Question 2: Solution

24 February 2022 15:28

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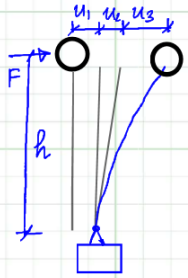
Part (a): (7)

Natural frequencies:

System-A: $\omega_n^A = \sqrt{k/m} = \sqrt{\frac{1.5\times10^9\ N/m}{1\times10^8\ kg}} = 3.87\ rad/sec$

$T_n^A = \frac{2\pi}{\omega_n^A} = 1.622\ sec.$ (+2)

System-B:



$u = u_1 + u_2 + u_3$

$u = F/k_{eq}$;

$u_1 = F/k_t$; $u_2 = \frac{F.h^2}{k_r}$; $u_3 = F/k$

$\therefore \frac{F}{k_{eq}} = F\left(\frac{1}{k_t} + \frac{h^2}{k_r} + \frac{1}{k}\right)$

$\Rightarrow \frac{1}{k_{eq}} = \frac{1}{1\times10^{11}} + \frac{(16)^2}{1\times10^{12}} + \frac{1}{1.5\times10^9}$

$\Rightarrow k_{eq} = 1.072\times10^9\ N/m$ (+5)

$\therefore \omega_n^B = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{1.072\times10^9\ N/m}{1\times10^8\ kg}} = 3.274\ rad/sec$

$T_n^B = \frac{2\pi}{\omega_n^B} = 1.919\ sec.$

Part (b): (5)

From the design spectrum,

For System-A

$A = 1.80(1.622)^{-1}g = 1.1097g \Rightarrow$ scale for PGA = 0.5g $\Rightarrow A^A = 0.555g$

$\therefore$ Design lateral force $f^A = mA = (10^8 kg)(0.555)(9.81 m/s^2) = 544.33\times10^3 kN$ (+2½)

For System-B

$A = 1.80(1.919)^{-1}g = 0.9379g \Rightarrow$ scale for PGA = 0.5g $\Rightarrow A^B = 0.469g$

$\therefore$ Design lateral force $f^B = (10^8 kg)(0.469)(9.81 m/s^2) = 460.08\times10^3 kN$ (+2½)

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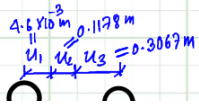
Part (c): (10)

Design lateral displacement

System-A

$u^A = f/k = \frac{544.33\times10^6\ N}{1.5\times10^9\ N/m} = 0.362\ m$ (+2)

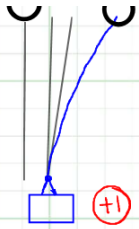


System-B

$u^B = f/k_{eq} = \frac{460.08\times10^6}{1.072\times10^9} = 0.429\ m$

$u_1^B = f/k_t = \frac{460.08\times10^6}{10^{11}} = 4.608\times10^{-3}\ m$

$u_2^B = \frac{fh^2}{k_r} = \frac{460.08\times10^6\times16^2}{10^{12}} = 0.1178\ m$ (+7)



$u_3^B = f/k = \frac{460.08 \times 10^6}{1.5 \times 10^9} = 0.3067\ m$

$u_1^B + u_2^B + u_3^B = 0.429\ m = u^B$ (checked)

Part (d) (3)

	System-A	System-B
Period	1.622 sec	1.919 sec
Design force (f)	544.33×10^3 lbf	460.08×10^3 lbf
Structural deformation	0.362 m	0.306 m
Displacement of mass	0.362 m	0.429 m

Base flexibility increases the period of the system and thereby reduces the design lateral force. Furthermore, base flexibility increases the displacement of the mass, but reduces the structural deformation.