CE723A: Finite Element Method
Homework 1 (Assigned: August 07, 2021; Due: August 28, 2021)

Name:
BUDHADITYA DE

Roll No.:
21103033

Problem	Points	Points awarded
1	10	
2	10	
3	35	
4	30	
5	25	
6	30	
7	30	
Total:	**170**	

Instructions:

[1] Please submit the necessary codes (MATLAB files), along with a scanned copy (in pdf format) of your report. Also attach the question paper to the front of your report. Submit all the files as a .zip folder via email to: ***iitk.ce723a@gmail.com***. In the subject line of the email, please write: *CE723A HW1 submission*.

[2] Name the files and folder using your roll number, the HW number, and the problem number. For example, if your roll no. is 12345: your submission file name should be *12345_HW1.zip*, the report should be named *12345_HW1.pdf*, the code for problem no. 3 should be named *12345_HW1_P3.m*, the code for problem no. 4 should be named *12345_HW1_P4.m* etc.

[3] Please show all numeric answers up to 4 decimal places.

[4] Please write neatly. Please maintain clarity in your solutions. If we do not understand what you have written, we will not grade your submission.

[5] You are encouraged to discuss amongst yourselves. But the submissions should be done individually, and you should be able to explain every single step that you have taken in solving any problem. Please do not use any unfair means.

[6] We may randomly call you in a viva and ask you to explain how you solved one/more of the problems. If we detect that you have used unfair means (e.g. you cannot explain a solution which you have included in your submission), even for a single problem, we will award a grade of zero to your entire submission.

1. For the bar shown in Figure 1.1:
   (a) Find analytically the expressions for the displacements and internal axial forces at $x = 0$, $L/2$ and L. Consider $x = 0$ at fixed end and $x = L$ at free end.
   (b) Obtain the expressions for nodal displacements using FEM, by discretizing the bar into two elements as shown in Figure 1.2.
   (c) Obtain the expressions for internal axial forces at nodes as $F^e = k^e u^e$, where k^e = element stiffness matrix, and u^e = element nodal displacement vector, for any element e.
   (d) How do the expressions obtained in (b) and (c) compare with those obtained in (a)?

(e) Now, obtain the expressions for the internal axial forces at nodes using $F^e = \{k^e u^e - f^e_{eq}\}$, where f^e_{eq} is the vector of equivalent nodal forces obtained from the distributed load $q(x)$ for any element e. How do these axial forces compare with the expressions obtained in (a)?

(f) Repeat (a) – (c) for the bar shown in Figure 1.3.

(g) Comment on your observations.

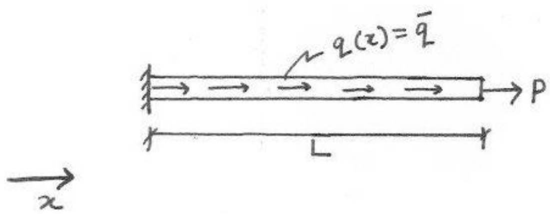


Figure 1.1

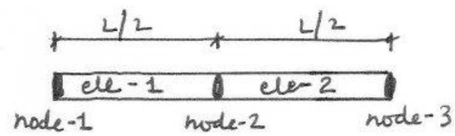


Figure 1.2

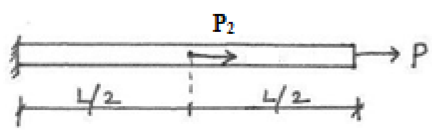


Figure 1.3

2. In the 3D – truss shown in Figure 2, all the members are made of the same material of E = 200 N/m^2, and all members have circular cross section of diameter D. Find the minimum necessary value of D, if the truss material has a maximum permissible stress in tension = 5 kN/m^2 and maximum permissible stress in compression = 3 kN/m^2. Use 3D – truss elements in your solution.

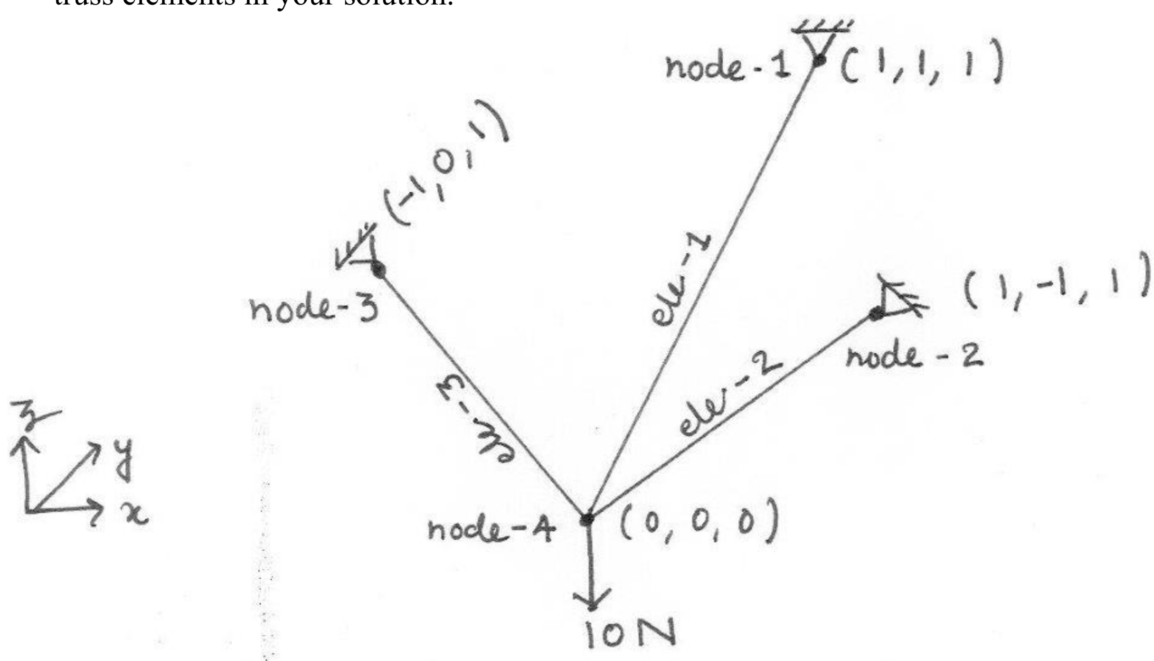
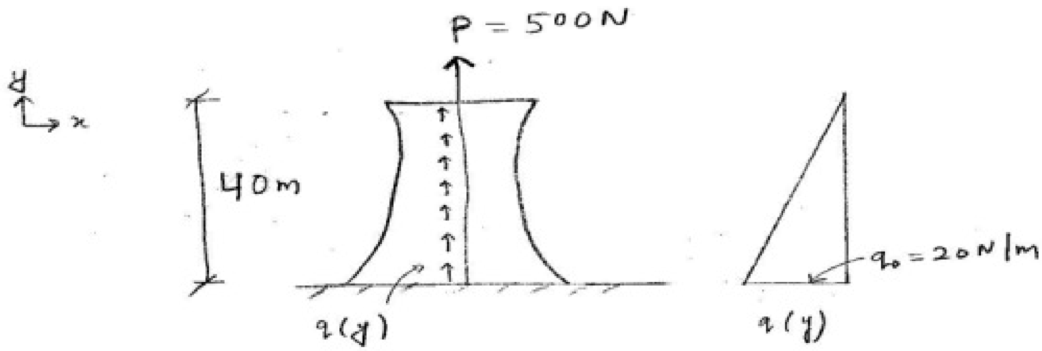


Figure 2

3. For the axially loaded bar with varying cross section shown in Figure 3:
   (a) Derive the analytical expression for the axial stress $\sigma(y)$
   (b) Derive the analytical expression for the axial displacement $u(y)$
   (c) Write a MATLAB code for finite element analysis of the bar, by discretizing it into *nel* bar elements. For any element, assume uniform cross-section (average of top and bottom cross-section of the element) and uniformly distributed $q(y)$ (such that the total distributed load in any element remains unchanged). Using *nel* = {2, 4, 10, 50, 100, 200, 500} in your code, compute the values of the following quantities:
      i. u at $y = 40m$ and u at $y = 20m$
      ii. σ at $y = 20m$ and σ at $y = 0m$

   and compare with their true values (from analytical expressions). Graphically present the comparison results, with the quantities $\{u(y = 40), u(y = 20), \sigma(y = 20), \sigma(y = 0)\}$ in the y-axis, and *nel* in the x-axis. Use different plots for u and σ.



radius at $y = 0$ $(R_0) = 5$ m

radius at $y = 40$m $(R_{40}) = 1$ m

variation of R along y

$$R(y) = 2.5 \times 10^{-3}\,(y^2 - 80y + 2000)$$

take $E = 1.3 \times 10^6$ N/m

Figure 3

4. (a) Write a MATLAB code to find the displacements at all nodes and the axial forces in all members of the truss shown in Figure 4, modeled using 2D – truss elements. Present your results in a tabulated format. Given: E= 2 x 107 N/m^2, A= 2.5 x 10-3 m^2, for all members.

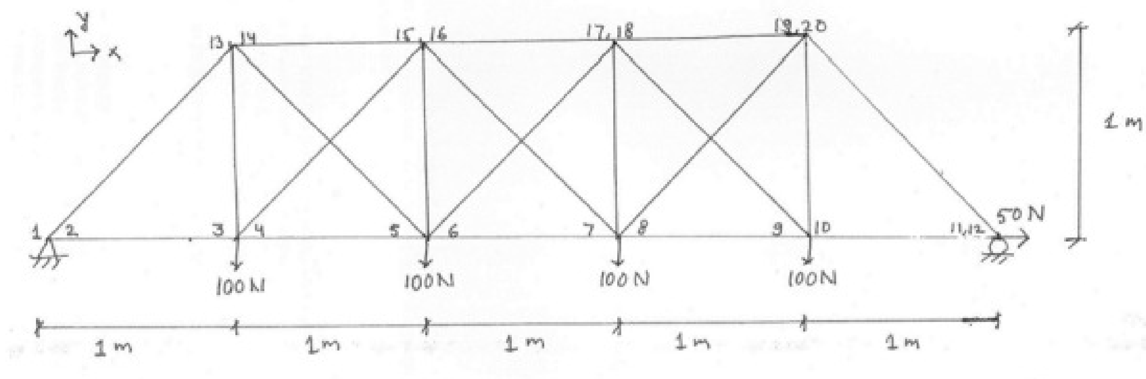


Figure 4

(b) For a truss made of a different material, but with the same geometry, loading and boundary conditions as in Figure 4, the measured nodal displacements corresponding to the different degrees of freedom (DoFs) shown in Figure 4 are as follows:
(Please note that the first DoF at each node corresponds to the x directional DoF.)

DoF	Disp(m)	DoF	Disp(m)	DoF	Disp(m)	DoF	Disp(m)
1	0	6	-0.0683	11	0.0294	16	-0.0672
2	0	7	0.0183	12	0	17	0.009
3	0.005	8	-0.0683	13	0.0303	18	-0.0672
4	-0.0478	9	0.0244	14	-0.0454	19	-0.0009
5	0.011	10	-0.0478	15	0.0204	20	-0.0454

Using the above data, find E of all the members, assuming that (i) all top chord members have same E ($= E^T$), (ii) all bottom chord members have same E ($= E^B$), (iii) all vertical members have same E ($= E^V$), and (iv) all diagonal members have same E ($= E^D$).

5. (a) For the beam shown in Figure 5.1, use FEM (consider 4 elements) to determine the nodal displacements, support reactions, and spring forces, and draw the shear force and bending moment diagrams. Given: $EI_a = 2\text{x}10^6$ N-m^2, $EI_b = 1\text{x}10^6$ N-m^2 , $L = 10$ m, $\Delta s = 0.01$ m, $P = 5000$ N, $w = 1$ KN/m.

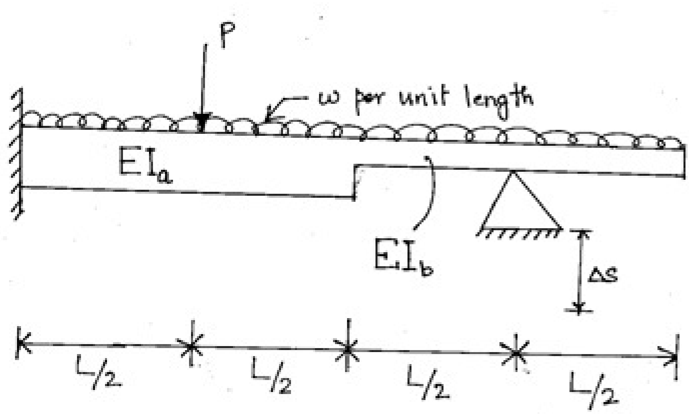


Figure 5.1

(b) The beam was originally designed without considering possible support settlement. To negate the effect of Δs, the supporting assembly shown in Figure 5.2 is proposed. Evaluate all spring constants of the supporting assembly. Do we actually need all the supporting springs? Comment on your answer.

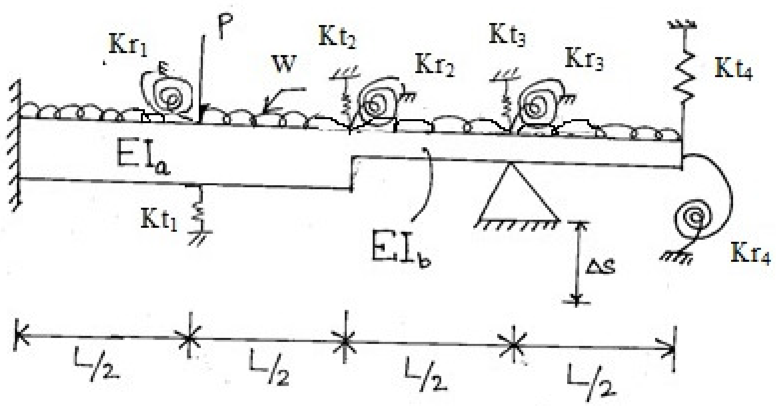


Figure 5.2

6. Model and analyze the frames shown in Figures 6.1 and 6.2 using four frame elements, to find: (a) Nodal displacements, (b) Support reactions, (c) Axial force, shear force, and bending moment diagrams of all members.
   Assume: E = 210 KN/mm^2 for all members, $I^{(1)} = I^{(4)} = 5.7\text{x}10^7$ mm^4, $I^{(2)} = I^{(3)} = 5.5\text{x}10^7$ mm^4, $L^{(1)} = L^{(4)} = 6$ m, $L^{(2)} = L^{(3)} = 4$ m, $A^{(1)} = A^{(4)} = 4500$ mm^2, $A^{(2)} = A^{(3)} = 4000$ mm^2, $w_1 = w_2 = 1$ N/mm.

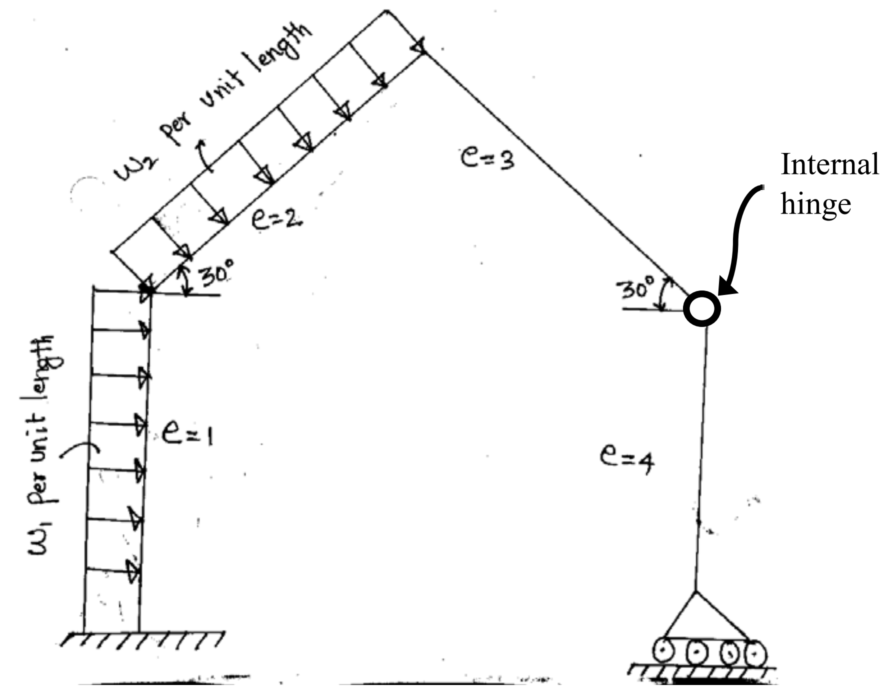


Figure 6.1

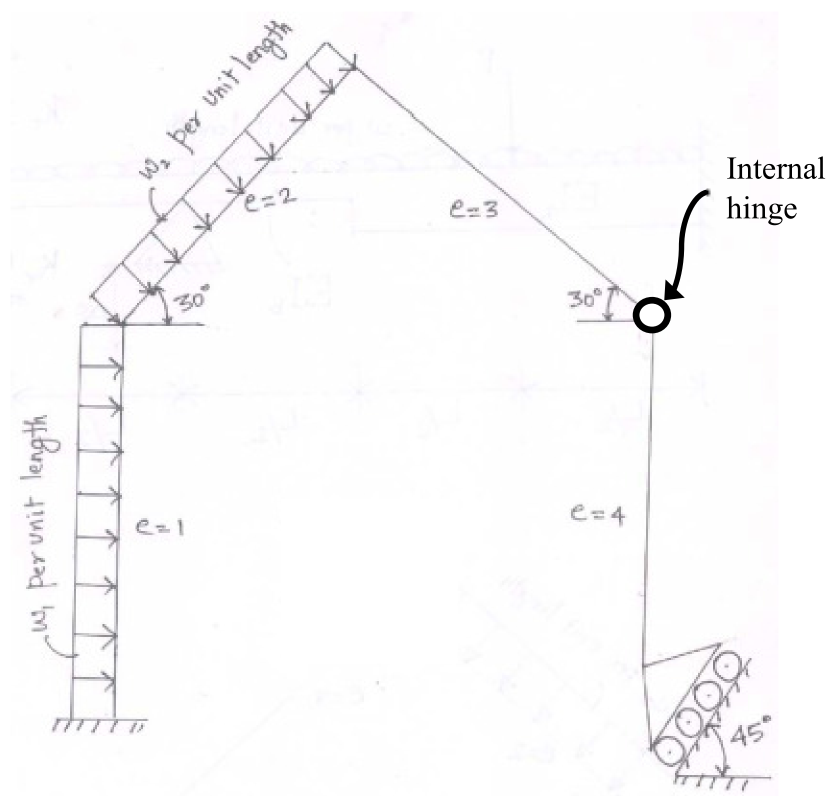


Figure 6.2

7. For the plane truss shown in Figure 7, determine the nodal displacements and support reactions, using finite element analysis. Let $E = 210$ GPa, $A = 6.0 \times 10^{-4} m^2$ for members 1 and 2, and $A = 6\sqrt{2} \times 10^{-4} m^2$ for member 3.

