Problem 1

11 January 2022 09:59 AM

88/90

Q1A) (i) EPP system:

EQ_HW3...

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```
%Assignment #3 P1(A)(i)-Constant-Ry Ductility & Residual Deformation
%Spectrum For EPP system for 44 different Ground Motions
%Central Difference Scheme
clc;clearvars;close all;
Tn=0.02:0.02:5;
[dt_GMi,GMi]=xlsread('GMName.xls','2:45'); % GMName.xls is in working folder
Num_GM=length(dt_GMi);
meu=zeros(length(Tn),Num_GM); %Matrix to store the ductility demands for each Tn↙
against given Ry
u_r=zeros(length(Tn),Num_GM); %Matrix to store the residual displacement for each Tn↙
againts given Ry
for p=1:Num_GM
path = ['C:\Users\User\MATLAB\',GMi{p}];
fid = fopen(path,'r');
data=textscan(fid,'%f64','HeaderLines',4);
a_g=data{:};
fclose(fid); N=length(a_g)-1;
dt1=dt_GMi(p);
t=0:dt1:(N*dt1+20);
t1=0:0.005:(N*dt1+20);
% Adding zero padding to the given Earthquake excitation data
a_g=[a_g;zeros((20/dt1),1)]; % appneding the a_g vector with zeros for the next 20↙
sec.
% interpolating the acceleration values within the refined time range
a_g1=interp1(t,a_g,t1);
Ry=6;
del_t=0.005;
dt=0.005;
    for x=1:length(Tn)
        %Producing System response data for Equivalent Linear Elastic system
        Z=0.05; %Damping ratio
        m=1; %Considering unit mass
        Wn=(2*pi)/Tn(x); %Natural Frequency
        k=m*Wn^2; %Linear elastic Stiffness
        Wd=Wn*sqrt(1-Z^2); %Damped Natural Frequency
        %Defining Parameters required A,B,C,D & A1,B1,C1,D1
        A=exp(-Z*Wn*del_t)*((Z/sqrt(1-Z^2))*sin(Wd*del_t)+cos(Wd*del_t));
        B=exp(-Z*Wn*del_t)*(sin(Wd*del_t)/Wd);
        C=(((2*Z)/(Wn*del_t))+exp(-Z*Wn*del_t)*(((1-2*Z^2)/(Wd*del_t)-(Z/sqrt(1-Z^2)))↙
*sin(Wd*del_t)-(1+((2*Z)/(Wn*del_t)))*cos(Wd*del_t)))/Wn^2;
        D=(1-((2*Z)/(Wn*del_t))+exp(-Z*Wn*del_t)*(((2*Z^2-1)/(Wd*del_t))*sin(Wd*del_t)↙
+((2*Z)/(Wn*del_t))*cos(Wd*del_t)))/Wn^2;
        A1=-exp(-Z*Wn*del_t)*((Wn/sqrt(1-Z^2))*sin(Wd*del_t));
        B1=exp(-Z*Wn*del_t)*(cos(Wd*del_t)-(Z/sqrt(1-Z^2))*sin(Wd*del_t));
        C1=((-1/del_t)+exp(-Z*Wn*del_t)*(((Wn/(sqrt(1-Z^2)))+(Z/(del_t*sqrt(1-Z^2))))↙
*sin(Wd*del_t)+(cos(Wd*del_t)/del_t)))/Wn^2;
        D1=(1-exp(-Z*Wn*del_t)*((Z/sqrt(1-Z^2))*sin(Wd*del_t)+cos(Wd*del_t)))/↙
(Wn^2*del_t);
        u=zeros(length(a_g1),1); %Initialising displacement response vector of the↙
SDOF system
        v=zeros(length(a_g1),1); %Initialising velocity response vector of the SDOF↙
system
        acc=zeros(length(a_g1),1);
        for i=1:length(a_g1)-1
            u(i+1)=A*u(i)+B*v(i)-C*a_g1(i)-D*a_g1(i+1);
```

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```
            v(i+1)=A1*u(i)+B1*v(i)-C1*a_g1(i)-D1*a_g1(i+1);
            acc(i+1)=-a_g1(i+1)-2*Z*Wn*v(i+1)-Wn^2*u(i+1);
        end
        a_t=a_g1+acc;
        %plot(t(1:1560),u);
        umax=max(abs(u));
        f_0=k*umax; %Max. Force for system to remain Linear Elastic
        % Performing Inelastic Response Analysis
        u_epp=zeros(length(a_g1),1);
```

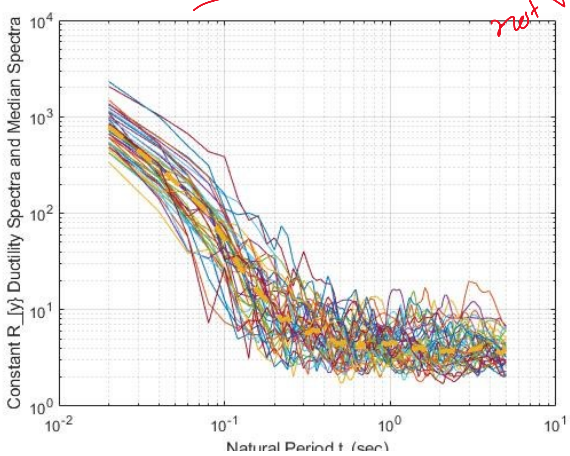
```
        v_epp=zeros(length(a_g1),1);
        a_epp=zeros(length(a_g1),1);
        %Initial calculations:
        fs=zeros(length(a_g1),1);
        fs(1)=k*u_epp(1);
        ry=Ry; %Yield Strength reduction factor
        fy=f_0/ry; %yield strength of the system
        a_epp(1)=(-m*a_g1(1)-2*m*Z*Wn*v_epp(1)-fs(1))/m; %Initial acceleration
        u_0=u_epp(1)-dt*v_epp(1)+0.5*dt^2*a_epp(1);
        k_hat=(m/dt^2)+((m*Z*Wn)/dt); %effective stiffness
        a=(m/dt^2)-((m*Z*Wn)/dt); %Integration parameter
        b=(2*m)/dt^2; %Integration parameter
        p_hat=0;du=0;fst=0;
        for i=1:length(a_g1)-1
            if i==1
                p_hat=-m*a_g1(1)-a*u_0-fs(1)+b*u_epp(1);
                u_epp(2)=p_hat/k_hat;
                v_epp(1)=(u_epp(2)-u_0)/(2*dt);
                a_epp(1)=(u_epp(2)-2*u_epp(1)+u_0)/dt^2;
            else
                p_hat=-m*a_g1(i)-a*u_epp(i-1)-fs(i)+b*u_epp(i);
                u_epp(i+1)=p_hat/k_hat;
                v_epp(i)=(u_epp(i+1)-u_epp(i-1))/(2*dt);
                a_epp(i)=(u_epp(i+1)-2*u_epp(i)+u_epp(i-1))/dt^2;
            end
            du=u_epp(i+1)-u_epp(i);
            fst=fs(i)+k*du;
            if abs(fst)>fy
                fs(i+1)=sign(fst)*fy;
            else
                fs(i+1)=fst;
            end
        end
        u_m=max(abs(u_epp)); %Maximum displacement
        meu(x,p)=abs(u_m/(fy/k));
        %Finding the residual displacement
        u_r(x,p)=abs(u_epp(end)-fs(end)/k)/(fy/k);
    end
end
median_meu=zeros(length(Tn),1);
median_ur=zeros(length(Tn),1);
for iter=1:length(Tn)
median_meu(iter,1)=median(meu(iter,:));
median_ur(iter,1)=median(u_r(iter,:));
end
figure(1)
```



```
loglog(Tn,meu)
hold on
loglog(Tn,median_meu,'-','linewidth',3)
xlabel('Natural Period t_{n}(sec)');
ylabel('Constant R_{y} Ductility Spectra and Median Spectra');
grid on
figure(2)
loglog(Tn,u_r)
hold on
loglog(Tn,median_ur,'-','linewidth',3)
grid on
xlabel('Natural Period t_{n}(sec)');
ylabel('Constant R_{y} Residual Deformation Spectra and Median Spectra');
```

