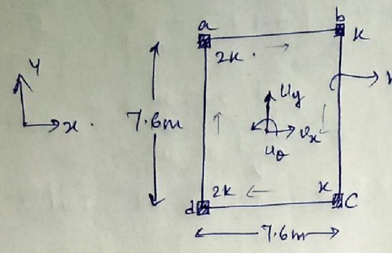
Problem 3

11 January 2022 09:59 AM

Q(3) :- Plan view :-

20/20



$m = 40.8 \times 10^3$ kg.

$\Uparrow \ddot{u}_{gy}(t)$

Stiffness along x-direction –

(i) Top = 2k + k = 3k = k_{xt}

(ii) Bottom = 2k + k = 3k = k_{xb}

Stiffness along y-direction –

(i) Left = 2k + 2k = 4k. = k_{yl}

(ii) Right = k + k = 2k. = k_{yr}.

Thus the rigid slab is unsymmetric about the y-axis –.

Developing the stiffness matrix –.

motion in x-direction –

$$k_{xx} = k_{xt} + k_{xb} = 3k + 3k = 6k.$$

$$k_{yx} = 0 \text{ (No stiffness along } y\text{)}.$$

$$k_{\theta x} = k_{xb} \times \frac{d}{2} - k_{xt} \times d/2 = 3k\cancel{d/2} - 3\cancel{kd/2} = 0.$$

Motion in y-direction,

$$k_{yy} = k_{yl} + k_{yr} = 4k + 2k = 6k.$$

$$k_{\theta y} = -k_{yl} \times \frac{b}{2} + k_{yr} b/2 = \cancel{2}k \times \frac{b}{\cancel{2}} - \cancel{4}k \times \frac{b}{\cancel{2}} = kb - 2kb = -kb.$$

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Motion along θ direction –.

$$k_{\theta\theta} = \frac{b^2}{4}(k_{yr} + k_{yl}) + \frac{d^2}{4}(k_{xt} + k_{xb}).$$

$$= \frac{b^2}{4}(6k) + \frac{d^2}{4}(6k)$$

$$= \frac{3}{2}kb^2 + \frac{3}{2}kd^2$$

~~as b = d ; $k_{\theta\theta}$ =~~

$$\therefore \underset{\sim}{k} = \begin{bmatrix} 6k & 0 & 0 \\ 0 & 6k & -kb \end{bmatrix}$$

$$\begin{bmatrix} & & \\ 0 & -kb & \frac{3}{2}kb^2+\frac{3}{2}kd^2 \end{bmatrix}.$$

$$\underset{\sim}{m} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & mr^2 \end{bmatrix} \text{ where } r^2 = \frac{1}{12}(b^2+d^2).$$

$\therefore$ equation of motion –

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & mr^2 \end{bmatrix} \begin{Bmatrix} \ddot{u}_x(t) \\ \ddot{u}_y(t) \\ \ddot{u}_\theta(t) \end{Bmatrix} + \begin{bmatrix} 6k & 0 & 0 \\ 0 & 6k & -kb \\ 0 & -kb & \left(\frac{3}{2}kb^2 + \frac{3}{2}kd^2\right) \end{bmatrix} \begin{Bmatrix} u_x \\ u_y \\ \theta u_\theta \end{Bmatrix} = -\underset{\sim}{m} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \ddot{u}_{gy}(t)$$

Thus, we see that motion along x is completely uncoupled and can be analysed separately –

$$m\ddot{u}_x(t) + 6ku_x = 0. \quad -(i)$$

Motion along y-θ direction are coupled –

$$\begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix} \begin{Bmatrix} \ddot{u}_y(t) \\ \ddot{u}_\theta(t) \end{Bmatrix} + \begin{bmatrix} 6k & -kb \\ -kb & \left(\frac{3}{2}kb^2 + \frac{3}{2}kd^2\right) \end{bmatrix} \begin{Bmatrix} u_y \\ u_\theta \end{Bmatrix} = -\begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix} \underbrace{\begin{Bmatrix} 1 \\ 0 \end{Bmatrix}}_{\iota} \ddot{x}_{gy}(t)$$

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$b = 7.6\,m \quad d = 7.6\,m.$