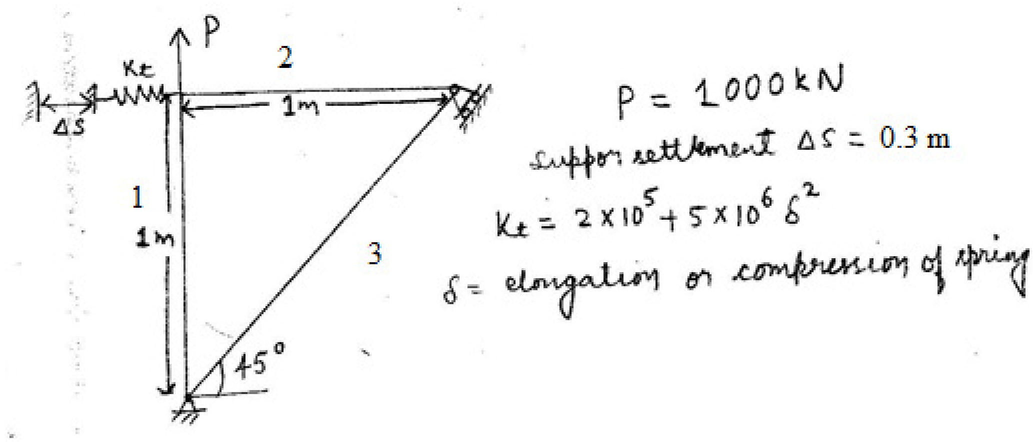


Figure 7

[Note: All hand written parts of the solutions (SFDs & BMDs) are present after the MATLAB outputs.]

Prob 1) Done in hand completely.

Prob 2) 3-D Truss MATLAB output:

```
Command Window

 NODAL DISPLACEMENTS:
  NODE          X-AXIS           Y-AXIS          Z-AXIS
    1         0.000e+00       0.000e+00       0.000e+00
    2         0.000e+00       0.000e+00       0.000e+00
    3         0.000e+00       0.000e+00       0.000e+00
    4         2.879e-03       0.000e+00      -6.783e-02
MEMBER END FORCES:
  MEMBER         AXIAL FORCE @ NODE 1        AXIAL FORCE @ NODE 2
    1                -4.330e+00                  4.330e+00
MEMBER END FORCES:
  MEMBER         AXIAL FORCE @ NODE 1        AXIAL FORCE @ NODE 2
    2                -4.330e+00                  4.330e+00
MEMBER END FORCES:
  MEMBER         AXIAL FORCE @ NODE 1        AXIAL FORCE @ NODE 2
    3                -7.071e+00                  7.071e+00
fx >>
```

Fig.1) Nodal displacements (in m) and member forces (in N)

Prob 3) Bar of varying thickness MATLAB output:

```
FOR NO.OF ELEMENTS=  2
NODAL DISPLACEMENT AT y=20m (in mm): 2.532970e-01
NODAL DISPLACEMENT AT y=40m (in mm): 1.330654e+00
NODAL STRESS AT y=0m (in N/m^2): 1.646430e+01
NODAL STRESS AT y=20m (in N/m^2): 4.324624e+01

FOR NO.OF ELEMENTS=  4
NODAL DISPLACEMENT AT y=20m (in mm): 3.346694e-01
NODAL DISPLACEMENT AT y=40m (in mm): 1.809294e+00
NODAL STRESS AT y=0m (in N/m^2): 1.454492e+01
NODAL STRESS AT y=20m (in N/m^2): 4.666969e+01

FOR NO.OF ELEMENTS= 10
NODAL DISPLACEMENT AT y=20m (in mm): 3.677144e-01
NODAL DISPLACEMENT AT y=40m (in mm): 1.981925e+00
NODAL STRESS AT y=0m (in N/m^2): 1.276866e+01
NODAL STRESS AT y=20m (in N/m^2): 4.758436e+01

FOR NO.OF ELEMENTS= 50
NODAL DISPLACEMENT AT y=20m (in mm): 3.744630e-01
NODAL DISPLACEMENT AT y=40m (in mm): 2.016824e+00
NODAL STRESS AT y=0m (in N/m^2): 1.172384e+01
NODAL STRESS AT y=20m (in N/m^2): 4.774011e+01
```

```
FOR NO.OF ELEMENTS= 100
NODAL DISPLACEMENT AT y=20m (in mm): 3.746778e-01
NODAL DISPLACEMENT AT y=40m (in mm): 2.017933e+00
NODAL STRESS AT y=0m (in N/m^2): 1.159155e+01
NODAL STRESS AT y=20m (in N/m^2): 4.774489e+01

FOR NO.OF ELEMENTS= 200
NODAL DISPLACEMENT AT y=20m (in mm): 3.747316e-01
NODAL DISPLACEMENT AT y=40m (in mm): 2.018211e+00
NODAL STRESS AT y=0m (in N/m^2): 1.152536e+01
NODAL STRESS AT y=20m (in N/m^2): 4.774609e+01

FOR NO.OF ELEMENTS= 500
NODAL DISPLACEMENT AT y=20m (in mm): 3.747466e-01
NODAL DISPLACEMENT AT y=40m (in mm): 2.018288e+00
NODAL STRESS AT y=0m (in N/m^2): 1.148564e+01
NODAL STRESS AT y=20m (in N/m^2): 4.774642e+01
fx >> |
```

Fig. 2) Nodal displacements and Stresses at the respective nodes for different no. of element

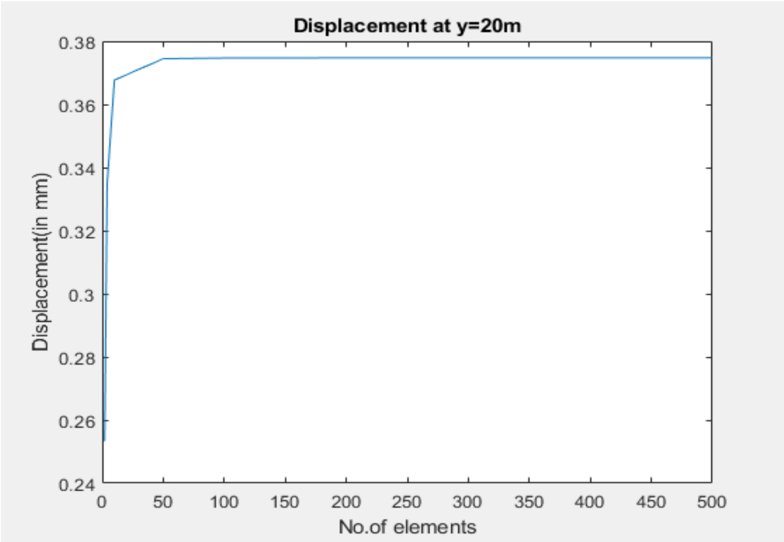


Fig 3) Variation of nodal displacement at y=20 m with increasing no of elements. (True value = 0.375 mm)

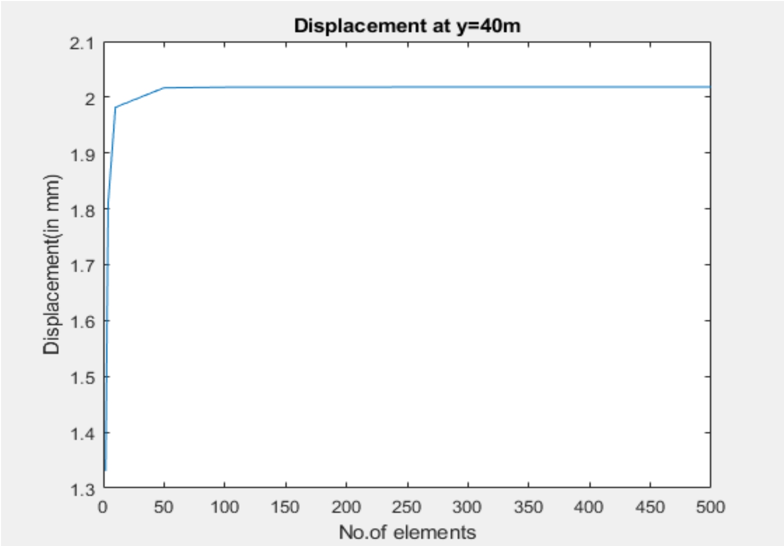


Fig 4) Variation of nodal displacement at y=40 m with increasing no of elements. (True value = 2.018 mm)

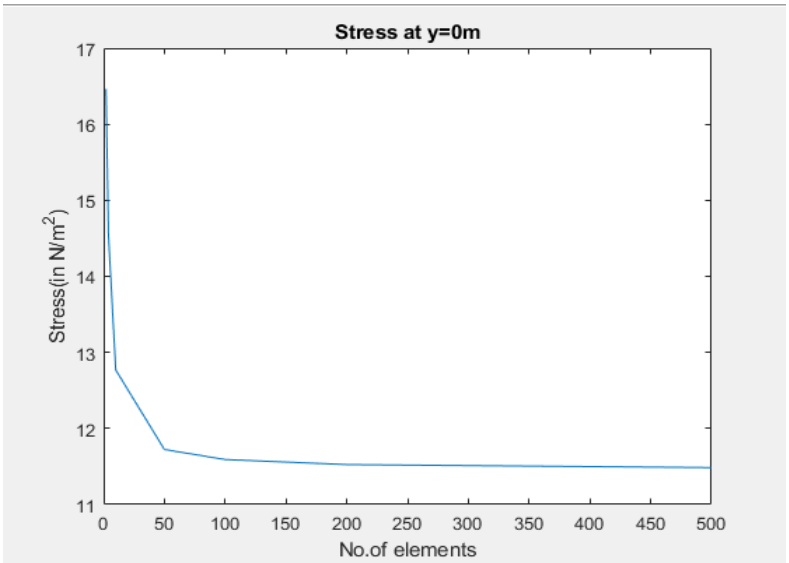


Fig 5) Variation of stress at y=0 m with increasing no of elements. (True value = 11.485 N/m^2)

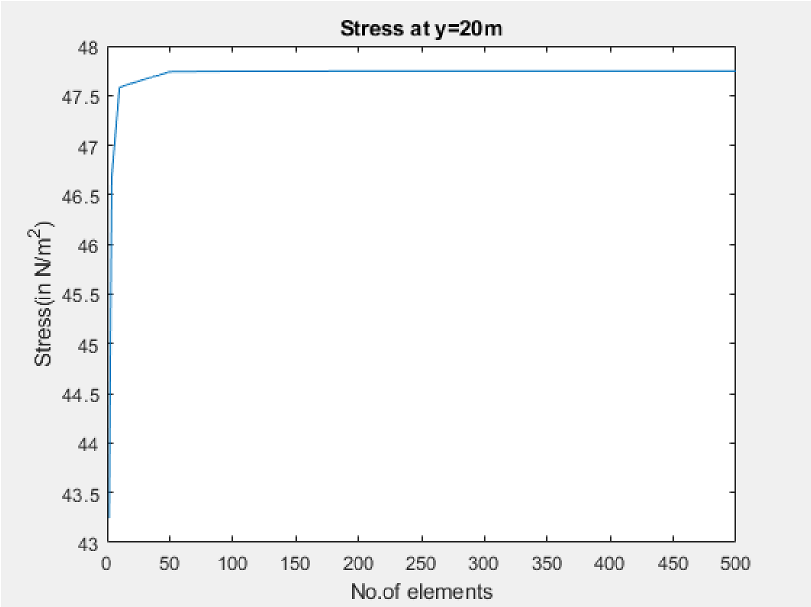


Fig 6) Variation of stress at y=20 m with increasing no of elements. (True value = 47.746 N/m^2)

Prob 4a) 2-D truss MATLAB output:

NODAL DISPLACEMENTS:

NODE	X-AXIS	Y-AXIS
1	0.000e+00	0.000e+00
2	5.000e-03	-3.632e-02
3	1.050e-02	-5.004e-02
4	1.730e-02	-5.004e-02
5	2.280e-02	-3.632e-02
6	2.780e-02	0.000e+00
7	2.250e-02	-3.381e-02
8	1.700e-02	-4.975e-02
9	1.080e-02	-4.975e-02
10	5.300e-03	-3.381e-02

Fig 7) Nodal displacements (in m)

MEMBER END FORCES:

MEMBER	AXIAL FORCE @ NODE 1	AXIAL FORCE @ NODE 2
1	-2.500e+02	2.500e+02
2	-2.752e+02	2.752e+02
3	-3.396e+02	3.396e+02
4	-2.752e+02	2.752e+02
5	-2.500e+02	2.500e+02
6	2.748e+02	-2.748e+02
7	3.104e+02	-3.104e+02
8	2.748e+02	-2.748e+02
9	-1.252e+02	1.252e+02
10	-1.479e+01	1.479e+01
11	-1.479e+01	1.479e+01
12	-1.252e+02	1.252e+02
13	2.828e+02	-2.828e+02
14	-1.058e+02	1.058e+02
15	3.562e+01	-3.562e+01
16	-1.470e+01	1.470e+01
17	-1.470e+01	1.470e+01
18	3.562e+01	-3.562e+01
19	-1.058e+02	1.058e+02
20	2.828e+02	-2.828e+02

fx >>

Fig 8) Member Forces along the axis of the members (in N) (-ve in the node 1 indicates Tension and +ve is Compression)

Prob 4(b): 2-D Truss Unknown Elastic Moduli of the members MATLAB output:

```
SL.No       MEMBER TYPE      ELASTIC MODULUS
  1        BOTTOM CHORD        1.956e+07
  2         TOP CHORD          9.815e+06
  3          VERTICAL          2.472e+07
  4          INCLINED          1.463e+07
>>
```

Fig 9) Unknown Elastic Moduli of the different category of members (in N/m^2)

Prob 5(a): Overhanging Beam MATLAB output:

```
 NODAL DISPLACEMENTS:
  NODE       VERTICAL(mm)     ROTATION(rads)
    1         0.000e+00        0.000e+00
    2        -1.135e+02       -2.465e-02
    3        -1.432e+02        1.265e-02
    4         0.000e+00        2.362e-02
    5         3.999e+01        2.790e-03
MEMBER END FORCES:
  NODE        Y-FORCE(N)      Z-MOMENT(Nm)
  MEMBER:1
    1         1.246e+01       3.683e+01
    2        -7.455e+00       1.295e+01
  MEMBER:2
    1         2.455e+00      -1.295e+01
    2         2.545e+00       1.272e+01
  MEMBER:3
    1        -2.545e+00      -1.272e+01
    2         7.545e+00      -1.250e+01
  MEMBER:4
    1         5.000e+00       1.250e+01
    2         1.364e-15      -4.093e-15
>>
```

Fig 10) Nodal displacements and member end forces in appropriate units

Prob 5(b): Unknown Stiffnesses of the Springs attached to the beam MATLAB output:

```
SL.No       SPRING       STIFFNESS
  1          Kt1         3.452e+06
  2          Kr1         3.902e+03
  3          Kt2        -1.303e+05
  4          Kr2        -4.073e+06
  5          Kt3         1.064e+06
  6          Kr3         2.034e+03
  7          Kt4        -2.410e+04
  8          Kr4         8.583e+05
>>
```

Fig 11) Unknown Stiffnesses of the Springs (translationals have unit N/m and rotationals have unit N/rads)

Prob 6(a): 2-D Frame with Internal Hinge MATLAB output:

```
NODAL DISPLACEMENTS:
  NODE       HORIZONTAL       VERTICAL        ROTATION

MEMBER NO. 1:
   1         0.000e+00        0.000e+00       0.000e+00
   2         1.535e+01       -8.436e-03      -2.609e-03

MEMBER NO. 2:
   2         1.535e+01       -8.436e-03      -2.609e-03
   3         1.768e+01       -4.048e+00       3.101e-04

MEMBER NO. 3:
   3         1.768e+01       -4.048e+00       3.101e-04
   4         2.001e+01       -1.356e-02       1.358e-03

MEMBER NO. 4:
   4         2.001e+01       -1.356e-02       0.000e+00
   5         2.001e+01        0.000e+00       0.000e+00
```

Fig 12) Nodal Displacements and rotations (in mm and rads resp.)

```
MEMBER END FORCES:
  NODE          AXIAL           SHEAR           MOMENT

MEMBER NO.1:
   1         1.329e+00        8.000e+00        2.321e+01
   2        -1.329e+00       -2.000e+00        6.795e+00

MEMBER NO.2:
   1        -1.068e+00        2.151e+00       -6.795e+00
   2         1.068e+00        1.849e+00        7.397e+00

MEMBER NO.3:
   1         1.068e+00       -1.849e+00       -7.397e+00
   2        -1.068e+00        1.849e+00        0.000e+00

MEMBER NO.4:
   1         2.135e+00        1.908e-16        0.000e+00
   2        -2.135e+00       -1.908e-16        2.567e-16
fx >>
```

Fig 13) Member end Forces and Moments (in KN and KNm resp.)

Prob 6(b): 2-D Frame with Internal Hinge and Inclined Support MATLAB output:

```
NODAL DISPLACEMENTS:
  NODE       HORIZONTAL       VERTICAL        ROTATION

MEMBER NO. 1:
   1         0.000e+00        0.000e+00       0.000e+00
   2         3.759e+01       -2.199e-02      -1.003e-02

MEMBER NO. 2:
   2         3.759e+01       -2.199e-02      -1.003e-02
   3         5.903e+01       -3.715e+01      -1.095e-02

MEMBER NO. 3:
   3         5.903e+01       -3.715e+01      -1.095e-02
   4         3.713e+01       -7.508e+01      -1.095e-02

MEMBER NO. 4:
   4         3.713e+01       -7.508e+01      -1.870e-02
   5        -7.508e+01       -7.508e+01      -1.870e-02
```

Fig 14) Nodal Displacements and rotations (in mm and rads resp.)

```
MEMBER END FORCES:
  NODE          AXIAL           SHEAR           MOMENT

MEMBER NO.1:
   1         3.464e+00       8.000e+00       3.800e+01
   2        -3.464e+00      -2.000e+00      -8.000e+00

MEMBER NO.2:
   1        -3.975e-12       4.000e+00       8.000e+00
   2         3.975e-12       4.080e-12       1.630e-11

MEMBER NO.3:
   1         6.994e-13      -4.068e-12      -1.627e-11
   2        -6.994e-13       4.068e-12       0.000e+00

MEMBER NO.4:
   1         1.422e-12       2.736e-15       0.000e+00
   2        -1.422e-12      -2.736e-15       1.641e-14
fx >>
```

Fig 15) Member end Forces and Moments (in KN and KNm resp.)