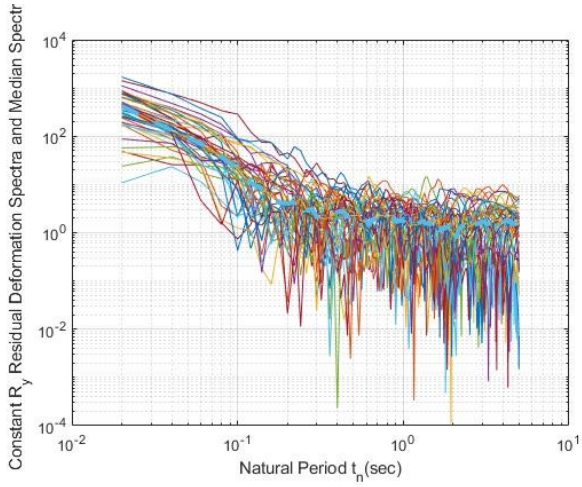
For Ry=4: (Thick Dotted line represents the Median Spectra)

not visible

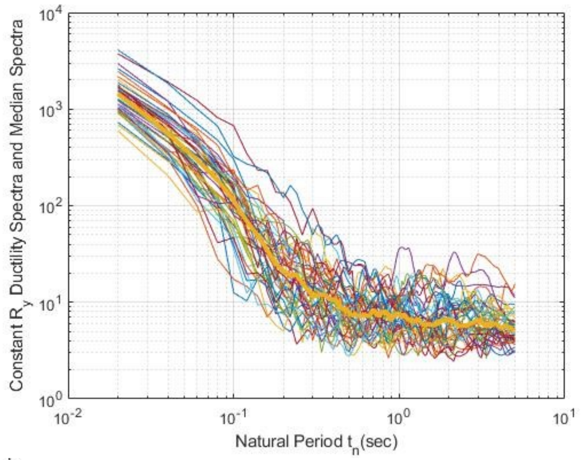


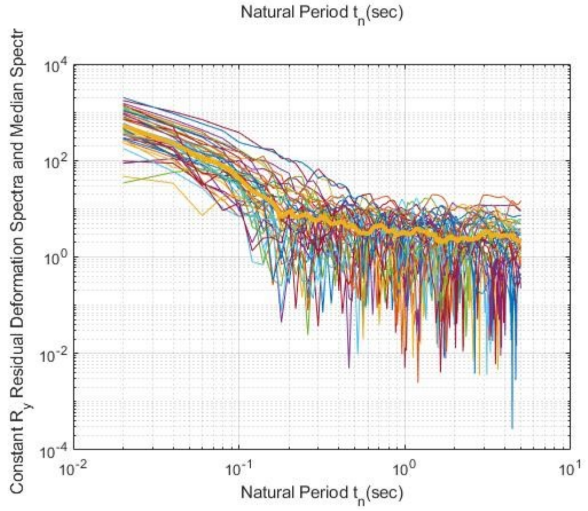
(-1)

Natural Period t_n(sec)



For Ry=6 (Thick Line is the Median Spectra)





Q1A) (ii) BP system:

EQ_HW3...

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```
%Assignment #3 P1(A)(ii)-Constant-Ry Ductility & Residual Deformation
%Spectrum For BP system for 44 different Ground Motions
%Central Difference Scheme
clc;clearvars;close all;
Tn=0.02:0.02:5;
[dt_GMi,GMi]=xlsread('GMName.xls','2:45'); % GMName.xls is in working folder
Num_GM=length(dt_GMi);
```