$r^2 = \frac{1}{12}(7.6^2 + 7.6^2) \Rightarrow I = mr^2 = 392768\ kg\,m^2$

$m = 40.8 \times 10^3\ kg$

$= 40800\ kg.$

Given $k =$ ~~267.2~~ $262.7 \times 10^3\ N/m$

$$\therefore \underset{\sim}{K} = \begin{bmatrix} 6\times 262.7\times 10^3 & -262.7\times 7.6\times 10^3 \\ -262.7\times 10^3 \times 7.6 & \left(\frac{3}{2}\times 262.7\times 10^3 \times 7.6^2 + \frac{3}{2}\times 262.7\times 10^3\times 7.6^2\right) \end{bmatrix}.$$

$$= \begin{bmatrix} 157.62\times 10^4 & -199.652\times 10^4 \\ -199.652\times 10^4 & 4552.066\times 10^4 \end{bmatrix}.$$

$$= 10^4 \times \begin{bmatrix} 157.62 & -199.652 \\ -199.652 & 4552.066 \end{bmatrix}.$$

Doing the eigen value analysis to get the natural frequencies and mode shapes –

$$|\underset{\sim}{K} - \omega^2 \underset{\sim}{m}| = 0.$$

$\omega_1 = \sqrt{35.8537} = 5.961\ rad/s.$

ω_2 (from motion along x)

$$= \sqrt{38.6324} = 6.215 \text{ rad/s}.$$

$$\omega_3 = \sqrt{118.9924} = 10.908 \text{ rad/s}.$$

5/5

the mode shapes concerning 'y' and 'θ' DOFs are :-

$$\phi_1 = \begin{Bmatrix} -0.0049 \\ -3.073\times10^{-4} \end{Bmatrix}$$

$$\phi_2 = \begin{Bmatrix} -9.5345\times10^{-4} \\ 0.0016 \end{Bmatrix}.$$

(b) $\underset{\sim}{s} = \underset{\sim}{m}\,\underset{\sim}{\overset{\circ}{i}} = \begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix}\begin{Bmatrix} 1 \\ 0 \end{Bmatrix}.$

$$= \begin{Bmatrix} 40800 \\ 0 \end{Bmatrix}.$$

$\underset{\sim}{s} = \sum_{n=1}^{2} \Gamma_n \underset{\sim}{m} \phi_n$. $\Gamma_n = \dfrac{L_n^h}{M_n}$; $M_n = \phi_n^T \underset{\sim}{m} \phi_n$.

$L_n^h = \phi_n^T \underset{\sim}{m}\,\overset{\circ}{\underset{\sim}{i}}$. $\overset{\circ}{i} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

$L_1^h = (-0.0049 \;\; -3.073\times10^{-4}) * \underset{\sim}{m} * \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$

$= \cancel{199.92}$. -198.2088

$L_2^h = (-9.5345\times10^{-4} \;\; 0.0016) * \underset{\sim}{m} * \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}.$

$= -38.9$.

$\Gamma_1 = -199.92$; $\Gamma_2 = -38.9$.

$s_1 = \Gamma_1 \underset{\sim}{m} \phi_1 = \overset{198.2088}{\cancel{-199.92}} * \begin{pmatrix} m & 0 \\ 0 & I \end{pmatrix}\begin{Bmatrix} -0.0049 \\ -3.073\times10^{-4} \end{Bmatrix}$

$= \cancel{\begin{pmatrix} 39968.006 \\ 24129.866 \end{pmatrix}}$. $\begin{pmatrix} 3.9287\times10^{4} \\ 2.3923\times10^{4} \end{pmatrix}$

$s_2 = \Gamma_2 \underset{\sim}{m} \phi_2 = -38.9 * \begin{pmatrix} m & 0 \\ 0 & I \end{pmatrix}\begin{Bmatrix} -9.5345\times10^{-4} \\ 0.0016 \end{Bmatrix}$

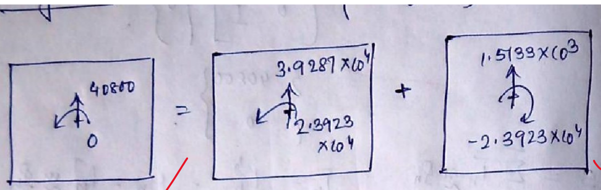
$= \begin{matrix} \cancel{1513.24} \\ \cancel{-24} \end{matrix} \;\; \begin{pmatrix} 1.5133\times10^{3} \\ -2.3923\times10^{4} \end{pmatrix}.$

$\underset{\sim}{s} = \begin{Bmatrix} 40800 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 3.9287\times10^{4} \\ 2.3923\times10^{4} \end{Bmatrix} + \begin{Bmatrix} 1.5133\times10^{3} \\ -2.3923\times10^{4} \end{Bmatrix}.$

$= \begin{Bmatrix} 40800.03 \\ 0 \end{Bmatrix}$ (verified)

Diagram —

5/10



(c) $\underset{\sim}{S} = \sum_{n=1}^{2N} \begin{Bmatrix} S_{yn} \\ S_{\theta n} \end{Bmatrix} = \begin{Bmatrix} S_{1y} = 3.9287\times10^4 \\ S_{1\theta} = 2.3923\times10^4 \end{Bmatrix} + \begin{Bmatrix} S_{2y} = 1.5133\times10^3 \\ S_{2\theta} = -2.3923\times10^4 \end{Bmatrix}$.

