Question 5: Solution

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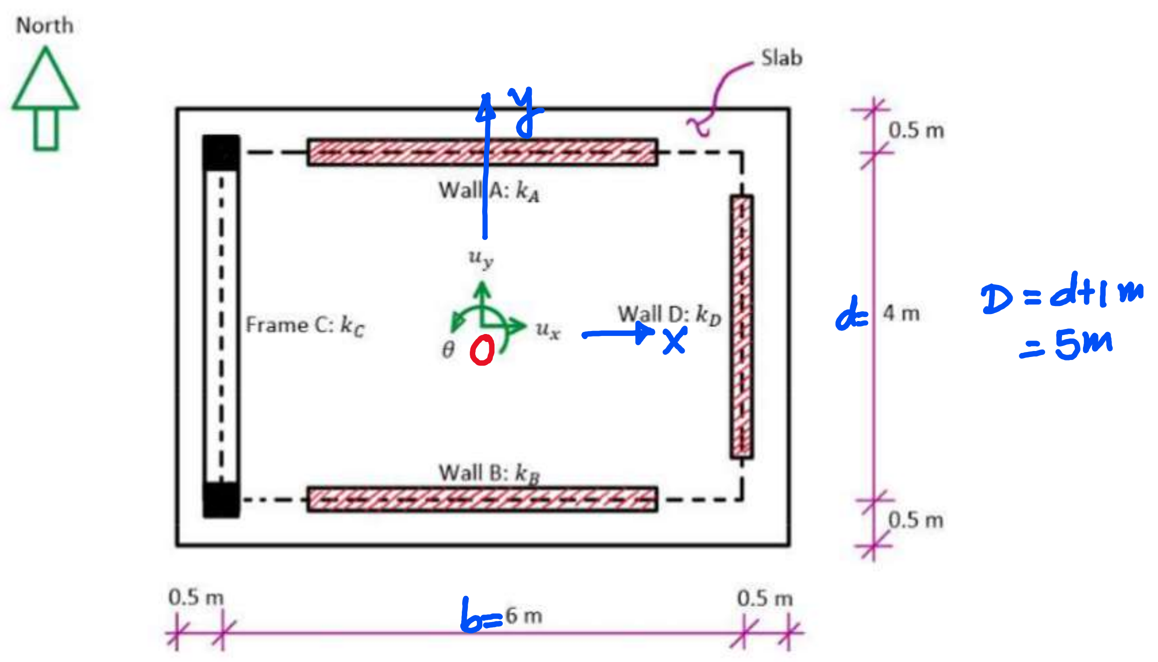