```
Num_GM=length(dt_GMi);
meu=zeros(length(Tn),Num_GM); %Matrix to store the ductility demands for each Tn
against given Ry
u_r=zeros(length(Tn),Num_GM); %Matrix to store the residual displacement for each Tn
againts given Ry
for p=1:Num_GM
path = ['C:\Users\User\MATLAB\',GMi{p}];
fid = fopen(path,'r');
data=textscan(fid,'%f64','HeaderLines',4);
a_g=data{:};
fclose(fid); N=length(a_g)-1;
dt1=dt_GMi(p);
t=0:dt1:(N*dt1+20);
t1=0:0.005:(N*dt1+20);
% Adding zero padding to the given Earthquake excitation data
a_g=[a_g;zeros((20/dt1),1)]; % appneding the a_g vector with zeros for the next 20
sec.
% interpolating the acceleration values within the refined time range
a_g1=interp1(t,a_g,t1);
Ry=6;
del_t=0.005;
dt=0.005;
    for x=1:length(Tn)
         %Producing System response data for Equivalent Linear Elastic system
        Z=0.05; %Damping ratio
        m=1; %Considering unit mass
        Wn=(2*pi)/Tn(x); %Natural Frequency
        k=m*Wn^2; %Linear elastic Stiffness
        Wd=Wn*sqrt(1-Z^2); %Damped Natural Frequency
        %Defining Parameters required A,B,C,D & A1,B1,C1,D1
        A=exp(-Z*Wn*del_t)*((Z/sqrt(1-Z^2))*sin(Wd*del_t)+cos(Wd*del_t));
        B=exp(-Z*Wn*del_t)*(sin(Wd*del_t)/Wd);
        C=(((2*Z)/(Wn*del_t))+exp(-Z*Wn*del_t)*(((1-2*Z^2)/(Wd*del_t)-(Z/sqrt(1-Z^2)))
*sin(Wd*del_t)-(1+((2*Z)/(Wn*del_t)))*cos(Wd*del_t)))/Wn^2;
        D=(1-((2*Z)/(Wn*del_t))+exp(-Z*Wn*del_t)*(((2*Z^2-1)/(Wd*del_t))*sin(Wd*del_t)
+((2*Z)/(Wn*del_t))*cos(Wd*del_t)))/Wn^2;
        A1=-exp(-Z*Wn*del_t)*((Wn/sqrt(1-Z^2))*sin(Wd*del_t));
        B1=exp(-Z*Wn*del_t)*(cos(Wd*del_t)-(Z/sqrt(1-Z^2))*sin(Wd*del_t));
        C1=((-1/del_t)+exp(-Z*Wn*del_t)*(((Wn/(sqrt(1-Z^2)))+(Z/(del_t*sqrt(1-Z^2))))
*sin(Wd*del_t)+(cos(Wd*del_t)/del_t)))/Wn^2;
        D1=(1-exp(-Z*Wn*del_t)*((Z/sqrt(1-Z^2))*sin(Wd*del_t)+cos(Wd*del_t)))/
(Wn^2*del_t);
        u=zeros(length(a_g1),1); %Initialising displacement response vector of the
SDOF system
        v=zeros(length(a_g1),1); %Initialising velocity response vector of the SDOF
system
        acc=zeros(length(a_g1),1);
        for i=1:length(a_g1)-1
            u(i+1)=A*u(i)+B*v(i)-C*a_g1(i)-D*a_g1(i+1);
```



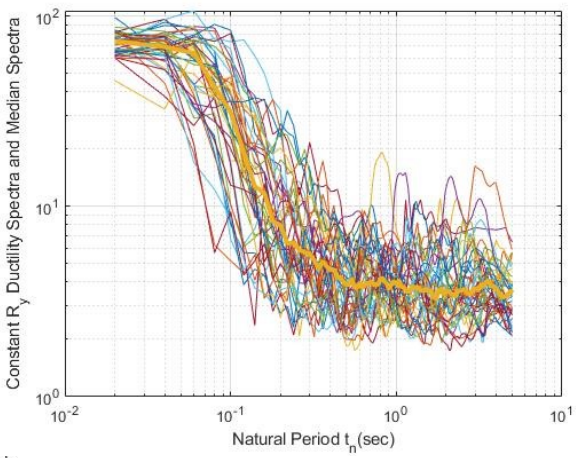
```
            v(i+1)=A1*u(i)+B1*v(i)-C1*a_g1(i)-D1*a_g1(i+1);
            acc(i+1)=-a_g1(i+1)-2*Z*Wn*v(i+1)-Wn^2*u(i+1);
        end
        a_t=a_g1+acc;
        %plot(t(1:1560),u);
        umax=max(abs(u));
        f_0=k*umax; %Max. Force for system to remain Linear Elastic
        % Performing Inelastic Response Analysis
        u_epp=zeros(length(a_g1),1);
        v_epp=zeros(length(a_g1),1);
        a_epp=zeros(length(a_g1),1);
        %Initial calculations:
        fs=zeros(length(a_g1),1);
        fs(1)=k*u_epp(1);
        ry=Ry; %Yield Strength reduction factor
        fy=f_0/ry; %yield strength of the system
        a_epp(1)=(-m*a_g1(1)-2*m*Z*Wn*v_epp(1)-fs(1))/m; %Initial acceleration
        alpha=0.05;
        k_lin=alpha*k;
        k_epp=(1-alpha)*k;
        fy_epp=(1-alpha)*fy;
        fs(1)=(k_lin+k_epp)*u_epp(1); %Initial resistive force
        u_0=u_epp(1)-dt*v_epp(1)+0.5*dt^2*a_epp(1);
        k_hat=(m/dt^2)+((m*Z*Wn)/dt); %effective stiffness
        a=(m/dt^2)-((m*Z*Wn)/dt); %Integration parameter
        b=(2*m)/dt^2; %Integration parameter
        p_hat=0;du=0;f_epp=0;f_lin=0;
        for i=1:length(a_g1)-1
            if i==1
                p_hat=-m*a_g1(1)-a*u_0-fs(1)+b*u_epp(1);
                u_epp(2)=p_hat/k_hat;
                v_epp(1)=(u_epp(2)-u_0)/(2*dt);
                a_epp(1)=(u_epp(2)-2*u_epp(1)+u_0)/dt^2;
            else
                p_hat=-m*a_g1(i)-a*u_epp(i-1)-fs(i)+b*u_epp(i);
```

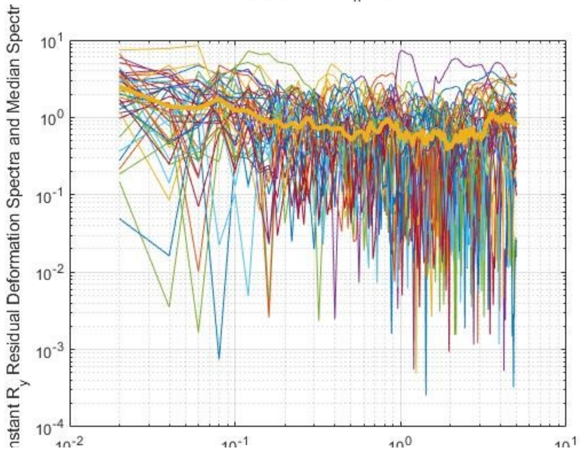
```
                u_epp(i+1)=p_hat/k_hat;
                v_epp(i)=(u_epp(i+1)-u_epp(i-1))/(2*dt);
                a_epp(i)=(u_epp(i+1)-2*u_epp(i)+u_epp(i-1))/dt^2;
            end
            du=u_epp(i+1)-u_epp(i);
            f_lin=f_lin+k_lin*du;
            f_epp=f_epp+k_epp*du;
            if abs(f_epp)>fy_epp
                f_epp=sign(f_epp)*fy_epp;
            end
            fs(i+1)=f_lin+f_epp;
        end
        u_m=max(abs(u_epp)); %Maximum displacement
        meu(x,p)=abs(u_m/(fy/k));
        %Finding the residual displacement
        u_r(x,p)=abs(u_epp(end)-fs(end)/k)/(fy/k);
    end
end
median_meu=zeros(length(Tn),1);
median_ur=zeros(length(Tn),1);
```



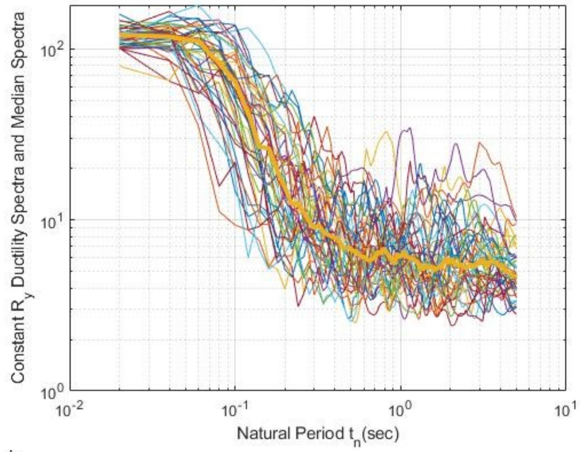
```
for iter=1:length(Tn)
median_meu(iter,1)=median(meu(iter,:));
median_ur(iter,1)=median(u_r(iter,:));
end
figure(1)
loglog(Tn,meu)
hold on
loglog(Tn,median_meu,'-','linewidth',3)
xlabel('Natural Period t_{n}(sec)');
ylabel('Constant R_{y} Ductility Spectra and Median Spectra');
grid on
figure(2)
loglog(Tn,u_r)
hold on
loglog(Tn,median_ur,'-','linewidth',3)
grid on
xlabel('Natural Period t_{n}(sec)');
ylabel('Constant R_{y} Residual Deformation Spectra and Median Spectra');
```

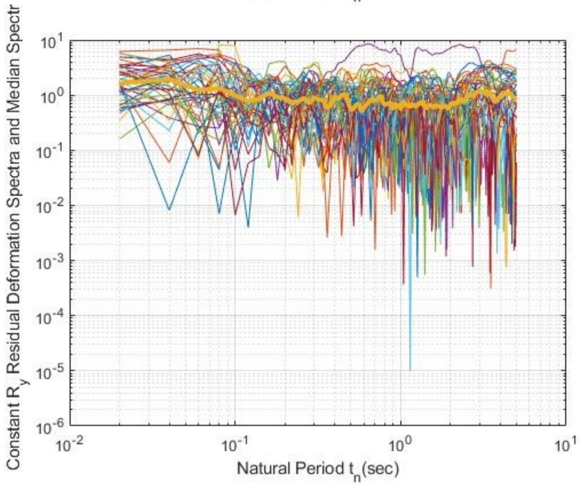
For Ry=4: (Thick Line is the Median Spectra)





For Ry=6: (Thick Line is the Median Spectra)





Q1A) (iii) BEL system:

EQ_HW3...

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```
%Assignment #3 P1(A)(iii)-Constant-Ry Ductility & Residual Deformation
%Spectrum For BEL system for 44 different Ground Motions
%Central Difference Scheme
clc;clearvars;close all;
Tn=0.02:0.02:5;
[dt_GMi,GMi]=xlsread('GMName.xls','2:45'); % GMName.xls is in working folder
Num_GM=length(dt_GMi);
meu=zeros(length(Tn),Num_GM); %Matrix to store the ductility demands for each Tn↙
against given Ry
u_r=zeros(length(Tn),Num_GM); %Matrix to store the residual displacement for each Tn↙
againts given Ry
for p=1:Num_GM
path = ['C:\Users\User\MATLAB\',GMi{p}];
fid = fopen(path,'r');
data=textscan(fid,'%f64','HeaderLines',4);
a_g=data{:};
fclose(fid); N=length(a_g)-1;
dt1=dt_GMi(p);
t=0:dt1:(N*dt1+20);
t1=0:0.005:(N*dt1+20);
% Adding zero padding to the given Earthquake excitation data
a_g=[a_g;zeros((20/dt1),1)]; % appneding the a_g vector with zeros for the next 20↙
sec.
% interpolating the acceleration values within the refined time range
a_g1=interp1(t,a_g,t1);
Ry=6;
del_t=0.005;
dt=0.005;
    for x=1:length(Tn)
        %Producing System response data for Equivalent Linear Elastic system
        Z=0.05; %Damping ratio
        m=1; %Considering unit mass
```