Prob 7) 2-D Truss with Non-Linear Spring MATLAB output:

```
NODAL DISPLACEMENTS:
 NODE          X-AXIS          Y-AXIS
   1         0.000e+00       0.000e+00
   2        -7.126e-04       7.937e-03
   3        -2.375e-04      -2.375e-04
   4        -3.000e-01       0.000e+00
```

Fig 16) Nodal displacements for the 1st guess value considering constant component of the spring stiffness (in m)

```
SUPPORT REACTIONS(in KN):
R1x=9.779e+01
R1y=-9.022e+02
R3y(in the inclined axes)=-1.383e+02
R4x=-1.935e+02
R4y=0.000e+00

 NODAL DISPLACEMENTS(in mm):
  NODE          HORIZONTAL       VERTICAL
    1           0.000e+00       0.000e+00
    2          -2.328e+00       7.937e+00
    3          -7.761e-01      -7.761e-01
    4          -3.000e+02       0.000e+00
fx >>
```

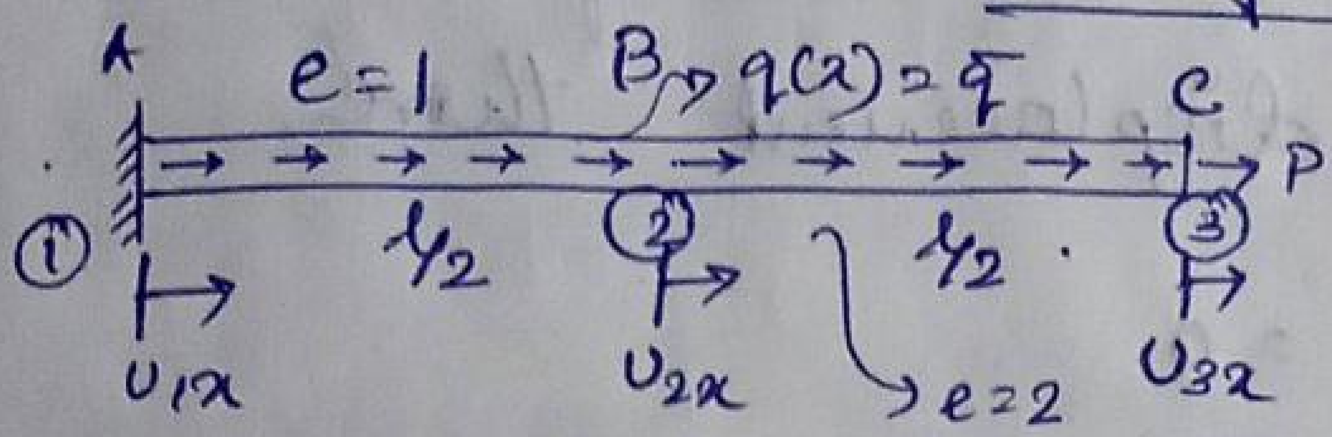
Fig 17) Final Nodal displacements (in mm) and Support Reactions (in KN)

8-08-2021 : FINITE ELEMENT ANALYSIS

Assignment I :- 21103033

Budhaditya De

Q:- 1

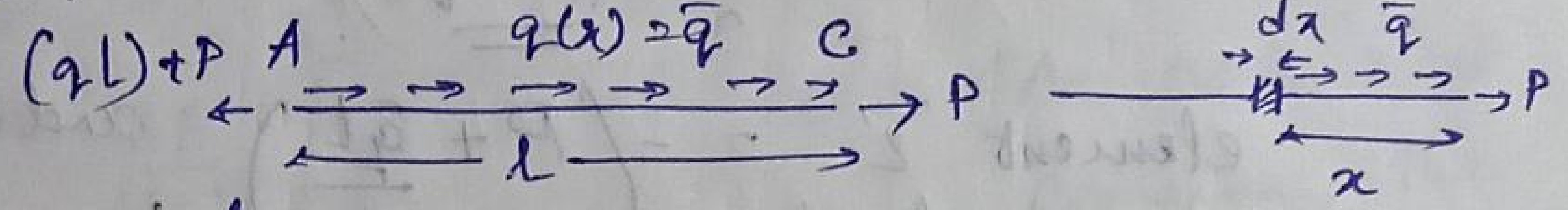


→ Analytical solution :-

(a) Displacement @ free end (i.e u_{3x}) →

– This is basically the displacement for the entire bar.

FBD –



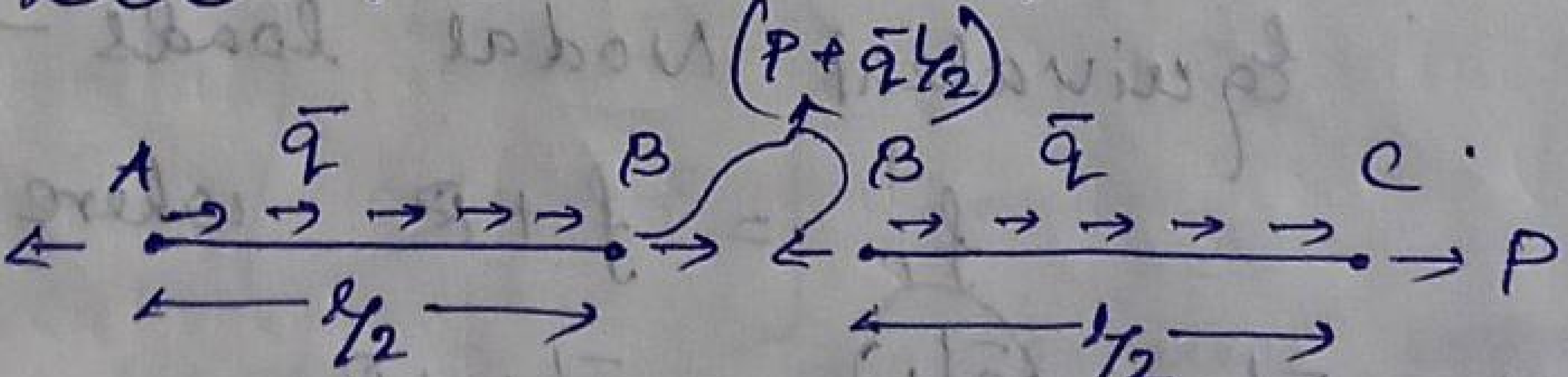
$$\therefore \quad u_{3x} = \int_0^L \frac{(P+\bar{q}x)\,dx}{AE}$$

$$= \left.\frac{Px}{AE}\right|_0^L + \left.\frac{\bar{q}x^2}{2AE}\right|_0^L$$

$$\Rightarrow u_{3x} = \frac{PL}{AE} + \frac{\bar{q}L^2}{\cancel{8}_2 AE}$$

(b) Displacement @ Midpoint (i.e u_{2x}) →

FBD –



Here the load P and distributed load $\bar{q}$ w/t over the length BC will act as point loads on AB,

$$\therefore \quad U_{2x} = \int_0^{l/2} \frac{\bar{q}(x)\,dx}{AE} + \int_0^{l/2} \frac{(P+\bar{q}l/2)\,dx}{AE}$$

$$= \left.\frac{qx^2}{2AE}\right|_0^{l/2} + \left.\frac{Px + \bar{q}\frac{l}{2}x}{AE}\right|_0^{l/2}$$

$$\Rightarrow U_{2x} = \frac{\bar{q}l^2}{8AE} + \frac{PL}{2AE} + \frac{\bar{q}l^2}{4AE} = \boxed{\frac{3\bar{q}l^2}{8AE} + \frac{PL}{2AE}}$$

(c) Displacement @ fixed end (u_{1x}) →

Since Node '1' is clamped, there wont be any displacement there,

$\therefore u_{1x} = 0$

The bar forces –

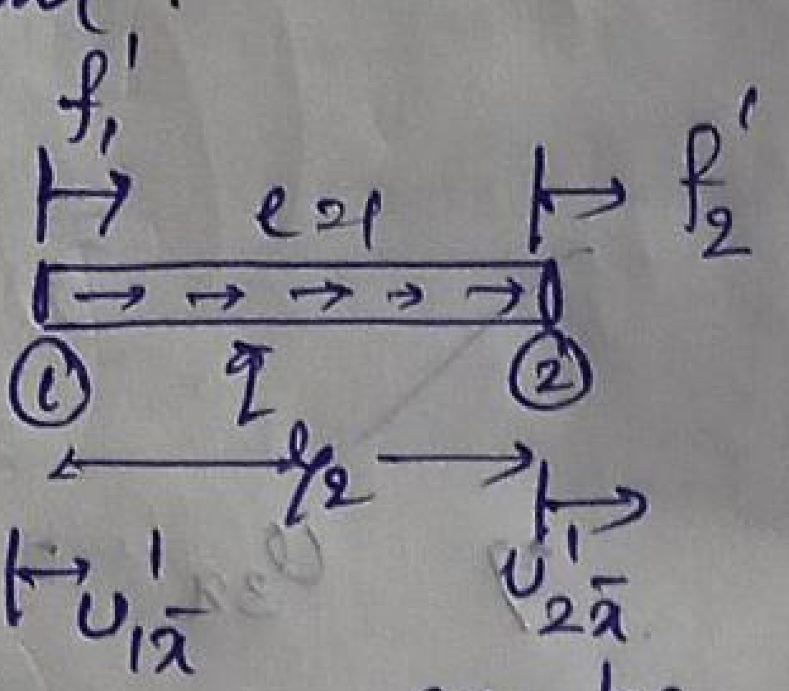
element '1' = $P + \bar{q}L/2 + \bar{q}L/2$

at node 1 = $-(P + \bar{q}L)$ and at node 2 = $(P + \bar{q}\frac{L}{2})$

element '2' at node 1 = $-\left(P + \frac{\bar{q}L}{2}\right)$. and at node '2' = 'P'.

(b) FEM analysis of the system.

(i) For element '1' →



Since the element and the global Co-ordinate system are same, $u_{1\bar{x}}, u_{2\bar{x}}$ --- can be written as u_{1x}, u_{2x} --- etc

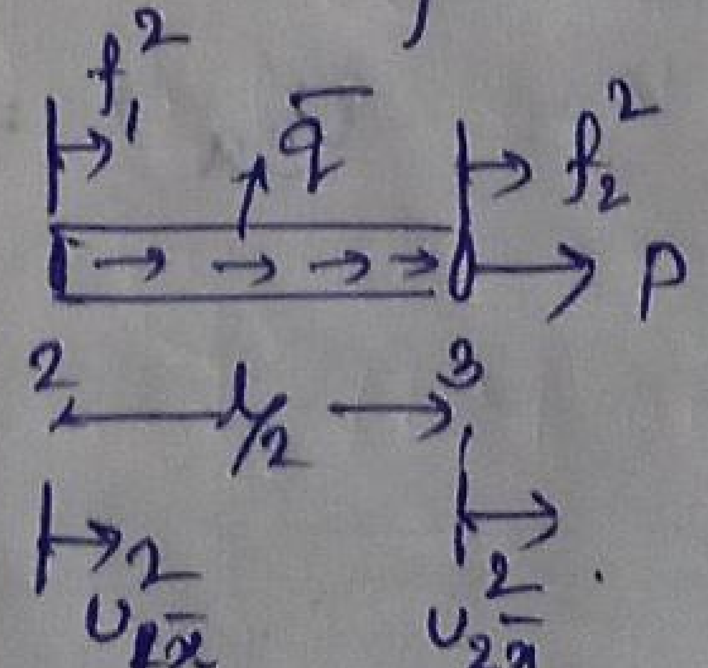
Equivalent Nodal loads –

$f_1^1 = f + R$ where $f = \frac{\bar{q}L/2}{2} = \frac{\bar{q}L}{4}$

$f_2^1 = \bar{q}L/(2\times2)\,(\bar{q}L/4) = \bar{q}L/4 + R$

We are equally distributing the distributed load at both the nodes of the element

(ii) For element 2 :-



like previous case,

$u_{1\bar{x}}^2 = u_{2x}$

$u_{2\bar{x}}^2 = u_{3x}$

$$f_1^2 = \frac{\bar{q}l}{2} \times \frac{1}{2} = \frac{\bar{q}l}{4}$$

$$f_2^2 = \frac{\bar{q}L}{4} + P.$$

Now, element stiffness matrix :–

$$K_{1/2} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} = \frac{AE}{(L/2)} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Global stiffness matrix :–

$$K_G = \frac{AE}{(L/2)} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

(As node '2' is common, the stiffness components of the elements will be combined there)

Global Force vector =

$$\begin{Bmatrix} f_1^1 \\ f_1^2 + f_2^1 \\ f_2^2 \end{Bmatrix} = \begin{Bmatrix} \bar{q}\frac{L}{4} + R \\ \bar{q}L/4 + \frac{\bar{q}L}{4} \\ \bar{q}L/4 + P \end{Bmatrix}.$$

Writting the global Force-displacement equation.

$$\frac{AE}{(L/2)} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} \cancel{u_{1x}} \\ u_{2x} \\ u_{3x} \end{Bmatrix} = \begin{Bmatrix} \bar{q}\frac{L}{4} + R \\ \bar{q}L/2 \\ \bar{q}L/4 + P \end{Bmatrix}$$

as $u_{1x} = 0$, we can eliminate 1st row and column of 'K_G' matrix and reduce the calculation to 2×2 matrix,

$$\therefore \quad \frac{2AE}{L}\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_{2x} \\ u_{3x} \end{Bmatrix} = \begin{Bmatrix} \bar{q}L/2 \\ \bar{q}L/4 + P \end{Bmatrix}.$$

$$\Rightarrow \begin{Bmatrix} u_{2x} \\ u_{3x} \end{Bmatrix} = \frac{L}{2AE}\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}^{-1}\begin{Bmatrix} \bar{q}L/2 \\ \bar{q}L/4 + P \end{Bmatrix}.$$

Let $A = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$

$$A^{-1} = \frac{Adj\,(A)}{|A|} \;; \quad |A| = 2\cdot 1 - (-1)(-1) = 2-1 = 1$$

$$Adj\,(A) = \begin{bmatrix} +1 & -(-1) \\ -(-1) & +2 \end{bmatrix}^T$$

$$= \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}^T$$

$$= \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}.$$

$$\therefore \begin{Bmatrix} u_{2x} \\ u_{3x} \end{Bmatrix} = \frac{L}{2AE}\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}\begin{Bmatrix} \bar{q}L/2 \\ \bar{q}L/4 + P \end{Bmatrix}$$

$$= \frac{L}{2AE}\begin{pmatrix} \bar{q}L/2 + \bar{q}L/4 + P \\ \bar{q}L/2 + \bar{q}L/2 + 2P \end{pmatrix}$$

$$\begin{Bmatrix} u_{2x} \\ u_{3x} \end{Bmatrix} = \begin{Bmatrix} \dfrac{3\bar{q}L^2}{(2\times 4)AE} + \dfrac{PL}{2AE} \\ \dfrac{\bar{q}L^2}{2AE} + \dfrac{PL}{AE} \end{Bmatrix}$$

$\therefore$ We see the Nodal displacement by both analytical and FEM are same.

(e) Elemental Internal Axial forces :—

(i) Element 1 →

following, $P^e = K^e u^e$

$$\begin{Bmatrix} f_1^1 \\ f_2^1 \end{Bmatrix} = \frac{2AE}{L}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{Bmatrix} 0 \\ \frac{3\bar{q}L^2}{8AE} + \frac{PL}{2AE} \end{Bmatrix}.$$

$$\Rightarrow f_1^1 = \frac{2AE}{L}\left[-\frac{3\bar{q}L^2}{8AE} - \frac{PL}{2AE}\right]$$

and $$f_2^1 = \frac{2AE}{L}\left[\frac{3\bar{q}L^2}{8AE} + \frac{PL}{2AE}\right].$$

$$\Rightarrow f_1^1 = -\frac{3\bar{q}L}{4} - P.$$

$$f_2^1 = +\frac{3\bar{q}L}{4} + P.$$

(ii) Element 2 :-

$$\begin{Bmatrix} f_1^2 \\ f_2^2 \end{Bmatrix} = \frac{2AE}{L}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{Bmatrix} \frac{3\bar{q}L^2}{8AE} + \frac{PL}{2AE} \\ \frac{\bar{q}L^2}{2AE} + \frac{PL}{AE} \end{Bmatrix}$$

$$\Rightarrow f_1^2 = \frac{2AE}{L}\left[\frac{3\bar{q}L^2}{8AE} + \frac{PL}{2AE} - \frac{\bar{q}L^2}{2AE} - \frac{PL}{AE}\right]$$

$$= \frac{2AE}{L}\left[-\frac{\bar{q}L^2}{8AE} - \frac{PL}{2AE}\right]$$

$$\Rightarrow f_1^2 = -\frac{\bar{q}L}{4} - P.$$

Scanned by TapScanner

And, similarly,

$$f_2^2 = P + \frac{\bar{q}l}{4}$$ (by equilibrium only we can say so)

d) So we see that the element internal axial forces for both the members actually differ in analytical and FEM computations

This maybe due to the fact that we are ~~d~~ dividing the dist. load $\bar{q}$ equally at the 2 nodes of the elements which may not be the true picture.

~~(e)~~ (e) Element force vectors subtracting the equivalent nodal loads from the obtained elemental force vectors —

(i) Element 1 —

$$\begin{Bmatrix} f_1^{1*} \\ f_2^{1*} \end{Bmatrix} = \begin{Bmatrix} -\left(\frac{3\bar{q}l}{4} + P\right) \\ +\left(\frac{3\bar{q}l}{4} + P\right) \end{Bmatrix} - \begin{Bmatrix} \frac{\bar{q}l}{4} \\ \frac{\bar{q}l}{4} \end{Bmatrix}$$

$$= \begin{Bmatrix} -\frac{\cancel{4}\bar{q}l}{\cancel{4}} - P \\ \frac{\bar{q}l}{2} + P \end{Bmatrix}$$

$$\Rightarrow \begin{pmatrix} f_1^{1*} \\ f_2^{1*} \end{pmatrix} = \begin{Bmatrix} -(\bar{q}l + P) \\ \frac{\bar{q}l}{2} + P \end{Bmatrix}$$

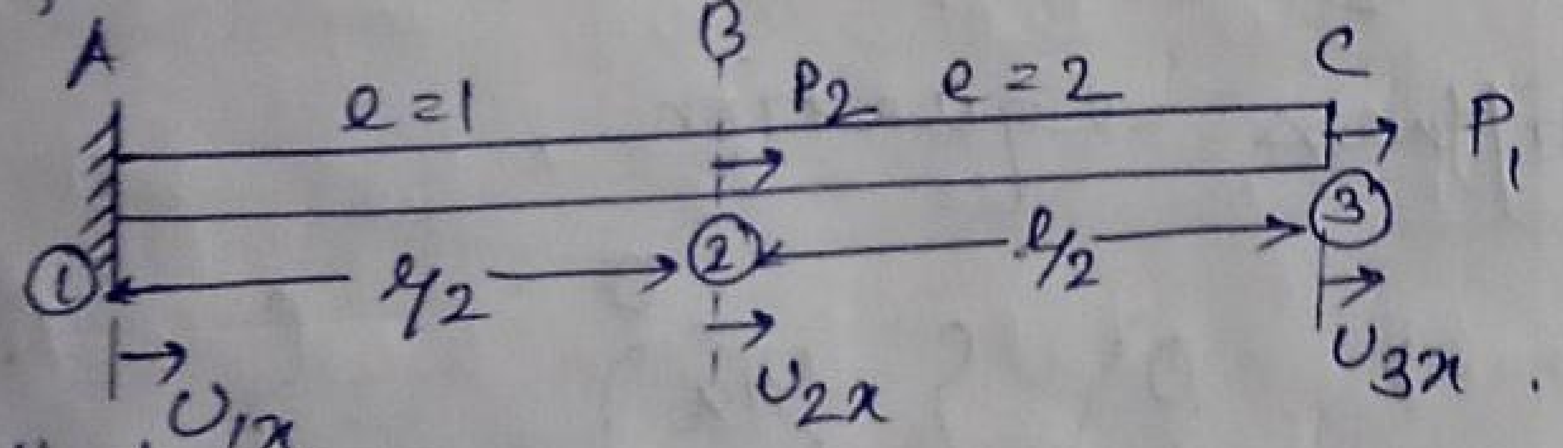
(ii) <u>Element 2</u> :-

$$\begin{Bmatrix} f_1^{2*} \\ f_2^{2*} \end{Bmatrix} = \begin{bmatrix} -\left(\frac{\bar{q}l}{4} + P\right) \\ +\left(\frac{\bar{q}l}{4} + P\right) \end{bmatrix} - \begin{bmatrix} \bar{q}l/4 \\ \frac{\bar{q}l}{4} \cancel{+P} \end{bmatrix}$$

$$\Rightarrow \begin{Bmatrix} f_1^{2*} \\ f_2^{2*} \end{Bmatrix} = \begin{Bmatrix} -\left(\frac{\bar{q}l}{2} + P\right) \\ P \end{Bmatrix}.$$

Now, we see that both the member end axial forces comes out to be similar in analytical as well as FEM. Since now the equivalent component of the external dist. load is taken out of the picture, we get the true display of the member end axial forces.

