static base shear for the 'n'
modes
U3n=zeros(length(A_t),3);
Vbn=zeros(length(A_t),3);
V3n=zeros(length(A_t),3);
%Roof Displacement Analysis
U3n(:,1)=A_t(:,1).*U3n_st(1);
U3n(:,2)=A_t(:,2).*U3n_st(2);
U3n(:,3)=A_t(:,3).*U3n_st(3);
U3=U3n(:,1)+U3n(:,2)+U3n(:,3);
U3n_max=[max(abs(U3n(:,1))),max(abs(U3n(:,2))),max(abs(U3n(:,3)))];
U3_max=max(abs(U3));
figure(3)
plot(t1(1:3000),U3n(1:3000,:).*1000,'linewidth',2);
hold on
plot(t1(1:3000),U3(1:3000).*1000,'k:','linewidth',3);
grid on
legend('Mode 1','Mode 2','Mode 3','Total');
xlabel('Time(sec)');
```

```
ylabel('Roof Displacement U_{3}(mm)');
%Base shear analysis
Vbn(:,1)=A_t(:,1).*Vbn_st(1);
Vbn(:,2)=A_t(:,2).*Vbn_st(2);
Vbn(:,3)=A_t(:,3).*Vbn_st(3);
Vb=Vbn(:,1)+Vbn(:,2)+Vbn(:,3);
Vbn_max=[max(abs(Vbn(:,1))),max(abs(Vbn(:,2))),max(abs(Vbn(:,3)))];
Vb_max=max(abs(Vb));
figure(4)
plot(t1(1:3000),Vbn(1:3000,:)./1000,'linewidth',2);
hold on
plot(t1(1:3000),Vb(1:3000)./1000,'k:','linewidth',3);