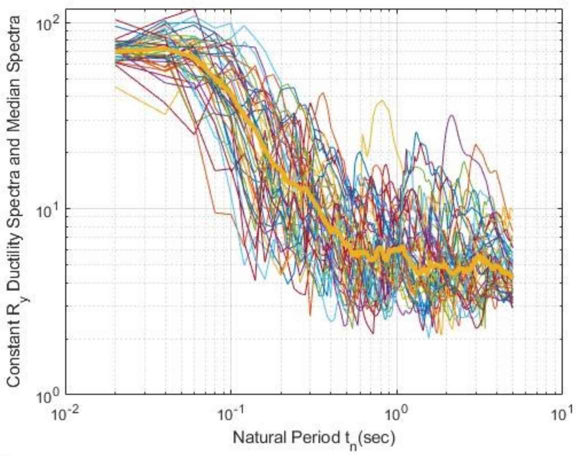
```
        Wn=(2*pi)/Tn(x); %Natural Frequency
        k=m*Wn^2; %Linear elastic Stiffness
        Wd=Wn*sqrt(1-Z^2); %Damped Natural Frequency
        %Defining Parameters required A,B,C,D & A1,B1,C1,D1
        A=exp(-Z*Wn*del_t)*((Z/sqrt(1-Z^2))*sin(Wd*del_t)+cos(Wd*del_t));
        B=exp(-Z*Wn*del_t)*(sin(Wd*del_t)/Wd);
        C=(((2*Z)/(Wn*del_t))+exp(-Z*Wn*del_t)*(((1-2*Z^2)/(Wd*del_t)-(Z/sqrt(1-Z^2)))↙
*sin(Wd*del_t)-(1+((2*Z)/(Wn*del_t)))*cos(Wd*del_t)))/Wn^2;
        D=(1-((2*Z)/(Wn*del_t))+exp(-Z*Wn*del_t)*(((2*Z^2-1)/(Wd*del_t))*sin(Wd*del_t)↙
+((2*Z)/(Wn*del_t))*cos(Wd*del_t)))/Wn^2;
        A1=-exp(-Z*Wn*del_t)*((Wn/sqrt(1-Z^2))*sin(Wd*del_t));
        B1=exp(-Z*Wn*del_t)*(cos(Wd*del_t)-(Z/sqrt(1-Z^2))*sin(Wd*del_t));
        C1=((-1/del_t)+exp(-Z*Wn*del_t)*(((Wn/(sqrt(1-Z^2)))+(Z/(del_t*sqrt(1-Z^2))))↙
*sin(Wd*del_t)+(cos(Wd*del_t)/del_t)))/Wn^2;
        D1=(1-exp(-Z*Wn*del_t)*((Z/sqrt(1-Z^2))*sin(Wd*del_t)+cos(Wd*del_t)))/↙
(Wn^2*del_t);
        u=zeros(length(a_g1),1); %Initialising displacement response vector of the↙
SDOF system
        v=zeros(length(a_g1),1); %Initialising velocity response vector of the SDOF↙
system
        acc=zeros(length(a_g1),1);
        for i=1:length(a_g1)-1
            u(i+1)=A*u(i)+B*v(i)-C*a_g1(i)-D*a_g1(i+1);
```