(f)

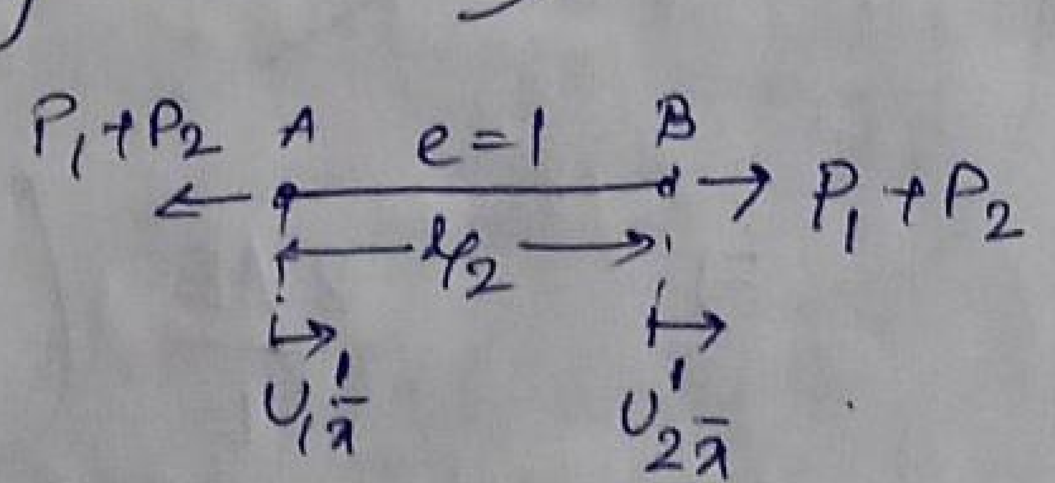


<u>Analytical</u> :-

<u>Nodal displacements</u> :-

(i) $u_{1x} = 0$ (due fixity of node '1')

(ii) $u_{2x} = \int_0^{l/2} \frac{(P_1 + P_2)\,dx}{AE}$

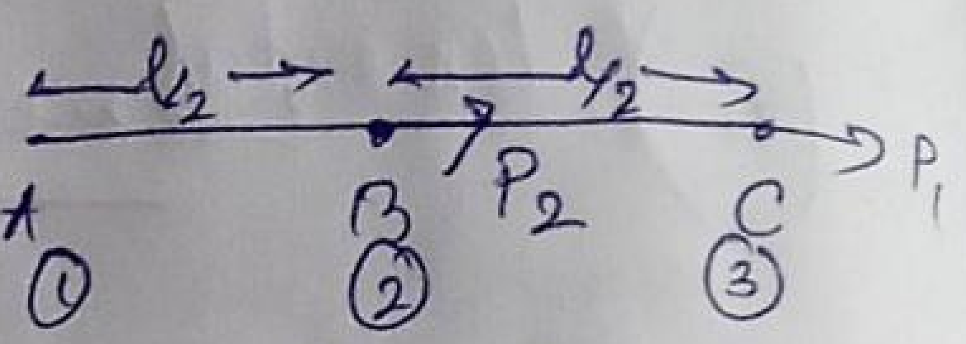


$$= \frac{(P_1 + P_2)\, l}{2AE}.$$

member force @ node 1 $= -(P_1 + P_2)$
@ node 2 $= (P_1 + P_2)$

Scanned by TapScanner

(iii) $$U_{3x} = \int_0^{l/2} \frac{P_2\,dx}{AE} + \int_0^{l} \frac{P_1\,dx}{AE}.$$



$$= \frac{P_2 L}{2AE} + \frac{P_1 L}{AE}.$$

~~Member~~ force ~~@ node~~ 1

$\cancel{0}$.

FEM :-

Here in this case, the difference will be in external load vector, as there is no distributed load along the member we do not have to add any equivalent ~~lod~~ load component to the external nodal loads already acting.

$\therefore \quad f_1^1 = 0 + R$

$f_1^2 = P_2/2$

Similarly, $f_2^1 = P_2/2$

$f_2^2 = P_1$.

$\therefore$ Using stiffness equation,

$$\frac{2AE}{L}\begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}\begin{Bmatrix} \cancel{u_{1x}}^{\,0} \\ u_{2x} \\ u_{3x} \end{Bmatrix} = \begin{Bmatrix} R \\ P_2/2 + P_2/2 \\ P_1 \end{Bmatrix}.$$

$$\Rightarrow \quad \frac{2AE}{L}\begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}\begin{Bmatrix} u_{2x} \\ u_{3x} \end{Bmatrix} = \begin{pmatrix} P_2 \\ P_1 \end{pmatrix}.$$

$$\Rightarrow \begin{pmatrix} u_{2x} \\ u_{3x} \end{pmatrix} = \frac{L}{2AE}\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} P_2 \\ P_1 \end{pmatrix}$$

$$\Rightarrow u_{2x} = \frac{L}{2AE}\left(P_2 + P_1\right) = \frac{(P_1+P_2)L}{2AE}.$$

$$u_{3x} = \frac{P_2 L}{2AE} + \frac{P_1 L}{AE}.$$

— Both the methods again yield same nodal displacements as the previous case.

Element force vectors —

(i) Element 1

$$\begin{pmatrix} f_1^1 \\ f_2^1 \end{pmatrix} = \frac{2AE}{L}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ \frac{(P_1+P_2)l}{2AE} \end{pmatrix}.$$

$$f_1^1 = -(P_1+P_2)$$

$$f_2^1 = +(P_1+P_2).$$

(ii) element 2 :-

$$\begin{pmatrix} f_{\cancel{2}1}^2 \\ f_2^2 \end{pmatrix} = \frac{2AE}{L}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} \frac{(P_1+P_2)l}{2AE} \\ \frac{P_2 L}{2AE} + \frac{P_1 L}{AE} \end{pmatrix}$$

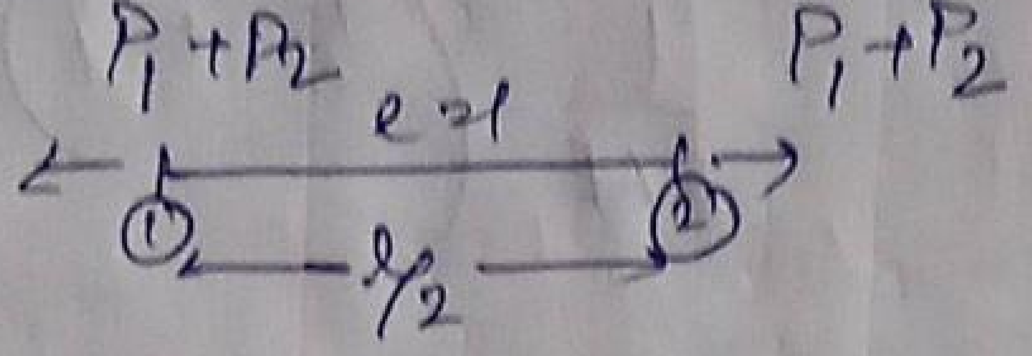
$$\Rightarrow f_2^1 = \frac{2AE}{L}\left(\frac{P_1 l + \cancel{P_2 l}}{2AE\ \cancel{2AE}} - \frac{\cancel{P_2 l}}{\cancel{2AE}} - \frac{P_1 l}{AE}\right)$$

$$= -\frac{P_1 l}{2AE}\times\frac{2AE}{l} = -P_1.$$

and $f_2^2 = +P_1$.

Analytical solution,

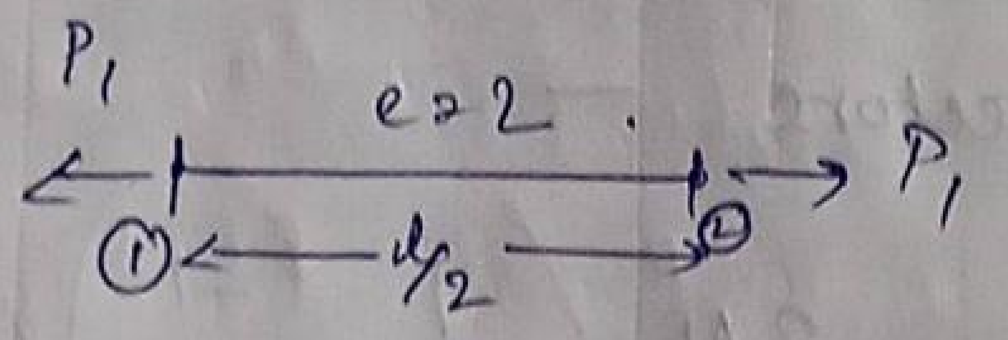
(i) for element 1 →



Member force @ node 1 $= -(P_1+P_2)$.

@ node 2 $= +(P_1+P_2)$.

(ii) for element 2 →

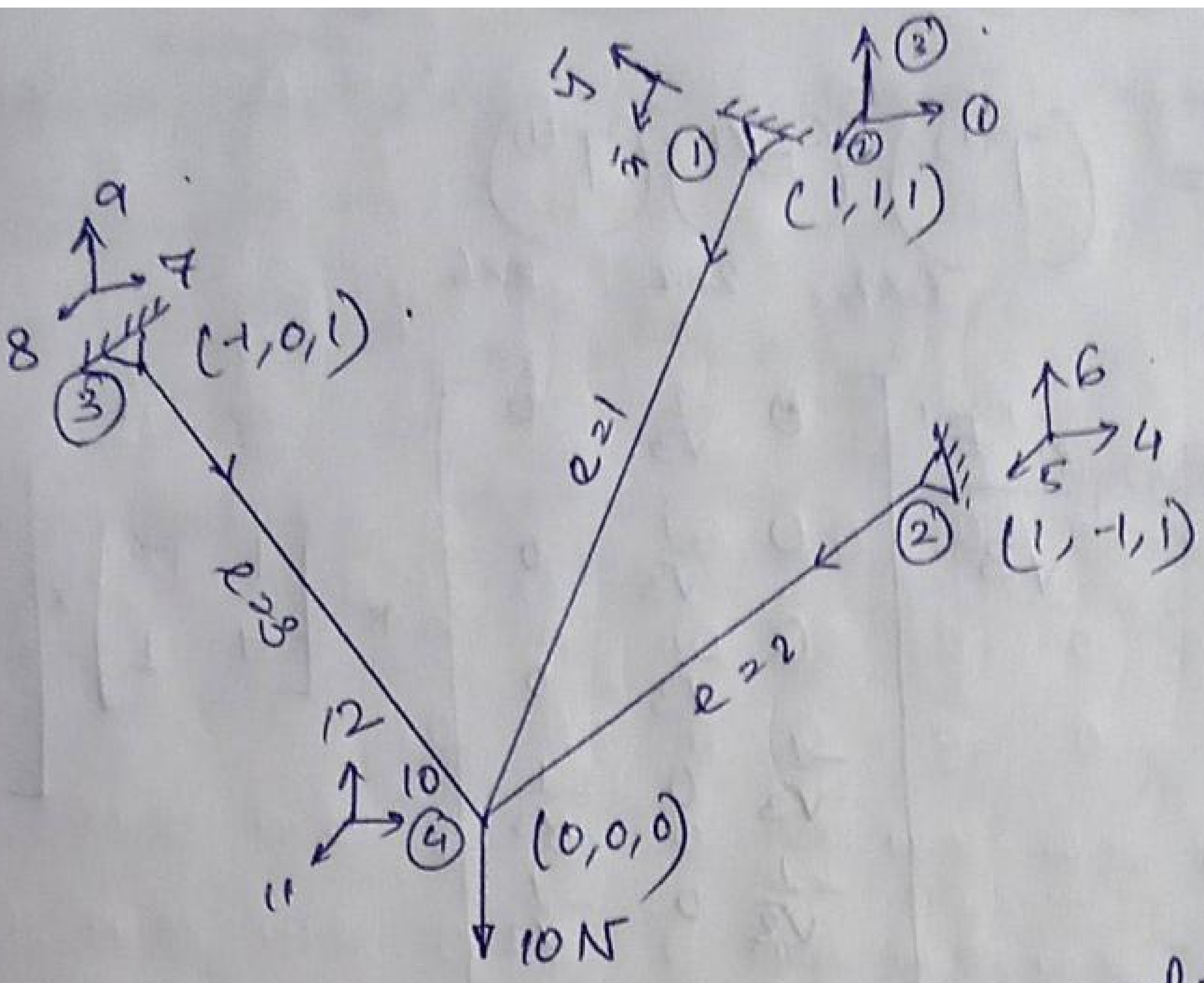


Member force @ node 1 $= -P_1$

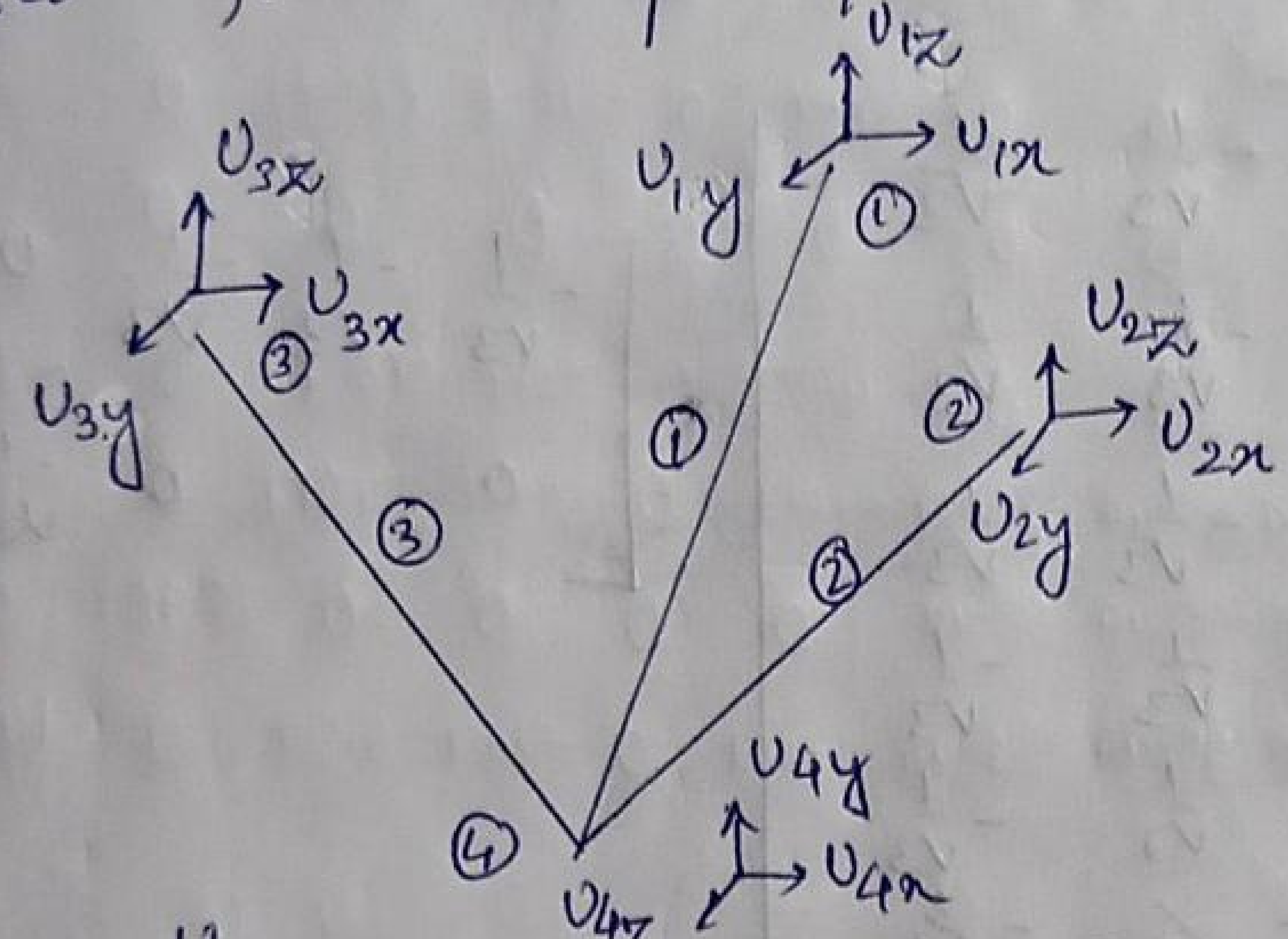
" " @ node 2 $= +P_1$.

g) So we observe that the member end forces are also the same for both elements which is due to the fact that no equivalent nodal load had to be incorporated at the respective nodes of the elements due to the absence of any load distributed along the length of the member. Hence as all the external load components were applied at the nodes of the system itself therefore analysis by both the methods yielded identical results.