grid on
legend('Mode 1','Mode 2','Mode 3','Total');
xlabel('Time(sec)');
ylabel('Base shear V_{b}(KN)');
%3rd Storey shear analysis
V3n(:,1)=A_t(:,1).*V3n_st(1);
V3n(:,2)=A_t(:,2).*V3n_st(2);
V3n(:,3)=A_t(:,3).*V3n_st(3);
V3n_max=[max(abs(V3n(:,1))),max(abs(V3n(:,2))),max(abs(V3n(:,3)))];
V3=V3n(:,1)+V3n(:,2)+V3n(:,3);
V3_max=max(abs(V3));
figure(5)
plot(t1(1:3000),V3n(1:3000,:)./1000,'linewidth',2);
hold on
plot(t1(1:3000),V3(1:3000)./1000,'k:','linewidth',3);
grid on
legend('Mode 1','Mode 2','Mode 3','Total');
xlabel('Time(sec)');
ylabel('3rd storey shear V_{3}(KN)');
```

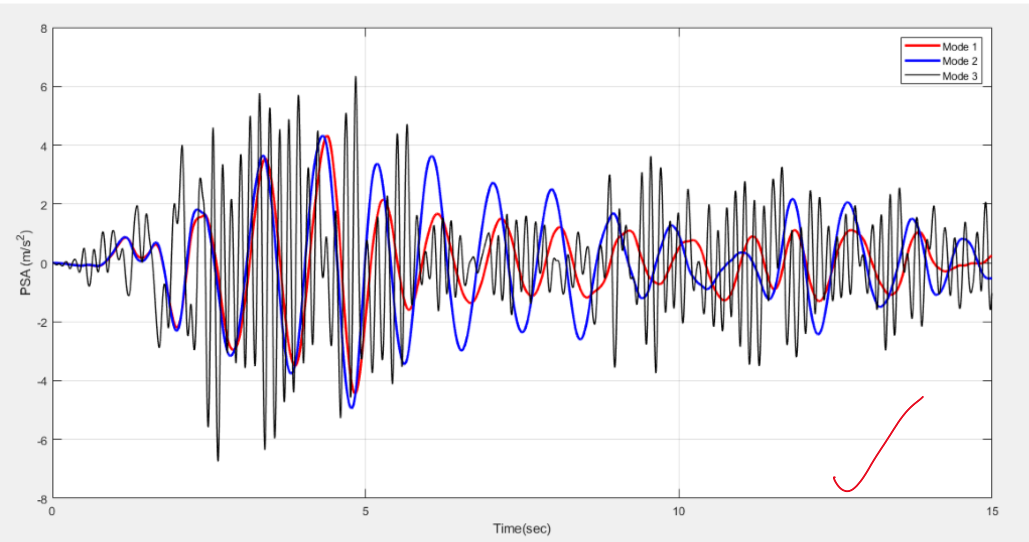

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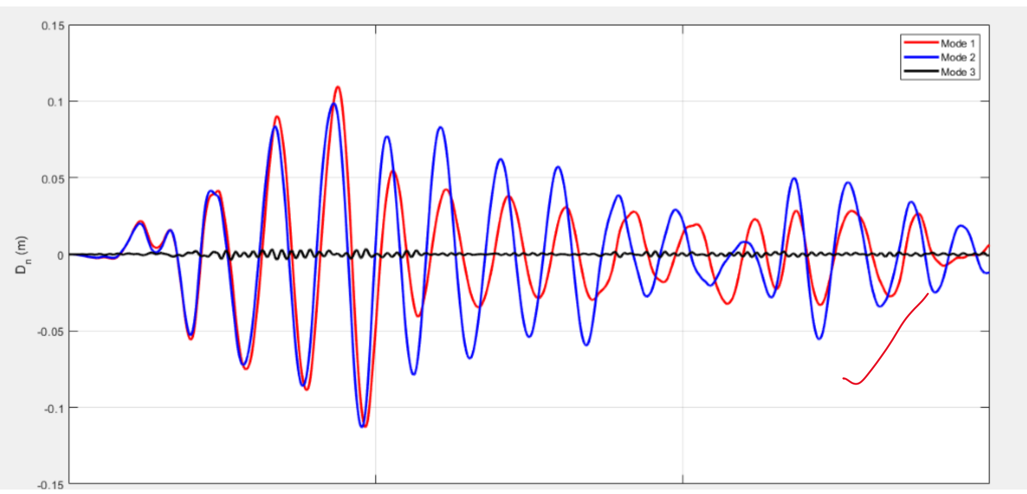
Fig 1. Plot of Pseudo Acceleration response vs time for all 3 modes



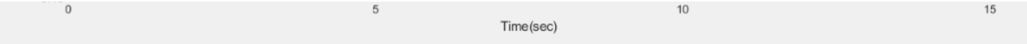


Fig 2. Plot of displacement response vs time for all modes