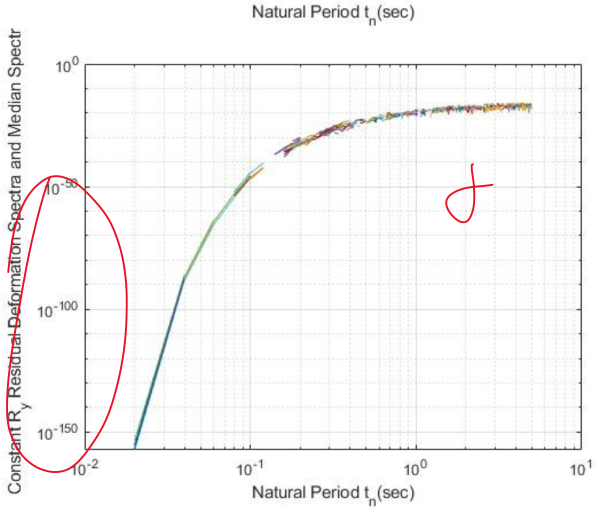


```
            v(i+1)=A1*u(i)+B1*v(i)-C1*a_g1(i)-D1*a_g1(i+1);
            acc(i+1)=-a_g1(i+1)-2*Z*Wn*v(i+1)-Wn^2*u(i+1);
        end
        a_t=a_g1+acc;
        %plot(t(1:1560),u);
        umax=max(abs(u));
        f_0=k*umax; %Max. Force for system to remain Linear Elastic
        % Performing Inelastic Response Analysis
        u_epp=zeros(length(a_g1),1);
        v_epp=zeros(length(a_g1),1);
        a_epp=zeros(length(a_g1),1);
        %Initial calculations:
        fs=zeros(length(a_g1),1);
        fs(1)=k*u_epp(1);
        ry=Ry; %Yield Strength reduction factor
        fy=f_0/ry; %yield strength of the system
        a_epp(1)=(-m*a_g1(1)-2*m*Z*Wn*v_epp(1)-fs(1))/m; %Initial acceleration
        alpha=0.05;
        k_lin=alpha*k;
        k_bel=(1-alpha)*k;
        fy_bel=(1-alpha)*fy;
        fs(1)=(k_lin+k_bel)*u_epp(1); %Initial resistive force
        u_0=u_epp(1)-dt*v_epp(1)+0.5*dt^2*a_epp(1);
        k_hat=(m/dt^2)+((m*Z*Wn)/dt); %effective stiffness
        a=(m/dt^2)-((m*Z*Wn)/dt); %Integration parameter
        b=(2*m)/dt^2; %Integration parameter
        p_hat=0;du=0;f_lin=0;f_bel=0;
        for i=1:length(a_g1)-1
            if i==1
                p_hat=-m*a_g1(1)-a*u_0-fs(1)+b*u_epp(1);
                u_epp(2)=p_hat/k_hat;
                v_epp(1)=(u_epp(2)-u_0)/(2*dt);
                a_epp(1)=(u_epp(2)-2*u_epp(1)+u_0)/dt^2;
            else
                p_hat=-m*a_g1(i)-a*u_epp(i-1)-fs(i)+b*u_epp(i);
                u_epp(i+1)=p_hat/k_hat;
                v_epp(i)=(u_epp(i+1)-u_epp(i-1))/(2*dt);
                a_epp(i)=(u_epp(i+1)-2*u_epp(i)+u_epp(i-1))/dt^2;
            end
            du=u_epp(i+1)-u_epp(i);
            if abs(u_epp(i+1))>(fy/k)
                f_bel=sign(u_epp(i+1))*fy_bel;
                f_lin=k_lin*u_epp(i+1);
            else
                f_bel=k_bel*u_epp(i+1);
                f_lin=k_lin*u_epp(i+1);
            end
                fs(i+1)=f_lin+f_bel;
        end
        u_m=max(abs(u_epp)); %Maximum displacement
        meu(x,p)=abs(u_m/(fy/k));
        %Finding the residual displacement
        if abs(u_epp(end))>(fy/k)
            if u_epp(end)<0
                u_r(x,p)=abs(u_epp(end)+((abs(fs(end))-fy)/k_lin)+(fy/k));
```

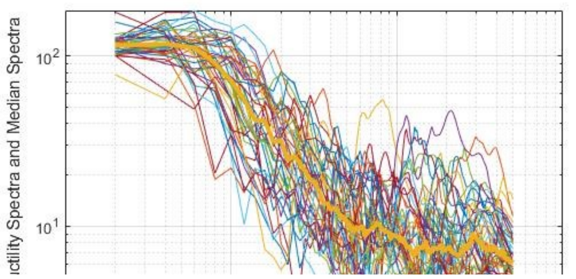
```
            else
                u_r(x,p)=abs(u_epp(end)-((fs(end)-fy)/k_lin)-(fy/k));
            end
        else
            u_r(x,p)=abs(u_epp(end)-fs(end)/k)/(fy/k);
        end
    end
end
median_meu=zeros(length(Tn),1);
median_ur=zeros(length(Tn),1);
for iter=1:length(Tn)
median_meu(iter,1)=median(meu(iter,:));
median_ur(iter,1)=median(u_r(iter,:));
end
figure(1)
loglog(Tn,meu)
hold on
loglog(Tn,median_meu,'-','linewidth',3)
xlabel('Natural Period t_{n}(sec)');
ylabel('Constant R_{y} Ductility Spectra and Median Spectra');
grid on
figure(2)
loglog(Tn,u_r)
hold on
loglog(Tn,median_ur,'-','linewidth',3)
grid on
xlabel('Natural Period t_{n}(sec)');
ylabel('Constant R_{y} Residual Deformation Spectra and Median Spectra');
```

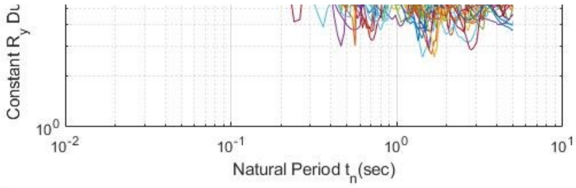
For Ry=4 (Thick Line is the Median Spectra)