Figure 5: Floor plan for Question 5

B = 6+1 = 7m

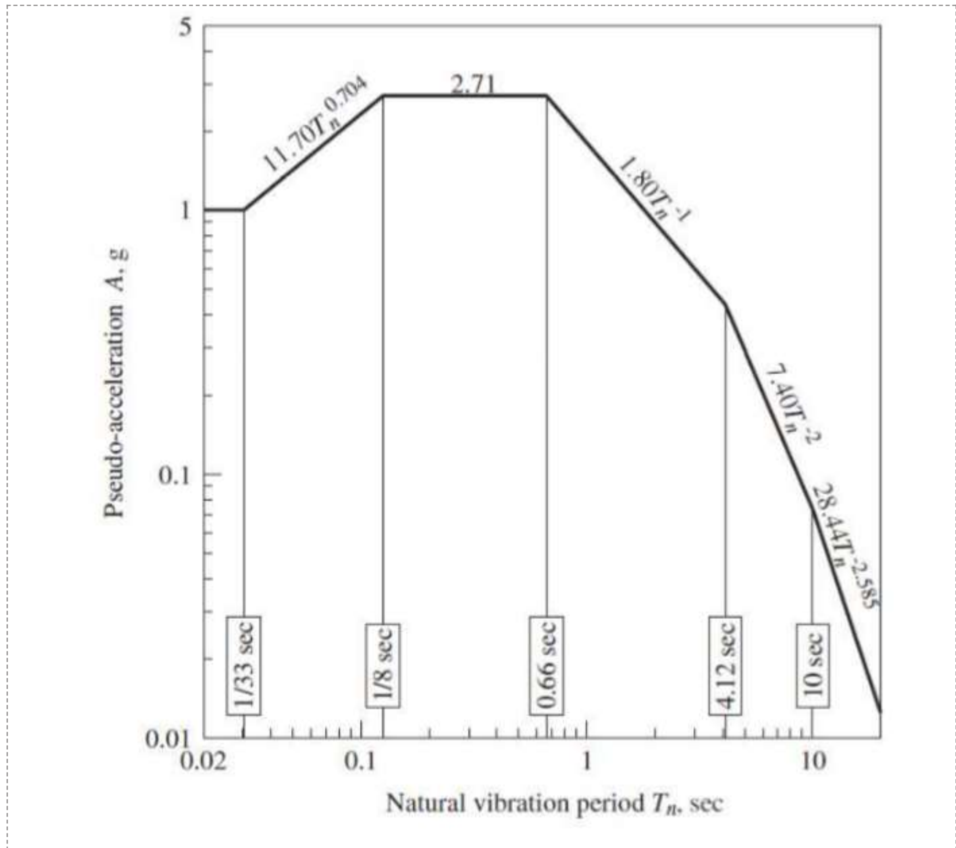

Figure 2: Elastic pseudo-acceleration design spectrum for ζ=5% for Questions 2 and 5

Part (a): Natural Frequencies and Mode Shapes [10]

Weight of roof slab $= (5\ \text{kN/m}^2)(7\ \text{m})(5\ \text{m}) = 175\ \text{kN}$

Mass, $m = \frac{W}{g} = 1.786 \times 10^4\ \text{kg}$

Mass moment of inertia about vertical axis passing through O,

$$I_O = m \cdot \left(\frac{B^2 + D^2}{12}\right) = 1.101 \times 10^5\ \text{kg-m}^2$$

The plan is symmetric about x-axis. Hence, gm in N-S direction will not cause any motion along X-(E-W) direction.

Motion along X-direction

$$m\ddot{u}_x + 2k_x u_x = 0$$

where $k_x = k_A = k_B = 500$ kN/m

$\Rightarrow$ Natural frequency $\omega_x = \sqrt{\frac{2k_x}{m}} = \sqrt{\frac{2\times 500\times 10^3 \text{ N/m}}{1.101\times 10^5 \text{ kg}}} = 7.483$ rad/sec

+2

" period $T_x = \frac{2\pi}{\omega_x} = 0.84$ sec

Motion along Y and θ directions:

Mass matrix $\underline{m} = \begin{bmatrix} m & 0 \\ 0 & I_0 \end{bmatrix} = \begin{bmatrix} 1.786\times 10^4 & 0 \\ 0 & 1.101\times 10^5 \end{bmatrix}$ in kg and m unit

+1

Determine stiffness matrix using direct stiffness method

$u_i = \underline{a}_i \underline{u}$ where u_i = displacement of i^{th} frame

$\underline{a}_i$ = transformation matrix

$\underline{u} = \begin{Bmatrix} u_y \\ \theta \end{Bmatrix}$

For frame-A,

$\underline{a}_A = \langle 0 \quad -d/2 \rangle = \langle 0 \quad -2 \rangle$

Similarly, $\underline{a}_B = \langle 0 \quad +d/2 \rangle = \langle 0 \quad 2 \rangle$

$\underline{a}_C = \langle 1 \quad -b/2 \rangle = \langle 1 \quad -3 \rangle$

$\underline{a}_D = \langle 1 \quad +b/2 \rangle = \langle 1 \quad 3 \rangle$

∴ Total stiffness matrix

$$\underline{k} = \sum_{i=1}^{N} \underline{a}_i^T k_i \underline{a}_i = a_A^T k_A a_A + \underline{a}_B^T k_B \underline{a}_B + \underline{a}_C^T k_C \underline{a}_C + \underline{a}_D^T k_D \underline{a}_D$$

$$= \begin{Bmatrix} 0 \\ -2 \end{Bmatrix} \underset{\times 10^3}{500} \langle 0 \quad -2 \rangle + \begin{Bmatrix} 0 \\ 2 \end{Bmatrix} \underset{\times 10^3}{500} \langle 0 \quad 2 \rangle + \begin{Bmatrix} 1 \\ -3 \end{Bmatrix} \underset{\times 10^3}{250} \langle 0 \quad -3 \rangle + \begin{Bmatrix} 1 \\ 3 \end{Bmatrix} \underset{\times 10^3}{400} \langle 0 \quad 3 \rangle$$

$$= \begin{bmatrix} 650 & 450 \\ 450 & 9.85 \times 10^3 \end{bmatrix} \times 10^3 \quad \text{in N and m unit}$$