Q(2)



Associated global dofs for each node.



$$\therefore U = \begin{Bmatrix} U_{1x} \\ U_{1y} \\ U_{1z} \\ U_{2x} \\ U_{2y} \\ U_{2z} \\ U_{3x} \\ U_{3y} \\ U_{3z} \end{Bmatrix} \begin{Bmatrix} U_{4x} \\ U_{4y} \\ U_{4z} \end{Bmatrix}$$

Element level stiffness matrix in $\bar{x}, \bar{y}$ system

$$k^{(e)} = \frac{A^{(e)}(E)^{(e)}}{L^{(e)}} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}.$$

Now for each member, we define the transformation matrix $T^{(e)}$ —

Element connectivity

$$T^{(1)} = \begin{bmatrix} \frac{x_4 - x_1}{L^{(1)}}, & \frac{y_4 - y_1}{L^{(1)}} & \frac{z_4 - z_1}{L^{(1)}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{x_4 - x_1}{L^{(1)}} & \frac{y_4 - y_1}{L^{(1)}} & \frac{z_4 - z_1}{L^{(1)}} \end{bmatrix}_{2\times 6}.$$

; for $e=1$, $-[1, 4]$

$L^{(1)} = \sqrt{1^2 + 1^2 + 1^2}$
$= \sqrt{3}$.

$$\Rightarrow T^{(1)} = \begin{Bmatrix} \frac{-1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} \end{Bmatrix}.$$

$\therefore$ Element stiffness matrix for $e=1$ in global x, y system —

Scanned by TapScanner

$$K^{(1)} = \underbrace{\left(T^{(1)}\right)^T}_{6\times 2} \underbrace{\left(k^{(1)}\right)}_{2\times 2} \underbrace{\left(T^{(1)}\right)}_{2\times 6}$$

$$= \frac{A'E'}{\sqrt{3}} \begin{bmatrix} -\frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{3}} & 0 \\ 0 & -\frac{1}{\sqrt{3}} \\ 0 & -\frac{1}{\sqrt{3}} \\ 0 & -\frac{1}{\sqrt{3}} \end{bmatrix} * \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \times \begin{bmatrix} -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \end{bmatrix}$$

$$= \frac{A'E'}{\sqrt{3}} \begin{bmatrix} -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \end{bmatrix} * \begin{bmatrix} -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \end{bmatrix}.$$

$$K^{1} = \frac{A'E'}{\cancel{k}\sqrt{3}} \begin{array}{c} \begin{array}{cccccc} u_{1x} & u_{y} & u_{1z} & u_{4x} & u_{4y} & u_{4z} \end{array} \\ \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \end{array} \begin{array}{c} \\ u_{1x} \\ u_{1y} \\ u_{1z} \\ u_{4x} \\ u_{4y} \\ u_{4x}. \end{array}$$

Similarly, Transformation matrix for $e=2$ 6×6.

$$T^{(2)} = \begin{bmatrix} \frac{x_4-x_2}{l^2} & \frac{y_4-y_2}{l^2} & \frac{z_4-z_2}{l^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{x_4-x_2}{l^2} & \frac{y_4-y_2}{l^2} & \frac{z_4-z_2}{l^2} \end{bmatrix}.$$

$$l^{(2)} = \sqrt{1+1+1} = \sqrt{3}$$

$$= \begin{bmatrix} -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \end{bmatrix}$$

Again stiffness matrix for e=2 in global x,y co-ordinate system :-

$$K^{(2)} = \left[T^{(2)}\right]^T * \left[R^{(2)}\right] * \left[T^{(2)}\right].$$

On solving the above multiplication,

$$K^{(2)} = \frac{A^2E^2}{L^2}\;\begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \end{array}
\begin{array}{cccccc} u_{2x} & u_{2y} & u_{2z} & u_{4x} & u_{4y} & u_{4z} \end{array}$$

$$\begin{array}{c} \\ \end{array}\left[\begin{array}{cccccc}
\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \\
-\frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \\
\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \\
-\frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \\
\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \\
-\frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & \frac{1}{3}
\end{array}\right]_{6\times 6}
\begin{array}{c} u_{2x} \\ u_{2y} \\ u_{2z} \\ u_{4x} \\ u_{4y} \\ u_{4z} \end{array}$$

and

$$T^{(3)} = \begin{bmatrix} \frac{x_4-x_3}{L^3} & \frac{y_4-y_3}{L^3} & \frac{z_4-z_3}{L^3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{x_4-x_3}{L^3} & \frac{y_4-y_3}{L^3} & \frac{z_4-z_3}{L^3} \end{bmatrix}.$$

$$L^{(3)} = \sqrt{1^2+1^2} = \sqrt{2}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \end{bmatrix}.$$

$$K^{(3)} = \left[T^{(3)}\right]^T * \left[R^{(3)}\right] * \left[T^{(3)}\right].$$

$$= \frac{A^3E^3}{\sqrt{2}}\;
\begin{array}{c} \begin{array}{cccccc} u_{3x} & u_{3y} & u_{3z} & u_{4x} & u_{4y} & u_{4z} \end{array} \\
\left[\begin{array}{cccccc}
\frac{1}{2} & 0 & -\frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} \\
0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & 0 & -\frac{1}{2} \\
-\frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & 0 & -\frac{1}{2} \\
0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{2} & 0 & -\frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2}
\end{array}\right]_{6\times 6} \end{array}
\begin{array}{c} \\ u_{3x} \\ u_{3y} \\ \cancel{8}\,u_{3z} \\ u_{4x} \\ u_{4y} \\ u_{4z}. \end{array}$$

Now as all members are of circular c/s with same Dia and same 'E'

$$\therefore A^{(1)}E^{(1)} = A^{(2)}E^{(2)} = A^{(3)}E^{(3)} = (AE).$$

Scanned by TapScanner

After arranging the stiffness matrix in the global Co-ordinate system — The force-displacement equation in the global system —

$$
A*\begin{array}{c}
\begin{array}{cccccccccccc}
\underline{1} & \underline{2} & \underline{3} & \underline{4} & \underline{5} & \underline{6} & \underline{7} & \underline{8} & \underline{9} & \underline{10} & \underline{11} & \underline{12}
\end{array}\\
\left[\begin{array}{ccccccccc|ccc}
38.49 & +38.49 & 38.49 & 0 & 0 & 0 & 0 & 0 & 0 & -38.49 & -38.49 & -38.49 \\
'' & '' & '' & 0 & 0 & 0 & 0 & 0 & 0 & '' & '' & '' \\
'' & '' & '' & 0 & 0 & 0 & 0 & 0 & 0 & '' & '' & '' \\
0 & 0 & 0 & 38.49 & -38.49 & 38.49 & 0 & 0 & 0 & -38.49 & 38.49 & -38.49 \\
0 & 0 & 0 & -38.49 & 38.49 & -38.49 & 0 & 0 & 0 & 38.49 & -38.49 & +38.49 \\
0 & 0 & 0 & 38.49 & -38.49 & 38.49 & 0 & 0 & 0 & -38.49 & 38.49 & -38.49 \\
0 & 0 & 0 & 0 & 0 & 0 & 70.7 & 0 & -(70.7) & -(70.7) & 0 & 70.7 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -70.7 & 0 & 70.7 & 70.7 & 0 & -70.7 \\
\hline
-38.49 & -38.49 & 38.49 & -38.49 & 38.49 & -38.49 & -70.7 & 0 & 70.7 & 147.7 & 0 & 6.27 \\
'' & '' & '' & 38.49 & -38.49 & 38.49 & 0 & 0 & 0 & 0 & 76.980 & 0 \\
'' & '' & '' & -38.49 & 38.49 & -38.49 & 70.7 & 0 & -70.7 & 6.27 & 0 & 147.7
\end{array}\right]_{12\times12}
\end{array}
\left[\begin{array}{c}
\cancel{u_{1x}}^{\,0} \\ \cancel{u_{1y}}^{\,0} \\ \cancel{u_{1z}}^{\,0} \\ \cancel{u_{2x}}^{\,0} \\ \cancel{u_{2y}}^{\,0} \\ \cancel{u_{2z}}^{\,0} \\ \cancel{u_{3x}}^{\,0} \\ \cancel{u_{3y}}^{\,0} \\ \cancel{u_{3z}}^{\,0} \\ \hline u_{4x} \\ u_{4y} \\ u_{4z}
\end{array}\right]_{12\times1} = f.
$$

Separating the unknown displacements,

$$
A*\begin{pmatrix} 147.69 & 0 & 6.27 \\ 0 & 76.98 & 0 \\ 6.27 & 0 & 147.69 \end{pmatrix}\begin{Bmatrix} u_{4x} \\ u_{4y} \\ u_{4z} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ -10 \end{Bmatrix}
$$

$$
\Rightarrow \begin{Bmatrix} u_{4x} \\ u_{4y} \\ u_{4z} \end{Bmatrix} = \frac{1}{A} * \begin{pmatrix} 147.69 & 0 & 6.27 \\ 0 & 76.98 & 0 \\ 6.27 & 0 & 147.69 \end{pmatrix}^{-1}\begin{Bmatrix} 0 \\ 0 \\ -10 \end{Bmatrix}
$$

Using calculator

$$
\begin{Bmatrix} u_{4x} \\ u_{4y} \\ u_{4z} \end{Bmatrix} = \frac{1}{A}\begin{bmatrix} 0.068 & 0 & -0.0003 \\ 0 & 0.013 & 0 \\ -0.0003 & 0 & 0.0068 \end{bmatrix}\begin{Bmatrix} 0 \\ 0 \\ -10 \end{Bmatrix}
$$

$$
\Rightarrow \begin{Bmatrix} u_{4x} \\ u_{4y} \\ u_{4z} \end{Bmatrix} = \frac{1}{A}\begin{Bmatrix} 2.874\times10^{-3} \\ 0 \\ -0.06\overset{7}{\cancel{8}}6 \end{Bmatrix}
$$

Member forces :-

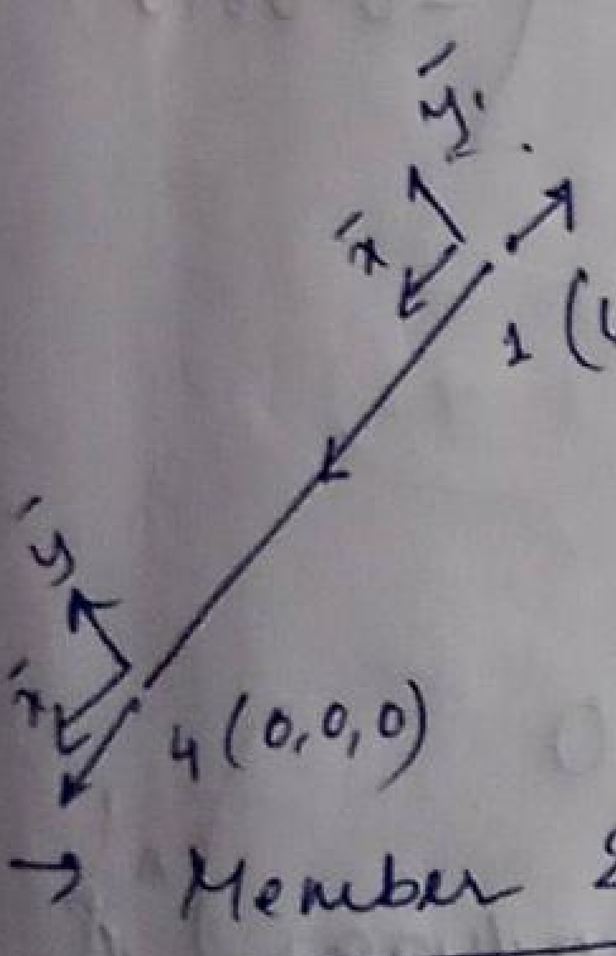
→ Member 1 :- Displacements associated with this element in global system —

$$u_1 = \begin{Bmatrix} u_{1x} \\ u_{1y} \\ u_{1z} \\ u_{4x} \\ u_{4y} \\ u_{4z} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 2.874\times10^{-3} \\ 0 \\ -0.0676 \end{Bmatrix} \times \frac{1}{A}.$$

Transforming into local $\bar{x},\bar{y}$ system.

$$\bar{u}_1 = \underset{2\times6}{[T^{(1)}]} * \underset{6\times1}{u_1} = \frac{1}{A}\begin{bmatrix} -\frac{1}{\sqrt3} & -\frac{1}{\sqrt3} & -\frac{1}{\sqrt3} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt3} & -\frac{1}{\sqrt3} & -\frac{1}{\sqrt3} \end{bmatrix}\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 2.874\times10^{-3} \\ 0 \\ -0.0676 \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} \bar{u}_{1x} \\ \bar{u}_{2x} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0.03737 \end{Bmatrix} \times \frac{1}{A}.$$



$$\begin{Bmatrix} \bar{f}_{1x} \\ \bar{f}_{2x} \end{Bmatrix} = \frac{\cancel{A}\times200}{\sqrt3}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} 0 \\ 0.03737 \end{Bmatrix}\frac{1}{\cancel{A}}$$

$$= \begin{Bmatrix} -4.315\,N \\ +4.315\,N \end{Bmatrix}$$ (Member is in tension).

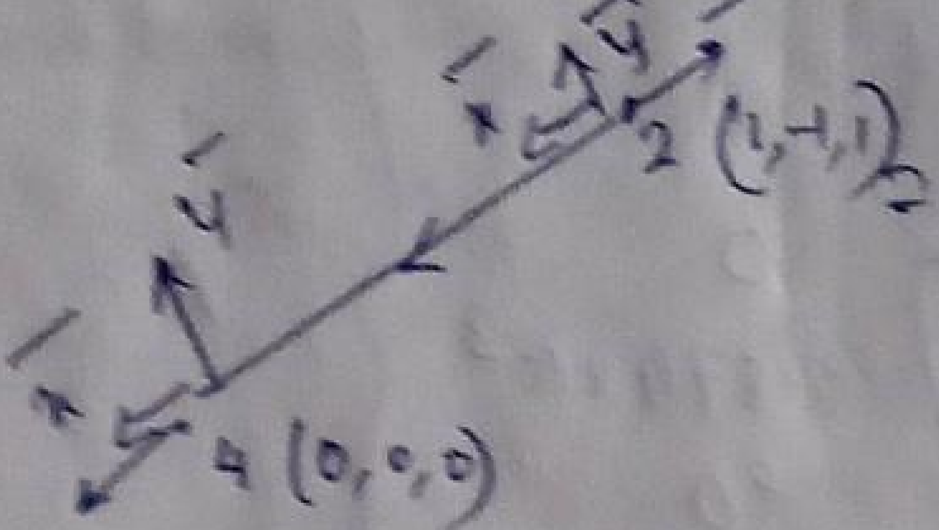
→ Member 2 :-

$$u_2 = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 2.874\times10^{-3} \\ 0 \\ -0.0676 \end{Bmatrix} \times \frac{1}{A}.$$

$$\therefore \bar{u}_2 = [T^{(2)}] * u_2 = \frac{1}{A}\begin{bmatrix} -\frac{1}{\sqrt3} & \frac{1}{\sqrt3} & -\frac{1}{\sqrt3} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt3} & \frac{1}{\sqrt3} & -\frac{1}{\sqrt3} \end{bmatrix}\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 2.874\times10^{-3} \\ 0 \\ -0.0676 \end{Bmatrix}$$

$$\begin{Bmatrix} \bar{u}_{1x} \\ \bar{u}_{2x} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0.03737 \end{Bmatrix} \times \frac{1}{A}.$$

$$\begin{Bmatrix} \bar{f}_{1x} \\ \bar{f}_{2x} \end{Bmatrix} = \frac{\cancel{A} \times 200}{\sqrt{3}} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.03737 \end{Bmatrix} \times \frac{1}{\cancel{A}}$$



$$= \begin{Bmatrix} -4.315N \\ +4.315N \end{Bmatrix}.$$ (Member is in tension).

Member 3 :-

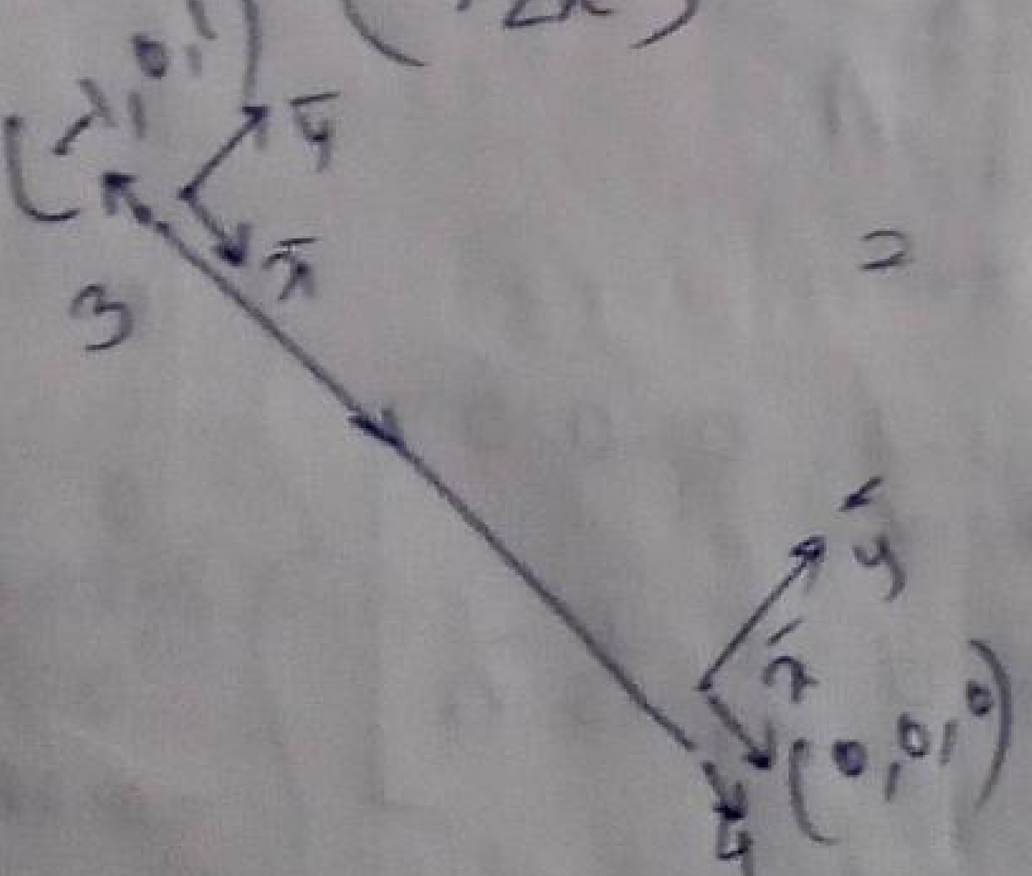
$$U_3 = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 2.874 \times 10^{-3} \\ 0 \\ -0.0676 \end{Bmatrix} * \frac{1}{A}$$

$$\bar{U}_3 \; \cancel{\bar{U}_1} = [T^{(3)}] * U_3$$

$$= \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 2.874 \times 10^{-3} \\ 0 \\ -0.0676 \end{Bmatrix} \times \frac{1}{A}$$

$$\Rightarrow \begin{pmatrix} \bar{u}_{1x} \\ \bar{u}_{2x} \end{pmatrix} = \begin{pmatrix} 0 \\ 0.04983 \end{pmatrix} \times \frac{1}{A}$$

$$\begin{Bmatrix} \bar{f}_{1x} \\ \bar{f}_{2x} \end{Bmatrix} = \frac{\cancel{A} \times 200}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{pmatrix} 0 \\ 0.04983 \end{pmatrix} \times \frac{1}{\cancel{A}}$$



$$= \begin{Bmatrix} -7.047N \\ +7.047N \end{Bmatrix}.$$ (Member in tension)

Now as max force in members is 7.047 N hence max area required will be corresponding to this element.