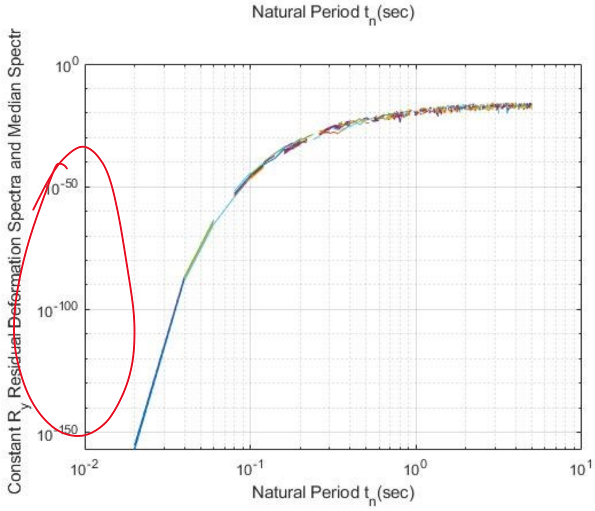




For Ry=6







Comments:

The Median Spectra embodies the general trend in the Constant Ry ductility and the Residual Deformation Spectra which we can observe here. The ductility demand is larger than Ry for small value of Tn when the response is in the acceleration sensitive region and then when it enters the velocity sensitive region, the ductility demand may or may not be greater at times with respect to Ry.
For the Residual Deformation median spectra in case of the Bilinear Elastic system, the graph is actually a straight line with zero ordinate, but in the Log scale it is not able plot it as it is in the form of discreet points. (here it is of the order of 10^-18)
[Note: The plots are done in log scale as in the normal scale no distinguishing could be made between the graphs as all were coinciding with one another]

Q1B)

EQ_HW3...

BilinearPl...

BilinearEl...

ElastoPla...

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```
%Assignment #3 P1(B)-Constant-Ry Ductility & Residual Deformation
%Spectrum For EPP system for 44 different Ground Motions
%Comparison of Median Spectra between EPP, BP, BEL systems
%Central Difference Scheme
clc;clearvars;close all;
Tn=0.02:0.02:5;
[dt_GMi,GMi]=xlsread('GMName.xls','2:45'); % GMName.xls is in working folder
Num_GM=length(dt_GMi);
meu_epp=zeros(length(Tn),Num_GM); %Matrix to store the ductility demands for each Tn
against given Ry for EPP system
u_r_epp=zeros(length(Tn),Num_GM); %Matrix to store the residual displacement for each
Tn against given Ry for EPP system
meu_bp=zeros(length(Tn),Num_GM);  %%Matrix to store the ductility demands for each Tn
against given Ry for BP system
u_r_bp=zeros(length(Tn),Num_GM); %Matrix to store the residual displacement for each
Tn againts given Ry for BP system
meu_bel=zeros(length(Tn),Num_GM);  %%Matrix to store the ductility demands for each Tn
against given Ry for BEL system
u_r_bel=zeros(length(Tn),Num_GM); %Matrix to store the residual displacement for each
Tn againts given Ry for BEL system
```

```
for p=1:Num_GM
path = ['C:\Users\User\MATLAB\',GMi{p}];
fid = fopen(path,'r');
data=textscan(fid,'%f64','HeaderLines',4);
a_g=data{:};
fclose(fid); N=length(a_g)-1;
dt1=dt_GMi(p);
t=0:dt1:(N*dt1+20);
t1=0:0.005:(N*dt1+20);
% Adding zero padding to the given Earthquake excitation data
a_g=[a_g;zeros((20/dt1),1)]; % appneding the a_g vector with zeros for the next 20
sec.
% interpolating the acceleration values within the refined time range
a_g1=interp1(t,a_g,t1);
Ry=6;
del_t=0.005;
dt=0.005;
for x=1:length(Tn)
        %Producing System response data for Equivalent Linear Elastic system
        Z=0.05; %Damping ratio
        m=1; %Considering unit mass
        Wn=(2*pi)/Tn(x); %Natural Frequency
        k=m*Wn^2; %Linear elastic Stiffness
        Wd=Wn*sqrt(1-Z^2); %Damped Natural Frequency
        %Defining Parameters required A,B,C,D & A1,B1,C1,D1
        A=exp(-Z*Wn*del_t)*((Z/sqrt(1-Z^2))*sin(Wd*del_t)+cos(Wd*del_t));
        B=exp(-Z*Wn*del_t)*(sin(Wd*del_t)/Wd);
        C=(((2*Z)/(Wn*del_t))+exp(-Z*Wn*del_t)*(((1-2*Z^2)/(Wd*del_t)-(Z/sqrt(1-Z^2)))
*sin(Wd*del_t)-(1+((2*Z)/(Wn*del_t)))*cos(Wd*del_t)))/Wn^2;
        D=(1-((2*Z)/(Wn*del_t))+exp(-Z*Wn*del_t)*(((2*Z^2-1)/(Wd*del_t))*sin(Wd*del_t)
+((2*Z)/(Wn*del_t))*cos(Wd*del_t)))/Wn^2;
        A1=-exp(-Z*Wn*del_t)*((Wn/sqrt(1-Z^2))*sin(Wd*del_t));
        B1=exp(-Z*Wn*del_t)*(cos(Wd*del_t)-(Z/sqrt(1-Z^2))*sin(Wd*del_t));
        C1=((-1/del_t)+exp(-Z*Wn*del_t)*(((Wn/(sqrt(1-Z^2)))+(Z/(del_t*sqrt(1-Z^2))))
*sin(Wd*del_t)+(cos(Wd*del_t)/del_t)))/Wn^2;
```



```
        D1=(1-exp(-Z*Wn*del_t)*((Z/sqrt(1-Z^2))*sin(Wd*del_t)+cos(Wd*del_t)))/
(Wn^2*del_t);
        u=zeros(length(a_g1),1); %Initialising displacement response vector of the
SDOF system
        v=zeros(length(a_g1),1); %Initialising velocity response vector of the SDOF
system
        acc=zeros(length(a_g1),1);
        for i=1:length(a_g1)-1
            u(i+1)=A*u(i)+B*v(i)-C*a_g1(i)-D*a_g1(i+1);
            v(i+1)=A1*u(i)+B1*v(i)-C1*a_g1(i)-D1*a_g1(i+1);
            acc(i+1)=-a_g1(i+1)-2*Z*Wn*v(i+1)-Wn^2*u(i+1);
        end
        a_t=a_g1+acc;
        %plot(t(1:1560),u);
        umax=max(abs(u));
        f_0=k*umax; %Max. Force for system to remain Linear Elastic
        % Performing Inelastic Response Analysis
        fy=f_0/Ry; %yield strength of the system
        %Defining parameters for EPP system
        u_epp=zeros(length(a_g1),1);
        v_epp=zeros(length(a_g1),1);
        a_epp=zeros(length(a_g1),1);
        fs_epp=zeros(length(a_g1),1);
        [meu_epp(x,p),u_r_epp(x,p)]=ElastoPlastic(m,Z,Wn,dt,a_g1,k,fy,u_epp,v_epp,
a_epp,fs_epp);
        %Defining parameters for BP system
        u_bp=zeros(length(a_g1),1);
        v_bp=zeros(length(a_g1),1);
        a_bp=zeros(length(a_g1),1);
        fs_bp=zeros(length(a_g1),1);
        [meu_bp(x,p),u_r_bp(x,p)]=BilinearPlastic(m,Z,Wn,dt,a_g1,k,fy,u_bp,v_bp,a_bp,
fs_bp);
        %Defining parameters for BEL system
        u_bel=zeros(length(a_g1),1);
        v_bel=zeros(length(a_g1),1);
        a_bel=zeros(length(a_g1),1);
        fs_bel=zeros(length(a_g1),1);
        [meu_bel(x,p),u_r_bel(x,p)]=BilinearElastic(m,Z,Wn,dt,a_g1,k,fy,u_bel,v_bel,
a_bel,fs_bel);
end
end
median_meu_epp=zeros(length(Tn),1);
median_ur_epp=zeros(length(Tn),1);
median_meu_bp=zeros(length(Tn),1);
median_ur_bp=zeros(length(Tn),1);
median_meu_bel=zeros(length(Tn),1);
median_ur_bel=zeros(length(Tn),1);
for iter=1:length(Tn)
```