+3

Perform eigenvalue analysis to get

$\omega_1 = 5.875$ rad/sec and $\omega_2 = 9.557$ rad/sec

$T_1 = 1.07$ sec and $T_2 = 0.658$ sec

$$\underline{\phi}_1 = \begin{Bmatrix} 1 \\ -0.074 \end{Bmatrix} \qquad \underline{\phi}_2 = \begin{Bmatrix} 0.459 \\ 1 \end{Bmatrix}$$

+4

Part (b): Response Spectrum Analysis [20]

Modal masses $\quad M_1 = \underline{\phi}_1^T \underline{m}\, \underline{\phi}_1 = 1.847 \times 10^4$ kg

$\quad M_2 = \underline{\phi}_2^T \underline{m}\, \underline{\phi}_2 = 1.139 \times 10^5$ kg

$$L_n^h = \underline{\phi}_n^T \underline{m}\, \underline{\imath} \quad \text{where } \underline{\imath} = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$$

$$= \underset{1\times 2}{\langle \phi_{yn} \ \ \phi_{\theta n} \rangle} \underset{2\times 2}{\begin{bmatrix} m & 0 \\ 0 & I_0 \end{bmatrix}} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} = \langle m\phi_{yn} \quad I_0 \phi_{\theta n} \rangle \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} = m\phi_{yn}$$

$\therefore\ L_1^h = 1.786\times10$, $L_2^h = 1.786\times10 \times 0.459 = 8.198\times10$ (+5)

$$\Gamma_n = \frac{L_n^h}{M_n} \Rightarrow \Gamma_1 = \frac{1.786\times10^4}{1.847\times10^4} = 0.967 \quad \& \quad \Gamma_2 = 0.072$$

$$\underline{u}_n = \underline{u}_n^{st} A_n \qquad \underline{u}_n^{st} = \frac{\Gamma_n}{\omega_n^2}\underline{\phi}_n \Rightarrow \underline{u}_1^{st} = \begin{Bmatrix} 0.028 \\ -2.084\times10^{-3} \end{Bmatrix}$$

$$\underline{u}_2^{st} = \begin{Bmatrix} 3.612\times10^{-4} \\ 7.876\times10^{-4} \end{Bmatrix}$$

$$\left.\begin{aligned} A_1 &= \frac{1.80}{1.07}\times0.3g = 0.505g \\ A_2 &= 2.71\times0.3g = 0.813g \end{aligned}\right\} \quad \underline{u}_1 = \begin{Bmatrix} 0.139 \\ 0.01 \end{Bmatrix}\begin{matrix} m \\ rad \end{matrix} \ \text{Mode-1} \quad \underline{u}_2 = \begin{Bmatrix} 2.877\times10^{-3} \\ 6.275\times10^{-3} \end{Bmatrix}\begin{matrix} m \\ rad \end{matrix} \ \text{Mode-2}$$

(+5)

$$\therefore\ \left.\begin{aligned} u_y &= \sqrt{0.139^2 + (2.877\times10^{-3})^2} = 0.139\,m \\ \theta &= \sqrt{(0.01)^2 + (6.275\times10^{-3})} = 0.012\,rad \end{aligned}\right\}$$

(+2)

Use transformation matrices to get u_i, then do $k_i u_i$ to get equivalent lateral force in each mode. Then combine using SRSS.

For frame-A:

$$u_{A1} = \underbrace{\langle 0 \quad -2 \rangle}_{a_A} \begin{Bmatrix} 0.139 \\ 0.01 \end{Bmatrix} = 0.021\,m \quad \text{Mode-1}$$

$$u_{A2} = \langle 0 \quad -2 \rangle \begin{Bmatrix} 2.877\times10^{-3} \\ 6.275\times10^{-3} \end{Bmatrix} = -0.013\,m$$

$$f_{A1} = k_A u_{A1} = 10.306\,kN \quad \text{Mode-1}$$

(+8)

$f_{A2} = k_A u_{A2} = -6.275 \text{ kN}$ Mode-1

$\therefore f_A = \sqrt{(10.306)^2 + (-6.275)^2} = 12.066 \text{ kN}$

Similarly, $f_B = 12.066 \text{ kN}$, $f_C = 42.554 \text{ kN}$, $f_D = 43.918 \text{ kN}$