Since all members are in tension

$$(\sigma_P)_t = 5000 \text{ N/m}^2.$$

and area of each member is circular

$$\therefore A = \frac{\pi}{4} d^2.$$

$$\therefore (\sigma_P)_t \times \frac{\pi}{4} d^2 = 7.047$$

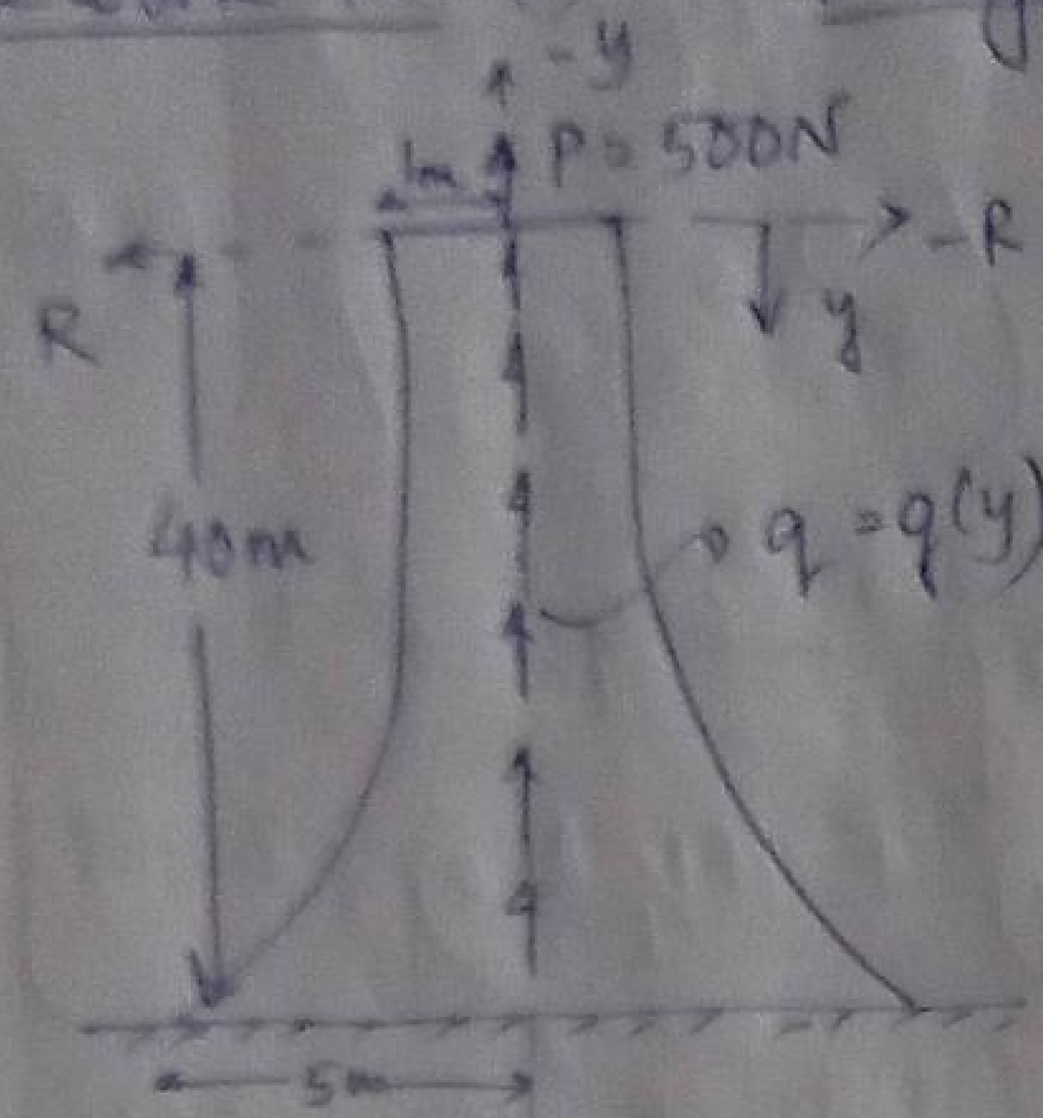
$$\Rightarrow 5000 \times \frac{\pi}{4} d^2 = 7.047$$

$$\Rightarrow d = \sqrt{\frac{7.047 \times 4}{\pi \times 5000}}$$

$$\Rightarrow d = 42.36 \text{ mm.}$$

$\therefore$ Required diameter of the members $\approx$ 4.24 cm.

Home work Prob(3) Analytical Solution :



for Convenience in Calculation let us take the origin to be the far end of the bar.

Original Equation of $R = 2.5 \times 10^{-3}(y^2 - 8y + 2000)$.

Now as origin has changed,

let $R = ay^2 + by + c$

at $y=0$, $R = 1$.

$1 = a + b + c$.

at $y = 40m$, $R = 5$

$5 = a(1600) + 40b + c$

at $y=0$, $dR/dy = 0$, $0 = 2a\overset{0}{y} + b$.

$\Rightarrow b = 0$,

$\therefore 5 = a(1600) + 1$.

$\Rightarrow a = \dfrac{4}{1600} = \dfrac{1}{400}$

$R = \dfrac{y^2}{400} + 1$.

(a) Axial stress at any point $= \dfrac{P_T}{A(y)}$

where $P_T = P + \int_0^y q(y)\,dy$.

$q = q_0\left(\dfrac{y}{40}\right) = \dfrac{20}{40}y = \left(\dfrac{1}{2}y\right)$

$= P + \int_0^y \dfrac{1}{2}y\,dy = 500 + \dfrac{y^2}{4}\ N$.

$\therefore \sigma' = \dfrac{500 + \frac{y^2}{4}}{\pi\left(\frac{y^2}{400}+1\right)^2}$, as $A(y) = \pi R(y)^2$
$= \pi\left(1+\frac{y^2}{400}\right)^2$.

$\therefore$ σ at $y = 40$, which is the original configurat-
-ions of fixed joint at the base,

$$\sigma_0 = \frac{500 + \frac{1600}{4}}{\pi\left(\frac{1600}{400}+1\right)^2} \ N/m^2$$

$$= \frac{900}{\pi(5)^2} = \underline{11.4592\ N/m^2}$$

σ at $y = 20$ m ; at midpoint

$$\sigma_{20} = \frac{500 + \left(\frac{400}{4}\right)}{\pi\left(\frac{400}{400}+1\right)^2}$$

$$= \frac{600}{\pi(2)^2} = 47.7465\ N/m^2.$$

(b) Analytical expression for displacement
(i) at free end, i.e $y = 40$m in original config.

$$\Delta y = \int_0^y \frac{P_T\, dy}{A(y)E}.$$

$$= \int_0^L \frac{\left(500 + \frac{y^2}{4}\right)dy}{\pi\left(1+\frac{y^2}{400}\right)^2 E}.$$

$$\Delta y = \int_0^L \frac{500\, dy}{\pi E\left(1+\frac{y^2}{400}\right)^2} + \frac{1}{\pi E}\int_0^L \frac{\frac{y^2}{4}\, dy}{\left(1+\frac{y^2}{400}\right)^2}$$

let $\frac{1}{400} = c$,

$$\therefore \Delta y = \frac{500}{\pi E}\int_0^L \frac{dy}{(1+cy^2)^2} + \frac{1}{4\pi E}\int_0^L \frac{y^2 dy}{(1+cy^2)^2}$$

$$\frac{1}{400} = c \Rightarrow \sqrt{c} = \frac{1}{20}$$

$$\therefore l\sqrt{c} = \frac{40}{20} = 2$$

Let $cy^2 = \tan^2\theta$.

$$\Rightarrow y = \frac{\tan\theta}{\sqrt{c}}.$$

$$dy = \frac{\sec^2\theta\, d\theta}{\sqrt{c}}$$

$$\therefore \Delta y = \frac{500}{\pi E\sqrt{c}}\int_0^{\tan^{-1}(l\sqrt{c})} \frac{\sec^2\theta\, d\theta}{\sec^4\theta} + \frac{1}{4\pi E}\int_0^{\tan^{-1}(l\sqrt{c})} \frac{\left(\frac{\tan^2\theta}{c}\right)\sec^2\theta\, d\theta}{\sec^4\theta\sqrt{c}}$$

$$= \frac{500}{\pi E\sqrt{c}}\int_0^{\tan^{-1}(l\sqrt{c})} \cos^2\theta\, d\theta + \frac{1}{4\pi E c}\int_0^{\tan^{-1}(l\sqrt{c})} \frac{\sin^2\theta\, d\theta}{\sqrt{c}}.$$

$$= \frac{500}{\pi E\sqrt{c}}\int_0^{1.107} \cos^2\theta\, d\theta + \frac{\cancel{400}\,100}{\cancel{4}\pi E}\int_0^{1.107} \frac{\sin^2\theta\, d\theta}{\sqrt{c}}.$$

$$= \frac{500}{2\pi E\sqrt{c}}\int_0^{1.107} (1+\cos 2\theta)\, d\theta + \frac{100}{2\pi E}\int_0^{1.107} \left(\frac{1-\cos 2\theta}{\sqrt{c}}\right) d\theta.$$

$$= \frac{500}{2\pi E\sqrt{c}}\left(1.107 + \frac{\sin 2\theta}{2}\Big|_0^{1.107}\right) + \frac{100}{2\pi E}\left(\frac{1.107 - \frac{\sin 2\theta}{2}\Big|_0^{1.107}}{\sqrt{c}}\right)$$

$$= \left(\frac{0.0923}{\sqrt{c}} + 0.1781\right) \text{mm}.$$

$$= \underline{2.0192} \text{ mm}$$

(iii) at $y = 20$ m, $R = 2$m.

lets now shift the origin to the midpoint of the bar, $R = ay^2 + by + c$

at $y = 0$, $R = 2$,

$$2 = a(0) + b(0) + c$$

$$\Rightarrow \underline{c = 2}$$

at $y = 20$, $R = \cancel{80}\ 5\,m$.

$$5 = a(400) + 20b + 2$$

at $y = 0$, $dR/dy = 0$, $0 = 2a\cancel{y}^{0} + b$

$\rightarrow b = 0$.

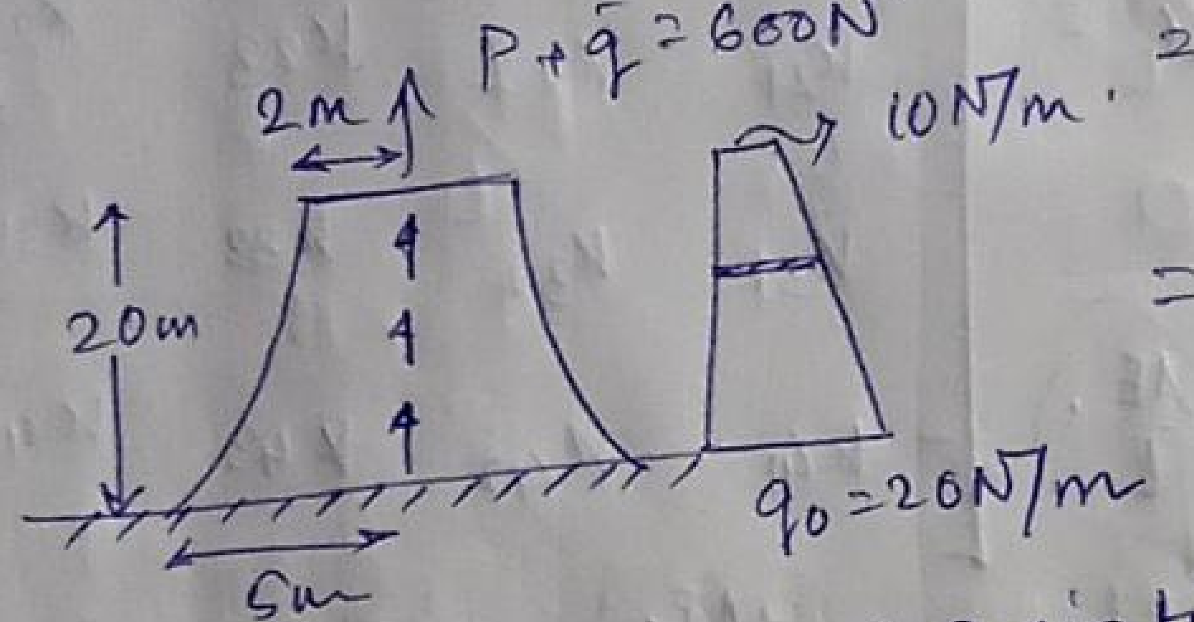
$\therefore a = \cancel{5}^{3}/400$

$\therefore R = \left(\frac{3}{400}y^2 + 2\right)\,m$.

Now, Total load appearing as point load at mid point $= P + \frac{1}{2} \times q(y)\Big|_{y=20} \times 20$

$= P + \frac{1}{\cancel{2}} \times \frac{\cancel{20}^{10}}{\cancel{40}_{\cancel{2}}} \times \cancel{20}^{10} \times 20$

$= 500 + 100 = 600\,N$.



Now variation of q over the length

$$q(y) = 10 + \frac{10}{20} \times y = \left(10 + \frac{y}{2}\right)$$

$\therefore$ Deflection at mid point —

$$\Delta y = \int_0^{20} \frac{\left\{600 + \frac{1}{2} \times \left(10 + \frac{y}{2} + 10\right) \times y\right\} dy}{\pi\left(2 + \frac{3}{400}y^2\right)^2 E}.$$

$$\Rightarrow \Delta y = \int_0^{20} \frac{600\, dy}{\pi E\left(2 + \frac{3}{400}y^2\right)^2} + \int_0^{20} \frac{\left(20 + \frac{y}{2}\right) \times \frac{y}{2}\, dy}{\pi E\left(2 + \frac{3}{400}y^2\right)^2}.$$

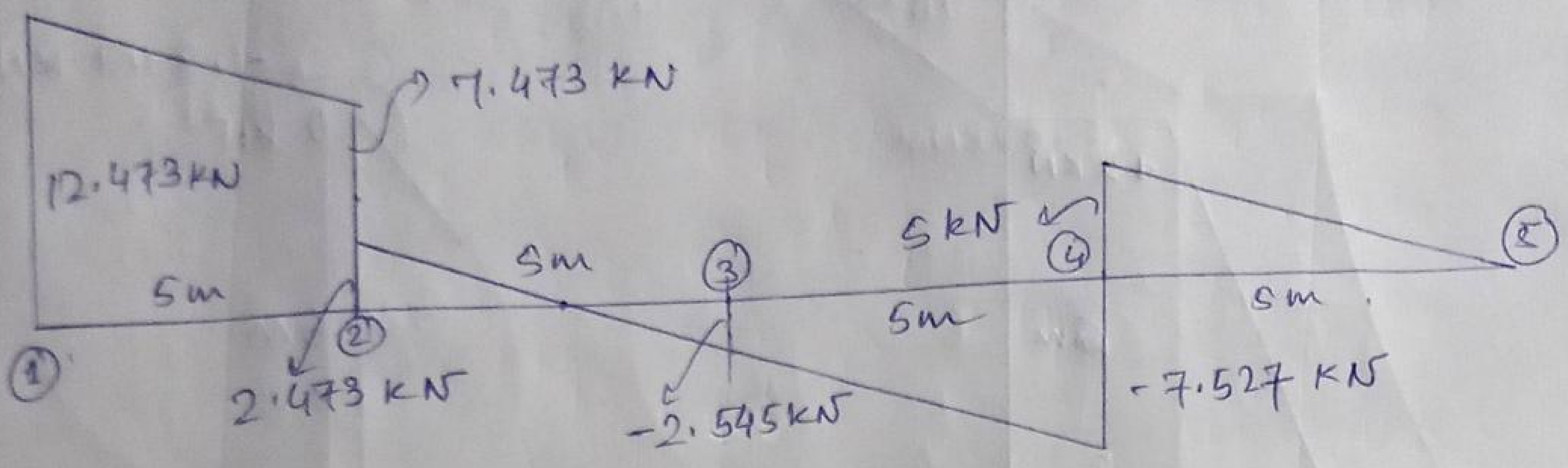
$$= \int_0^{20} \frac{600\, dy}{\pi E\left(2 + \frac{3}{400}y^2\right)^2} + \int_0^{20} \frac{10y + \frac{y^2}{4}\, dy}{\pi E\left(2 + \frac{3}{400}y^2\right)^2}.$$

Since the integral is abit time consuming to compute — we did this integrals in Wolfram Alpha and the results obtained are ; $y_{20} = \underline{\underline{0.3748\,mm}}$

Scanned by TapScanner

Prob 5(a) :-

(a) Shear Force diagram :-

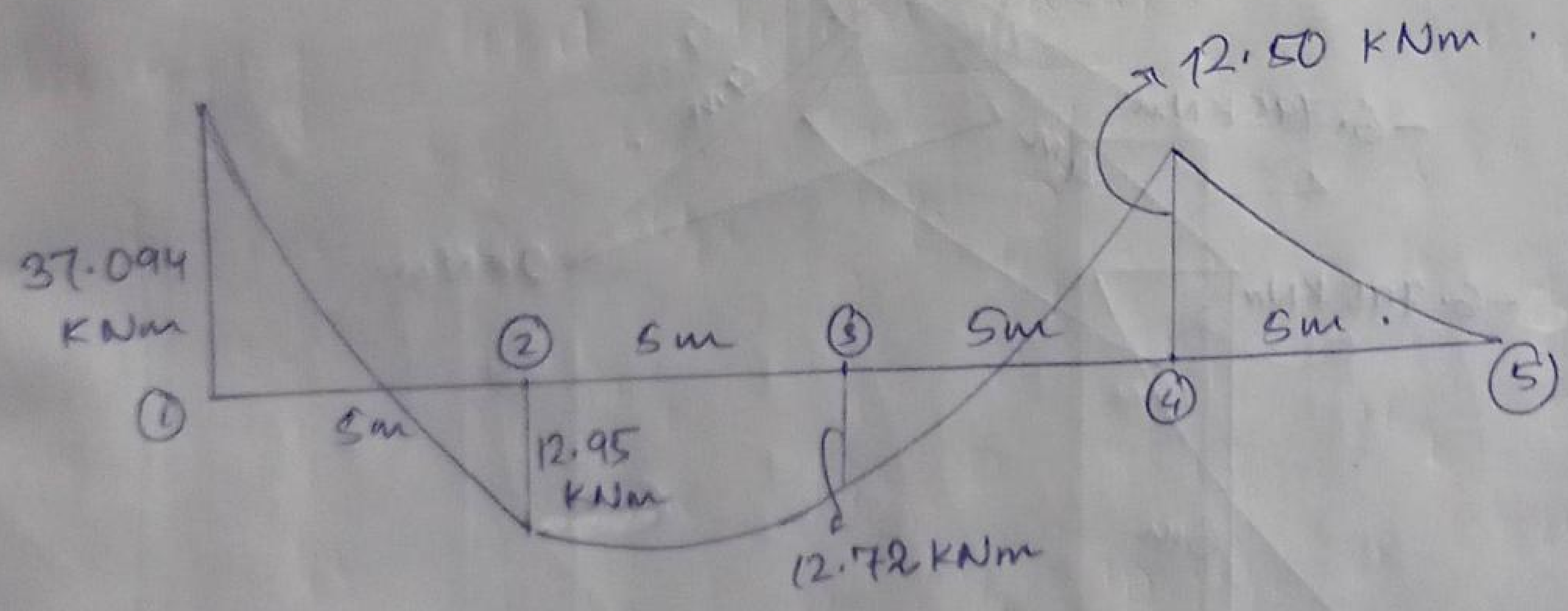


Support Reactions :-

$R_{1y} = 12.473$ KN

$R_{4y} = 12.527$ KN.