```
median_meu_epp(iter,1)=median(meu_epp(iter,:));
median_ur_epp(iter,1)=median(u_r_epp(iter,:));
median_meu_bp(iter,1)=median(meu_bp(iter,:));
median_ur_bp(iter,1)=median(u_r_bp(iter,:));
median_meu_bel(iter,1)=median(meu_bel(iter,:));
median_ur_bel(iter,1)=median(u_r_bel(iter,:));
end
```

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```
figure(1)
loglog(Tn,median_meu_epp)
hold on
loglog(Tn,median_meu_bp)
hold on
loglog(Tn,median_meu_bel)
grid on
xlabel('Natural Period T_{n}(sec)');
ylabel('Constant R_{y} median ductility Spectra');
legend('EPP system','BP system','BEL system');
figure(2)
loglog(Tn,median_ur_epp)
hold on
loglog(Tn,median_ur_bp)
hold on
loglog(Tn,median_ur_bel)
grid on
xlabel('Natural Period T_{n}(sec)');
ylabel('Constant R_{y} Median Residual Deformation Spectra');
legend('EPP system','BP system','BEL system');
```

The Functions for the 3 different systems used in the code are given as follows:

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```
function [meu,u_r] = ElastoPlastic(m,Z,Wn,dt,a_g1,k,fy,u_epp,v_epp,a_epp,fs_epp)
%Inelastic SDOF response analysis for EPP system using CDM
fs_epp(1)=k*u_epp(1);
a_epp(1)=(-m*a_g1(1)-2*m*Z*Wn*v_epp(1)-fs_epp(1))/m; %Initial acceleration
u_0=u_epp(1)-dt*v_epp(1)+0.5*dt^2*a_epp(1);
k_hat=(m/dt^2)+((m*Z*Wn)/dt); %effective stiffness
a=(m/dt^2)-((m*Z*Wn)/dt); %Integration parameter
b=(2*m)/dt^2; %Integration parameter
p_hat=0;du=0;fst=0;
for i=1:length(a_g1)-1
    if i==1
        p_hat=-m*a_g1(1)-a*u_0-fs_epp(1)+b*u_epp(1);
        u_epp(2)=p_hat/k_hat;
        v_epp(1)=(u_epp(2)-u_0)/(2*dt);
        a_epp(1)=(u_epp(2)-2*u_epp(1)+u_0)/dt^2;
    else
        p_hat=-m*a_g1(i)-a*u_epp(i-1)-fs_epp(i)+b*u_epp(i);
        u_epp(i+1)=p_hat/k_hat;
        v_epp(i)=(u_epp(i+1)-u_epp(i-1))/(2*dt);
        a_epp(i)=(u_epp(i+1)-2*u_epp(i)+u_epp(i-1))/dt^2;
    end
    du=u_epp(i+1)-u_epp(i);
    fst=fs_epp(i)+k*du;
    if abs(fst)>fy
        fs_epp(i+1)=sign(fst)*fy;
    else
        fs_epp(i+1)=fst;
    end
end
u_m=max(abs(u_epp)); %Maximum displacement
meu=abs(u_m/(fy/k));
%Finding the residual displacement
u_r=abs(u_epp(end)-fs_epp(end)/k)/(fy/k);
end
```

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```
function [meu,u_r] = BilinearPlastic(m,Z,Wn,dt,a_g1,k,fy,u_bp,v_bp,a_bp,fs_bp)
%Inelastic SDOF response analysis for BP system using CDM
a_bp(1)=(-m*a_g1(1)-2*m*Z*Wn*v_bp(1)-fs_bp(1))/m; %Initial acceleration
alpha=0.05;
k_lin=alpha*k;
```

```
k_epp=(1-alpha)*k;
fy_epp=(1-alpha)*fy;
fs_bp(1)=(k_lin+k_epp)*u_bp(1); %Initial resistive force
u_0=u_bp(1)-dt*v_bp(1)+0.5*dt^2*a_bp(1);
k_hat=(m/dt^2)+((m*Z*Wn)/dt); %effective stiffness
a=(m/dt^2)-((m*Z*Wn)/dt); %Integration parameter
b=(2*m)/dt^2; %Integration parameter
p_hat=0;du=0;f_epp=0;f_lin=0;
for i=1:length(a_g1)-1
    if i==1
        p_hat=-m*a_g1(1)-a*u_0-fs_bp(1)+b*u_bp(1);
        u_bp(2)=p_hat/k_hat;
        v_bp(1)=(u_bp(2)-u_0)/(2*dt);
        a_bp(1)=(u_bp(2)-2*u_bp(1)+u_0)/dt^2;
    else
        p_hat=-m*a_g1(i)-a*u_bp(i-1)-fs_bp(i)+b*u_bp(i);
        u_bp(i+1)=p_hat/k_hat;
        v_bp(i)=(u_bp(i+1)-u_bp(i-1))/(2*dt);
        a_bp(i)=(u_bp(i+1)-2*u_bp(i)+u_bp(i-1))/dt^2;
    end
    du=u_bp(i+1)-u_bp(i);
    f_lin=f_lin+k_lin*du;
    f_epp=f_epp+k_epp*du;
    if abs(f_epp)>fy_epp
        f_epp=sign(f_epp)*fy_epp;
    end
    fs_bp(i+1)=f_lin+f_epp;
end
u_m=max(abs(u_bp)); %Maximum displacement
meu=abs(u_m/(fy/k));
%Finding the residual displacement
u_r=abs(u_bp(end)-fs_bp(end)/k)/(fy/k);
end
```