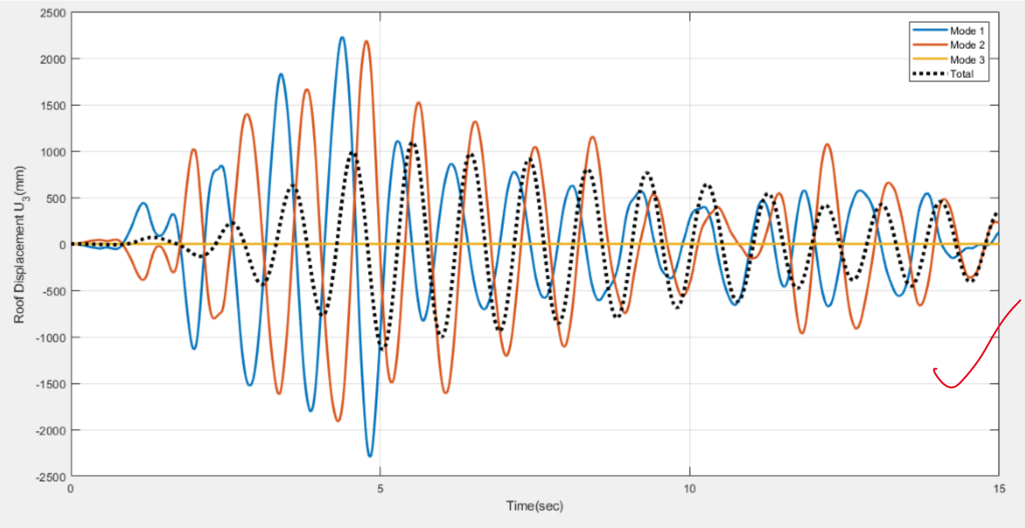


Fig 3. Plot of Roof displacement for all three modes and the total Roof Displacement vs time

The maximum values of the roof displacements of the three modes:
U3n_max=[2.2947, 2.188, 1.2433*10^-4] m

U3_max= 1.1408 m (maximum roof displacement considering all modes)

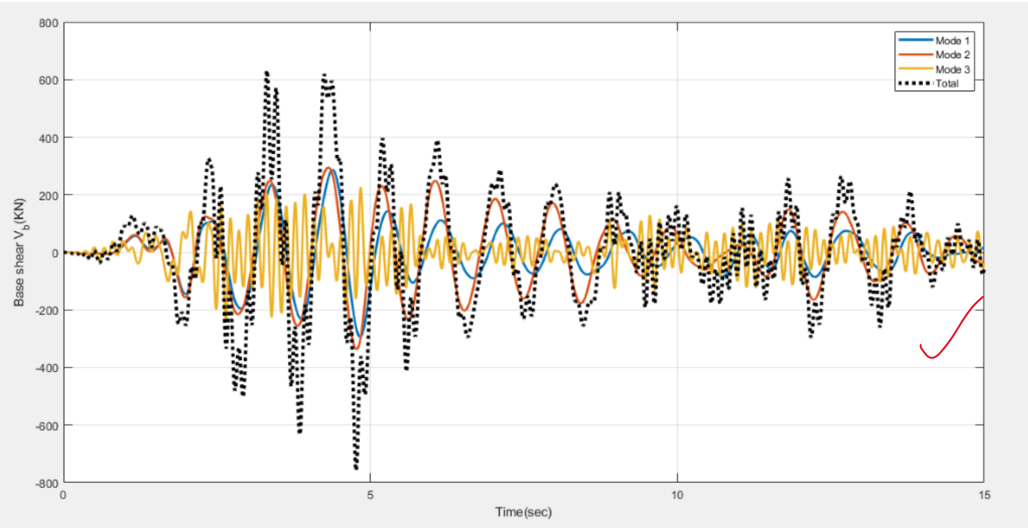


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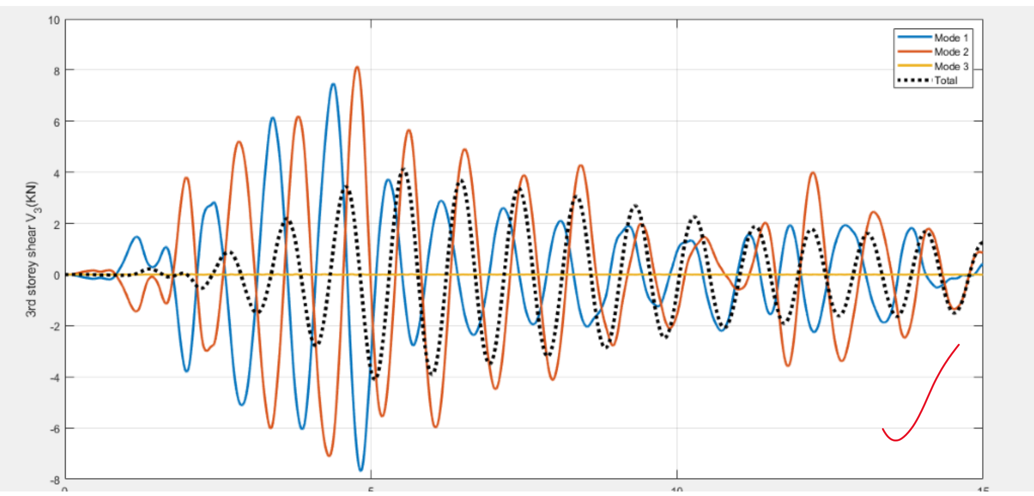
Fig 4. Plot of base shear of the tower for all modes and the total Base shear vs Time

The maximum base shear for all 3 modes:
Vbn_max=[2.9484*10^5, 3.3691*10^5, 2.3992*10^5] N

Vb_max=7.5829*10^5 N (considering all 3 modes)



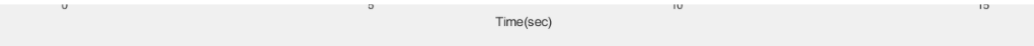


Fig5. Plot of Appendage shear for all 3 modes and the Total Appendage shear vs time

Maximum Appendage shear for the 3 modes :
V3n_max=[7.6939*10^3, 8.1373*10^3, 19.4177] N

V3_max= 4.13.17*10^3 N (considering all 3 modes)