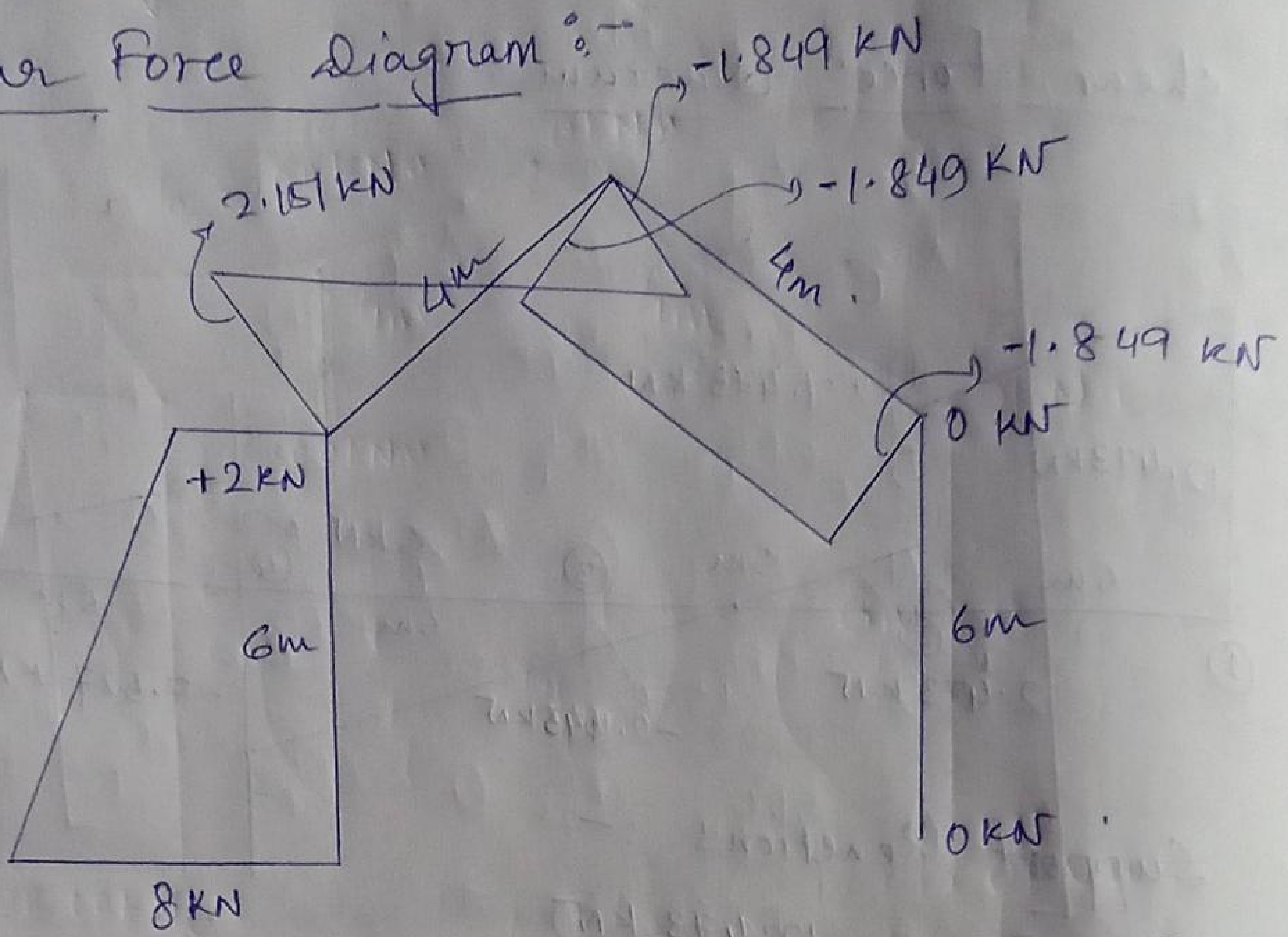
(b) Bending Moment diagram :-



Support Moment = 37.094 KNm

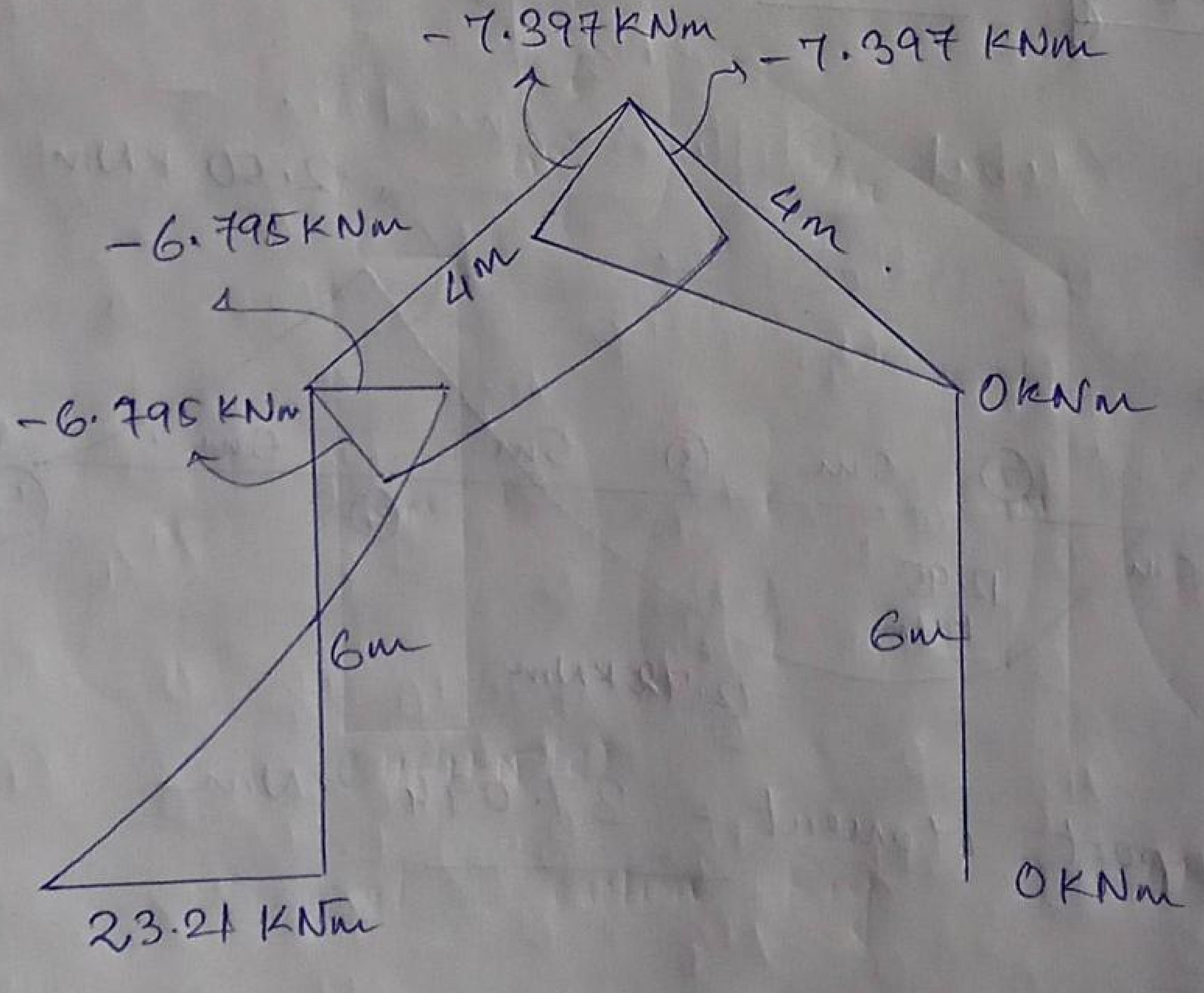
Prob 6(a) :- · Frame with internal hinge

(a) Shear Force Diagram :-



Support Reaction R_{1x} = − 8.00 kN.

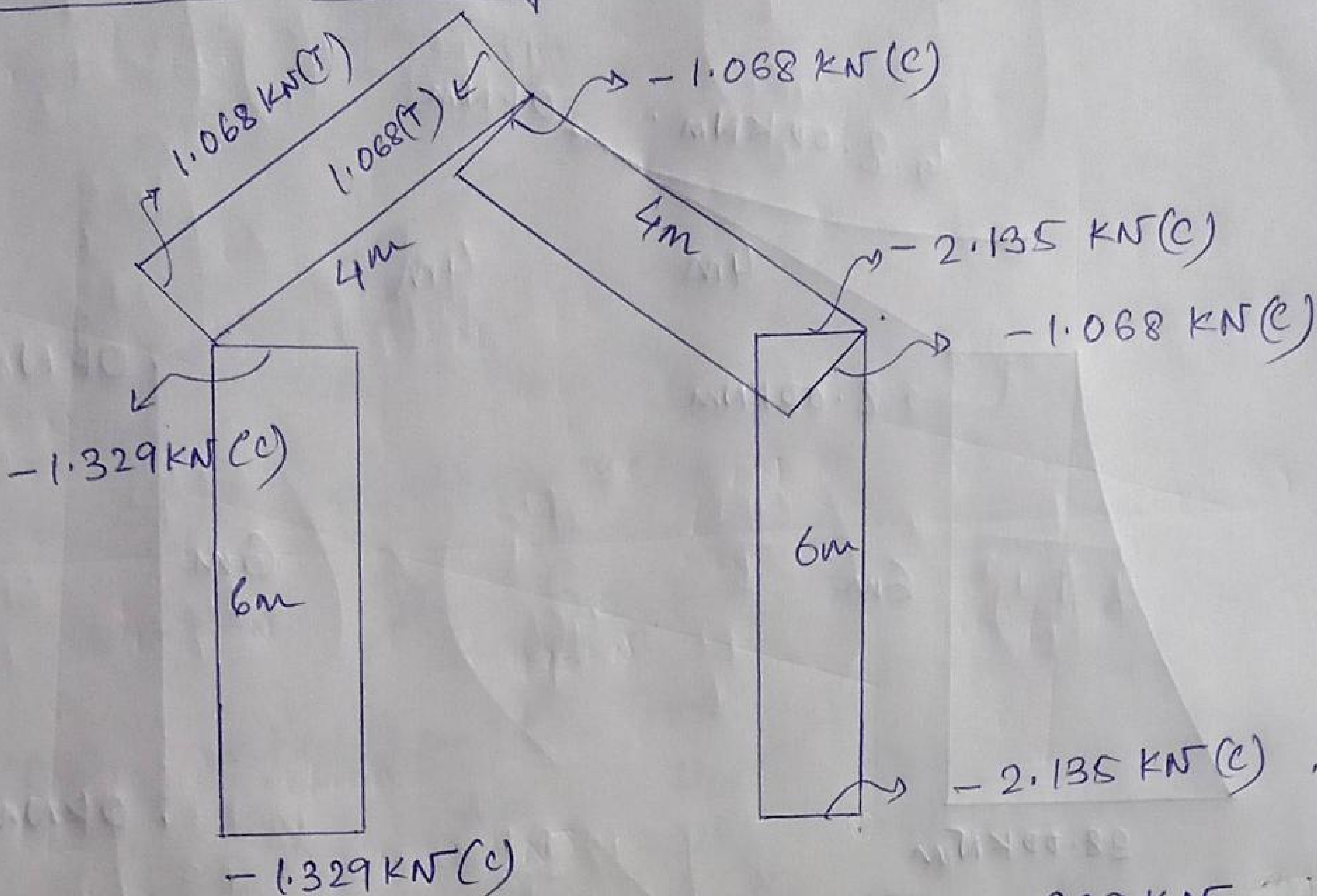
(b) Bending Moment Diagram :-



Support Moment M_{1a} = +23.21 KNm.

P.T.O

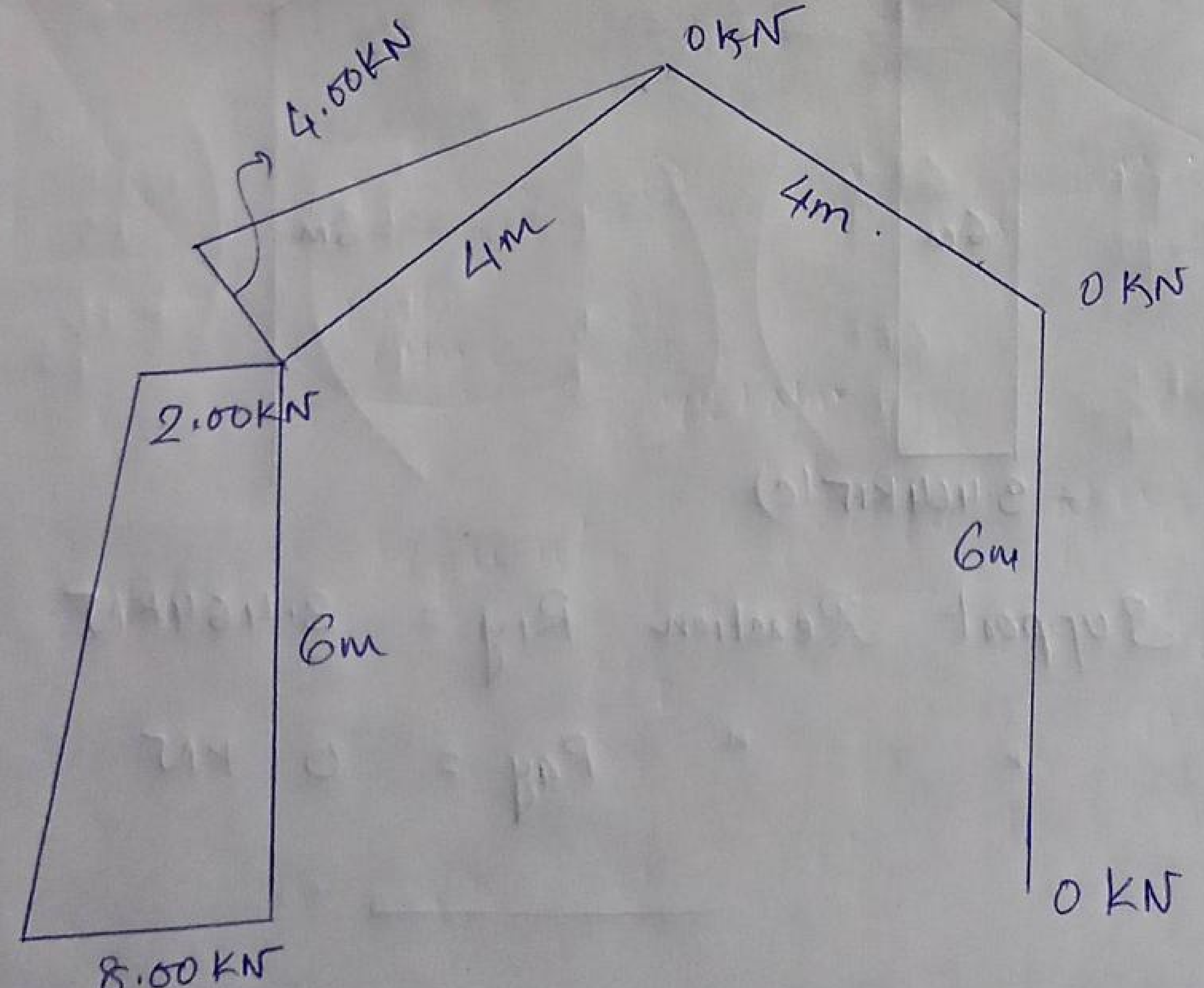
(e) Axial Force Diagram :-



Support Reactions :- R_{1y} = 1.329 kN

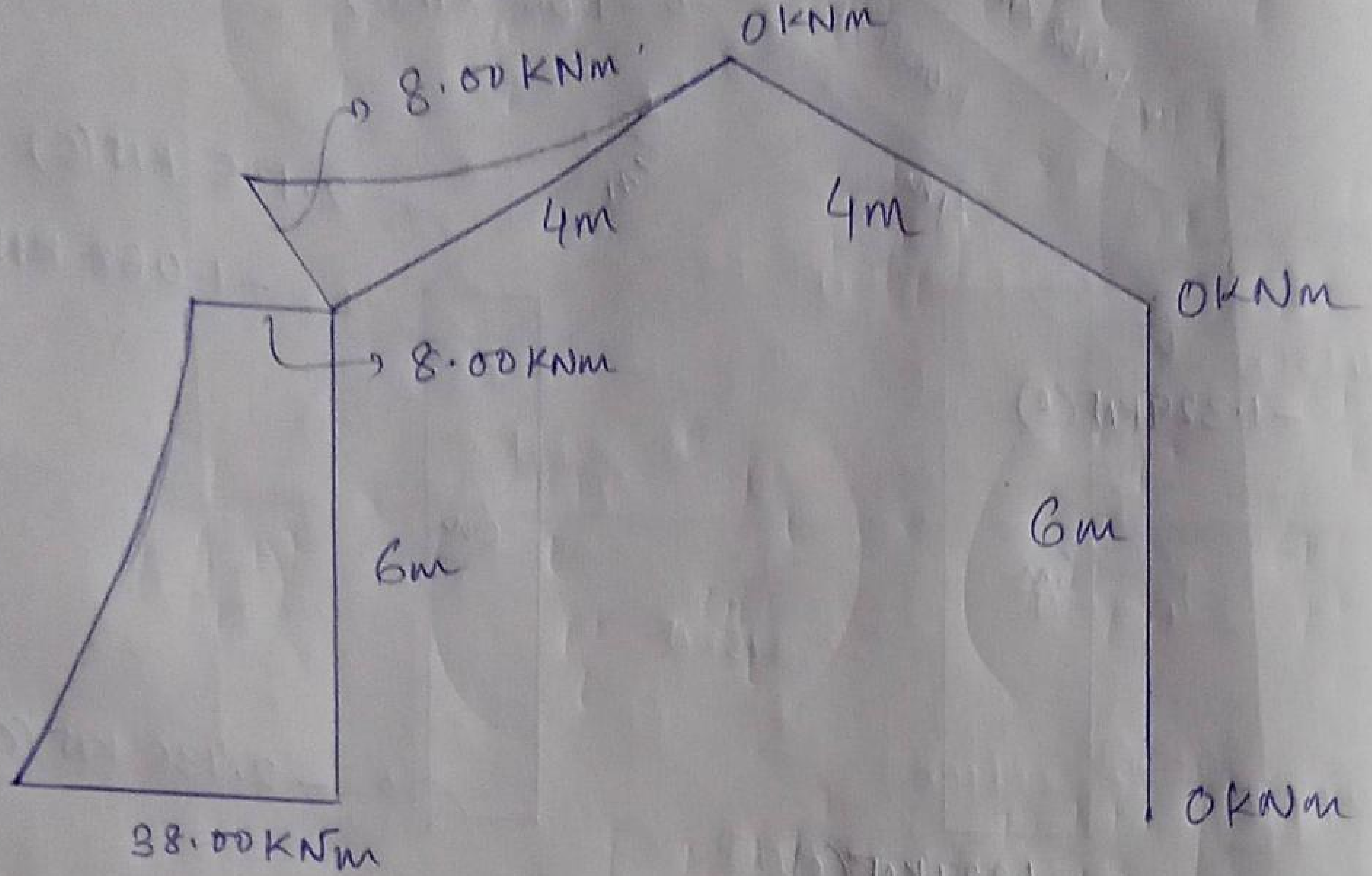
R_{4y} = 2.135 kN

Prob 6(b) :- Frame with internal hinge and inclined support.