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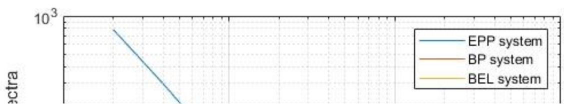
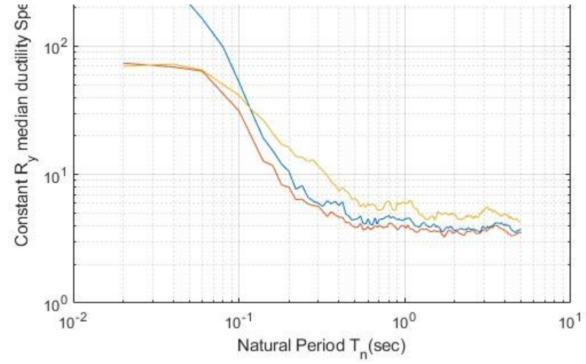
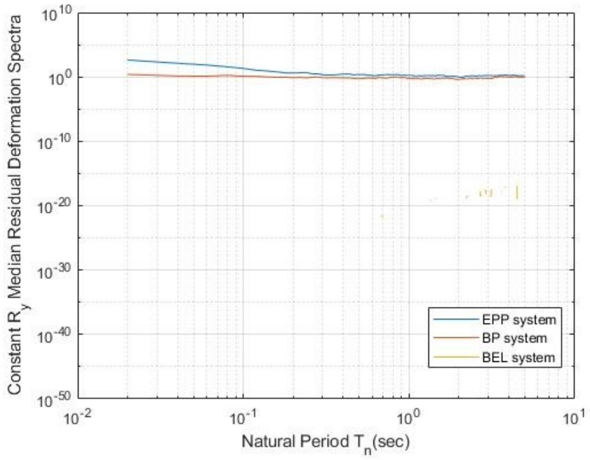
```
function [meu,u_r] =BilinearElastic(m,Z,Wn,dt,a_g1,k,fy,u_bel,v_bel,a_bel,fs_bel)
%Inelastic SDOF response analysis for BEL system using CDM
a_bel(1)=(-m*a_g1(1)-2*m*Z*Wn*v_bel(1)-fs_bel(1))/m; %Initial acceleration
alpha=0.05;
k_lin=alpha*k;
k_bel=(1-alpha)*k;
fy_bel=(1-alpha)*fy;
fs_bel(1)=(k_lin+k_bel)*u_bel(1); %Initial resistive force
u_0=u_bel(1)-dt*v_bel(1)+0.5*dt^2*a_bel(1);
k_hat=(m/dt^2)+((m*Z*Wn)/dt); %effective stiffness
a=(m/dt^2)-((m*Z*Wn)/dt); %Integration parameter
b=(2*m)/dt^2; %Integration parameter
p_hat=0;du=0;f_lin=0;f_bel=0;
for i=1:length(a_g1)-1
    if i==1
        p_hat=-m*a_g1(1)-a*u_0-fs_bel(1)+b*u_bel(1);
        u_bel(2)=p_hat/k_hat;
        v_bel(1)=(u_bel(2)-u_0)/(2*dt);
        a_bel(1)=(u_bel(2)-2*u_bel(1)+u_0)/dt^2;
    else
        p_hat=-m*a_g1(i)-a*u_bel(i-1)-fs_bel(i)+b*u_bel(i);
        u_bel(i+1)=p_hat/k_hat;
        v_bel(i)=(u_bel(i+1)-u_bel(i-1))/(2*dt);
        a_bel(i)=(u_bel(i+1)-2*u_bel(i)+u_bel(i-1))/dt^2;
    end
    du=u_bel(i+1)-u_bel(i);
    if abs(u_bel(i+1))>(fy/k)
        f_bel=sign(u_bel(i+1))*fy_bel;
        f_lin=k_lin*u_bel(i+1);
    else
        f_bel=k_bel*u_bel(i+1);
        f_lin=k_lin*u_bel(i+1);
    end
    fs_bel(i+1)=f_lin+f_bel;
end
u_m=max(abs(u_bel)); %Maximum displacement
meu=abs(u_m/(fy/k));
if abs(u_bel(end))>(fy/k)
    if u_bel(end)<0
        u_r=abs(u_bel(end)+((abs(fs_bel(end))-fy)/k_lin)+(fy/k));
    else
        u_r=abs(u_bel(end)-((fs_bel(end)-fy)/k_lin)-(fy/k));
    end
else
    u_r=abs(u_bel(end)-fs_bel(end)/k)/(fy/k);
end
end
```

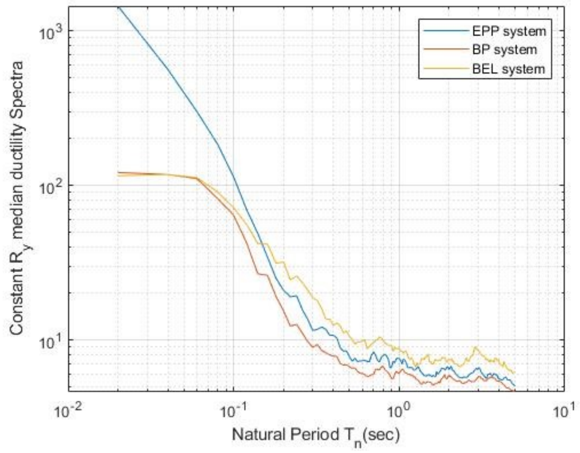
For Ry=4

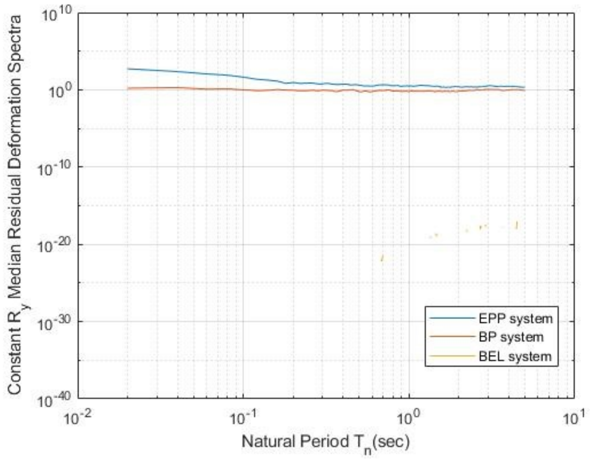






For Ry=6





Comments:

For the acceleration sensitive region, the median ductility spectra for the EPP system is the highest as in that region the PSA experienced and in turn the seismic force to be resisted by the structure is higher and hence the system, by entering the plastic deformation stage demands to deform to a much larger extend in order to resist the higher magnitude of earthquake force.
In the Velocity sensitive region however, the Bilinear Elastic system has higher ductility median spectra.
The residual deformation median spectra for the Bilinear Elastic system is once again not been able to plot in the log scale as it is practically zero, the system being elastic nonetheless, comes

back to its original configuration after the withdrawal of the earthquake excitation.