$\therefore V_{bn}^{st} = \sum_{j=1}^{N} S_{jn}$

$\Rightarrow V_{b1}^{st} = S_{1y} = 3.9287\times10^4$ kg.

$V_{b2}^{st} = S_{2y} = 1.5133\times10^3$ kg.

Now $M_n^* = V_{bn}^{st} \Rightarrow M_1^* = 3.9287\times10^4$ kg

$M_2^* = 1.5133\times10^4$ kg.

5/5

$\sum M_n^* = 40860.3$ kg $= \sum_{j=1}^{N} m_j$

$\therefore$ verified. $= 40.8\times10^3$ kg.

Also, $T_{b1}^{st} = S_{1\theta} = 2.3923\times10^4$ kgm. $= I_{01}^*$

$T_{b2}^{st} = S_{2\theta} = -2.3923\times10^4$ kgm $= I_{02}^*$

$\therefore I_{01}^* + I_{02}^* = 0 \rightarrow$ verified

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(d) $U_{yn}^{st} = \frac{\Gamma_n \phi_{yn}}{\omega_n^2}$ $\quad \underset{\sim}{\phi} = [\phi_1 \ \phi_2]$.

$\Rightarrow U_{y1}^{st} = \frac{\Gamma_1 \phi_{1y}}{\omega_1^2}$ $\quad \phi_1 = \begin{Bmatrix} \phi_{1y} \\ \phi_{1\theta} \end{Bmatrix} = \begin{Bmatrix} -0.0049 \\ -3.073\times10^{-4} \end{Bmatrix}$.

$= \frac{-199.92\times(-0.0049)}{35.537}$ $\quad \phi_2 = \begin{Bmatrix} \phi_{2y} \\ \phi_{2\theta} \end{Bmatrix} = \begin{Bmatrix} -9.5345\times10^{-4} \\ 0.0016 \end{Bmatrix}$

$= 0.0276\ s^2. \rightarrow$ in terms of $D_n(t) \Rightarrow U_{y1} = \Gamma_1 \phi_{1y} D_1(t)$
$= 0.979\, D_n(t)$

$U_{2y}^{st} = \frac{\Gamma_2 \phi_{2y}}{\omega_2^2} \Rightarrow$ in terms of $D_n(t)$

$U_{2y} = \Gamma_2 \phi_{2y} D_2(t)$

$= -38.9\times -9.5345\times10^{-4} D_n(t)$

$= 0.0371\, D_2(t)$.

3/3

$\therefore U_y = U_{1y} + U_{2y}$

$= 0.979\, D_1(t) + 0.0371\, D_2(t)$.

$U_{1\theta} = \Gamma_1 \phi_{1\theta} D_1(t) = -199.92\times(-3.073\times10^{-4})\, D_n(t)$

$= 0.0614\, D_1(t)$.

$$u_{2\theta} = \Gamma_2 \phi_{2\theta} D_2(t) = -38.9 \times 0.0016\, D_n(t)$$
$$= -0.0622\, D_2(t)$$

$$v_\theta = u_{1\theta} + u_{2\theta} = 0.0614\, D_1(t) - 0.0622\, D_2(t).$$

(e)
$$V_{b1} = V_{b1}^{st} A_1(t) = 3.9287 \times 10^4 A_1(t)$$
$$V_{b2} = V_{b2}^{st} A_2(t) = 1.5133 \times 10^3 A_2(t).$$
$$V_b = V_{b1} + V_{b2} = 3.9287 \times 10^4 A_1(t) + 1.5133 \times 10^3 A_2(t).$$

3/3

$$T_{b1} = T_{b1}^{st} A(t) = 2.3923 \times 10^4 A_1(t)$$
$$T_{b2} = T_{b2}^{st} A_2(t) = -2.3923 \times 10^4 A_2(t)$$
$$T_b = T_{b1} + T_{b2} = 2.3923 \times 10^4 A_1(t) - 2.3923 \times 10^4 A_2(t).$$

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EQ_HW4...


```
clc
clearvars
m=10^3*[40.8   0     0
        0    40.8   0
        0     0   392.768];
k=10^4*[157.62     0        0
         0     157.62  -199.652
         0    -199.652  4552.066];
[V,D]=eig(k,m);
M=V'*m*V;
Lnh=V'*m*[0;1;0];
S1=Lnh(1,1)*m*V(:,1);
S2=Lnh(2,1)*m*V(:,2);
S3=Lnh(3,1)*m*V(:,3);
```