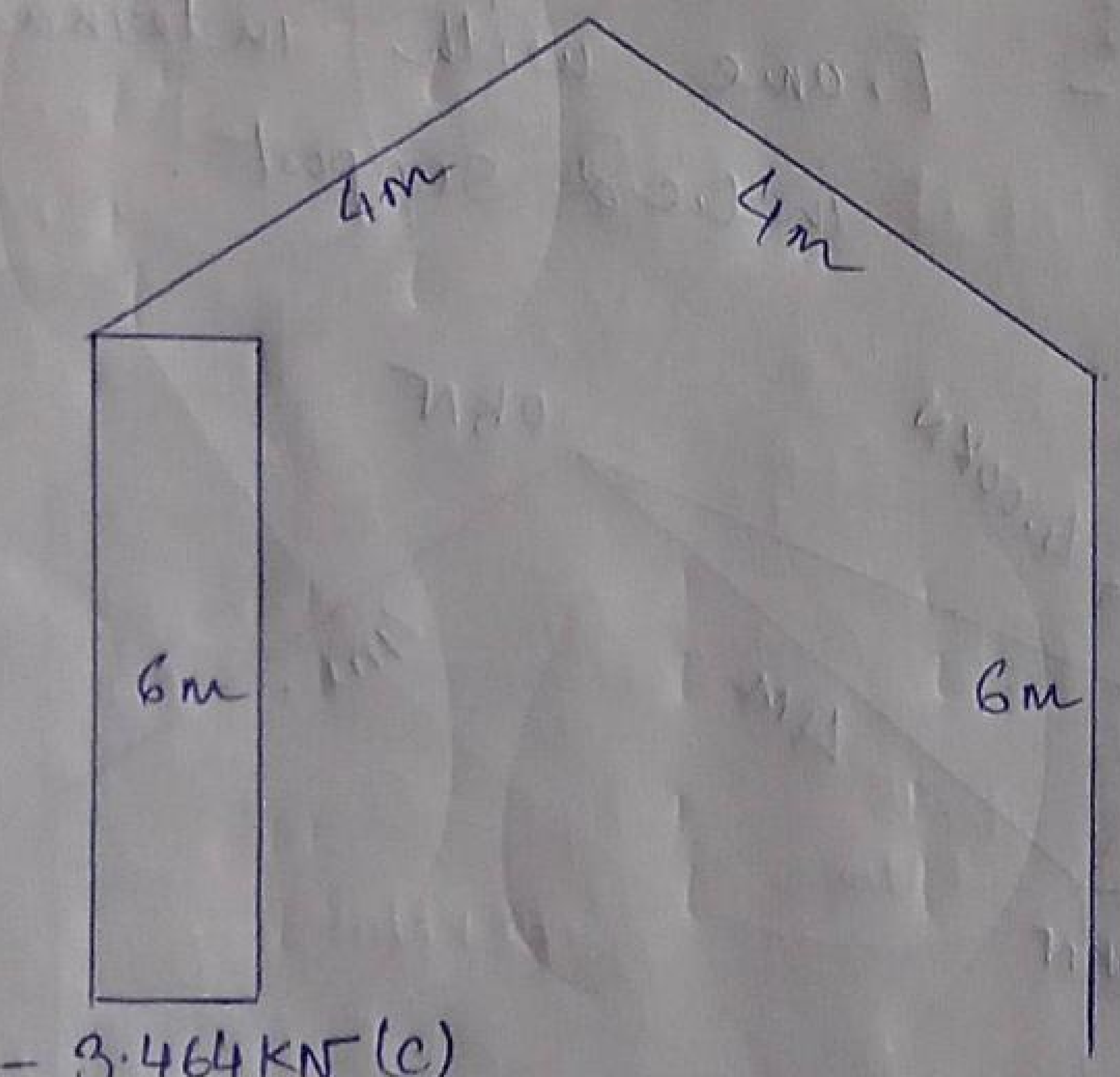
Support Reaction R_{1x} = -8.00 kN.

(b) Bending Moment Diagram :-



Support Moment M_{10} = 38.00 KNm

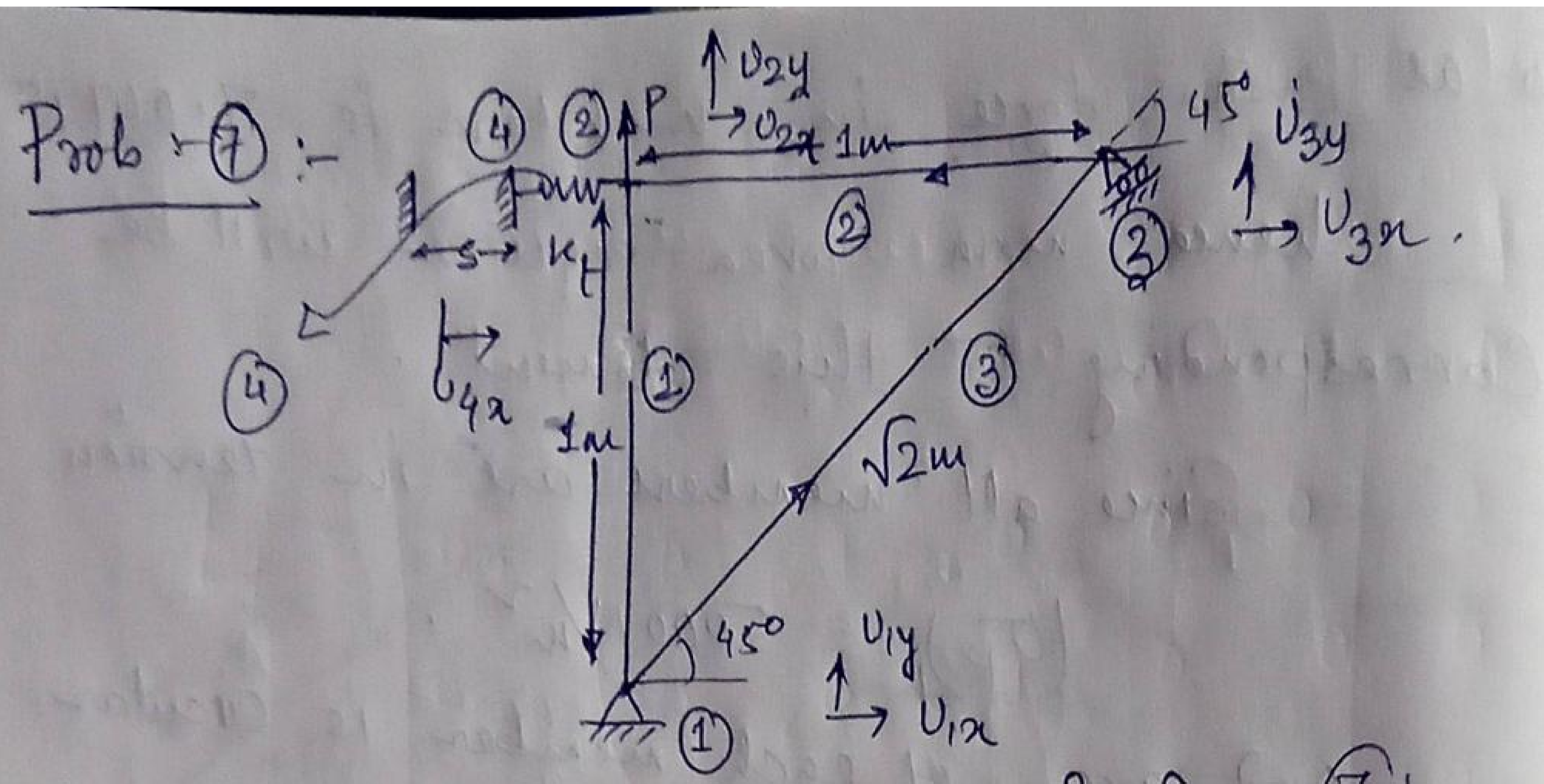
(c)



Support Reaction R_{1y} = 3.464KN

" " R_{4y} = 0 KN

Prob - 7 :-



Total No. of degrees of freedom = 7.

No. of unrestrained dofs – U_{2x}, & U_{2y} and U_{3x} (S) → in the inclined support along its orientation.

Element Connectivity –

for e=1 ;	start node	end node
e=1	1	2.
e=2	2	3
e=3	1	3.
e=4	2	4.

For element 1 :- Element Conn. = $(1 \to 2)$

angle of orientation = ~~45°~~ 90°.

length = ~~√2~~ 1 m.

Associated dofs = $\{U_{1x}, U_{1y}, U_{2x}, U_{2y}\}$

Area of C/S = 6×10^{-4} m^2

$E = 210 \times 10^9$ Pa.

Transformation matrix for e=1.

$$T_1 = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \end{bmatrix}_{2\times4}$$

$$\Rightarrow T_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$K_{elem} = k_1 \rightarrow$ element level stiffness matrix in local system —

$$\Rightarrow k_1 = \frac{A_1 E}{4}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}.$$

$$= 126 \times 10^6 \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}.$$

$K_1 \rightarrow$ element level stiffness in global system.

$$K_1 = T_1^T * k_1 * T_1$$

$$= 126 \times 10^6 * \underset{4\times2}{\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}} * \underset{2\times2}{\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}} * \underset{2\times4}{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

$$K_1 = \begin{array}{c|cccc} & u_{1x} & u_{1y} & u_{2x} & u_{2y} \\ \hline u_{1x} & 0 & 0 & 0 & 0 \\ u_{1y} & 0 & 126 & 0 & -126 \\ u_{2x} & 0 & 0 & 0 & 0 \\ u_{2y} & 0 & -126 & 0 & 126 \end{array} \times 10^6$$

Gather matrix for element 1

$$L_1 = \begin{array}{c|ccccccc} & U_{1x} & U_{1y} & U_{2x} & U_{2y} & U_{3x} & U_{3y} & U_{4x} \\ \hline U_{1x} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ U_{1y} & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ U_{2x} & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ U_{2y} & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{array}_{4\times 7}$$

Now position of K_1 in the overall global stiffness matrix is given by.

$= L_1' * K * L_1$

$$\therefore K_G = 10^6 * \begin{array}{ccccccc|c} U_{1x} & U_{1y} & U_{2x} & U_{2y} & U_{3x} & U_{3y} & U_{4x} & \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & U_{1x} \\ 0 & 126 & 0 & -126 & 0 & 0 & 0 & U_{1y} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & U_{2x} \\ 0 & -126 & 0 & 126 & 0 & 0 & 0 & U_{2y} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & U_{3x} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & U_{3y} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & U_{4x}. \end{array}_{7\times 7}$$

— This will keep on adding up for every element.

For Element 2 :- Element conn = $(2 \to 3)$

Angle of orientation = 0°

Associated dofs = $\{U_{2x}, U_{2y}, U_{3x}, U_{3y}\}^T$.

length = 1 m

Area of C/s A_2 = 6×10^{-4} m^2

$E = 210\times10^9$ Pa.

Transformation matrix T_2

$$= \begin{bmatrix} \cos 0^{\circ} & \sin 0^{\circ} & 0 & 0 \\ 0 & 0 & \cos 0^{\circ} & \sin 0^{\circ} \end{bmatrix}.$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

$$k_2 = \frac{A_2 E}{L_2} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$= 126 \times 10^6 * \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}.$$

$$K_2 = \left(T_2' * k_2 * T_2 \right).$$

$$= 126 \times 10^6 * \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$K_2 = 10^6 \times \begin{array}{c} \begin{array}{cccc} \underline{U_{2x}} & \underline{U_{2y}} & \underline{U_{3x}} & \underline{U_{3y}} \end{array} \\ \begin{pmatrix} 126 & 0 & -126 & 0 \\ 0 & 0 & 0 & 0 \\ -126 & 0 & 126 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array} \begin{array}{c} U_{2x} \\ U_{2y} \\ U_{3x} \\ U_{3y}. \end{array}$$

Gather matrix for element 2 :—

$$L_2 = \begin{array}{c} \begin{array}{ccccccc} \underline{U_{1x}} & \underline{U_{1y}} & \underline{U_{2x}} & \underline{U_{2y}} & \underline{U_{3x}} & \underline{U_{3y}} & \underline{U_{4x}} \end{array} \\ \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array} \begin{array}{c} u_{2x}. \\ u_{2y} \\ u_{3x} \\ u_{3y} \end{array}$$

Scanned by TapScanner

Updated K_G will be $= K_G + L_2' * K_2 * L_2$.

After updating —

$$K_G = 10^6 * \begin{array}{c|c} \begin{array}{ccccccc} U_{1x} & U_{1y} & U_{2x} & U_{2y} & U_{3x} & U_{3y} & U_{4x} \end{array} & \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 126 & 0 & -126 & 0 & 0 & 0 \\ 0 & 0 & 126 & 0 & -126 & 0 & 0 \\ 0 & -126 & 0 & 126 & 0 & 0 & 0 \\ 0 & 0 & -126 & 0 & 126 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{7\times 7} & \begin{array}{c} U_{1x} \\ U_{1y} \\ U_{2x} \\ U_{2y} \\ U_{3x} \\ U_{3y} \\ U_{4x} \end{array} \end{array}$$

For element 3 :- element connectivity $1 \rightarrow 3$

angle of orientation $= 45^\circ$.

Length $= \sqrt{2}$ m

Area of c/s $A_3 = 6\sqrt{2} \times 10^{-4}\ m^2$

$E = 210$ GPa. $= 210 \times 10^9$ Pa.

Associated dofs $= \{U_{1x}, U_{1y}, U_{3x}, U_{3y}\}^T$

Transformation matrix for e=3.

$$T_3 = \begin{bmatrix} \cos 45^\circ & \sin 45^\circ & 0 & 0 \\ 0 & 0 & \cos 45^\circ & \sin 45^\circ \end{bmatrix}.$$

$$= \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

Scanned by TapScanner

$$k_3 = \frac{A_3 E}{L_3}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}.$$

$$= \frac{6\cancel{\sqrt{2}} \times 10^{-4} \times 210 \times 10^{9}}{\cancel{\sqrt{2}}\ m}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= 126 \times 10^{6} * \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}.$$

$$\therefore K_3 = T_3' * k_3 * T_3.$$

$$= 126 \times 10^{6} * \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} \end{bmatrix} * \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} * \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\Rightarrow K_3 = 10^6 \times \begin{array}{c} \begin{array}{cccc} U_{1x} & U_{1y} & U_{3x} & U_{3y} \end{array} \\ \begin{bmatrix} 63 & 63 & -63 & -63 \\ 63 & 63 & -63 & -63 \\ -63 & -63 & 63 & 63 \\ -63 & -63 & 63 & 63 \end{bmatrix} \end{array} \begin{array}{c} U_{1x} \\ U_{1y} \\ U_{3x} \\ U_{3y} \end{array}$$

Gather matrix for $e = 3$,

$$L_3 = \begin{array}{c} \begin{array}{ccccccc} U_{1x} & U_{1y} & U_{2x} & U_{2y} & U_{3x} & U_{3y} & U_{4x} \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} U_{1x} \\ U_{1y} \\ U_{3x} \\ U_{3y} \end{array}$$

Therefore position of K_3 in the overall global stiffness matrix will be given by $K_G = K_G + L_3' * K_3 * L_3$.

Scanned by TapScanner

Updated

$$K_g = 10^6 * \begin{array}{c} \\ \\ \left[\begin{array}{ccccccc} U_{1x} & U_{1y} & U_{2x} & U_{2y} & U_{3x} & U_{3y} & U_{4x} \\ 63 & 63 & 0 & 0 & -63 & -63 & 0 \\ 63 & 189 & 0 & -126 & -63 & -63 & 0 \\ 0 & 0 & 126 & 0 & -126 & 0 & 0 \\ 0 & -126 & 0 & 126 & 0 & 0 & 0 \\ -63 & -63 & -126 & 0 & 189 & 63 & 0 \\ -63 & -63 & 0 & 0 & 63 & 63 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array} \begin{array}{c} U_{1x} \\ U_{1y} \\ U_{2x} \\ U_{2y} \\ U_{3x} \\ U_{3y} \\ U_{4x} \end{array}$$

For element (4) [spring] element connectivity (2-4).

Associated degrees of freedom $\{U_{2x}, U_{4x}\}$ U_{2y} U_{4y}

$$\text{stiffness} = 2\times10^5 + 5\times10^6\,\delta^2$$

$$= (0.2 + 5\delta^2) * 10^6$$

$\delta \rightarrow$ elongation in the spring

From the intuitive deformation diagram of the frame, it is evident that $U_{2x} = 0$ as the load at that point is acting perfectly vertical.

$\therefore$ The net deflection of the spring.

$$= \sqrt{(U_{2y})^2 + (U_{4x})^2}$$

$$\delta = \sqrt{(U_{2y})^2 + (0.3)^2}$$

$$\therefore \text{stiffness} = [0.2 + 5(U_{2y}^2 + 0.09)] * 10^6$$

Here onwards, done entirely on matlab.