Question 5

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(c) Yes it is possible to determine the peak modal base shear and base overturning moments without determining time histories of $A_n(t)$. We can use the response spectrum for this purpose. We only need the peak pseudo acceleration response values.

equivalent static force $= f_n(t) = s_n A_n(t)$

$$f_n = s_n A_n$$

$$\Rightarrow f_{jn} = s_{jn} A_n$$

We can do any analysis for this force now. instead of s_n in RHA.

Question 5 :-

Roof slab dimensions $= 7m \times 5m$.

Weight $= 5\ kN/m^2$

Total weight $= (5 \times 7 \times 5)\ kN$.

$= 175\ kN. = \frac{175 \times 10^3}{10}$

$= 17500\ kg$

$k_A = k_B = 500\ kN/m$.

$k_C = 250\ kN/m$

$k_D = 400\ kN/m$.

$PGA = 0.3g$.

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$$\underset{\sim}{m} = \begin{bmatrix} 17500 & 0 & 0 \\ 0 & 17500 & 0 \\ 0 & 0 & \frac{17500}{12}(7^2+5^2) \end{bmatrix} kg.$$

$$\underset{\sim}{k} = \begin{bmatrix} (k_A + k_B) & 0 & 2(k_B - k_A) \\ 0 & (k_C + k_D) & 3(k_C - k_D) \\ 2(k_B - k_A) & 3(k_C - k_D) & 3^2(k_C + k_D) + 2^2(k_A + k_B) \end{bmatrix}$$

$$\Rightarrow \underset{\sim}{K} = \begin{bmatrix} 1000 & 0 & 0 \\ 0 & 650 & -450 \\ 0 & -450 & 9850 \end{bmatrix} \times 10^3 \text{ kN/m}.$$

Now for modal frequencies, eigen values →.

$$|\underset{\sim}{K} - \omega^2 \underset{\sim}{m}| = 0.$$

$$\begin{vmatrix} 1000\times10^3 - \omega^2(17500) & 0 & 0 \\ 0 & 650\times10^3 - \omega^2(17500) & -450 \\ 0 & -450 & 9850\times10^3 - \omega^2(107916.67) \end{vmatrix} = 0$$

$\Rightarrow$ Solving, we get $\omega^2 = 35.23, 57.143, 93.1874$ (in the order) rad/s

$\omega = 5.935, 7.56$ rad/s and 9.65 rad/s.

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for Mode 1 :-

$$\phi_1 = \begin{Bmatrix} 0 \\ -0.0074 \\ -0.006 \end{Bmatrix}.$$

$$\phi_2 = \begin{Bmatrix} 0.0076 \\ 0 \\ 0 \end{Bmatrix}.$$

$$\phi_3 = \begin{Bmatrix} 0 \\ -0.0014 \\ 0.0030 \end{Bmatrix}.$$

We see that the x-DOF is completely independent, so we can separate it out.

$$\underset{\sim}{m}\, \ddot{u}_x + (k_A + k_B) u_x = 0 \quad -(i)$$

and

$$\begin{pmatrix} m & 0 \\ 0 & I \end{pmatrix} \begin{Bmatrix} \ddot{u}_y \\ \ddot{u}_\theta \end{Bmatrix} + \begin{bmatrix} k_C + k_D & 3(k_C - k_D) \\ 3(k_C - k_D) & 9(k_C + k_D) + 4(k_A + k_B) \end{bmatrix} \begin{Bmatrix} u_y \\ u_\theta \end{Bmatrix} = -m\ddot{u}_g(t)$$

Now, $\phi = \begin{bmatrix} -0.0074 & -0.0014 \\ -0.006 & 0.0030 \end{bmatrix}$

Now, $S_n = \Gamma_n \, \underset{\sim}{m} \phi_n$.

$$M_n = \phi^T \underset{\sim}{m} \phi.$$

$$= \begin{pmatrix} -0.0074 & -0.006 \\ -0.0014 & 0.003 \end{pmatrix} \begin{pmatrix} 17500 & 0 \\ 0 & 107916.67 \end{pmatrix} \times \phi$$

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Which gives $M_n = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Next, $L_n^h = \sum_{j=1}^{N} m_j \phi_{jn}$.

$$= \phi^T \underset{\sim}{m} \, i.$$

$$i = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

$$\therefore L_n^h = \begin{pmatrix} -0.0074 & -0.006 \\ -0.0014 & 0.003 \end{pmatrix} \times M \times \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}.$$

$$= \begin{pmatrix} -130.086 \\ -24.035 \end{pmatrix} \begin{matrix} \to L_1^h \\ \to L_2^h \end{matrix}$$

$$\Gamma_n = \frac{L_n^h}{M_n} = L_n^h \text{ as } M_n = 1.0.$$

$$\therefore \Gamma_n = \begin{Bmatrix} -130.086 \\ -24.035 \end{Bmatrix}.$$

$$\therefore S = \begin{Bmatrix} \underset{\sim}{m} 1 \\ 0 \end{Bmatrix} = \sum_{n=1}^{2N} S_n = \sum_{n=1}^{2N} \Gamma_n \begin{Bmatrix} m \phi_{yn} \\ r^2 m \phi_{\theta n} \end{Bmatrix}.$$

~~for~~ $S_1 = \Gamma_1 \underset{\sim}{m} \phi_1$

$$= -130.086 \begin{pmatrix} 17500 & 0 \\ 0 & 107916.67 \end{pmatrix} \begin{Bmatrix} -0.0074 \\ -0.006 \end{Bmatrix}$$

$$= 10^4 \begin{Bmatrix} 1.685 \\ 8.4231 \end{Bmatrix}.$$

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You don't need S_n

$$S_2 = \Gamma_2 \underset{\sim}{m} \phi_2$$

$$= -24.035 \begin{pmatrix} 17500 & 0 \\ 0 & 107916.67 \end{pmatrix} \begin{Bmatrix} -0.0014 \\ +0.003 \end{Bmatrix}.$$

$$= 10^4 \begin{Bmatrix} 0.589 \\ -8.4231 \end{Bmatrix}.$$

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